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We compute both seigniorage rate and welfare cost of inflation rate in Colombia using a Sidrauski-type model in which preferences are non-separable functions of the service flows of non-durable goods and money holdings. The set of the estimated parameters imply sizeable welfare cost of inflation and seigniorage rates. However, even though for low inflation rates seigniorage rate markedly increases with the rate of inflation, for very high inflation rates it reaches an asymptote.

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I. INTRODUCTION

Reduction of welfare cost of inflation has been one of the goals of monetary policy in Colombia. However, estimates assessing quantitatively the welfare losses associated with different rates of inflation have shown quite different results, which means that further research in this sense would be useful.

Among the first approximations to this measure for Colombia is the paper by Carrasquilla-Galindo-Patron (1994). Their estimates of welfare loss for an increase in inflation rate from 5 per cent to 20 percent reach the sizable figure of 7 percent of the *GDP*. On the other hand, the estimates made by Posada (1995) and Riascos (1997) for an inflation rate of 20 percent are around 3.9 percent and 1.5 percent of *GDP*, respectively. The paper by Carrasquilla-Galindo-Patron has more information about the Colombian economy and a richer econometric framework that the one used by Riascos (1997) and Posada (1995) (which is a calibration exercise), but the estimates of their model are quite large in comparison with previous results for countries with inflation rates higher or lower than the Colombian, such as Israel or the United States, respectively.

Carrasquilla-Galindo-Patron used an approach similar to the one used by Eckstein and Leiderman (1992), but they did not take into account the service flow of purchases of goods for more than one period, even though the consumption variable used by them included consumption of durable goods. Besides, the monetary aggregate used was *M2*, which would not be a good aggregate to analyze the effects of the inflation rate on welfare because it includes deposits that earn interests which some times are even higher than inflation rates.

In this paper, we try one approach closer to the one used by Eckstein and Leiderman than the one used by Carrasquilla-Galindo-Patron. The first part of the paper deals with estimation -on quarterly time series for Colombia- of the parameters of a model that treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in the modern monetary theory (Sidrausky, 1967).

After obtaining estimates for the key parameters, the second and main part of our work consist of comparing steady states of the model assuming different rates of inflation to determine both, the welfare loss associated with different steady states rates of inflation and the relationship between inflation rate and seigniorage revenue predicted by the model. We calculate that the steady state welfare cost of a moderate inflation of 10 percent per year at 1.3 percent of *GNP*, quite similar to the one of

Israel, 1.4 per cent and more than twice as big as the available estimates for the United States - given the same inflation rate. The welfare cost of an inflation rate around 20 percent per year is about 2.4 per cent of *GDP* according to our estimates, which are in between the estimates by Posada and Riascos.

The remainder of this paper is organized as follows. In Section II we describe the model and discusses some steady state implications of the model. In Section III we describe the data and econometric methodology used in estimation. Section IV presents the parameter estimates and tests of the over-identifying restrictions. In Section V, we use parameter estimates and auxiliary assumptions about hypothetical steady state are used to determine the model's quantitative implications for the relation between seigniorage and the rate of inflation and for the welfare cost of inflation. Concluding remarks are presented in Section VI.

II. THE MODEL

A. DESCRIPTION OF THE MODEL

This model treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in modern monetary theory (see e.g. Eichenbaum, Hansen and Singleton (1988) for similar specifications).

Following Eckstein and Leiderman (1992), the economy is populated by infinitely lived families, with population growing at rate n. Suppose that consumers rank alternative sequences of consumption of services from goods and real money balances using the utility functional:

$$(1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \; U(m_t, c_t^*),$$

In (1), c_t^* is the consumption per-capita of services from goods at date t, m_t denotes real money balances per capita, $\beta \in (0,1)$ is a subjective discount factor and $U(\bullet)$ is a concave utility function that is increasing in both its arguments. The indirect utility function defined over consumption purchases and real money balances is temporally nonseparable. E_0 denoted expectations conditional on information available at time $t = 0$.

The household can hold its wealth in the form of either money or capital. Its budget constraint in per capita real units is given by

$$(2) \quad b_t = b_{t-1}\frac{(1+r_{t-1})}{(1+n_t)} + \frac{m_{t-1}}{(1+n_t)(1+\pi_t)} + y_t - m_t - c_t$$

where b_t, m_t, and c_t are, respectively, the real per capita values of one-period financial assets, money balances, and consumption. n_t and π_t denote population growth and the rate of inflation, respectively, and the real interest factor $(1+r_{t-1})$ is equal to $(1+R_{t-1})(1+\pi_t)$, where R_{t-1} denotes the nominal return on assets. y_t is real per capita income from other sources.

The consumption percapita of services from goods, c_t^* , is not measured and, therefore, it is necessary to specify a technology for transforming goods into services in order to proceed with the empirical analysis. Following Telser and Graves (1972), all consumers are assumed to have access to linear technologies that transform consumption goods purchased today into services flows in the future. The service flow c_t^* is assumed to be given by

$$(3) \quad c_t^* = \delta_0 c_t + \delta_1 c_{t-1}$$

where c_t denotes actual purchases of consumer goods, $\delta_0=1$[1]. Thus consumption purchases at time t directly affect consumption services in both t and $t+1$.

The period utility function is one of the more frequently used in intertemporal optimizing models, the constant relative risk-averse utility function (*CRRA*),

$$(4) \quad U(m_t, c_t^*) = \frac{\left[m_t^{\gamma} c_t^{*1-\gamma}\right]^{\theta} - 1}{\theta} \quad 0<\gamma<1 \;\; \theta<1$$

where γ and θ are preferences parameters, with coefficient of relative risk aversion $\rho = (1-\theta)$[2]. The parameter θ must be less than unity in order to obtain a concave utility that represents a risk averse representative agent. The lower θ the higher the relative risk aversion coefficient and the lower the intertemporal elasticity of

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1 For durable goods, see Dunn and Singleton (1985) for an alternative specification that take into account a different technology for transforming goods into services.

2 The relative risk aversion coeficient is defined as $\rho = -\left(u''(y_t)^* y_t\right)/u'(y_t)$, where is a composite good, (in this case $y_t = [m_t^{\gamma} c_t^{1-\gamma}]$). Therefore, for the *CRRA* utility function, $\rho = -\left((\theta-1)y^{\theta-2}\right)^* y/y^{\theta-1} = (1-\theta)$.

substitution[3]. This utility function is also called isoelastic function because the elasticity of substitution between consumption at any two points in time, t and s, is constant and equal to $(1/\rho)$. Utility is non-separable across the decision variables c_t^* and m_t, so long as $\theta \neq 0$. If $\theta = 0$ then preferences are assumed to take the logarithmic form $U(\bullet) = \theta \log m_t + (1-\theta)\log c_t^*$.

Substituting the specification about the relation between consumption services and purchases into (1) we have to solve the problem

$$(5) \quad \underset{b_t m_t}{Max} \; E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(m_t, (c_t + \delta c_{t-1})\right).$$

subject to the budget constraint (2), that becomes,

$$(6) \quad c_t = b_{t-1} \frac{(1+r_{t-1})}{(1+n_t)} + \frac{m_{t-1}}{(1+n_t)(1+\pi_t)} + y_t - m_t - b_t, \text{ and}$$

$$(7) \quad c_{t+1} = b_t \frac{(1+r_t)}{(1+n_{t+1})} + \frac{m_t}{(1+n_{t+1})(1+\pi_{t+1})} + y_{t+1} - m_{t+1} - b_{t+1}$$

Therefore, differentiating with respect to b_t and m_t and dividing by $U_{c^*}(t)$ we obtain the Euler equations,

$$(8) \quad \beta E_t\left[\frac{U_c^*(t+1)}{U_c^*(t)}\left(\frac{(1+r_t)}{(1+n_{t+1})} - \delta\right)\right] + \beta^2 \delta E_t\left[\frac{U_c^*(t+2)}{U_c^*(t)}\,\frac{(1+r_t)}{(1+n_{t+1})}\right] - 1 = 0$$

(9)

$$\frac{Um(t)}{U_c^*(t)} + \beta E_t\left[\frac{U_c^*(t+1)}{U_c^*(t)}\left(\frac{1}{(1+n_{t+1})(1+\pi_{t+1})} - \delta\right)\right] + \beta^2 \delta E_t\left[\frac{U_c^*(t+2)}{U_c^*(t)}\,\frac{1}{(1+n_{t+1})(1+\pi_{t+1})}\right] - 1 = 0$$

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[3] Now, remember that the elasticity of intertemporal substitution between two dates in time t and s is $\sigma(y_t) = \frac{-u'(y_s)/u'(y_t)}{y_s/y_t}\,\frac{d(y_s/y_t)}{d[u'(y_s)/u'(y_t)]}$ and taking the limit as $s \to t$, $= \lim_{s \to t} \sigma(y_t) = \frac{-u'(y_t)}{u''(y_t) * y_t}$. This is just the inverse of the Arrow-Pratt coefficient of relative risk aversion. Note that as $u''(y_t)$ gets small (as with nearly linear utility, intertemporal substitution becomes large.

Euler eq.(8) relates the disutility of giving up one unit of consumption at date t to the present value of the utility from shifting that unit of consumption in the next period. Euler eq.(9) relates the expected utility cost of giving up one unit of consumption at date t to the expected benefits from allocating in money holdings the foregone consumption during one period.

The marginal utilities with respect to m_t and c_t^* appearing in eqs (8) and (9) are given by

(10) $U_m(t) = \gamma (m_t)^{\gamma\theta-1}(c_t + \delta c_{t-1})^{\theta(1-\gamma)}$

(11) $U_{c^*}(t) = (1-\gamma)(m_t)^{\gamma\theta}(c_t + \delta c_{t-1})^{\theta(1-\gamma)-1}$

Notice that marginal utility of consumption in (11) is not uniquely related to c because it depends on m. Similarly, marginal utility of money is related not only to m but to c. The marginal utilities of goods in (11) involve the expected values of the marginal utilities of future services, because goods purchased at date t provide services in both current and future periods.

B. IMPLICATIONS OF THE MODEL FOR SEIGNIORAGE AND THE WELFARE COST OF INFLATION

The implications of the model for seigniorage and welfare cost of inflation are derived by comparing steady states of the model assuming different rates of inflation.

It is assumed that per capita consumption and real money balances grow in steady states at a constant rate Φ, that population grows at a constant rate n, and that all real variables do not change with respect to steady state changes in the rate of inflation.

With this assumptions and rearranging Euler equation (9), we obtain a steady state 'demand for money' which depends on explicit preference parameters,

(12) $$m = \frac{\left[\frac{\gamma}{(1-\gamma)}\right]\left[1+\frac{\delta}{(1-\Phi)}\right]^{*}c}{1+\alpha 1 - \frac{\alpha 2}{(1+\pi)}}$$

where c and π denotes the steady state values of consumption per capita and rate of inflation and,

$$\alpha_1 = \beta\delta(1+\Phi)^{\theta-1}$$

$$\alpha_2 = (1+n)^{-1}(1+\alpha_1)(1+\Phi)^{\theta-1}\beta$$

As expected, the steady state 'demand for money' is inversely related to the inflation rate, $\partial m/\partial\pi<0$, and positively related to the preference parameter γ, $\partial m/\partial\gamma>0$. Moreover, assuming that the parameters in eq. (12) are invariant with respect to steady state changes in the rate of inflation, we calculate the absolute value of the elasticity of money demand with respect to a steady state change in the inflation rate as $(\partial m/\partial\pi)(\pi/m) = \left[(1+\pi)(1+n)(1+\Phi)^{1-\theta}\beta^{-1}-1\right]^{-1}(\pi/1+\pi)$. This elasticity first increases with the rate of inflation, reaches a maximum and then decreases with further increases in inflation. We find that the higher the degree of risk aversion, the lower is the inflation elasticity of money demand.

Welfare cost of various steady state levels of inflation are calculated by substituting eq. (12) into (4) and compute the percentage decrease in consumption per capita that would generate the same welfare loss as that from moving from $\pi=0$ to $\pi>0$. This welfare loss is expressed as a percentage of *GNP* and given by,

$$(13) \quad WL\left[\left(\frac{1+\alpha_1-\alpha_2\,(1+\pi)^{-1}}{1+\alpha_1-\alpha_2}\right)^{\gamma}-1\right]\Psi$$

Where Ψ is the ratio of consumption to *GDP*[4]. *WL* depends positively on the parameter preference γ. The higher the preferences for money balances, the higher will be the welfare loss of moving from $\pi=0$ to a given $\pi>0$. Similarly, given the other parameter values, if the representative agent becomes less risk averse, the welfare loss of a higher inflation rate will be higher, and the larger the discount factor, β, the higher the welfare loss will be.

Finally, seigniorage per capita is given by

$$(14) \quad S_t \equiv \left(\frac{dM/d_t}{M}\right)m_t = \left(1-\frac{M_{t-1}}{M_t}\right)m_t = \mu m_t$$

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4 "Den Haan (1990) shows that a welfare measure based on an expression such as eq.(13) leads to very similar answers as the measure that calculates the area under the steady-state money demand function of the structural model" (Eckstein and Leiderman, 1992).

where M is the monetary base and m denotes the monetary base in real per capita units. In steady state equilibrium the gross rate of change of the monetary base, $\frac{M_t}{M_{t-1}}$, is equal to $(1+n)(1+\Phi)(1+\pi)$. Substituting for m_t the derived demand for real, monetary base from eq. (12) and dividing by *GDP* per capita we obtain the ratio of *seigniorage to GDP* in steady state.

$$(15)\quad SR = \left[1 - \frac{1}{(1+n)(1+\Phi)(1+\pi)}\right] \frac{\left[\frac{\gamma}{(1-\gamma)}\right]\left[1+\frac{\delta}{(1-\Phi)}\right]^{*}\Psi\kappa}{1+\alpha_1 - \frac{\alpha_2}{(1+\pi)}}$$

Where Ψ is the ratio of consumption to *GDP* and k is the inverse of the money supply multiplier. As in the standard literature about seigniorage, there are two components of *SR*: the inflation-tax rate, that increases when inflation accelerates, and the tax base which is the demand for real balances and it decreases when inflation accelerates. Notice that the higher the degree of relative risk aversion, $(1-\theta)$ in the parameters α, the lower is the ratio of *seigniorage* to *GNP*. On the other hand, the higher is the preference for holding real money balances, γ, the higher will be the amount of revenue from *seigniorage* that the government can obtain.

In order to obtain an increasing *seigniorage rate*, *SR*, with respect to π it must be the case that $\left[1-\beta(1+\Phi)^{\theta}\right] > 0$. This condition would be met if $\beta<1$, $\Phi \geq 0$, and $\theta \leq 0$. In this case *SR* would not exhibit a Laffer curve that arises from a model based on a Cagan-type money demand.

III. THE ECONOMETRIC MODEL AND THE DATA

The Euler equations can be used to construct orthogonality conditions for use in estimation and inference. If we set

$$(16)\quad u_{1t+2}(\sigma) \equiv \beta E_t\left[\frac{U_c^*(t+1)}{U_c^*(t)}\left(\frac{(1+r_t)}{(1+n_{t+1})} - \delta\right)\right] + \beta^2\delta E_t\left[\frac{U_c^*(t+2)}{U_c^*(t)}\,\frac{(1+r_t)}{(1+n_{t+1})}\right] - 1$$

(17)

$$u_{2t+2}(\sigma\equiv)\frac{Um(t)}{U_c^*(t)} + \beta E_t\left[\frac{U_c^*(t+1)}{U_c^*(t)}\left(\frac{1}{(1+n_{t+1})(1+\pi_{t+1})} - \delta\right)\right] + \beta^2\delta E_t\left[\frac{U_c^*(t+2)}{U_c^*(t)}\,\frac{1}{(1+n_{t+1})(1+\pi_{t+1})}\right] - 1$$

Where

$$\frac{Um(t)}{U_c^*(t)} = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{c_t+\delta c_t}{m_t}\right)$$

$$\frac{U_c^*(t+1)}{U_c^*(t)} = \left(\frac{m_{t+1}}{m_t}\right)^{\theta\gamma}\left(\frac{c_{t+1}+\delta c_t}{c_t+\delta c_{t-1}}\right)^{\theta(1-\gamma)-1}$$

$$\frac{U_c^*(t+2)}{U_c^*(t)} = \left(\frac{m_{t+2}}{m_t}\right)^{\theta\gamma}\left(\frac{c_{t+2}+\delta c_{t+1}}{c_t+\delta c_{t-1}}\right)^{\theta(1-\gamma)-1}$$

Clearly, with this specification there are four parameters to be estimated, namely $\sigma = (\beta,\delta,\gamma,\theta)$. Notice that the Euler equations (8) and (9) in conjunction with an iterated conditional expectations argument can be used to show that

$$(18) \quad E\left(u_{it+2}\,(\sigma_0)\bullet z_{ij}\right) = E\left[E\,[u_{it+2}\,(\sigma_0)\,|\,\Omega_t]\bullet z_{ij}\right] = 0,$$

for any vector z_t that belongs to the information set, Ω_t, at time t. In this context, z_t is a vector of instruments that might include a constant which amounts to taking the unconditional expectation of the Euler equation, and variables such as c_t/c_{t-n}, or indeed any other macroeconomic variables contained in the representative agent´s information set.

The moment conditions in (18) provide the basis for *GMM* estimation of the parameters $(\beta,\delta,\gamma,\theta)$. This type of conditions are sometimes refered to as "orthogonality conditions" because it states that z_t is statistically orthogonal to $u_{i,t+k}$ (remember that orthogonality is the sample analog of absence of correlation).

Hansen's (1982) Generalized Method of Moments (*GMM*) provides a convenient framework for estimating nonlinear system of simultaneous equations like the one we have from equations (16) and (17). Suppose that the goal is to estimate a system of nonlinear equations of the form

$$y_t = f\,(\sigma,\, x_t) + u_t$$

For x_t a $(k\times1)$ vector of explanatory variables and σ a $(p\times1)$ vector of unknown parameters. Let z_{it} denote a vector of instruments that are uncorrelated with the ith element of u_t. The q orthogonality conditions for this model are

$$f(\sigma, w_t) = \begin{bmatrix} [y_{1t} - f_1(\sigma, x_t)]z_{1t} \\ [y_{2t} - f_2(\sigma, x_t)]z_{2t} \\ . \\ . \\ . \\ [y_{qt} - f_q(\sigma, x_t)]z_{qt} \end{bmatrix},$$

Hansen (1982a) shows that the estimator of σ_0 with the smallest asymptotic covariance matrix given our choice of instruments is obtained by minimizing the criterion function

$$(19) \quad J_T(\sigma) = g_T(\sigma)' S_T^{-1} g_T(\sigma)$$

where $g_T(\sigma) = T^{-1}\Sigma z_t u_t(\sigma)$, and S is the asymptotic variance of $T^{1/2} g(\bullet)$ (a consistent estimator of the population weighting matrix). Identification requires an order condition ($q \geq p$) and that the columns of $\partial f(\sigma, w_t / \partial\sigma')$ be linearly independent.

In our model, due to the presence of a two-period-ahead forecast error in the Euler equations, the type of covariance matrix estimate that would be more suitable would be the Heterocedasticity Autocorrelation Consistency Covariance Matrix. Finally, according with Hansen (1982), the vector of observed variables, w_t, must be strictly stationary, then the expressions in (16) and (17) are scaled by $U_c^* = \left[(1-\gamma)(m_t)^{\gamma\theta}(c_t + \delta c_{t-1})^{\theta(1-\gamma)-1}\right]$ in order that the disturbances will be strictly stationary processes in the presence of certain types of real growth in purchases of goods and money balances.

For the empirical analysis of equations (16) and (17), the sample period is 1977.2 thorough 1997.4. Quarterly data on total private consumption were obtained from National Planning Department. We also used a measure for real purchases of non-durable plus services based on the classification made by Alejandro López (1996). Money is defined as the standard *M1*. The nominal interest rate is the quarterly lending rate charged by banks. The inflation rate is measured by the percentage change in the *GDP* price deflactor.

IV. PARAMETER ESTIMATES AND TEST RESULTS

The results from estimating the model are displayed in Table 1. We report two set of estimates corresponding to two alternative definitions of consumption, total private consumption, *C*, and consumption of non-durable plus services, *CN*.

Table 1
Parameter Estimates and *t*-Values

Parameters	C	CN
β	0,959 (235,67)	0,959 (245,5)
γ	0,052 (36,29)	0,055 (35,46)
θ	-1,310 (-3,04)	-1,102 (-2,79)
δ	0,717 (1,71)	0,697 (2,66)
	$J(\sigma)$ = 8,08	$J(\sigma)$ = 8,25

C: Is aggregate consumption.
CN: Is aggregate consumption of nondurables.
J(s): Is the value of the criterion function.

In addition to the parameter estimates and their respective *t* - values, we report a statistic for testing the validity of the over-identifying restrictions implied by the model, the J_T statistics. The instrument vector, z_t, associated with the disturbance u_{1t} and u_{2t} included the constant unity, the first lagged value of the growth rates of consumption and real money balances per capita, the inflation rate and the real interest factor. With these five instruments and two equations there are ten orthogonality conditions, q=10. Since there are four parameters to be estimated, p=4, there are six overidentifying restrictions.

The parameter estimates displayed in Table 1 are qualitatively similar for the two alternative definitions of consumption. The point estimates of the concavity parameter, θ, is lower than zero, which means a relatively high risk aversion coefficient and a low intertemporal elasticity of substitution; the consumer has strong preferences for present consumption over future consumption given its high relative risk aversion coefficient, $\rho = (1-\theta) = 2.1$. Estimates of γ and β are between zero and one, as expected. The parameter $\gamma = 0.055$ is small, which means that the utility derived from holding money is low relatively to the utility derived form consumption goods. The estimates of the lag for consumption of non-durable, δ, are economically plausible and similar to the estimates for Israel (between 0.3-0.6) and the United States (0.6 according to Dunn and Singleton estimates).

The test statistic, $J_t(\sigma)$, is equal to 8.08 when total consumption is used and to 8.25 when the proxy for non-durable and services variable is used. The null hypothesis is that the model is correctly specified, and one compares the test statistic to a χ^2_{q-p}. In this case, the critical χ^2_6 is 12.6 at 5 percent significance level. So we do not reject the null hypothesis and assume that the model is correctly specified.

V. IMPLICATIONS FOR SEIGNIORAGE AND WELFARE COST OF INFLATION.

Based on the parameter estimates obtained in the previous section, and based on eq. (13) and (15), we made some estimates from the seigniorage as a percentage of *GDP* and from the welfare cost of inflation for Colombia. Table 3 report the results for seigniorage as a percentage of *GDP* and for the welfare cost of inflation for Colombia and also for Israel according to the estimates obtained by Eckstein and Leiderman (1992). In Table 2 we have the parameter values used for each country.

Table 2
Parameter Values

	Israel	**Colombia**
β	0,980	0,960
γ	0,050	0,055
θ	-1,500	-1,100
δ	0,300	0,600
ψ	0,610	0,700
n	0,580	0,510
ϕ	0,008	0,005

Source: Eckstein and Leiderman (1992) for Israel. Estimated parameters with *CN* on Table 1, previous section for Colombia.

The values for Ψ, n, ϕ correspond to the quarterly sample means of the share of consumption in *GDP*, the rate of change of population, and the rate of change of consumption per capita, respectively[5].

The results for *seigniorage rate*, Table 3, show that it is an increasing function of the rate of inflation. That is, government can raise more revenue by increasing monetary base growth and inflation. However, notice that when inflation rate reaches levels of hyperinflation (20% quarterly), *SR* remained around 3% of *GNP* for Israel

Table 3
Seignorage Ratio and welfare cost of inflation for Israel and Colombia (as percentage of *GDP*)

π(quartely)	*SR* - Israel	*SR* - Colombia	*WL* - Israel	*WL* - Colombia
0,000	0,012	0,010	0,000	0,000
0,012	0,017	0,018	0,008	0,007
0,024	0,020	0,024	0,014	0,013
0,050	0,023	0,031	0,025	0,023
0,100	0,027	0,039	0,038	0,037
0,150	0,028	0,043	0,046	0,046
0,200	0,029	0,045	0,053	0,053
0,280	0,030	0,048	0,060	0,062
0,320	0,030	0,048	0,063	0,066
0,500	0,031	0,051	0,072	0,077
0,700	0,031	0,052	0,079	0,085
0,900	0,031	0,052	0,083	0,090
1,000	0,032	0,053	0,085	0,092
1,200	0,032	0,053	0,088	0,095
1,300	0,032	0,053	0,089	0,097

Source: The figures in this table were calculated based on parameter values in Table 2 and Eckstein and Leiderman (1992) for Israel's data.

[5] The quaterly rate of change in population for Colombia is the average growth of population between 1978 and 1997. To obtain quaterly data we use the anual grwoth rate divided by four.

and 5% for Colombia. In both cases, for low rates of inflation *SR* markedly increases with growing inflation, but then *SR* reaches an asymptote.

The asymptate is reached faster in the case of Israel due apparently to a higher level of the parameter β, the discount factor.

The results suggest that in Colombia an inflation rate of 20 percent per year would result in a *seigniorage rate* of about 3.1 percent of *GDP* (Table 3). This result correspond well with the actual figures. Average inflation rate during the period 1977-1997 was about 23 percent and the observed *seigniorage rate* was 2.9 percent of *GNP* according a document of the Department of Monetary and Reserve from the Central Bank of Colombia (1995).

In order to compute the decrease in per capita consumption (expressed as percent of *GNP*) that would generate the same *welfare loss* as that from increasing inflation from zero to a given rate, we use eq. (13). As showed before, *welfare cost of inflation*, *WL*, is negatively related to the degree of relative risk aversion, $(1-\theta)$ from the representative agent and its preferences towards money holdings, γ, and positively related to the discount factor, β. From Table 3, we see that a shift from zero inflation to an annual rate of inflation of 10 percent (i. e., 2.41 per quarter) results in a loss in utility equivalent to about 1.4 percent of *GDP* in Israel and 1.3 percent of *GDP* in Colombia. Even though Israel has a relative risk aversion coefficient higher than Colombia, its marginal utility from holding money balances is lower[6], and its discount factor is higher, the result is that welfare loss of inflation is higher in Israel than Colombia, though the difference is not quite important.

The estimate of Welfare Loss of inflation that we found here is much lower than the estimate for Colombia obtained by Carrasquilla-Galindo-Patron (1994). In their estimates, a decrease of the inflation rate from 20 to 5 percent represents a decrease in the welfare cost of inflation of the sizable amount of 7 percent of *GDP*. Our estimates for the welfare loss due to an increase in the inflation rate from 10% to 20% are equivalent to about 1.0 per cent of *GDP*, this result is similar to the estimate of 1.2 per cent of *GDP* found by Posada (1995)[7].

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6 Probably due to the fact that inflation rate in Israel reached very high levels during the eighties.

7 The welfare cost calculations of Posada (1995) allow the possibility of endogenous production and capital acumulation. Here, the calculation of seigniorage and of welfare cost of inflation are based on the assumption of neutrality.

Finally, comparing the estimate of *welfare loss* of around 1.3 percent of *GDP* when inflation increases from zero to 10 percent in Colombia and Israel, to the estimates found for the United States, it is more than twice as big as the welfare loss for the United States. For example, for the same inflation rate the estimated *welfare loss* for the United States are 0.28% of *GDP* according to McCallum (1989), 0.3% of *GDP* according to Fisher (1981), and 0.39 percent of *GDP* computed by Cooley and Hansen (1989)[8]. It its not straight forward to explain why WL is higher in Israel and Colombia than in the United States because the calculations are based in different methodologies. However, we have done an experiment by plugging the parameter estimates for the United States found in different studies in the equation of *WL*, (β is found to be 0.99, θ is around -0.5 and δ is around 0.56[9]), and the result was that the explanation for a lower *WL* is that the representative agent in the United States has a lower preference for holding money balances, that is, it is lower than in Israel and Colombia. This is a result consistent with the fact that transaction technology is much more developed in the United States and therefore, the utility that a representative agent derive from holding real balances is low given that he can keep his money balances in saving accounts and cash them at a lower transaction cost.

VI. SUMMARY

Based on an optimizing model with money in the utility function, me have presented estimates for the underling parameter vales and the seignorage rate and welfare loss of inflation in Colombia.

The point estimates of the concavity parameter, θ is lower than zero and has the expected sign, which means a relatively high risk aversion coefficient and low intertemporal elasticity of substitution. The discount factor estimated was around 0.96, the preference parameter γ is around 0.05. Finally, the parameter that captures the service flow of hte consumption goods, δ, is around 0.7, a little higher than the estimates found in previous studies for the *USA*. and Israel which are around 0.6.

The second part of this paper consisted of comparing steady states of the model assuming different rates of inflation to determine both, the *welfare loss* associated

8 McCallum (1989) and Fisher (1981) based their calculations on an approximation of the area under the demand curve for money, while Cooley and Hansen (1989) based their calculations on a real bussiness cycle model in which money is introduced via cash-in-advance constraints.

9 Dunn and Singleton (1984).

with different steady states of inflation and the relationship between inflation rate and the *seigniorage revenue*. The results show that the *welfare loss* due to an increase in the inflation from 5% to 20% is no higher than 2.3% of the *GDP*, much lower estimates than the estimates from Carrasquilla-Patron-Galindo; around 7% of *GDP*.

On the other hand, our estimates for the welfare loss due to an increase in the inflation rate from 10% to 20% are equivalent to about 1% of the *GDP*, similar to the 1.2% of *GDP* found by Posada (1995).

Finally, the results on *seigniorage rates* shows that *seigniorage rate* is an increasing function of inflation rate, but it reaches an asymptote. It does not have the shape of the Laffer curve. Besides, seigniorage rate is higher in Colombia than in Israel; an inflation rate of 20% per year in Colombia would result in a seigniorage rate of about 3.1% of *GDP*, while in Israel it would be around 2.3%.

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