Foreign reserves' strategic asset allocation

Por:
Carlos León y Daniel Vela

Borradores de ECONOMÍA

Núm. 645
2011

- Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia - Bogotá - Colombia -

Foreign reserves' strategic asset allocation[†]

Carlos León[♠] and Daniel Vela[♣]
Banco de la República (Colombia)

Abstract

Despite foreign reserves' strategic asset allocation relies mainly on Modern Portfolio Theory (MPT), the unique characteristics of central banks obliges them to articulate and reconcile typical optimization procedures with reserves' management objectives such as providing confidence regarding the ability to meet the country's external commitments. Moreover, further involvedness come from broad economic factors as diverse as the openness of capital and current accounts, external debt's maturity and currency composition, and exchange rate regime.

Therefore, in order to alleviate the divergence from theory and practice regarding foreign reserves' strategic asset allocation, this paper describes the methodologies and procedures developed and employed by the Foreign Reserves Department of Banco de la República.

The mainstay of the paper is a long-term-dependence-adjusted and non-loss-constrained version of the Black-Litterman model for obtaining the efficient frontier from a set of investments complying with safety, liquidity and return criteria, where the choice of the portfolio which maximizes utility makes use of an estimation of the Board of Directors' risk aversion.

Results exhibit the effects of the unique nature of foreign reserves management for emerging markets. Typical features of foreign reserves management by central banks, such as non-loss restrictions due to capital preservation objectives, result in increased complexity in the optimization process and in asset allocations significantly distant from standard MPT's optimality.

Keywords: foreign reserves, Black-Litterman, strategic asset allocation.

JEL classification: G11, E58, C11, C61.

[†] We are grateful to Marco Ruiz and Alejandro Reveiz, who provided valuable comments and suggestions. Authors are grateful to the staff involved in this process, especially to Andrés Ramirez and Camilo Botía. This document relies on years of research and development at the Foreign Reserves Department of Banco de la República. The herein presented model is still under implementation, and some assumptions are made for illustrative purposes only. As usual, the opinions and statements are the sole responsibility of the authors and do not necessarily represent neither those of the Banco de la República nor of its Board of Directors.
[♠] Research and Development Section Manager, Financial Infrastructure Oversight Department, Banco de la República, cleonrin@banrep.gov.co / carlosleonr@hotmail.com. Involvement in this document occurred while working as Researcher for the Foreign Reserves Department, Banco de la República. [Corresponding author]
[♣] Professional, Research and Analysis Group, Foreign Reserves Department, Banco de la República, dvelabar@banrep.gov.co.

1. Introduction

The process of constructing portfolios based on Modern Portfolio Theory (MPT) is rather widespread. It mainly consists of assessing the expected risk and return of a set of eligible individual investment opportunities and their contribution to the aggregated risk and return of an overall portfolio, followed by the definition of the efficient portfolios which are reasonable within the investor's risk tolerance. This process usually ends with the choice of the efficient portfolio which maximizes investor's utility function.

Regardless of the chosen methodological approach for undertaking the mentioned process, portfolio construction initiates with the definition of key inputs such as (i) the investor's risk tolerance; (ii) the set of eligible investments; (iii) the investment horizon, and (iv) the *numeraire* or unit of measurement. Central banks and other sovereign investors also have to define the aforesaid inputs, but they can't do it in isolation from issues related to their core objectives or functions.

For example, regarding foreign exchange reserves management by central banks, the size of the portfolio and the weights assigned to individual investments depend on broad economic factors as diverse as the openness of capital and current accounts, external debt's maturity and currency composition, and exchange rate regime, just to name a few. According to Borio *et al.* (2008) this sort of additional sources of complexity is what makes foreign reserves manager's tasks qualitatively harder.

Moreover, unlike other types of wealth managed by traditional asset managers, foreign reserves management has to meet a critical range of objectives. The International Monetary Fund (IMF, 2004) identifies some of these objectives: (i) supporting and maintaining the confidence in the monetary and exchange rate policies; (ii) limiting external vulnerability (resilience to shocks); (iii) providing confidence to markets regarding the country's ability to meet its external obligations; (iv) demonstrating the backing of domestic currency by external assets, and (v) maintaining a reserve for national disasters or emergencies. Furthermore, because central banks display severe reluctance towards attaining financial losses, it is a common practice to include non-loss constraints to the portfolio optimization procedure, thus exposing asset allocation to additional complexities and yielding asset allocations which would be regarded as significantly suboptimal from standard MPT's view.

Consequently, foreign reserves' portfolio construction by a central bank is more involved than MPT's literature commonly describes. Therefore, in order to address the foreign reserves' asset allocation topic from a comprehensive point of view, this paper describes most of the complexities confronted by the Foreign Reserves Department of Banco de la República (BR) while defining its long-term theoretical portfolio.[1] Because none of the sources of complexity or the solutions implemented work on isolation, special attention has been drawn to a structured examination of the methodological approaches developed and articulated for the strategic asset allocation process.

[1] The herein presented analysis focuses on the definition of the long-term benchmark for the foreign reserves portfolio, not the optimal level of reserves for the central bank.

This paper is divided in five chapters. The first one is this introduction. The second describes the modified version of the Black-Litterman asset allocation model employed for constructing the efficient frontier, which includes a non-loss-constraint and a long-term dependence adjustment of the covariance matrix. The third depicts the approach implemented for selecting the investment portfolio from the efficient frontier, where customary utility maximization relies on the estimation of Board of Directors' risk aversion. The fourth presents the main results of the approach and some comparisons which capture the effects of different constraint scenarios and different macroeconomic environments. The fifth presents some insights regarding resulting allocations to a traditional investment within reserves management: gold. Finally, the sixth makes some concluding remarks.

2. Methodological approach

Reserves' management is often referenced to a theoretical portfolio or benchmark, which serves as the structure of the portfolio for the long-term and is constructed *via* what is commonly known as strategic asset allocation. As frequently documented (Solnik and McLeavey, 2003), strategic asset allocation is derived by constructing an optimization using long-term capital market expectations, where it usually takes the form of an investable benchmark that is assigned as an objective to the manager(s).

Solnik and McLeavey (2003) emphasize three main issues for appropriately defining a benchmark: (i) scope of the benchmark; (ii) attitude towards currency risk, and (iii) set of weights chosen.

The first issue corresponds to the choice of asset classes, which will ultimately define the set of eligible investment opportunities for constructing the benchmark. As acknowledged by Reveiz (2004), the choices of liquidity, market and credit risks are subordinated to the objectives and risk aversion of the central bank, where the universe of assets and the maximum risk exposures are to be defined by the top decision-making body (hereafter referred as the Board of Director's for practical purposes). Accordingly, the objectives of holding foreign reserves often oblige to limit investment opportunities to the safest and most liquid in the global market.

About the attitude towards currency risk, different approximations are used by central banks. Some choose to minimize portfolio's and balance sheet's volatility, usually in terms of local currency. Others, as assumed in this document for illustrative purposes, maintain reserves to meet the country´s expected obligations in foreign currency (i.e. trade in goods and services and capital transactions), where the *numeraire* for asset allocation purposes consists of the currency composition of those liabilities[2]; that is, no currency return or risk is considered since there is a *natural hedge* resulting from *asset-liability management*. According to Borio *et al.* (2008), this characterization implies that in response to changes in exchange rates the value of the *numeraire* would change in proportion to that of the corresponding liabilities, thereby stabilizing the value of reserves in relation to them.

[2] There are other choices for *numerarire* (Borio et al., 2008). For example, if reserves objective is to ensure access to imports under stress conditions the natural choice would be the corresponding basket of imports measured in real terms. If the objective is minimizing the effects of reserves' mark-to-market on the profitability and capital of the central bank, the domestic currency may be a natural choice for *numerarie*.

The third issue, the definition of the benchmark's weights, has spurred a substantial number of approaches (Solnik and McLeavey, 2003). The most common approach consists of using a published global market index (*e.g.* MSCI or S&P World indexes) with weights proportional to market capitalization, where some variants substitute capitalization for GDP figures, whilst more advanced portfolio managers may prefer using optimization techniques.

Portfolio optimization techniques aim to achieve a superior performance for a given level of risk within a quantitative framework. The most celebrated framework is the quadratic optimization in the mean-variance space[3], in which expected return and risk are conveniently estimated as the first and second moments of the distribution of the returns of the individual asset or the overall portfolio, where introducing linear and non-linear constraints is rather straightforward. In this way the optimization routine delivers an efficient frontier: a set of portfolios in which each level of attainable expected return is obtained with the minimum risk.

Despite being the most simple and recognized, the customary mean-variance portfolio optimization procedure entails several shortcomings. First and most widespread, the assumption of normality of asset's or portfolio's returns, which ultimately holds the mean-variance framework together. Second, as depicted by Black and Litterman (1991), small changes in expected returns can lead to large reallocation of weights (*i.e.* high sensitivity of the allocations to the inputs). Third, the presence of extreme portfolio weights or "corner solutions" (Zimmermann *et al.* 2003; He and Litterman, 1999). Fourth, long-term portfolios' excessive risk taking (Valdés, 2010; Reveiz *et al.* 2010; Pastor and Stambaugh, 2009; Schotman *et al.* 2008).

The most well-known method to overcome some of the previously mentioned shortcomings, namely the second and third, is the Black-Litterman (BL) approach. BL uses the global Capital Asset Pricing Model (CAPM) equilibrium as a neutral reference point or center of gravity, and investors' views to construct intuitive and convenient portfolios, where the former corresponds to the strategic asset allocation and the latter to the tactical asset allocation, respectively; because this document focuses on strategic asset allocation, investors' views will not be addressed hereafter.

BL model is based on a rather simple idea, which is the main and distinctive feature of the model: expected returns ought to be consistent with market equilibrium (*e.g.* CAPM equilibrium), except to the extent that the investor explicitly states otherwise (Black and Litterman, 1991). Therefore, because the market equilibrium is embraced as resulting of optimal portfolios approximating market capitalization weights, BL model allows the investor to start from the market equilibrium portfolio and an estimated covariance matrix in order to solve for the expected returns for which this portfolio is optimal.

Because it consists of a somewhat contrary procedure, converse to the traditional mapping from expected returns to optimal portfolios, solving for expected returns within BL approach is known as *inverse optimization*; as defined by Black and Litterman (1991), the investor starts with the portfolio and lets the model solve for the expected returns for which the portfolio is optimal. Accordingly, let Π_{mkt} be the *n*-dimensional vector of market-implied expected excess returns; λ_{mkt} the coefficient of relative risk aversion compatible with market's equilibrium;

[3] Alternative optimization spaces have been proposed. For example, Reveiz and León (2010) and de Beaufort and Berkelaar (2010) suggest using the maximum drawdown as the risk metric for portfolio optimization purposes; authors find that these methods help overcoming some of the drawbacks of traditional mean-variance optimization.

ω_{mkt} the market capitalization weights, and Σ the covariance matrix of historical excess returns, market's expected excess return by *inverse optimization* is given by *[F1]*. [4]

$$\Pi_{mkt} = \lambda_{mkt}\Sigma\omega_{mkt} \qquad \textit{[F1]}$$

Π_{mkt} is the set of expected returns that would clear the market if all investors had identical views. The covariance matrix (Σ) and market weights (ω_{mkt}) are to be estimated from historical prices and market capitalization, respectively. Market's risk aversion (λ_{mkt}) may be approximated by estimating the observed (historical) balance of additional excess return against additional risk.[5]

The vector of market-implied excess returns (Π_{mkt}) serve as a neutral starting point, close to what might be called a normal behavior of an average investor, and reflects a hypothetical passive manager who tracks a benchmark portfolio (Zimmerman *et al.*, 2003). Afterwards, the investor is able to create a set of deviations from market's equilibrium (*i.e.* views) which BL model will conveniently blend according to the degrees of confidence in these views and in the CAPM equilibrium. This is the main feature of BL model: it enables investors to combine their unique views regarding the performance of various assets with the market equilibrium in a manner that results in intuitive, diversified portfolios (Idzorek, 2002).

In order to consider investor's view, for *k*-different views on linear combinations of expected returns on *n*-assets composing the portfolio, the vector of *n*-revised expected excess returns ($\overline{\Pi}$) is calculated as in *[F6]*,

$$\overline{\Pi} = [(\tau\Sigma)^{-1} + P'\Omega^{-1}P]^{-1}\,[(\tau\Sigma)^{-1}\Pi_{mkt} + P'\Omega^{-1}Q] \qquad \textit{[F6]}$$

---

[4] BL model implicitly assumes that portfolio's utility *[F2]* is a function of the expected excess return of the portfolio (Π), investor's risk aversion (λ), covariance matrix (Σ) and portfolio's weights (ω). Utility maximization problem is solved by setting the first derivative of *[F2]* with respect to portfolio's weights (ω) equal to zero, which yields *[F3]*, where ω^* is the optimal weight assigned to each asset within the portfolio.

$$U_p = \Pi\omega - 0.5\lambda(\omega\Sigma\omega') \qquad \textit{[F2]}$$

$$\omega^* = \frac{\Pi}{\lambda\Sigma} \qquad \textit{[F3]}$$

Please note that rearranging *[F3]*, and using market capitalization weights (ω_{mkt}) as optimal weights and markets' relative risk aversion coefficient (λ_{mkt}) yields market's expected excess return (Π_{mkt}) in *[F1]*.

[5] Following Bodie *et al.* (2001), let *[F2]* be expressed in terms of excess returns as in *[F4]*, where portfolio's utility is a function of the expected return of the market's portfolio of risky assets (μ_r); the expected return of the market's risk-free asset (μ_{rf}); the risky asset's covariance matrix (Σ_r); the market's portfolio of risky assets' weights (ω_r); the market's risk aversion (λ_{mkt}), and the markets' preference (weight) for risky assets (Φ_{mkt}). Utility maximization problem is solved by setting the first derivative of *[F4]* with respect to Φ_{mkt} equal to zero, which is conveniently solved for λ_{mkt} as in *[F5]*.

$$U_p = \mu_{rf} + (\mu_r - \mu_{rf})\Phi - 0.5\lambda_{mkt}(\Phi_{mkt}^2\omega_r\Sigma_r\omega_r{}') \qquad \textit{[F4]}$$

$$\lambda_{mkt} \cong \frac{\mu_r - \mu_{rf}}{\Phi_{mkt}\omega_r\Sigma_r\omega_r{}'} \qquad \textit{[F5]}$$

Regarding foreign reserves' management, BR's choice of inputs for *[F5]* consists of using the observed (historical) returns of the U.S. Treasury Notes and Bonds, and the U.S. Treasury Bills to estimate the expected returns of the risky and risk-free assets (e.g. μ_r and μ_{rf}), respectively; estimating Σ_r as the U.S. Treasury Notes and Bonds' covariance matrix; using market capitalization as ω_r; and estimating market's preference (weight) for risky assets (Φ_{mkt}) from market capitalization. This particular choice will be addressed in the third chapter.

where Ω is the (k x k) covariance matrix of views' errors[6]; τ is a scalar measuring the uncertainty of the market equilibrium as a neutral reference or prior[7]; P is a (k x n) matrix with each row representing a view portfolio, where an element of P is nonzero if the respective asset is involved in the view and zero otherwise[8]; and Q is a k-vector that contains investor's views on each asset's expected returns. Following Idzorek (2002), the intuition behind *[F6]* is that the BL model is a weighted average of the market-implied excess returns (Π_{mkt}) and the views vector (Q), in which the relative weightings are a function of the uncertainty of the market equilibrium as a neutral reference (τ) and the uncertainty of the views (Ω).

As stated before, because this document focuses on strategic asset allocation, investors' views are not considered; in this case investors should hold the market portfolio. Therefore the –equilibrium- distribution of expected excess returns which serve as inputs to the BR's quadratic optimization in the mean-variance space is given by *[F7]*.

$$\overline{\Pi} \sim \mathrm{N}(\Pi_{mkt}, \tau\Sigma) \qquad \textit{[F7]}$$

Despite using the equilibrium expected excess returns provides a stable input which avoids large reallocation of weights overtime and extreme portfolio weights or "corner solutions", long-term portfolios' excessive risk taking issues still remain. Following León and Reveiz (2010) this excessive risk taking in strategic asset allocation may rise from concealed risk taking due to the long-term-independence-of-returns assumption.

Based on the evidence of long-term dependence of financial returns León and Reveiz (2010) develop an adjusted version of the Hurst exponent[9], which they exploit to estimate the true long-term-serial-dependence of returns and to adjust the estimated covariance matrix according to the investment horizon for optimization purposes (*e.g.* one year). In other words, instead of relying on the square-root-of-time rule to scale the estimated covariance matrix, which entails a time-invariant covariance matrix, they estimate the observed pace at which variance and covariance of returns scale over time.

Results by León and Reveiz (2010) verify that the long-term independence assumption is inappropriate, where the longer the investment horizon the greater the error resulting from relying on such assumption. They conclude that ignoring long-term dependence within the mean-variance portfolio optimization results in concealed risk taking, especially for long-term investors (*e.g.* central banks, pension funds and sovereign wealth managers), and in a tendency to provide extreme weights due to sharp differences between return/risk ratios

[6] BL original model assumes that all views are independent from each other, which would result in Ω being a diagonal matrix where non-zero terms correspond to the variance of the errors of each view; please note that for certain (*e.g.* 100% confidence) views Ω contains zeros only.

[7] A small value of τ corresponds to a high confidence in the equilibrium return (CAPM) estimates. According to Idzorek (2002) it is customary to use a τ value close to zero (*e.g.* $0.01 < \tau < 0.5$). This document uses $\tau = 0.025$.

[8] When a relative view is expressed the elements of a row sum up to zero; when an absolute view is expressed, the corresponding row consists of a 1 in the place of the asset under view and zeros everywhere else (*e.g.* the sum of its elements is 1).

[9] Hurst exponent (H) is named after British physicist H.E. Hurst (1880–1978), whose analysis suggested that numerous natural phenomena significantly diverged from being long-term independent as assumed by Geophysics at that time. Hurst developed a methodology for estimating the empirical exponent which fits how the random variable behaves with respect to time. A revised version of Hurst's methodology was developed by Mandelbrot and Wallis (1969) under the name of Rescaled Range Analysis (R/S); in this document this methodology is used for estimating H and an adjusted version of H, as presented in León and Reveiz (2010) or León and Vivas (2010). The unfamiliar reader may refer to the previously mentioned documents and Peters (1996).

resulting from inadequate covariance scaling. Ultimately León and Reveiz (2010) suggest adjusting the covariance matrix (Σ) according to the investment horizon as in *[F8]*, where $\hat{\sigma}^2_{\{(i,j),hf\}}$ and $\hat{\sigma}^2_{\{(i,j),lf\}}$ are the high-frequency (*e.g.* daily) and low-frequency (*e.g.* annual) estimated covariance between assets *i* and *j*, respectively; $\breve{H}_i$ and $\breve{H}_j$ are the adjusted Hurst exponents[10] for assets *i* and *j*; and m is the number of high-frequency periods which compose a low-frequency period (*e.g.* 252 days in a year).

$$\hat{\sigma}^2_{\{(i,j),lf\}} = \left(m^{\breve{H}_i+\breve{H}_j}\right)\left(\hat{\sigma}^2_{\{(i,j),hf\}}\right) \qquad [F8]$$

Using this adjustment within the BL framework allows the strategic asset allocation procedure to deal to some extent with (i) large reallocation of weights due to sharp changes of the inputs; (ii) the presence of extreme portfolio weights or "corner solutions"; and (iii) long-term portfolios' excessive risk taking. Consequently, the main unattended assumption underlying the model is the normality of asset's or portfolio's returns, an assumption inevitably shared by all mean-variance portfolio models.

Finally, the model comprises two major restrictions devoted to matching the resulting asset allocation with risk management goals.[11] First, as aforesaid, this paper assumes –for illustrative purposes- the existence of an implicit *natural hedge* goal in the currency composition of the foreign reserves, which should replicate the expected balance of payments outflows.

Second, given the reluctance to attain losses in foreign reserves' management, the quadratic optimization procedure also includes a non-linear constraint which limits asset allocations to those not resulting in losses with a 95% confidence level; this is akin to maintaining efficient portfolios' expected losses below or equal to 5% of the occurrences. This kind of constraint is common for central bank's asset allocation procedures, and usually consists of limiting the probability of a negative return over 12 months to a small figure, such as 1% or 5% (de Beaufort and Berkelaar, 2010).[12]

3. Selecting the investment (optimal) portfolio

The adjusted and constrained BL approach yields an efficient frontier which is compatible with foreign reserves management objectives. Selecting the single asset allocation which will provide investment portfolio's weights requires identifying the optimal or utility maximizing efficient portfolio.

In order to determine the utility of each portfolio comprising the efficient frontier, a proper utility function has to be chosen. The asset allocation model uses the classical quadratic utility function, which is based on the intuition that utility increases with higher expected returns and decreases with increased volatility (Litterman, 2003a). Let μ_p and σ_p^2 be a portfolio's expected return and variance, respectively, and let λ_{CB} be the central bank's relative risk aversion parameter, the utility function for selecting the investment portfolio is the following:

---

[10] Estimated adjusted Hurst exponents are presented in Figure A1 (Annex).

[11] Other customary constraints, such as non-negativity of portfolio weights and the sum of portfolio weights equal to 1 are also included.

[12] In Colombia's foreign reserves management framework such constraint is calculated using a delta-normal (e.g. variance-covariance) Value at Risk approach with a 95% confidence level and a holding period equal to the investment horizon (e.g. one year).

$$U_p = \mu_p - 0.5\lambda_{CB}\sigma_p^2 \qquad \textit{[F9]}$$

This choice follows several considerations. First, the selected utility function is the basis for the mean-variance optimization approach, for the equilibrium theory (CAPM) and for MPT (Litterman, 2003a), where all these theories or models add up to the BL approach herein employed.[13] Second, the main insights of MPT are likely to be robust with respect to alternative functions deemed to be more accurate[14] (Litterman, 2003a). Third, this function represents the assumption which states that as risk increases there is an increasing aversion – in the form of willingness to forgo expected return- to additional increases in risk (Litterman, 2003b). The third consideration is rather important: it may be a desirable characteristic since it could capture the prominent asymmetric exposure to risk of a central bank's utility function (Pihlman and van der Hoorn, 2010; Reveiz, 2004), where the rewards from high returns are much smaller than the negative consequences of having to report losses[15] and where capital losses are to be avoided in the investment horizon.

Calculating the utility for each of the efficient frontier's portfolios is straightforward except for the central bank's risk aversion parameter (λ_{CB}). Analogous to the procedure employed for estimating the market's risk aversion parameter in *[F1]*, and due to the convenience of the chosen utility function, λ_{CB} can be estimated from the appraisal of the Board of Directors' or the top-decision making body's risk aversion. Let μ_{UST} and σ_{UST}^2 be the observed (historical) return and variance of a portfolio containing a set of U.S. Treasury Notes and Bonds (*i.e.* the risky assets), respectively, where portfolio weights correspond to their market capitalization; μ_{USB} the observed U.S. Treasury Bills' return (*i.e.* the risk-free asset); Φ_{CB} the Board of Directors' preference (weight) for the U.S. Treasury Notes and Bonds portfolio, λ_{CB} may be estimated as in *[F10]*:

$$\lambda_{CB} \cong \frac{\mu_{UST} - \mu_{USB}}{\Phi_{CB}\sigma_{UST}^2} \qquad \textit{[F10]}$$

The intuition behind this estimation of λ_{CB} is that it allows capturing the Board of Directors' trade-off between return and risk for what can be regarded as a simplified traditional foreign reserves' portfolio. Each *j*-member of the Board is presented a wide spectrum of return and risk alternatives, which correspond to different combinations of risky (*e.g.* U.S. Treasury Notes and Bonds) and risk-free (*e.g.* U.S. Treasury Bills), where each combination corresponds to a different Φ_{CB} value. Because the choice made by each *j*-member of the Board entails a particular choice of Φ_{CB_j}, the overall preference for risky assets (Φ_{CB}) results from the arithmetic mean of such *j*-preferences.

This particular choice of risky and risk-free parameters results from the benchmark and investment horizon. First, both U.S. Treasury Bills and U.S. Treasury Notes and Bonds are

[13] Please note that the *inverse optimization* procedure in *[F1]*, which is the distinctive feature of the BL model, assumes that investors' utility is precisely the quadratic function in *[F9]*.

[14] Alternative functions are often preferred because the quadratic exhibits a non-intuitive behavior with respect to investor's wealth level: as he becomes wealthier, he reduces his investments in risky assets. This is referenced as an Increasing Absolute Risk Aversion (IARA) utility function (Danthine and Donaldson, 2001).

[15] In this sense Pihlman and van der Hoorn (2010) document that at the beginning of the 2008's global financial crisis the vast majority of central banks joined the flight-to-quality, and reduced their banking sector and agencies exposures far more than their decrease in their reserves; even some banks that reduced those exposures experienced an increase in their reserves. According to the authors this pattern is a clear sign of a shift in their risk assessment or their risk tolerance, also supported by central bank's shift to safe-haven assets such as government bills and bonds from reserve-issuing countries.

natural choices for asset allocation within a central bank, where reserves have traditionally been held in liquid, short-duration government bills and bonds, denominated in a handful of reserve currencies, predominantly the U.S. dollar and euro (Pihlman and van der Hoorn, 2010), which is also representative of central bank's preference for capital preservation (Reveiz, 2004).

Second, U.S. Treasury Bills is an intuitive proxy for the risk-free asset because its price-risk is negligible along the chosen investment horizon (*e.g.* one year), and the U.S. Treasury Notes and Bonds is a natural choice for non-risk-free assets within a traditional foreign reserves framework. Restraining the risk-free and non-risk-free assets to U.S. dollar denominated instruments in the illustrated case is also insightful and convenient since the search for a *natural hedge* between foreign reserves and the balance of payments biases reserves' currency composition towards U.S. dollar.

4. Main results

The herein presented exercise aims to illustrate the asset allocation process from different scenarios and environments, which serve the purpose of identifying the effects resulting from the unique nature of foreign reserves management and from different time-varying macroeconomic conditions, respectively. Assumptions are for illustrative purposes; the intuition and analysis is applicable to several central banks, especially those pertaining to non-reserve-issuing countries.

The exercise consists of the application of the methodology depicted in the second and third chapters for a 27-risk factors weekly database from December 31st 1998 to November 30th 2010. As exhibited in Figure 1, risk factors comprise treasury Bills from U.S., Germany and Japan, which will serve as risk-free assets; treasury Bonds and Notes from U.S., Germany and Japan; U.S. Treasury Inflation-Protected Securities (TIPS); U.S. agencies; U.S. corporates; supranationals; equity and gold. Several terms-to-maturities are considered where deemed appropriate, and prices are considered in each asset's currency of issuance[16], with gold considered as an U.S. dollar denominated asset currency constraint calculations[17]. The preference for risky assets (Φ_{CB}) is assumed to be 75%, which results in a 10.78 risk aversion parameter for all scenarios.

Five different scenarios were designed in order to appreciate the effects of the restrictions inherent to defining the optimal strategic asset allocation for foreign reserves, where each scenario corresponds to applying or ignoring the settings considered (Figure 2).

[16] As stated before, this is a key feature of the exercise since the objective of BR is to hold exposures to the selected currencies in order to obtain a *natural hedge* with the balance of payments' expected outflows.

[17] Alternatively the gold could also be included as another currency of the currency composition; therefore no consideration should be made when optimizing the universe of risk factors. This approach requires further research due to the effects that it can have on the strategic asset allocation policies.

Figure 1
Universe of risk factors

<table>
<tr><th colspan="2">Risk Factor</th><th>Market</th><th>Currency</th><th>Description</th></tr>
<tr><td rowspan="3">Risk-free</td><td rowspan="3">Treasury Bills</td><td>U.S.</td><td>USD</td><td rowspan="3">Bills indexes from selected reserve-issuing countries. [Bloomberg TKR G0BA, G0DB and G0YB]</td></tr>
<tr><td>GER.</td><td>EUR</td></tr>
<tr><td>JAP.</td><td>JPY</td></tr>
<tr><td rowspan="10">Risky</td><td rowspan="3">Treasury Notes & Bonds</td><td>U.S.</td><td>USD</td><td rowspan="3">0-5Y, 5-10Y and 10Y+ Bonds and Notes indexes from selected reserve-issuing countries. [(U.S.) GVQA, G6O2 and G9O2; (GER.) G1DB, G6D0 and G9D0; (JAP.) GVYA, G6Y0 and G9Y0]</td></tr>
<tr><td>GER.</td><td>EUR</td></tr>
<tr><td>JAP.</td><td>JPY</td></tr>
<tr><td>Treasury TIPS</td><td rowspan="4">U.S.</td><td rowspan="7">USD</td><td>1-10Y and 10Y+ U.S. Treasury Inflation Protected Securities indexes. [G5QI and G9QI]</td></tr>
<tr><td>U.S. Agencies</td><td>1-5Y, 5-10Y and 10Y+ indexes for unsubordinated bonds issued by agencies; 36 issuers [GVP0, G6P0 and G9P0]</td></tr>
<tr><td>U.S. Corporates</td><td>1-5Y, 5-10Y and 10Y+ indexes for companies rated from AAA to BBB. [CVB0, C6B0, C9B0, CVC0, C6C0 and C9C0]</td></tr>
<tr><td>U.S. Mortgages</td><td>U.S. fixed MBS; 3 government-sponsored enterprises (Fannie Mae, Freddie Mac, Ginnie Mae) [M0AN]</td></tr>
<tr><td>Supranationals</td><td rowspan="3">Global</td><td>1-3Y AAA supranational bonds index; 9 issuers from several countries or regions. [GS1S]</td></tr>
<tr><td>Gold</td><td>Bloomberg Spot price. [GOLDS]</td></tr>
<tr><td>Equity</td><td>S&P's capitalization weighted index based on 100 highly capitalized stocks selected from the S&P 500. [OEX]</td></tr>
</table>

Source: Bloomberg. Unless otherwise stated, Bank of America – Merrill Lynch, provided total return indexes.

Figure 2
Scenarios

Settings	**Scenarios**				
	1	**2**	**3**	**4**	**5**
Excludes Currency Composition Constraints	✓	✗	✗	✗	✗
Excludes 95% Confidence Non-Loss Constraints	✓	✓	✗	✗	✗
Includes Equity	✓	✓	✓	✗	✗
Includes Corporates, Supranational, Agencies and Mortgages	✓	✓	✓	✓	✗
Includes U.S., GER and JAO Treasury Bills, Bonds and Gold	✓	✓	✓	✓	✓

Source: authors' design.

The first scenario follows the ordinary strategic asset allocation case, in which a global benchmark should include all asset classes or risk factors[18], where no non-loss restrictions or currency composition objectives are considered. The second scenario includes a currency composition constraint, which consists of limiting the currency composition of the efficient portfolios to those complying with U.S. dollar 85%, Euro 12% and Japanese Yen 3%[19]; this is for illustrative purposes only. The third scenario not only includes the currency composition constraint, but adds a 95% confidence non-loss constraint, calculated as described in the previous chapter. The fourth scenario considers the currency composition and non-loss constraints, and excludes equity as a risk factor. The fifth, the most restrictive scenario, considers the currency composition and non- loss constraints, and limits the risk factors to gold and Bills, Bonds and Notes from selected reserve-issuing countries.

These scenarios were chosen for two reasons: (i) they illustrate the main active constraints for foreign reserves management at BR, and (ii) they reproduce some of the documented stages of reserves management. Regarding the second reason, central banks' practice in the last

[18] Despite all risk factors are included the currency risk issue is not addressed in this scenario. As stated before, this is a key feature of the exercise since the objective of BR is to hold exposures to the selected currencies in order to obtain a *natural hedge* with the balance of payments' expected outflows. In order to considerate exchange rate risk Black (1989) introduced the Universal Hedging concept.

[19] This reproduces BR's currency composition constraint, where the reserves' currency composition replicates the balance of payments' outflows in the previous three years. Although there are transactions with many countries, currencies that have low transaction costs and high liquidity in its financial markets are selected only. It also takes into account that the U.S. Dollar is, so far, the only currency used for intervention in the local market.

decades makes it possible to identify four main periods which characterize the way foreign reserves have been managed, where the attitude towards credit and market risks are the main attributes for each period. According to McCauley and Fung (2003), Borio *et al.* (2008b) and Pihlman and van der Hoorn (2010), these periods or stages may be summarized as follows:

Figure 3
Four periods of foreign reserves management

Period	Description
1970s	Reserve managers shifted most of their short-term holdings out of Treasury securities and into short-term bank deposits and private money market instruments. This came with the acceptance of additional credit risk. (Compatible with Scenario 5)
1980s and most of 1990s	As the bond market entered a long bull period in the 1980s reserve managers found that they could enhance returns by extending maturities, and continued to do so into the 1990s. Managers increased their portfolios' duration via shifting from Bills into Bonds and Notes. This came with the acceptance of more market risk. (Compatible with Scenario 5)
End of 1990s	In the last years of the 1990s reserve managers decided to enhance returns on their long-term holdings. Treasury securities holdings decreased in favor of debt securities issued by government sponsored enterprises (*e.g.* Fannie Mae and Freddie Mac), by corporate entities, and equity. Diversification of reserves was driven primarily by a search for yield, where the traditional diversification argument –risk reduction- played only a minor role. Additional risk exposures from non-traditional reserves assets came with the acceptance of additional credit risk, but also of other types of risk (*e.g.* model and prepayment risks) arising from non-linear or complex exposures. (Compatible with Scenario 3 and 4)
After 2008 crisis	Reserve managers decided to imitate other investors' flight-to-quality, which results in a significant reduction in diversification. Foreign reserves allocation shifted to safe-haven assets such as government Bills, Notes and Bonds from reserve-issuing countries. This shift is a signal of renewed reluctance to credit and market risk. (Compatible with Scenario 5)

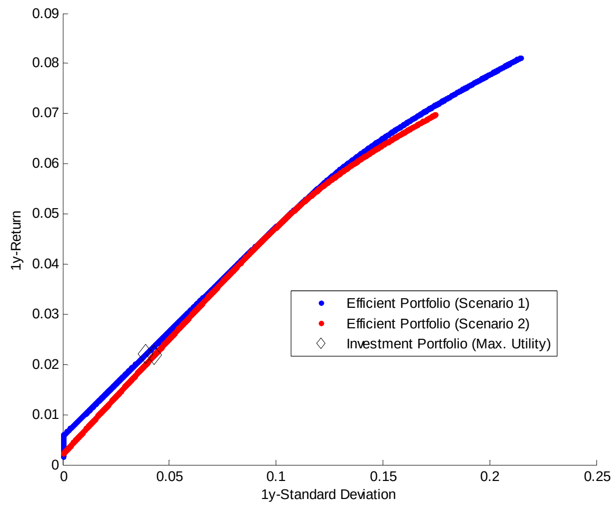
Source: based on McCauley and Fung (2003), Borio *et al.* (2008b) and Pihlman and van der Hoorn (2010).

Figure 4 exhibits the efficient frontiers obtained for the first and second scenarios, where the investment portfolio for each scenario corresponds to the maximum utility portfolio. Both frontiers appear to be somewhat similar, with investment portfolios not-very-distant from each other.

The consolidated[20] asset allocation resulting from Scenario 1 and Scenario 2 is depicted in Figure 5. The first scenario focuses on Germany Bills, Notes and Bonds (68.8%) and equity (16.9%), with significantly lower allocations to U.S. Bills, Notes and Bonds, agencies and corporates. The expected loss-probability of this particular allocation is rather high (28.5%), with 95% Conditional Value at Risk or Expected Shortfall about 2.4% and maximum drawdown of 7.6%. This Scenario's duration is about three years.

[20] Due to simplicity issues, detailed results are provided in an annex (Figure A1).

Figure 4
Efficient frontiers for scenarios 1 and 2



Source: authors' calculations.

Figure 5
Asset allocations and main features of investment (optimal) portfolios
Scenarios 1 and 2

SCENARIO	Allocations (Weights)												*Main Features*							
	TREASURY BILLS			TREASURY NOTES & BONDS			U.S. AGENCIES	U.S. CORPORATES	SUPRA	U.S. MORTGAGES	GOLD	EQUITY	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	U.S.	GER.	JAP.														
1	0,00%	41,89%	0,00%	2,83%	26,95%	0,61%	3,34%	7,52%	0,00%	0,00%	0,00%	16,86%	*2,2%*	*3,9%*	*0,57*	*3,05*	*28,5%*	*1,39*	*2,4%*	*7,6%*
2	22,23%	3,49%	0,00%	19,50%	8,63%	3,03%	3,18%	9,15%	0,00%	12,78%	0,04%	17,97%	*2,2%*	*4,3%*	*0,51*	*3,59*	*30,4%*	*1,21*	*2,7%*	*8,7%*

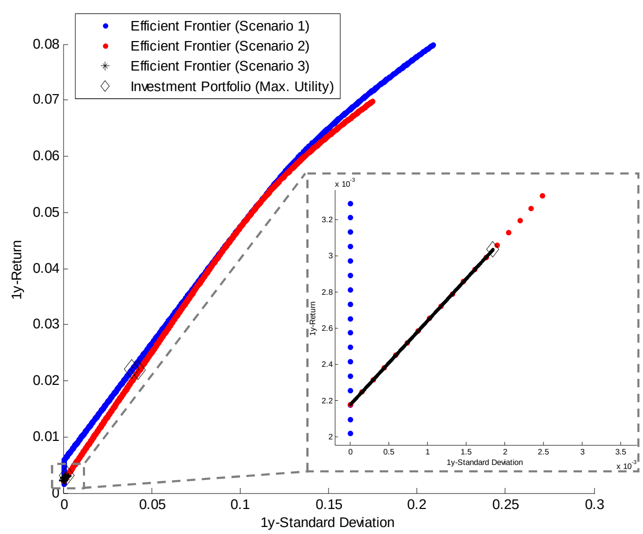
Source: authors' calculations.

The second scenario, which includes the *natural hedge* oriented currency composition constraint, exhibits major changes in the resulting allocation. Despite the efficient frontiers and the investment portfolios' main features (in Figure 5) seem not-too-distant from each other, the weights assigned in the second scenario's investment portfolio concentrate in U.S. Bills, Notes and Bonds (41.7%) and equity (18.0%), whereas agencies, corporate, mortgages and supranationals account for 25.1% of the portfolio, whilst gold receives an insignificant but positive allocation. Due to a significant decrease in Bills along with a significant rise in U.S. Notes, Bonds and mortgages, Scenario 2's duration increases from 3.1 to 3.6 years.

As expected, the shift from the first scenario's investment portfolio to the second results in a suboptimal allocation, characterized by assuming higher risk (*e.g.* standard deviation, CVaR and maximum drawdown) for a similar level of return, with a lower return/standard deviation ratio and expected utility. Nevertheless, it is noteworthy that introducing the currency constraint resulted in a major change in the allocation, but in a mild change in the properties of the investment portfolios, where the only significant change is the aforementioned 6-month increase in duration.

The third scenario, comprising currency and non-loss constraints, is presented in Figure 6, where the area containing Scenario 3's frontier is magnified for simplicity purposes. Unlike the addition of currency constraints, it is clear that including a 95% confidence non-loss constraint has a noteworthy impact on the resulting frontier: efficient portfolios which comply with the non-loss constraint are restricted to a rather small fraction of those contained in the second scenario.

Figure 6
Efficient frontiers for Scenarios 1, 2 and 3



Source: authors' calculations.

Figure 7
Asset allocations and main features of investment (optimal) portfolios
Scenarios 1 to 3

SCENARIO	Allocations (Weights)												*Main Features*							
	TREASURY BILLS			TREASURY NOTES & BONDS			U.S. AGENCIES	U.S. CORPORATES	SUPRA	U.S. MORTGAGES	GOLD	EQUITY	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	U.S.	GER.	JAP.														
1	0,00%	41,89%	0,00%	2,83%	26,95%	0,61%	3,34%	7,52%	0,00%	0,00%	0,00%	16,86%	*2,2%*	*3,9%*	*0,57*	*3,05*	*28,5%*	*1,39*	*2,4%*	*7,6%*
2	22,23%	3,49%	0,00%	19,50%	8,63%	3,03%	3,18%	9,15%	0,00%	12,78%	0,04%	17,97%	*2,2%*	*4,3%*	*0,51*	*3,59*	*30,4%*	*1,21*	*2,7%*	*8,7%*
3	82,16%	11,96%	2,17%	0,86%	0,16%	0,80%	0,14%	0,40%	0,01%	0,56%	0,00%	0,78%	*0,3%*	*0,2%*	*1,64*	*0,47*	*5,0%*	*0,30*	*0,1%*	*0,2%*

Source: authors' calculations.

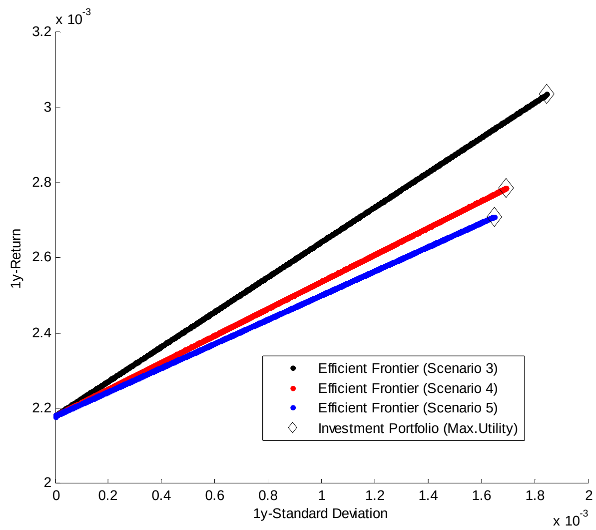
As aforementioned, non-loss constraints are common for particularly risk adverse agents such as central banks, where there is a preference for liquidity and safety, with a clear capital preservation goal (Reveiz, 2004; de Beaufort and Berkelaar, 2010). As expected, the resulting asset allocation significantly deviates from previous scenarios (Figure 7), where the expected loss-probability corresponding to Scenario 3 is –as required- 5%; about a sixth of any of the previous scenarios. Such decrease in loss-probability comes with the acceptance of significantly lower return and risk, along with lower utility, but a higher return/risk ratio.

It is clear that the non-loss constraint is responsible for an allocation heavily reliant on risk-free investments such as Treasury Bills (96.3%), which only allow for minor allocations in U.S. Notes and Bonds (1.8%), and marginal allocations in other risk factors (1.9%), where gold assignment continues to be null. Non-loss constraint also explains the investment portfolio's low duration.

It is important to highlight that Scenario 3's result may be dominated by the current macroeconomic environment, where historically low yields and volatile corporate and equity markets confine non-loss-complying allocations to a narrow portion of scenarios 1 and 2 frontiers; it is likely that under different market conditions the asset allocation shifts from Bills to other risk factors, especially to Treasury Notes and Bonds, with significant increases in duration.[21]

Figure 8 exhibits scenarios 3, 4 and 5; frontiers corresponding to scenarios 1 and 2 are not presented for simplicity. As expected, excluding equity (Scenario 4) from the set of eligible risk factors makes the attainable efficient frontier and investment portfolio farther from optimality. However, as evident in Figure 9, because non-loss constraints introduced in Scenario 3 discards equity as an effective eligible risk factor, changing from Scenario 3 to Scenario 4 has marginal effects in the resulting investment portfolio. Likewise, Scenario 5 (*e.g.* excluding equity, corporates, agencies, mortgages and supranationals) results in an unimportant change in the main features of the resulting investment portfolio.

Figure 8
Efficient frontiers for Scenarios 3, 4 and 5



Source: authors' calculations.

Finally, Scenario 5 considers the effect of limiting the universe of risk factors to those that may be considered as strictly traditional in foreign reserves management (*e.g.* Bills, Notes and Bonds from reserve-issuing countries, and gold), along with currency and non-loss constraints. As expected, allocations concentrate on Treasuries according to the currency constraint.

[21] Using a different macroeconomic environment (e.g. June 2006) the authors corroborated such statement. Dominated by higher yields, which allow the optimization for more maneuver space to comply with the non-loss constraint, the asset allocation resulted in a 1.76 years duration (compared to 0.47), where Bills were assigned 53.46% (compared to 96.3%), Bonds and Notes 15.88% (compared to 1.82%), equity 16.23% (0.78%), and corporate, agencies, supranationals and mortgages 14.43% (1.11%).

Figure 9

Asset allocations and main features of investment (optimal) portfolios

Scenarios 1 to 5

SCENARIO	Allocations (Weights)												Main Features							
	TREASURY BILLS			TREASURY NOTES & BONDS			U.S. AGENCIES	U.S. CORPORATES	SUPRA	U.S. MORTGAGES	GOLD	EQUITY	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	U.S.	GER.	JAP.														
1	0,00%	41,89%	0,00%	2,83%	26,95%	0,61%	3,34%	7,52%	0,00%	0,00%	0,00%	16,86%	*2,2%*	*3,9%*	*0,57*	*3,05*	*28,5%*	*1,39*	*2,4%*	*7,6%*
2	22,23%	3,49%	0,00%	19,50%	8,63%	3,03%	3,18%	9,15%	0,00%	12,78%	0,04%	17,97%	*2,2%*	*4,3%*	*0,51*	*3,59*	*30,4%*	*1,21*	*2,7%*	*8,7%*
3	82,16%	11,96%	2,17%	0,86%	0,16%	0,80%	0,14%	0,40%	0,01%	0,56%	0,00%	0,78%	*0,3%*	*0,2%*	*1,64*	*0,47*	*5,0%*	*0,30*	*0,1%*	*0,2%*
4	81,94%	11,88%	1,77%	1,30%	0,24%	1,20%	0,21%	0,61%	0,01%	0,84%	0,00%	0,00%	*0,3%*	*0,2%*	*1,64*	*0,54*	*5,0%*	*0,28*	*0,1%*	*0,1%*
5	82,65%	11,71%	0,87%	2,26%	0,41%	2,10%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	*0,3%*	*0,2%*	*1,64*	*0,58*	*5,0%*	*0,27*	*0,1%*	*0,1%*

Source: authors' calculations.

As with the change from Scenario 3 to 4, the shift from 4 to 5 has marginal effects on return, risk (*e.g.* standard deviation, CVaR or Maximum Drawdown) or utility. Again, as in scenarios 3 and 4, it is important to highlight the macroeconomic environment this exercise is structured in, where worldwide low yields in presence of a rather rigorous non-loss constraint discards risk factors which would otherwise allow for higher returns.

Regarding the behavior of the herein presented approach for strategic asset allocation through different macroeconomic conditions or environments, figures 10 and 11 exhibit the 2002-2010 resulting weights and features for Scenario 4; this is the one that illustrates BR's active restrictions and assumptions the closest. Each year's asset allocation considers the data effectively available at that moment, from December 1998 onwards (*e.g.* November 2003's allocation considers data from December 1998 to October 2003), whereas the only parameters held constant are the adjusted Hurst exponents used in *[F8]* and the Board of Director's preference (weight) for risk-free assets ($1 - \Phi_{CB} = 0.25$).[22]

Figure 10

Investment portfolio's asset allocations and main features

(Scenario 4, 2002-2010)

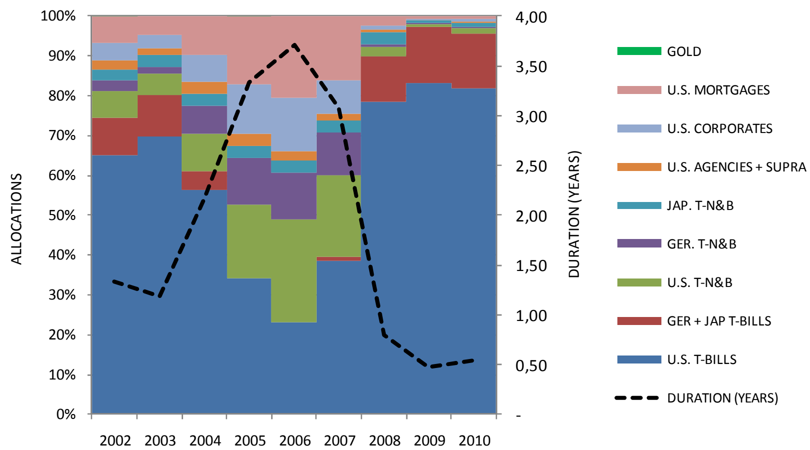
Year	Allocations (Weights)											Main Features							
	TREASURY BILLS			TREASURY NOTES & BONDS			U.S. AGENCIES	U.S. CORPORATES	SUPRA	U.S. MORTGAGES	GOLD	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	U.S.	GER.	JAP.													
2002	64,91%	9,59%	0,00%	6,65%	2,53%	2,97%	2,12%	4,47%	0,00%	6,75%	0,00%	*1,7%*	*1,1%*	*1,64*	*1,34*	*5,0%*	*1,69*	*0,4%*	*0,5%*
2003	69,72%	10,37%	0,00%	5,28%	1,75%	2,97%	1,66%	3,39%	0,00%	4,85%	0,00%	*1,4%*	*0,8%*	*1,64*	*1,18*	*5,0%*	*1,32*	*0,3%*	*0,7%*
2004	56,30%	4,68%	0,00%	9,34%	7,20%	2,97%	2,97%	6,67%	0,00%	9,88%	0,00%	*2,7%*	*1,6%*	*1,64*	*2,15*	*5,0%*	*2,53*	*0,7%*	*1,6%*
2005	34,06%	0,00%	0,00%	18,60%	11,88%	2,97%	2,80%	12,38%	0,00%	17,30%	0,01%	*4,5%*	*2,7%*	*1,64*	*3,33*	*5,0%*	*4,13*	*1,1%*	*2,6%*
2006	23,17%	0,00%	0,00%	25,70%	11,88%	2,97%	2,48%	13,18%	0,00%	20,63%	0,00%	*5,6%*	*3,1%*	*1,78*	*3,71*	*3,7%*	*5,13*	*1,2%*	*3,0%*
2007	38,57%	1,12%	0,00%	20,23%	11,00%	2,97%	1,48%	8,38%	0,00%	16,26%	0,00%	*3,8%*	*2,3%*	*1,64*	*3,08*	*5,0%*	*3,57*	*1,0%*	*2,4%*
2008	78,38%	11,44%	0,17%	2,32%	0,68%	2,86%	0,60%	1,24%	0,03%	2,28%	0,00%	*0,6%*	*0,4%*	*1,64*	*0,80*	*5,0%*	*0,60*	*0,2%*	*0,2%*
2009	83,09%	11,95%	2,32%	0,67%	0,17%	0,71%	0,15%	0,36%	0,01%	0,57%	0,00%	*0,2%*	*0,1%*	*1,64*	*0,47*	*5,0%*	*0,17*	*0,0%*	*0,1%*
2010	81,94%	11,88%	1,77%	1,30%	0,24%	1,20%	0,21%	0,61%	0,01%	0,84%	0,00%	*0,3%*	*0,2%*	*1,64*	*0,54*	*5,0%*	*0,28*	*0,1%*	*0,1%*

Source: authors' calculations.

[22] Adjusted Hurst parameters are estimated with all data available (daily time-series from December 1998 to November 2010); they are held constant (Figure A2 in the Annex) because the estimation requires at least ten years of daily time-series to be adequate (León and Vivas, 2010). Risk aversion parameter (λ_{CB}) is held constant since results for the estimation herein suggested are available for recent years only; nevertheless, the other constituents of the estimation of λ_{CB} vary over time, which results in a time-varying risk aversion parameter (please refer to Figure A3 in the Annex).

As asserted before, the results presented previously are particularly dependent on the macroeconomic environment in which the asset allocation calculation takes place. The concentration on Treasury Bills due to the non-loss constraint varies during the analyzed period: allocation to Treasury Bills from selected reserves-issuing countries averages 66.16%, with a minimum of 23.17% in 2006 and a maximum of 97.36% in 2009. Consequently, the main features of each year's asset allocation vary accordingly, where investment portfolio's duration conveniently summarizes its exposure to market risk.

Figure 11
Investment portfolio's asset allocations and duration
(Scenario 4, 2002-2010)



Source: authors' calculations.

The time-varying effects of the non-loss constraint exhibits that market conditions allow for allocations which could effectively diversify and enhance the risk – return trade-off. For example, during times of high yields and low risk, namely by mid-2000s, U.S. Mortgages and U.S. Corporates were decisive in the increase of the investment portfolio's duration, return and utility.

Regarding this results, despite a one-year investment horizon may pertain to what a traditional investor considers as long-term, some reserves' managers are nowadays considering the usage of longer horizons. Such considerations are critical since the –theoretical- definition of asset allocation entails investment horizons of 10 years or more (Winklemann, 2003), and because the existence of long-term dependence results in varying optimal portfolios across different investment horizons (León and Reveiz, 2010). Therefore, for illustrative purposes, Figure 12 and Figure 13 present the 2002-2010 resulting weights and features for Scenario 4, where the investment horizon is three-years[23].

[23] Some changes to the original universe of risk factors (Figure 1) were necessary, as depicted in Figure A3 in the annex. The main change is the usage of 1-3 years Treasury Bonds indexes as the risk-free assets (instead of Treasury Bills), and the modification of the indexes used for the risky assets. The rest of the assumptions remain.

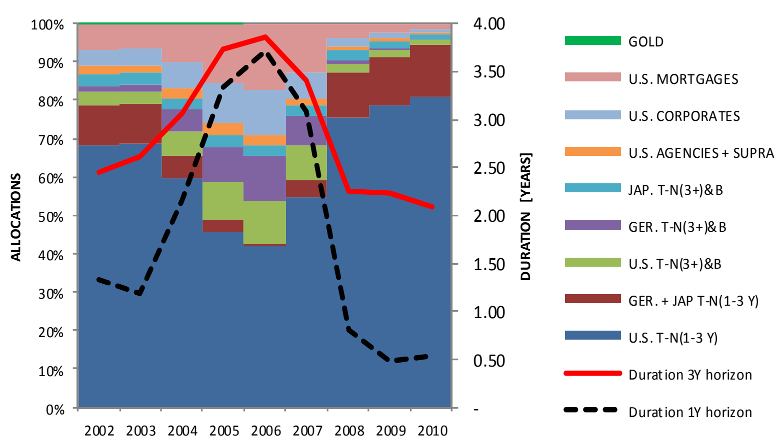
Figure 12
Investment portfolio’s asset allocations and main features
(Scenario 4, 2002-2010, 3-years horizon)

Year	Allocations (Weights)											*Main Features*							
	TREASURY NOTES [1-3 YEARS]			TREASURY NOTES (3+) & BONDS			U.S. AGENCIES	U.S. CORPORATES	SUPRA	U.S. MORTGAGES	GOLD	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	U.S.	GER.	JAP.													
2002	68.03%	10.66%	0.00%	3.55%	1.46%	2.97%	2.16%	4.40%	0.00%	6.76%	0.00%	*2.1%*	*1.3%*	*1.64*	*2.45*	*5.0%*	*2.10*	*0.5%*	*1.3%*
2003	68.73%	10.23%	0.00%	3.35%	1.89%	2.97%	2.06%	4.41%	0.00%	6.36%	0.00%	*2.1%*	*1.3%*	*1.64*	*2.62*	*5.0%*	*2.12*	*0.5%*	*1.7%*
2004	59.66%	6.01%	0.00%	6.03%	5.87%	2.97%	2.70%	6.80%	0.00%	9.95%	0.01%	*3.2%*	*1.9%*	*1.64*	*3.07*	*5.0%*	*3.14*	*0.8%*	*2.2%*
2005	45.76%	2.98%	0.00%	10.21%	8.90%	2.97%	3.08%	10.71%	0.00%	15.38%	0.01%	*4.7%*	*2.8%*	*1.64*	*3.74*	*5.0%*	*4.55*	*1.2%*	*2.8%*
2006	41.99%	0.52%	0.00%	11.46%	11.36%	2.97%	2.70%	11.51%	0.00%	17.48%	0.00%	*5.1%*	*3.1%*	*1.64*	*3.86*	*5.0%*	*4.89*	*1.3%*	*3.0%*
2007	54.80%	4.47%	0.00%	8.80%	7.65%	2.97%	1.63%	6.66%	0.00%	13.02%	0.00%	*3.6%*	*2.2%*	*1.64*	*3.41*	*5.0%*	*3.50*	*0.9%*	*2.5%*
2008	75.67%	11.42%	0.00%	2.39%	0.70%	2.97%	0.99%	2.05%	0.04%	3.77%	0.00%	*1.2%*	*0.7%*	*1.64*	*2.26*	*5.0%*	*1.20*	*0.3%*	*1.6%*
2009	78.72%	11.68%	1.04%	1.69%	0.44%	1.93%	0.62%	1.49%	0.03%	2.35%	0.00%	*0.8%*	*0.5%*	*1.64*	*2.23*	*5.0%*	*0.83*	*0.2%*	*1.5%*
2010	80.89%	11.88%	1.68%	1.31%	0.24%	1.29%	0.34%	0.99%	0.02%	1.37%	0.00%	*0.6%*	*0.4%*	*1.64*	*2.08*	*5.0%*	*0.63*	*0.2%*	*1.5%*

Source: authors’ calculations.

As expected, because the risk-free assets increased their duration (i.e. due to the shift from Treasury Bills’ to 1-3 years Treasury Bonds’), the exposure to market risk increased during most of the period under analysis (2002-2004 and 2008-2010), but remained almost the same during the 2005-2007 phase; the latter occurs as a consequence of non-loss restriction, which imposes an implicit ceiling for market exposure. It is also noteworthy that the resulting allocation is more concentrated on the new risk-free asset, which yields a higher return than the former one, without increasing the portfolio’s risk; thus all the risky assets decrease their weight.

Figure 13
Investment portfolio’s asset allocations and duration
(Scenario 4, 2002-2010, 3-years horizon)



Source: authors’ calculations.

Finally, the absence of gold allocations across scenarios, environments or investment horizons is remarkable. Despite being conventionally considered as a reserve asset, none of the different settings or macroeconomic conditions assigned weights significantly different from zero. Because of its relevance, the next chapter briefly presents some insights about this issue.

5. Insights on gold's absence from asset allocations

A more comprehensive look at gold's properties may be taken if it is possible to identify its role when added to a traditional foreign reserves portfolio. The decision to add an asset to a portfolio usually follows a risk reduction *rationale*, where such reduction results from the asset being a *hedge*, a *diversifier* or a *safe haven*. According to Baur and Lucey (2009), a *hedge* corresponds to an asset that is uncorrelated or negatively correlated with the portfolio on average, and does not have the specific property of reducing losses in times of market stress since the asset may even exhibit a positive correlation during those events; a *diversifier* is an asset positively correlated (but not perfectly correlated) with the portfolio on average, and does not have the specific property of reducing losses in times of market stress either; a *safe haven* is an asset that is uncorrelated or negatively correlated with the portfolio in times of market stress, where its main characteristic is its ability to reduce portfolio's losses in extreme market conditions.

In order to identify gold's asset type (e.g. *hedge*, *diversifier* or *safe haven*) Figure 14 exhibits gold's and 1-3 years treasuries' monthly returns from 1985 to 2010.[24] According to the data, correlation coefficient (0.1) confirms that on average gold is positively correlated, but not perfectly correlated; therefore, according to Baur and Lucey (2009), it could be regarded as a diversifier, but not as an asset providing protection during times of market stress (a *safe haven*) or as an asset which is uncorrelated or negatively correlated on average (a *hedge*).

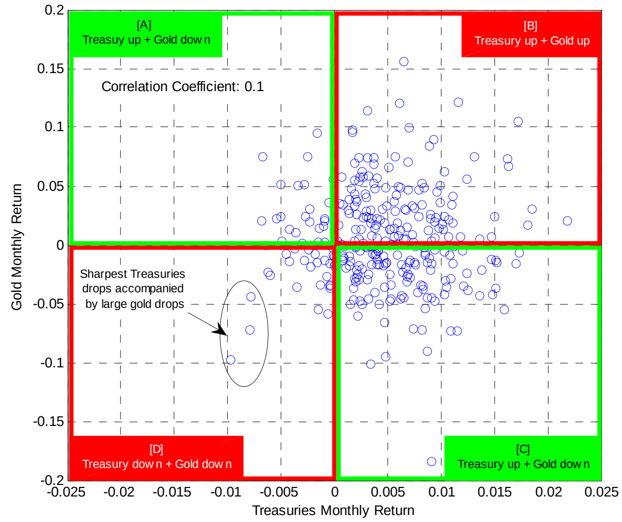
The information provided by the correlation coefficient is also available by means of graphic inspection. Figure 14 consists of 297 observations, each one belonging to a specific quadrant; quadrants A and C contain observations in which gold and treasuries returns compensate each other (48.1% of the observations), whereas quadrants B and D contain those in which both risk factors moved in tandem (51.9% of the observations). The density of observations inside each quadrant confirms that there is some diversification effect, but on average it's almost equally likely to find periods in which gold compensate treasuries' behavior and periods in which gold reinforce treasuries' behavior. Moreover, regarding treasuries' episodes of large losses, available data shows that the sharpest drop in treasuries (-0.97%) was accompanied by the third largest decrease in gold (-9.8%), which is a pattern that recurs for treasuries' second and third most acute losses. Therefore, concurrent with Baur and Lucey (2009)[25], gold may be classified as a source of diversification, but not as a *hedge* or a *safe haven* for a traditional foreign reserves portfolio.

However, despite being a diversifying asset, gold's volatility tends to exclude allocations to this commodity. Gold's monthly standard deviation (maximum drawdown) for the 1985-2010 series previously used is 4.2% (48.1%), whilst 1-3 years treasuries' is 0.5% (1.5%); this fact complicates gold's allocation within an optimization procedure, either in MPT's mean-variance or Reveiz and León (2010)'s total return-maximum drawdown space. Furthermore, as is the case with BR's foreign reserves management framework, strategic asset allocations including non-loss constraints make even more difficult to consider gold as an effectively eligible asset.

[24] The same analysis was developed with Germany and Japan treasuries. The conclusions herein presented also apply to those cases.

[25] Using a different methodology Baur and Lucey (2009) conclude that gold acts as a *safe haven* for equity indexes, but not for bonds in the U.S., U.K. and German markets. Additionally, Baur and Lucey estimate that gold being a *safe haven* for stock is limited to short intervals of 15 days approximately, which results from investors' increasing demand for the metal amid extreme negative returns, and sell it after market participants regained confidence and volatility decreased.

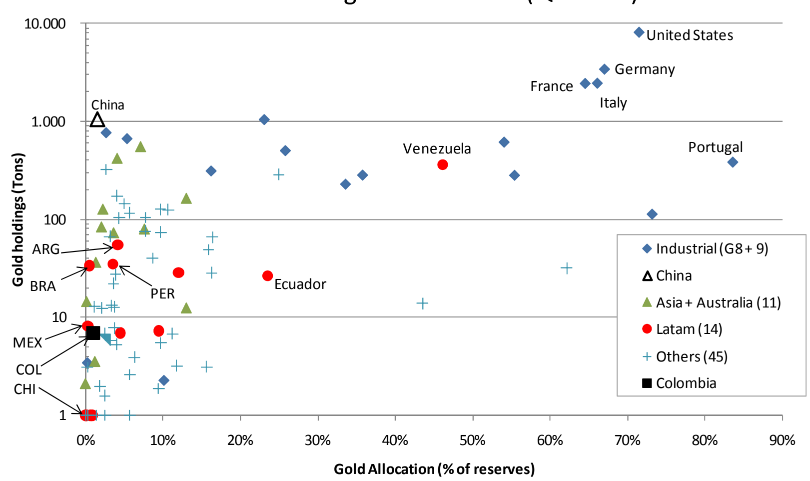
Figure 14
Gold and 1-3-years U.S. treasuries monthly returns
(December 1985 – September 2010)



Source: authors' calculations.

Some other central banks maintain minor allocations to gold. As exhibited in Figure 15, as of the first quarter of 2010, industrialized countries[26] tend to invest a significant part of their foreign exchange reserves in gold (40.5% on average), whilst China, Asia & Australia and Latin America hold around 1.5%, 4.6% and 7.1%[27], respectively, with all-countries average around 10.1%.

Figure 15
Central banks' gold allocations (Q1-2010)



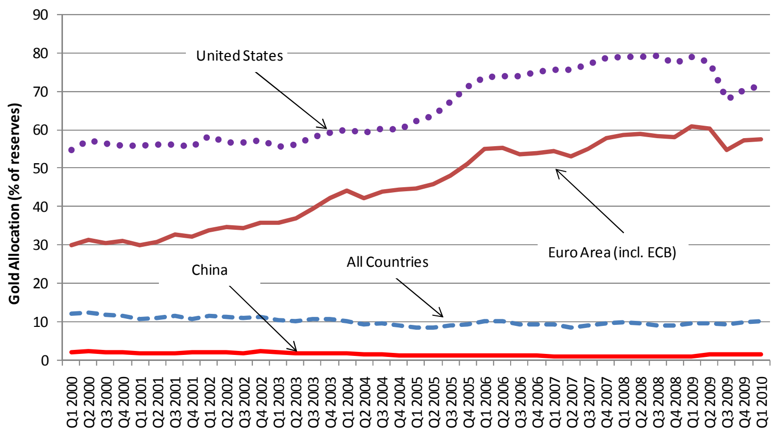
Source: authors' calculations, based on data from the World Gold Council

[26] Industrialized countries correspond to: France, Germany, Italy, Japan, U.K, U.S., Canada, Russia, Switzerland, Netherlands, European Central Bank, Portugal, Spain, Greece, Austria, Belgium, and Luxemburg. Asia & Australia correspond to India, Taiwan, Filipinas, Singapore, Thailand, Australia, Indonesia, Malaysia, South Korea, Cambodia and Hong Kong. Latin America corresponds to Colombia, Venezuela, Argentina, Peru, Brazil, Bolivia, Ecuador, México, Salvador, Guatemala, Paraguay, Dominican Republic, Uruguay, Chile and Costa Rica.

[27] Notably, if Venezuela, Bolivia and Ecuador are excluded, the average drops from 7.1% to 2.1%.

As depicted in Figure 16, the pattern of Figure 15 has not changed during the last decade: large gold allocations are typical of the United States and the euro area, whilst other regions and countries (e.g. Latin America, China) prefer undersized holdings of the metal. Besides historical reasons (Borio *et al.* 2008), an intuitive rationale for reserve-issuing countries' gold allocations may be their narrow set of eligible investments (i.e. due to credit risk constraints), which oblige them to hold the metal amid a limited asset universe; on the other hand, non-reserve-issuing countries face less strict credit constraints and may access a broader and more efficient range of allocations.

Figure 16
Central banks' gold allocations (Q1-2000 – Q1-2010)



Source: authors' calculations.

6. Final remarks

This paper illustrated BR's approach for strategic asset allocation of foreign reserves. According to the literature and the author's experience, this approach is able to deal to some extent with some of MPT shortcomings, namely (i) large reallocation of weights due to sharp changes of the inputs; (ii) the presence of extreme portfolio weights or "corner solutions"; and (iii) long-term portfolios' excessive risk taking. However, because of using the mean-variance space for portfolio optimization, the normality of asset's or portfolio's returns assumption still remains.

As expected, results confirm the effects of the unique nature of foreign reserves management for emerging markets, where some particularities, such as non-loss restrictions due to capital preservation objectives, result in increased complexity in the optimization process and in asset allocations significantly distant from standard MPT's optimality.

As emphasized in the two previous chapters, It is remarkable to find that gold is left aside in all scenarios and all environments considered in this asset allocation exercise, which may be explained to some extent by it being significantly riskier than most of the risk factors herein considered, especially when compared to Treasuries[28]. Either measured by standard deviation

[28] Borio *et al.* (2008b) affirm that despite being common among central banks, gold holdings are better explained by historical reasons, and suggest three main supporting arguments for such statement: (i) the management of gold reserves is not closely integrated with that of the rest of the portfolio; (ii) the trends in the range of investable

or maximum drawdown, either considered individually or as a contributor to the aggregated risk of an overall portfolio, authors conclude that gold fails to enhance the main features of a reserves portfolio, even if the latter is limited to Bills, Notes and Bonds from reserve-issuing countries; this is intuitive for non-reserve-issuing countries, which face less strict credit constraints and may access a broader and more efficient range of allocations.

Nevertheless, some reasons still justify allocating gold within reserves portfolios. Besides historical reasons for holding gold (Borio *et al.* 2008), the most recent financial turmoil episode highlights the metal's market liquidity, and its well-known characteristics as lacking credit risk, being indestructible and the ultimate store of value (Dampster, 2010). Therefore, gold's allocation, despite being discarded by conventional and non-conventional optimization procedures, should be carefully regarded within an eclectic approach, which may include considering gold as an additional currency.

Finally, results impose challenges for the future. The 95% confidence non-loss constraint restriction affects the asset allocation process in a markedly manner, especially in presence of low yields and high volatility environments, where the likelihood of attaining losses is the highest; this is the case for the years that followed 2008's financial turmoil, where Treasury Bills concentrated about 95% of the allocation.

Therefore, as argued by de Beaufort and Berkelaar (2010), focusing on the probability of a negative return may be shortsighted (e.g. it may be simultaneously forcing suboptimal allocations and foregoing the true concept of capital preservation), and should be addressed properly. Some alternatives encompass using maximum drawdown based optimization (Reveiz and León, 2010; de Beaufort and Berkelaar 2010) or the distribution of drawdowns (Sornette, 2003); these are research projects for the near future.

---

assets have not typically influenced decision on gold reserves; (iii) gold reserves vary significantly across central banks, where emerging markets gold holdings are minor or even non-existent when compared with the size of the reserves or with reserves-issuing countries, and (iv) standard risk-return analysis is unable to justify gold holdings. Regarding the last argument, the results herein presented agree with it.

7. References[29]

Banco de la República, *Foreign reserve management*, 2009. [Available online: http://www.banrep.gov.co/documentos/economic_reports/Reserve/2009_March.pdf]

Baur, D. and Lucey, B., "Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold", *Working Paper*, presented at the Joint Bank for International Settlements/European Central Bank/World Bank Public Investors Conference (Basel, November 2-3, 2010), 2009. [Available online: http://ssrn.com/abstract=952289]

Black, F. and Litterman, R., "Asset allocation: combining investor views with market equilibrium", *The Journal of Fixed Income*, September, 1991.

Black, F., "Universal hedging: optimizing currency risk and reward in international equity portfolios", *Financial Analysts Journal,* July/August, 1989.

Bodie, Z.; Kane, A.; Marcus, A., *Investments*, McGraw Hill/Irwin, 2001.

Borio, C.; Ebbesen, J.; Galati, G.; Heath, A., "FX reserve management: elements of a framework", *BIS Papers*, No.38, BIS, 2008.

Borio, C.; Galati, G.; Heath, A., "FX reserve management: trends and challenges", *BIS Papers*, No.40, BIS, 2008b.

Dampster, N., "Gold should form an essential part of central banks' liquidity portfolio", *Central Banking*, July, 2010.

Danthine, J-P. and Donaldson, J.B., *Intermediate Financial Theory*, Prentice Hall, 2001.

de Beaufort, R. and Berkelaar, A., "Managing tail risk in reserve portfolios", *Sovereign Risk Management* (Ed. Rietveld, M.), Central Banking Publications, 2010.

He, G. and Litterman, R., "The intuition behind Black-Litterman model portfolios", *Investment Management Research*, Goldman Sachs Quantitative Resources Group, 1999.

Idzorek, T., "A step-by-step guide to the Black-Litterman model", Zephyr Associates, 2002.

IMF, *Guidelines for foreign exchange reserve management*, IMF, Washington, 2004.

León, C. and Reveiz, A., "Portfolio optimization and long-term dependence", *Borradores de Economía*, No.622, Banco de la República, 2010. [Presented at the Joint Bank for International Settlements/European Central Bank/World Bank Public Investors Conference (Basel, November 2-3, 2010)]*

León, C. and Vivas, F., "Dependencia de largo plazo y la regla de la raíz del tiempo para escalar la volatilidad en el mercado colombiano", *Borradores de Economía*, No. 603, Banco de la República, 2010.*

Litterman, R., "Risk measurement", *Modern Investment Management*, Wiley, 2003a.

Litterman, R., "The Capital Asset Pricing Model", *Modern Investment Management*, Wiley, 2003b.

Mandelbrot, B. and Wallis, J., "Robustness of the rescaled range R/S in the measurement of noncyclic long-run statistical dependence", *Water Resources Research*, No.5, 1969.

McCauley, R. and Fung, B., "Choosing instruments in managing dollar foreign exchange reserves", *BIS Quarterly Review*, BIS, March, 2003.

[29] Authors' preliminary versions of published documents (*) are available online (http://www.banrep.gov.co/publicaciones/pub_borra.htm).

Pastor, L. and Stambaugh, R.F., “Are stocks really less volatile in the long run?”, *NBER Working Paper Series*, No.14757, 2009.

Peters, E.E., *Chaos and Order in the Capital Markets*, John Wiley & Sons, 1996.

Pihlman, J. and van der Hoorn, H., “Procyclicality in central bank reserve management: evidence from the crisis”, *IMF Working Paper*, No.WP/10/150, 2010.

Reveiz, A. and León, C., “Efficient portfolio optimization in the wealth creation and maximum drawdown space”, *Interest Rate Models, Asset allocation and Quantitative Techniques for Central Banks and Sovereign Wealth Funds* (Eds. Berkelaar, B., Coche, J. and Nyholm, K.), Palgrave Macmillan, 2010a.*

Reveiz, A., “The case for macro risk budgeting and portfolio tranching in reserves management”, *Borradores de Economía*, No. 290, 2004.*

Reveiz, A.; León, C.; Castro, F.; Piraquive, G., “Modelo de simulación del valor de la pensión de un trabajador en Colombia”, *Pensiones y Portafolio: La Construcción de una Política Pública* (Eds. Laserna, J.M. y Gómez, M.C.), Banco de la República and Universidad Externado de Colombia, 2010.*

Schotman, P.C.; Tschering, R.; Budek, J., “Long memory and the term structure of risk”, *Journal of Financial Econometrics*, Vol.6, No.4, 2008.

Solnik, B. and McLeavey, D., *International Investments*, Pearson, 2003.

Sornette, D., *Why Stock Markets Crash*, Princeton, 2003.

Valdés, S., “Acciones, plazo de inversión y multifondos”, *Estudios Públicos*, No.117, Centro de Estudios Públicos, Chile, 2010.

Winkelmann, K., “Issues in Strategic Asset Allocation”, *Modern Investment Management*, Wiley, 2003.

Zimmermann, H.; Drobetz, W.; Oertmann, P., *Global Asset Allocation*, Wiley Finance, 2003.

8. Annex

Figure A1

Asset allocations and main features of investment (optimal) portfolios[30]

Scenarios 1 to 5

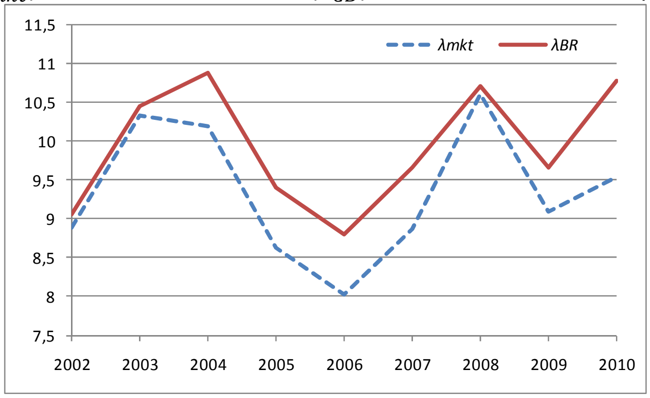
SCENARIO	Allocations (Weights)																									*Main Features*							
				U.S.												GERMANY			JAPAN														
	TREASURY BILLS			TREASURY N. & B.			TIPS		AGENCIES			CORPORATES AAA-->BBB			MORTGAGES	TREASURY NOTES & BONDS						SUPRA	GOLD	EQUITY	*RETURN*	*STANDARD DEVIATION*	*RETURN / STRD.DEV.*	*DURATION (YEARS)*	*LOSS PROBABILITY*	*EXPECTED UTILITY*	*CVaR (EXP. SHORTFALL)*	*MAXIMUM DRAWDOWN*
	U.S.	GER.	JAP.	0-5Y	5-10Y	10Y+	1-10y	1-10Y+	1-5y	5-10Y	10Y+	1-5y	5-10Y	10Y+		0-5Y	5-10Y	10Y+	0-5Y	5-10Y	10Y+											
1	0,00%	41,89%	0,00%	0,00%	0,00%	2,27%	0,00%	0,56%	0,00%	0,73%	2,61%	0,00%	3,29%	4,23%	0,00%	18,24%	8,71%	0,00%	0,00%	0,00%	0,61%	0,00%	0,00%	16,86%	*2,21%*	*3,89%*	*0,57*	*3,05*	*28,50%*	*1,39*	*2,42%*	*7,58%*
2	22,23%	3,49%	0,00%	8,11%	8,79%	0,57%	1,35%	0,68%	2,59%	0,59%	0,00%	4,61%	0,69%	3,85%	12,78%	6,73%	0,86%	1,03%	0,00%	0,00%	3,03%	0,00%	0,04%	17,97%	*2,18%*	*4,26%*	*0,51*	*3,59*	*30,42%*	*1,21*	*2,72%*	*8,73%*
3	82,16%	11,96%	2,17%	0,47%	0,22%	0,10%	0,05%	0,02%	0,10%	0,02%	0,01%	0,15%	0,15%	0,10%	0,56%	0,09%	0,04%	0,03%	0,42%	0,20%	0,17%	0,01%	0,00%	0,78%	*0,30%*	*0,18%*	*1,64*	*0,47*	*5,00%*	*0,30*	*0,08%*	*0,18%*
4	81,94%	11,88%	1,77%	0,71%	0,33%	0,15%	0,08%	0,04%	0,16%	0,03%	0,02%	0,23%	0,23%	0,15%	0,84%	0,13%	0,06%	0,05%	0,63%	0,31%	0,26%	0,01%	0,00%	0,00%	*0,28%*	*0,17%*	*1,64*	*0,54*	*5,00%*	*0,28*	*0,07%*	*0,09%*
5	82,65%	11,71%	0,87%	1,24%	0,57%	0,26%	0,13%	0,06%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,23%	0,10%	0,08%	1,11%	0,53%	0,46%	0,00%	0,00%	0,00%	*0,27%*	*0,16%*	*1,64*	*0,58*	*5,00%*	*0,27*	*0,07%*	*0,09%*
Duration (years)	0,31	0,43	0,32	2,23	6,48	13,93	4,83	14,23	2,07	5,21	11,02	2,82	6,21	12,36	2,85	1,30	6,62	14,26	2,29	7,13	15,73	1,97	N/A	N/A								
Adj. Hurst exp.	0,50	0,50	0,50	0,58	0,59	0,58	0,55	0,56	0,59	0,60	0,58	0,60	0,59	0,58	0,56	0,56	0,57	0,55	0,57	0,56	0,56	0,55	0,52	0,53								

Source: authors' calculations.

[30] The adjusted Hurst exponent is estimated as in León and Vivas (2010) and León and Reveiz (2010); for Treasury Bills, which are regarded as risk-free assets, adjusted Hurst exponent corresponds to the independence of returns assumption.

Figure A2

Market's (λ_{mkt}) and Board of Director's (λ_{CB}) risk aversion estimated parameters[31]



Source: authors' calculations.

Figure A3

Universe of risk factors (3-year horizon)

	Risk Factor	Market	Currency	Description
Risk-Free	Treasury Notes	U.S.	USD	1-3Y Notes indexes from selected reserve-issuing countries. [Bloomberg TKR G1O2, G1D0 and G1Y0]
		GER.	EUR	
		JAP.	JPY	
Risky	Treasury Notes & Bonds	U.S.	USD	3-5Y, 5-7Y, 7-10Y, 10Y+ Bonds and Notes indexes from selected issuing countries. [(U.S.) G2O2, G3O2, G4O2 and G9O2; (GER.) G2D0, G3D0, G4D0 and G9D0; (JAP.) G2Y0, G3Y0, G4Y0 and G9Y0]
		GER.	EUR	
		JAP.	JPY	
	Treasury TIPS	U.S.	USD	1-10Y and 10Y+ U.S. Treasury Inflation Protected Securities indexes. [G5QI and G9QI]
	U.S. Agencies			1-5Y, 5-10Y and 10Y+ indexes for unsobordinated bonds issue by agencies; 36 issuers [GVP0, G6P0 and G9P0]
	U.S. Corporates			1-5Y, 5-10Y and 10Y+ indexes for companies rated AAA to BBB. [CVB0, C6B0, C9B0, CVC0, C6C0 and C9C0]
	U.S. Mortgages			U.S. fixed MBS; 3 government-sponsored enterprises (Fannie Mae, Freddie Mac, Ginnie Mae) [M0AN]
	Supranationals	Global		1-3Y AAA supranational bonds index; 9 issuers from several countries regions. [GS1S]
	Gold			Bloomberg Spot Price. [GOLDS]

Source: Bloomberg. Unless otherwise stated, Bank of America – Merrill Lynch, provided total return indexes.

[31] Please note that since Board of Director's preference for risky assets (Φ_{CB}) is held constant whilst the market's change according to market capitalization, the behavior of the former is less volatile than the latter's. For example, it is intuitive to think that during 2008's financial market's turmoil the Board of Director's preference for risky assets (Φ_{CB}) could have decreased, making the risk aversion parameter even higher than presented, but, as stated before, no estimations for BR's Φ_{CB} are available at that time. What is most important is to realize that in all considered periods the risk aversion parameter (λ_{CB}) exceeds market's parameter (λ_{mkt}), which is a desirable feature.