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Output Gap Estimation, Estimation Uncertainty and its Effect on Policy Rules

Juan Manuel Julio R. and Javier Gómez P.[*]

The authors propose a short run model for the monetary transmission mechanism in which the output gap is modelled as an unobserved variable. By estimating this model using maximum likelihood on a Kalman Filter, the authors find an estimate of the unobserved output gap as well as its estimation uncertainty. The performance of monetary rules is studied both with certainty on the output gap values as well as with estimation uncertainty.

Although the estimated gap is more reasonable than some other estimates proposed for Colombia, it is estimated with a sizable degree of uncertainty. In fact, the gap is not significantly differente from zero in all but five quarters. This result amounts to say that we can not be sure about the sign or value of the gap except when the economy faces an unusual rate of growth. Moreover, we found that potential output does not differ statistically from a linear trend, thus, the gap may be understood as deviations from a linear trend, being money surprises the source of these deviations. This result may be due to the sample length.

In addition, we estimated the optimal linear policy rule with and without uncertainty and used it as a benchmark to evaluate the Taylor rule and the historical data. By introducing output gap estimation uncertainty the variance of the target variables increases, and so the reaction of the authority is smaller. Finally, Colombian historical results seen to resemble those of an economy under a Taylor rule with uncertainty.

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I. INTRODUCTION

In this paper we propose a short run model for the monetary transmission mechanism in which the output gap is an unobserved variable. By estimating this model using maximum likelihood on a Kalman Filter, we find an estimate of the unobserved output gap as well as its estimation uncertainty. The performance of monetary rules is studied both with certainty on the output gap values and with estimation uncertainty.

We split the relationship between money and inflation into short and long run. The long run transmission mechanism from money to prices as well as the transition to international levels of inflation was already dealt with in Gómez and Julio (1999) while the short run relationship is the subject matter of this paper.

Our short run model consists of an aggregate demand equation and an expectations augmented Phillips curve. In the first equation the output gap depends on its own lags and the monetary surprise while in the second the short run inflation movements depend on lagged inflation and output gap. Unanticipated money determines the output gap on impact, and the output gap determines inflation with a two year lag.

There are several ways to filter out the short or long run components of a time series. Lucas (1998), for example, extracts the long run component of money and inflation with a two tailed exponential filter, while Hodrick and Prescott (1980) extract the long run component solving a smoothing minimization problem. For this paper we measure the short run inflation as the deviation from the core, the core being measured as the Hodrick Prescott filtered inflation. We abstract from long run "core" inflation since its behavior may better be explained by a long run model as in Gómez and Julio (1999). On the other hand, the deviations of inflation from the core are explained in our model by the short run mechanics of unanticipated money and the output gap. Where the monetary surprises are defined as in Barro (1977), i.e. the residual of an autoregressive process on the money growth.

Once the model is set up in state space form, it is straightforward to obtain the likelihood function and the estimated parameters. Using the estimated parameters, the Kalman filter provides filtered and smoothed estimates of the unobserved gap as well as its estimation uncertainty. The difference between these two being the sample update effect.

Our estimated gap is more reasonable than some other estimates proposed for Colombia. In particular, our estimate captures well the slowdown of the eighties along with the subsequent recovery in the early nineties. It also depicts well the deflationary pressures in the nineties. Associated with the estimated gap we found a sizable degree of uncertainty in its estimation. In fact, the gap is not significantly different from zero in all but five quarters. This result amounts to say that we can not be sure about the sign or value of the gap except when the economy faces an unusual rate of growth. Hence, the gap could not be a good leading indicator of inflation or inflationary pressures.

Another interesting finding is that potential output does not differ statistically from a linear trend, thus, the gap may be understood as deviations from a linear trend, being the money surprises the source of these deviations. This result may be due to the sample length.

We estimated the optimal linear policy rule with and without uncertainty on the past and present values of the gap and used it as a benchmark to evaluate the Taylor rule and the historical data. By introducing output gap estimation uncertainty the variance of the target variables increases, and hence the reaction of the authority is smaller. Finally, Colombian historical results resemble those of an economy under a Taylor rule with uncertainty.

The plan of the paper is as follows. In the second part we explain the short run transmission mechanism of monetary policy. Following Smets (1998), in the third part we estimate the output gap as a variable that is unobserved. Following Rudebusch and Svensson (1998), in the fourth part we use the short run model to evaluate the performance of the Taylor rule and the actual monetary policy. Finally we present some conclusions.

II. THE SHORT RUN MONETARY TRANSMISSION MECHANISM

A. THE SHORT RUN MODEL

The model consists of an expectations-augmented aggregate demand, and an expectational Phillips curve. We abstract from the open economy dimension of the model because, as explained in Gómez (1999), in an economy that is relatively closed like Colombia, the exchange rate channels of monetary transmission are relatively weak.

The aggregate demand equation is intended to capture the relationship between economic activity and a measure of the stance of monetary policy measured as the unanticipated money growth. If we denote z to be the output gap in percentage points and m the unanticipated component of money growth, the aggregate demand equation is:

$$(1) \qquad z_t = \varphi(B)z_{t-1} + \lambda(B)m_t + \varepsilon_t^z$$

where $\varphi(B)$ and $\lambda(B)$ are finite polynomials on the backshift operator B.

The Phillips curve relates the real side of the economy, the gap z, with inflation π :

$$(2) \qquad \pi_t = \alpha(B)\pi_{t-1} + \beta(B)z_{t-1} + \varepsilon_t^\pi$$

where $\alpha(B)$ and $\beta(B)$ are two finite polynomials on the backshift operator B.

To complete the model we need a law of motion of unobserved potential output:

$$(3) \qquad y_{t+1}^p = \mu + y_t^p + \varepsilon_t^y$$

and the definition of the output gap:

$$(4) \qquad y_t = y_t^p + z_t$$

and ε_t^z, ε_t^π and ε_t^y are the model residuals, which are assumed to be jointly normal with zero mean and diagonal variance covariance matriz with elements σ_z^2, σ_p^2, and σ_y^2.

B. FEATURES OF THE MODEL

1. Monetary Surprises

Following a long tradition in monetary studies, as in Barro (1977), and (1978), Grossman (1981), and Kormendi and Maguire (1984), we decompose money growth in anticipated and unanticipated:

$$(5) \qquad \Delta \log M_t = c + \sum_{i=1}^{\infty} \beta_i \Delta \log M_{t-i} + m_t$$

In (5) M is the adjusted monetary base. $c+\sum_{i=1}^{\infty}\beta_i\Delta\log M_{t-i}$ is anticipated money growth, and the residual term, m_t is the unanticipated money growth or the surprise in the growth of money[1].

2. *Expectations Augmented Aggregate Demand*

While the anticipated component of money growth does not cause movements in output, the unanticipated growth does determine real activity. That is, output is neutral to anticipated money growth and not neutral to unanticipated money growth $(\lambda(1)\neq 0)$.

3. *Expectations Augmented Phillips Curve*

Since lagged inflation helps explain current inflation, the Phillips curve is expectational. If the coefficients of the inflation lags sum to one, there is full inflation persistence.

4. *A Lag in the Effect of Monetary Policy*

While the unanticipated money determines the output gap on impact, the output gap affects short run inflation rate with a lag. Thus, our model includes a lag in the effect of monetary policy on output. In Gómez and Julio (1999) we argue that money does not have a discernible impact on prices in the short run but causes a 1 to 1 response on prices in the long run, thus, there is a lag in the effect of monetary policy on prices that for the case of Colombia we estimate to be 11 quarters.

5. *A Transmission Mechanism*

The proposed model explicitly includes a monetary transmission mechanism.

6. *A Specification of the Evolution of Potential Output*

In (3), if the variance of the change in potential output is not zero, $\sigma_y > 0$, potential output is not stationary and follows a random walk with drift. However, if $\sigma_y = 0$, the potential output is stationary around a deterministic trend.

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[1] Another possible definition of monetary surprises would be the difference between actual and announced monetary base, however, Colombia targets money, hence the deviations of the monetary base with respect to the announced target are small. We then prefer to define monetary surprises as in the literature.

III. THE ESTIMATED MODEL AND OUTPUT *GAP*

In order to estimate the model and the corresponding Output gap we write the model equations in a State Space representation. This representation along with the Kalman filter help us compute the likelihood function for the model, from which we can obtain the parameter estimates. Once we get the parameter estimates, the Kalman filter provides the filtered output gap, an estimate of the gap based only on past information, and a smoothed estimate which is based on the whole sample.

Technical details on the current state space representation of the model and the estimation of its parameters can be found in the appendix.

A. THE DATA

Our measure of inflation is the four quarter difference of the logarithm of the consumer price index: $\pi_t = 100 * (p_t - p_{t-4})$ expressed as deviation from the Hodrick Prescott Filter with $\lambda = 1.600$. Figure 1 shows observed inflation and the core, while Figure 2 shows the short run component of inflation, the deviation of inflation from the core.

Figure 3 illustrates unanticipated money growth where the measure of money is the adjusted monetary base. The figure reveals the large swings of monetary policy in the nineties: the expansion of 1991-1992, the contraction of 1995-1996, and the contraction of 1998. Money growth in equation (5) is measured as the first logarithmic difference. Money surprises in Figure 3 are not defined as in Eq. (5) but as $\Delta \tilde{m}_t = 100 * \left(\sum_{j=1982:1}^{t} m_j - \sum_{j=1982:1}^{t-4} m_j \right)$

B. RESULTS

The final estimated model corresponds to the following equations in which the non significant parameters have been removed

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \lambda m_t + \varepsilon_t^z$$

$$\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \beta z_{t-8} + \varepsilon_t^\pi$$

$$y_{t+1}^p = \mu + y_t^p + \varepsilon_t^y$$

$$y_t = y_t^p + z_t$$

Figure 1
Observed and Long Run Inflation

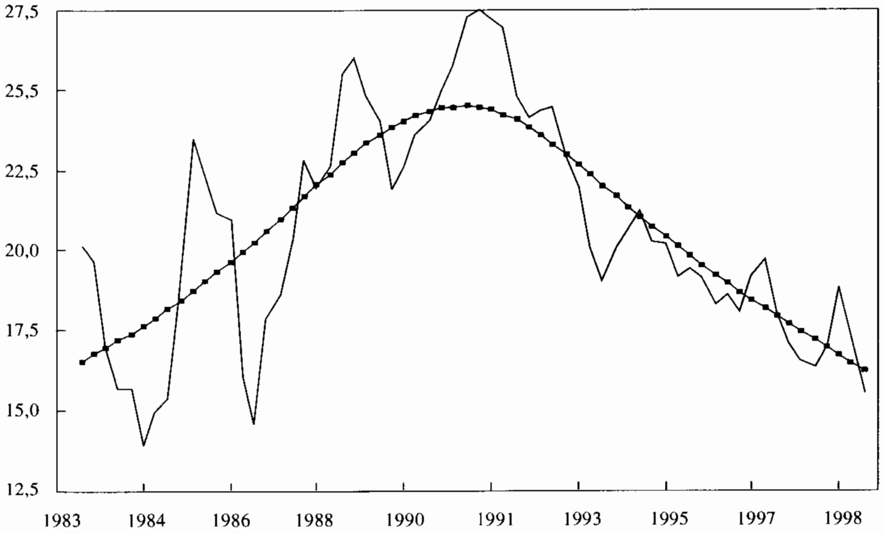


Figure 2
Deviation of Inflation from Core

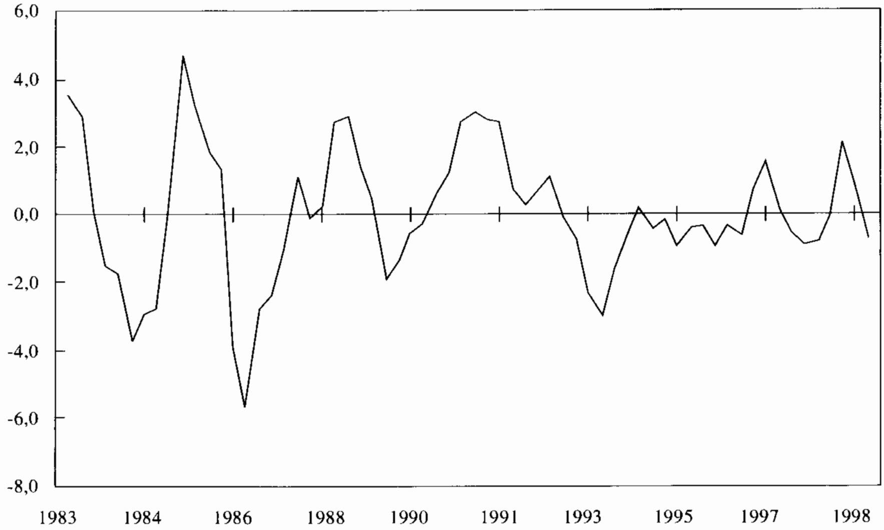


Figure 3

Unanticipated Money Growth

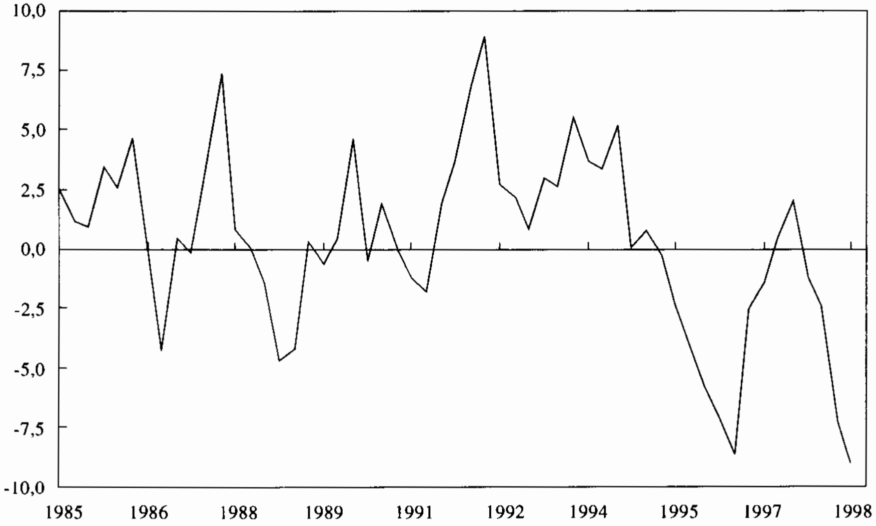


Table 1 presents the estimated coefficients for the model. The table reveals a significant contemporaneous effect of unanticipated money on the output gap $\lambda \neq 0$, a significant lagged effect of the output gap on inflation $\beta \neq 0$, a relatively high output persistence $\varphi_1 + \varphi_2 \equiv \varphi(1) = 0.873$, and low inflation gap inertia $\alpha(1) = 0.189$.

Figure 4 shows the estimated (smoothed) output gap along with the two standard deviation confidence band. This estimated gap depicts some interesting features not shown by earlier estimates. During the second half of the eighties it remains above zero while during the first half of the nineties it remains below zero except for the 1994-1995 period. Afterwards the gap goes below zero and then goes above during 1997 after which the gap goes far below zero.

Even though this facts were expected for the behavior of a gap for Colombia, strikingly there is considerable uncertainty in the estimation of the output gap, a matter not studied in earlier works. In fact, the gap is not statistically different from zero in all but five quarters in the sample. It means that for a point of time in which the gap is not different from zero, it is equally likely that it corresponds to inflationary or deflationary pressures. Moreover, the gap is significantly different from zero only when the economy faces an unusually high or low rate of growth. Since the band is

Table 1
Estimated Coefficients

	Coefficient	Standard Error	T Statistic
φ_1	0.513	0.142	3.607
φ_2	0.359	0.138	3.603
β	0.343	0.160	2.144
α_1	0.821	0.118	6.948
α_2	-0.414	0.150	-2.762
α_3	0.259	0.152	1.700
α_4	-0.476	0.111	-4.302
λ	0.224	0.075	2.983
σ_y^2	0.278	0.170	1.639
σ_p^2	1.020	0.225	4.541
σ_z^2	0.944	0.265	3.565

about four percentage points wide, the gap is significantly different from zero when it is above two percentage points or below minus two percentage points.

Since for most of the periods the estimated gaps are inside the minus two plus two interval, this result implies that the gap could not be a good leading indicator of inflationary pressures in most circumstances. However, it could shed valuable information whenever its value is outside the non significance interval.

Table 1 shows that the variance of potential output, σ_y^2 is not statistically different from zero. The asymptotic likelihood ratio test for the null $\sigma_y^2 = 0$ takes a value of 1.309, with a p-vale of 0.252. Hence we cannot reject the hypothesis that potential output is a deterministic trend (see equation (3)) and that the output gap is the deviation of output from this trend. As the gap is simply the deviation of output from a linear trend, the growth of the gap equals the growth of output except for a constant.

Figure 4
Estimated Output Gap and Two Standard
Deviation Confidence Band

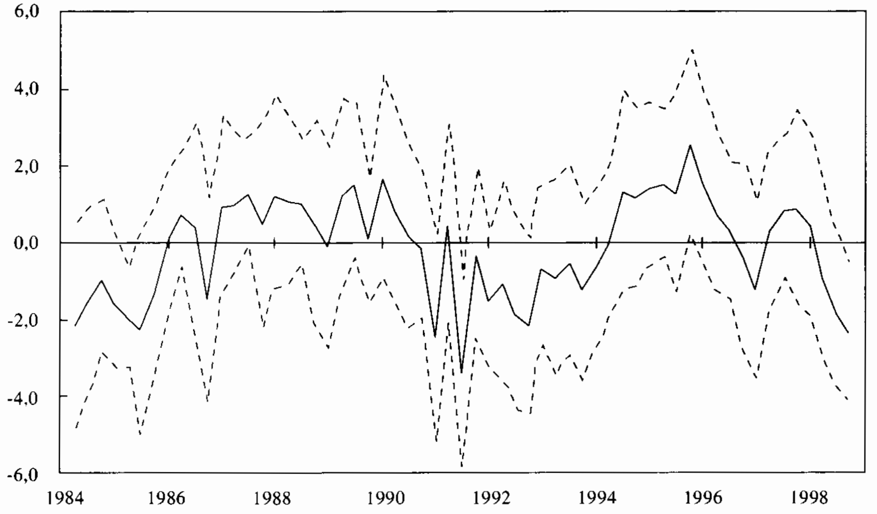


Table 2 presents the growth of output (or the growth of the gap) as a function of lagged output and the monetary surprise for different monetary aggregates. The table shows that, for all definitions of money, unanticipated money growth has a significant effect on output.

IV. INFLATION TARGETING

We follow Rudebush and Svensson (1998) in the distinction between instrument rules and targeting rules. An instrument rule dictates the behavior of the monetary instrument as a function of the available information, for instance, as a function of the state of the economy, the vector $\mathbf{X}_t$. Central banks would hardly constrain themselves to follow instrument rules. Instrument rules, however, may provide a benchmark for the policy actually followed. An inflation targeting strategy, in turn, minimizes deviations of inflation from target (strict inflation targeting), and also deviations of other objectives from target (flexible inflation targeting). As we show below, a targeting rule combined with a model of the workings of the economy is an implicit instrument rule. Below we compare two instrument rules implicit in the following (flexible) inflation targeting problem that minimizes a weighted avera-

Table 2

Unanticipated Money Growth and Output

Dependent variable *GDP* growth

Period 1984:2-1998:4

	Adjusted Monetary Base	**Monetary Base**	***M1***	***M3* plus bonds**
Constant	0.015	0.015	0.014	0.015
	(5.509)	(5.747)	(5.146)	(5.912)
Seasonal (*t*-2)	-0.039	-0.039	-0.036	-0.040
	(-4.968)	(-5.213)	(-4.615)	(-5.414)
Saesonal (*t*-1)	0.018	0.015	0.023	0.014
	(-1.465)	(1.277)	(1.876)	(1.240)
Seasonal (*t*)	0.014	0.013	0.017	0.014
	(1.358)	(1.263)	(1.603)	(1.390)
GDP growth (*t*-1)	-0.274	-0.274	-0.221	-0.294
	(-2.206)	(-2.242)	(-1.741)	(-2.489)
GDP growth (*t*-2)	-0.336	-0.361	-0.291	-0.352
	(-2.748)	(-3.005)	(-2.390)	(-2.997)
Money Surprise	0.278	0.103	0.277	0.289
	(2.624)	(2.307)	(2.756)	(2.815)
R^2	0.809	0.806	0.811	0.814
D. W.	2.430	2.363	2.285	2.303
S. E.	0.016	0.016	0.016	0.015

ge of the unconditional variance of the goal variables. The problem is to minimize the expected loss:

$$(6) \qquad \mathrm{E}[L_t] = \gamma\, Var[\pi_t] + (1-\gamma) Var[z_t] + \upsilon\, Var[m_t]$$

Once the output gap is estimated, the monetary authority has two options; first, the authority uses the estimated gap as an observed variable and does not take into account the estimation uncertainty, and second, the authority targets the estimated gap as unobserved.

A. MONETARY POLICY WITH CERTAINTY ABOUT THE GAP

Given this setup, if the authority chooses to leave aside the estimation uncertainty, the state and observation equations are given by

(7) $\mathbf{X}_t = \mathbf{A}\,\mathbf{X}_{t-1} + \mathbf{B}\,m_{t-1} + \mathbf{v}_t$

(8) $\mathbf{Y}_t = \mathbf{C}_x \mathbf{X}_t + \mathbf{C}_m m_t$

where

$$\mathbf{X}_t = \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ z_t \\ z_{t-1} \\ z_{t-2} \\ z_{t-3} \\ z_{t-4} \\ z_{t-5} \\ z_{t-6} \\ z_{t-7} \end{bmatrix}, \mathbf{A} = \begin{bmatrix} \sum\limits_{j=1}^{4} \alpha_j \mathbf{e}_j \beta \mathbf{e}_{12} \\ \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \sum\limits_{j=5}^{6} \varphi_j \mathbf{e}_j \\ \mathbf{e}_5 \\ \mathbf{e}_6 \\ \mathbf{e}_7 \\ \mathbf{e}_8 \\ \mathbf{e}_9 \\ \mathbf{e}_{10} \\ \mathbf{e}_{11} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \lambda \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_t = \begin{bmatrix} \varepsilon_t^{\pi} \\ 0 \\ 0 \\ 0 \\ \varepsilon_t^{z} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\mathbf{Y}_t = \begin{bmatrix} \pi_t \\ z_t \\ m_t \end{bmatrix}, \mathbf{C}_X = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_5 \\ 0 \end{bmatrix}, \mathbf{C}_m = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

where $\mathbf{e}_j$ is a 1 X 12 vector with a 1 in position $(1, j)$ and zeros elsewhere.

1. Efficient Instrument Rules

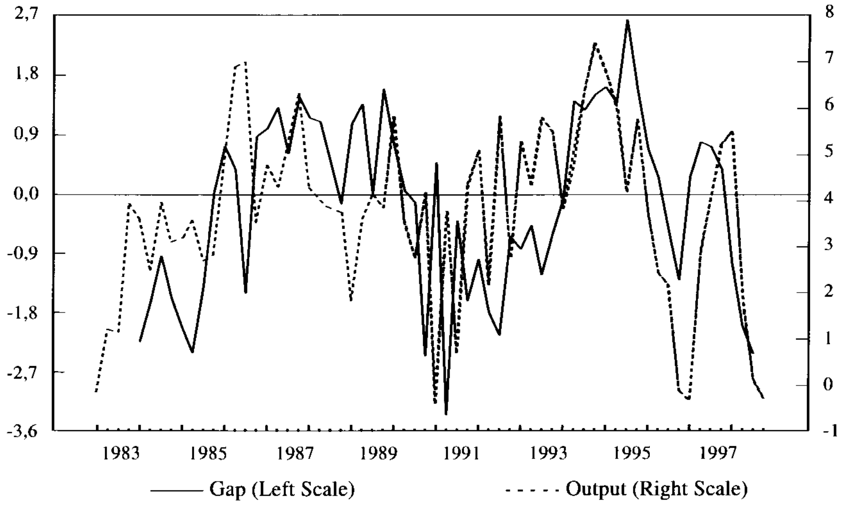
An instrument rule expresses the monetary instrument m_t as a function of the state variables in the economy:

(9) $m_t = \mathbf{g}\mathbf{X}_t$

The efficient feedback coefficients or elements of $\mathbf{g}$ may be obtained minimizing the weighted sum of the unconditional variance of the goal variables:

Figure 5

Unanticipated Money Growth and Change in the Output Gap subject to Eqs. (7) and (8)



$$E[L_t] = [\mathbf{Y}'\mathbf{K}\mathbf{Y}] = trace\left(\Sigma_{yy}\right)$$

where $\mathbf{K} = diag\,(\gamma, 1-\gamma, v)$ is a diagonal matrix,

$$\Sigma_{yy} = \mathbf{C}_X \Sigma_{xx} \mathbf{C}'_X\, , vec(\Sigma_{xx}) = [I - (\mathbf{M} \otimes \mathbf{M})]^{-1} vec\,(\Sigma_{vv}), \text{ and } \mathbf{M} = \mathbf{A} + \mathbf{B}\mathbf{g}.$$

We will consider the following two instrument rules.

a. The Optimal Linear Feedback Rule

Following a standar procedure presented in Sargent (1987), the optimal linear feedback rule is the solution to the problem:

$$\underset{\{m_t\}_{t=0}^{\infty}}{min}\; E_t \sum_{t=0}^{\infty} \mathbf{Y}_t'\mathbf{K}\mathbf{Y}_t$$

subject to (7) and (8).

The Bellman equation is:

$$(10) \quad \mathbf{X}_t'\mathbf{V}\mathbf{X}_t = \overset{min}{\underset{m}{}}\left\{\mathbf{Y}_t'\mathbf{K}\mathbf{Y}_t + \mathbf{X}_{t+1}'\mathbf{V}\mathbf{X}_{t+1}\right\}$$

Replacing (7) and (8) into (10), taking the derivative and solving for m_t, we write the optimal feedback rule:

$$(11) \quad m_t = -(\mathbf{R} + \mathbf{B}'\mathbf{V}\mathbf{B})^{-1}(\mathbf{U} + \mathbf{B}'\mathbf{V}\mathbf{A})\mathbf{X}_t$$

The optimal feedback rule is a linear function, $\boldsymbol{g}$, of the state vector $\mathbf{X}_t$ where

$$(12) \quad \mathbf{g} = -(\mathbf{R} + \mathbf{B}'\mathbf{V}\mathbf{B})^{-1}(\mathbf{U} + \mathbf{B}'\mathbf{V}\mathbf{A})$$

Inserting the optimal rule into the right hand side of equation (10), it is possible to find the Ricatti equation

$$(13) \quad \mathbf{V} = \mathbf{Q} + \mathbf{U}\mathbf{f} + \mathbf{f}'\mathbf{U}' + \mathbf{f}'\mathbf{R}\mathbf{f} + \mathbf{M}'\mathbf{V}\mathbf{M}$$

where $\mathbf{Q} = \mathbf{C}_x\mathbf{K}\mathbf{C}_x$, $\mathbf{U} = \mathbf{C}_x\mathbf{K}\mathbf{C}_m$, $\mathbf{R} = \mathbf{C}_m'\mathbf{K}\mathbf{C}_m$.

With an initial value of $\mathbf{V}$, the optimal feedback parameters can be found by iterating the Ricatti equation (13), and the optimal feedback rule (12).

For instance, if $\lambda = 0.5$ and $\nu = 0.25$, the optimal feedback rule is:

$$m_t = \mathbf{-0.194}\pi_t + 0.010\pi_{t-1} - 0.014\pi_{t-2} + 0.076\pi_{t-3}$$

$$-\mathbf{0.737}z_t - 0.307z_{t-1} + 0.076z_{t-2} + 0.098z_{t-3}$$

$$+0.079z_{t-4} + 0.029z_{t-5} - 0.020z_{t-6} - 0.054z_{t-7}$$

Of particular interest are the coefficients on contemporaneous inflation and output gap, -0.194 and -0.737, as they will serve as a basis for the comparison of the Taylor rule and the historical data.

b. The Taylor Rule

In the Taylor rule the monetary instrument depends exclusively on inflation and the output gap. In (9) the ***g*** vector takes the form:

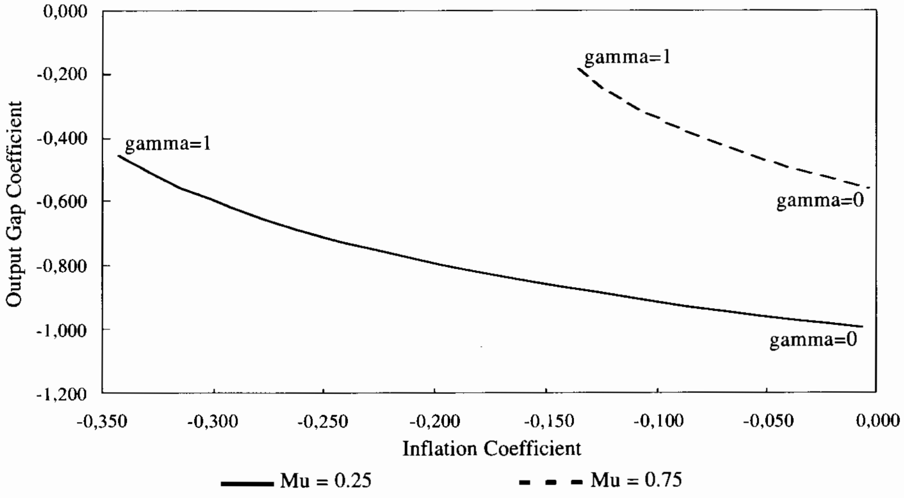
$$\mathbf{g} = [g_\pi \quad 0 \quad 0 \quad 0 \quad g_z \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

where g_π and g_z are the efficient inflation and output gap coefficients.

The efficient Taylor rule coefficients estimated minimizing the loss function are $g_\pi = -0.194$ and $g_z = -0.803$. These coefficients are quite similar to the corresponding elements of the optimal linear rule, also, the zero constraint on the remaining elements of vector ***g*** does not imply extreme changes to the optimal feedback parameters.

The efficient Taylor rule coefficients appear in Figure 6. The efficient coefficients are negative because monetary policy is a setting for the monetary base instead of a setting for the interest rate as if often understood in a Taylor rule. The negative sign means that in response to a upwards shock to inflation or output, money should

Figure 6
Inflation and Output Gap Coefficients in the Taylor Rule
for Different Preferences



decrease. The figure shows that for a given weight on the variance of unanticipated money (for a given ν), as the weight of inflation in the loss function increases (the bigger the γ), the higher the response of monetary policy to inflation (the larger the absolute value of g_{π}), and the lower the response of monetary policy to the output gap (the lower the absolute value of g_z). Figure 6 also shows that the higher the weight of the variance of unanticipated money in the loss function (the higher the ν), the lower the response of monetary policy to both inflation and the output gap (the lower in absolute value g_{π} and g_z).

B. MONETARY POLICY WITH UNCERTAINTY ABOUT THE GAP

In this case the monetary authority minimizes the loss function 6 to target the inflation rate and the unobserved gap. Now the state of the economy is known with uncertainty and is given as in 7, but the observation equation becomes

$$\mathbf{Y}_t = \mathbf{C}_x \mathbf{X}_t + \mathbf{C}_m m_t$$

where

$$\mathbf{Y}_t = \begin{bmatrix} \pi_t \\ \Delta y_t \\ m_t \end{bmatrix}, \mathbf{C}_x = \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_5 \\ 0 \end{bmatrix}, \mathbf{C}_m = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The central banks's estimate of the current state of the economy is given by $[\mathbf{X}_t] = [\mathbf{X}_t / \mathbf{Y}_t]$, and the optimal linear feedback rule is given by

$$m_t = -(\mathbf{R} + \mathbf{B}'\mathbf{V}\mathbf{B})^{-1}(\mathbf{U} + \mathbf{B}'\mathbf{V}\mathbf{A})\mathrm{E}[\mathbf{X}_t]$$

That is, the same 11 but this time in terms of $\mathrm{E}[\mathbf{X}_t]$ instead of. $\mathbf{X}_t$, where $\mathbf{V}$ can be found by means of 13, and thus leading to the same set of optimal feedback parameters as before. However, since the state of the economy is known with uncertainty, the variability of the goal variables will increase and so will the loss.

The minimized expected loss is given in this case by

$$trace\,(\mathbf{V}\Sigma_{vv}) + trace\,((\mathbf{g}'\mathbf{R}\mathbf{g} + 2\mathbf{g}'\mathbf{U}' + \mathbf{g}'\mathbf{B}'\mathbf{V}\mathbf{B}\mathbf{g})\Sigma_s)$$

where Σ_s is the forecast error variance in the Kalman filter of the appendix.

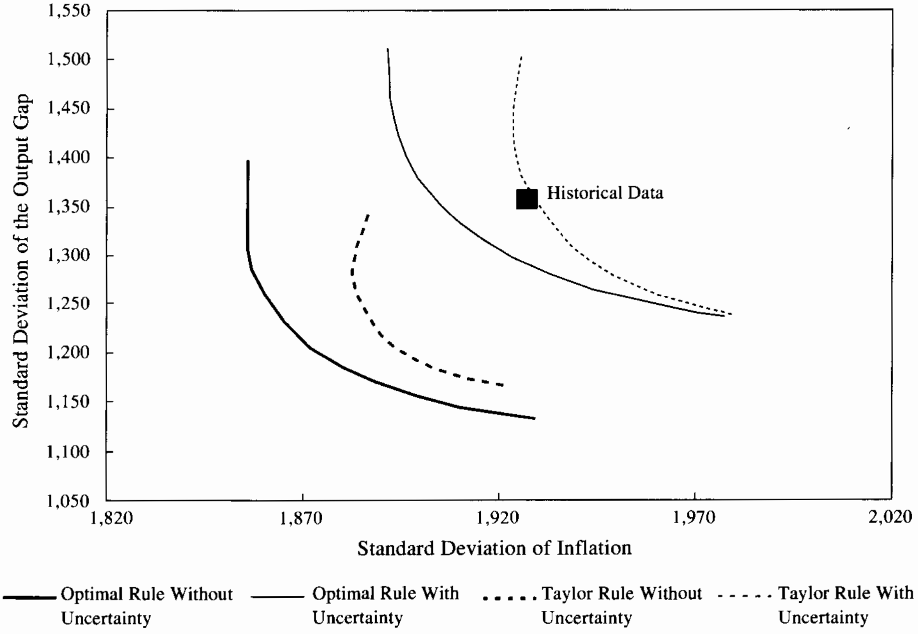
The efficient feedback parameters for the case of the Taylor rule are found by numerical minimization of 6, taking into account the change in the covariance matrices of the goal variables due to the uncertainty about the gap.

C. COMPARISON OF THE VARIANCE OF THE TARGET VARIABLES

Figure 7 compares the variance of inflation and the output gap in the optimal rule, the Taylor rule and the historical data. The comparison is made both for the certainty and uncertainty cases. The variance of the goal variables is the lowest in the optimal rule. As expected, for both rules the variance of the goal variables is also higher under uncertainty. The variance of the historical data is quite close to the variance of the goal variables under the Taylor rule under uncertainty with weights: 0.7 on inflation, 0.3 on the output gap, and 0.25 on money. This does not mean that the authorities followed a Taylor rule, what this menas is that the authorities behaved *as if* they had followed a Taylor rule under uncertainty about the output gap and with the above weights.

Figure 7

Output Gap - Inflation Variability Frontier



1. Impulse Response Functions

Figures 8 to 13 show the response of money, inflation, and the output gap to a one percentage point innovation in inflation (Figures 8 to 10) and the output gap (Figures 11 to 13). As shown in Figure 8 in response to a one percentage point increase in inflation money decreases. The contraction in money generates a contraction in the output gap (Figure 9). As the persistence of the output gap is relatively high $(\varphi(1) = 0.873)$, the gap does not return to zero rapidly but it decreases by 0.127 every two quarters. Figure 10 shows the response of inflation to the inflation shock. The Figure shows the small persistence of inflation. $(\alpha(1) = 0.189)$

The response of monetary policy to a positive innovation in the output gap is also a contraction (Figure 11). The contraction of monetary policy makes the initial one percentage point innovation in output to shorten because output responds to monetary policy on impact. Thus, the response of money makes output increase by $1.0 - \lambda m_t < 1.0$ (Figure 12). Figure 13 shows that inflation responds to the output gap with a two year lag.

The behavior of monetary policy under the optimal and the Taylor rules is remarkably similar, the differences being explained by the rather different response of money to lagged output in the optimal rule.

V. CONCLUSIONS

We showed and estimated a short run transmission mechanism of monetary policy for the case of Colombia. The unanticipated component of money growth determines output and the output gap on impact, and output and the output gap determine deviations of inflation from the core with an eight quarter lag.

We estimated the output gap as a variable that is unobserved and found that although there is considerable uncertainty in the measurement of the output gap, it is significantly different from zero for a few quarters in the sample. This result throws some doubt of the usefullnes of the output gap as a measure of inflationary pressures or as a leding indicator of inflation. More specifically, we find no compelling reason for a monetary authority to use an inflation forecast based on a poorly measured output gap or to react to a very uncertain leading indicator of inflation pressures.

Figure 8
Response of Money to an Inflation Shock

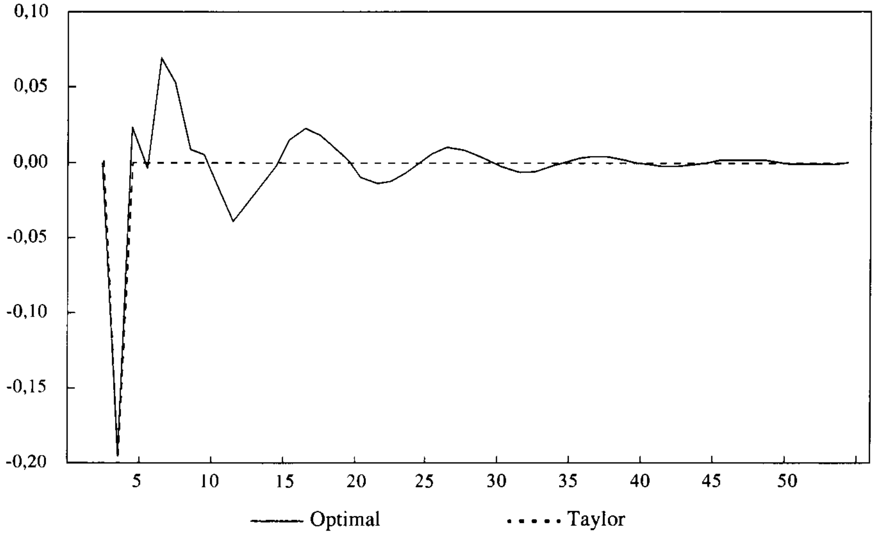


Figure 9
Response of Output to an Inflation Shock

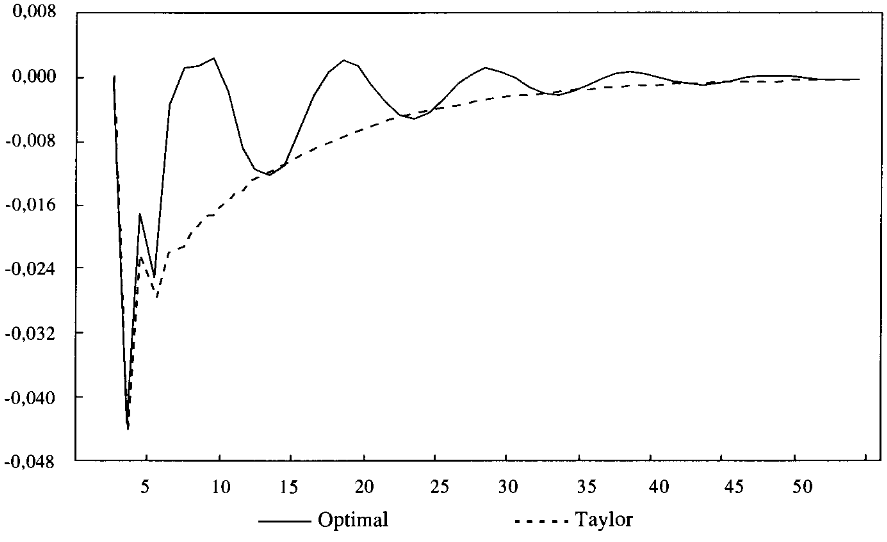


Figure 10
Response of Inflation to an Inflation Shock

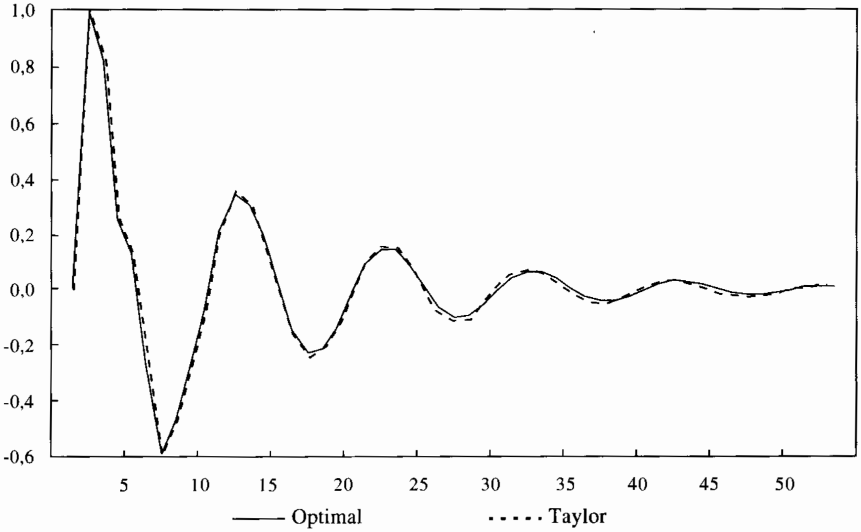


Figure 11
Response of Money to an Output Shock

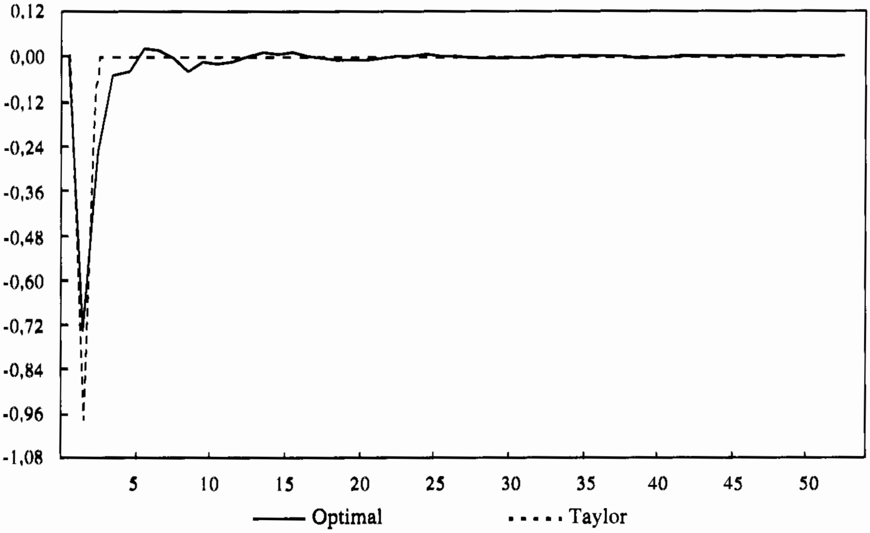


Figure 12
Response of Output to an Output Shock

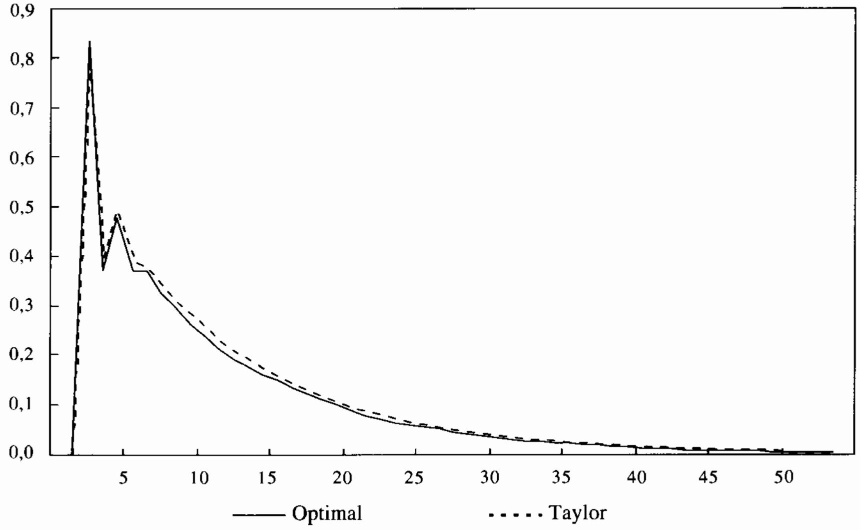
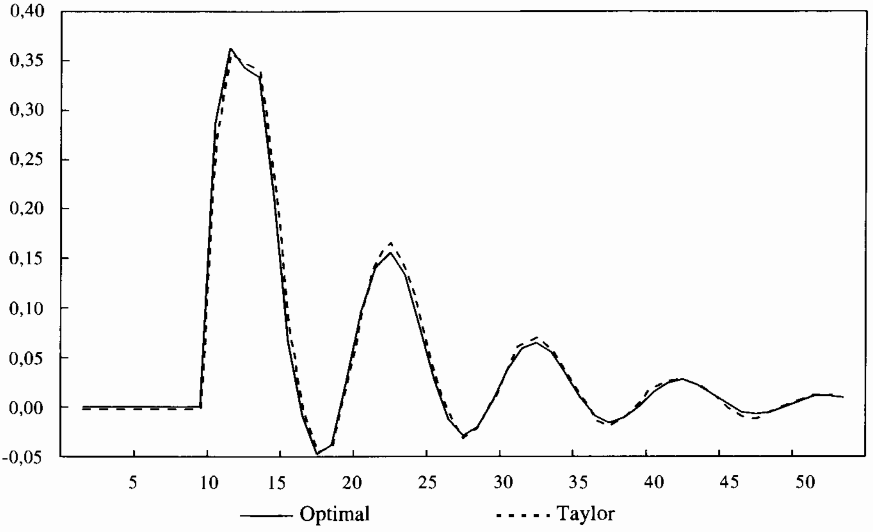


Figure 13
Response of Inflation to an Output Shock



As the variance of the first difference of potential output is not statistically different from zero, we do not reject the hypothesis that potential output follows a linear trend, thus, the output gap is the deviation of output from trend. We end up with a Keynesian view of the transmission mechanism of monetary policy in Colombia where unanticipated money growth determines the gap, and the gap determines deviation of inflation from core. However, this result may be due to the sample length.

The short run mechanism of monetary policy is as follows. Deviations of money from anticipated money explain, on impact, deviations of output from long run trend. Deviations of output from trend explain, with a two year lag, deviations of inflation from core.

Stating a loss function defined on the variance of inflation, output, and money, we found that the behavior of the goal variables under the Taylor rule is not too far from the behavior of the goal variables under the optimal rule. Compared to the optimal rule with certainty, the variance of the goal variables is higher for the Taylor rule, as well as for the case of uncertainty. The variance of the goal variables in the historical data is quite close to the uncertainty case when the instrument rule is the one of Taylor.

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APPENDIX

The State Space Representation and estimation

The description in this appendix refers to the final estimated model in which non significant parameters have been removed. Equations 1 to 4 are finally written as

$$
\begin{aligned}
& z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \lambda m_t + \varepsilon_t^z \\
& \pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + \alpha_4 \pi_{t-4} + \beta\, z_{t-8} + \varepsilon_t^\pi \\
\text{(A.1)}\quad & y_{t+1}^p = \mu + y_t^p + \varepsilon_t^y \\
& y_t = y_t^p + z_t
\end{aligned}
$$

.

where $V\left(\varepsilon_t^z\right) = \sigma_z^2, V\left(\varepsilon_t^\pi\right) = \sigma_z^2 \; and \; V\left(\varepsilon_t^y\right) = \sigma_y^2$.

In order to find the estimated parameter values from the observed data we can write these four equations in a state space representation where the observation equation is

$$\mathbf{Y}_t = \mathbf{H}\mathbf{X}_t + \mathbf{D}\mathbf{u}_t + \varepsilon_t$$

and the state equation is

$$\mathbf{X}_t = \mathbf{F}\mathbf{X}_{t-1} + \mathbf{G}\mathbf{u}_t + \mathbf{P}\boldsymbol{\eta}_t$$

where

$\mathbf{Y}_t = \begin{bmatrix} y_t - y_{t-1} \\ \pi_t \end{bmatrix}$ is the vector of observed variables,

$\mathbf{X}_t = \begin{bmatrix} z_t \\ z_{t-1} \\ z_{t-2} \\ z_{t-3} \\ z_{t-4} \\ z_{t-5} \\ z_{t-6} \\ z_{t-7} \\ z_{t-8} \end{bmatrix}$ is the vector of unobserved components,

$$\mathbf{u}_t = \begin{bmatrix} \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ \pi_{t-4} \\ m_t \end{bmatrix}$$ contains pre-determined variables at time t,

$\varepsilon_t = \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \end{bmatrix}$ and $\boldsymbol{\eta}_t = \left[\varepsilon_t^z\right]$ are the residual vectors, $V(\varepsilon_t) = diag\left(\sigma_y^2, \sigma_p^2\right)$and $V(\boldsymbol{\eta}_t) = \left[\sigma_z^2\right]$

$\mathbf{H}_{2x9} = \begin{bmatrix} \mathbf{e}_1 - \mathbf{e}_2 \\ \beta\,\mathbf{e}_9 \end{bmatrix}$ where $\mathbf{e}_i$ is the i-th elementary row vector of dimension 9,

$\mathbf{D}_{2x5} = \begin{bmatrix} \mathbf{0}_{1x5} \\ \sum_{j=0}^{4} \alpha_j \mathbf{e}_j \end{bmatrix}$ where $\mathbf{e}_i$ is the i-th elementary row vector of dimension 5,

$$\mathbf{F} = \begin{bmatrix} \sum_{j=1}^{2} \varphi_j \mathbf{e}_j \\ \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \\ \mathbf{e}_5 \\ \mathbf{e}_6 \\ \mathbf{e}_7 \\ \mathbf{e}_8 \end{bmatrix}$$ where $\mathbf{e}_i$ is the i-th elementary row vector of dimension 9,

$$\mathbf{G} = \begin{bmatrix} \mathbf{e}_5\lambda \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \\ \mathbf{0}_{1x5} \end{bmatrix} \; and \; \mathbf{P} = \begin{bmatrix} \mathbf{1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

For estimation all unknown parameters were arranged in a row vector of the form,

$$\Theta = \lfloor \varphi_1 \quad \varphi_2 \quad \beta \quad \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4 \quad \lambda \quad \sigma_y^2 \quad \sigma_p^2 \quad \sigma_z^2 \rfloor$$

Given initial values of the parameters, the state variables at time zero, and forecast error variance covariance matrix at time zero, the Kalman filter provides residuals from which we calculate the likelihood function. The maximization of the likelihood function is carried out by using standard numerical procedures. The value of the Hessian matrix was provided in each step, and from the last step Hessian we obtain the estimated the asymptotic variance covariance matrix of the estimated coefficients. See Harvey (1989), Watson and Engle (1983), and Burmeister & Wall (1982) for further details.

Our actual implementation of the Kalman filter and maximum likelihood estimation checks out the eigenvalues of the matrix **F** every time the parameter values changed, and no signs of non stationarity were found by using different starting values for the parameters. Given this result, initial conditions setup for Kalman filtering follow the lines of Harvey (1989, pages 120-122) and Hamilton (1994, page 378).

Using the estimated parameters, the Kalman filter produces the "filtered" state variables, that is the forecast of the output gap along with its forecasts error variance using information up to the last period of time. Final estimated gaps correspond to the fixed interval "smoothed" version which uses at each period of time all the sample information available. See Harvey (1989) for further details.

By using Doornik and Hansen's (1994) omnibus test for multivariate normality we find no evidence against this assumption.

Moreover, by using a Ljung-Box type test we find no evidence of multivariate autocorrelation in the residual series.

Residual Multivariate Normality

Test Stat.	**P Value**
1,0682	0,8992

Residual Autocorrelation

Lags	Test Stat.	P Value
14	47,7594	0,1865

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