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Por: Juan Manuel Julio

Borradores de ECONOMÍA

Núm. 546

2009

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Principal-Agent Problem with Minimum Performance Insurance: The Case of Mandatory Individual Pension Accounts*

Juan Manuel Julio†

Abstract

A minimum performance insurance in the Principal-Agent problem is wealth reducing to the principal. This result points to further inefficiencies in mandatory individual Pension Funds' contracts, particularly the one established in the 1993's 100th Law in Colombia.

1 Introduction

Starting with the 100th Law of 1993 and during the last 15 years the Colombian Central Government enacted a series of laws and constitutional amendments to reform the national pension system. Aside from few exceptions, all retirement benefits offered by public and private firms were banned and a parallel *mandatory* system was established. Currently, every working age Colombian chooses between the Defined Benefits system managed by the

*The author indebts very much the valuable comments and suggestions of Javier G. Gómez and Juan D. Prada from Banco de la República. However, any remaining errors as well as the conclusions and comments contained in this note are the sole responsibility of its author and do not compromise Banco de la República, its Board of Governors or the Universidad Nacional de Colombia.
JEL: D86, D82, G23.
Key Words: Incentives, Agency Theory, Pensions

†J. M. Julio is a researcher from the Macro Economic Modelling Department, Banco de la República and Associate Professor, Department of Statistics, Universidad Nacional de Colombia, Bogotá D. C., Colombia, jjulioro@banrep.gov.co.

Government's Social Security Office, and the Individual Accounts with Solidarity system managed by Private *Pension Fund* firms.

The key difference between the two systems is the risk over the monthly pension their members are entitled to. In the defined benefits system the mandatory savings go to a collective fund and the pension is a fixed percentage of the reported historical income. In the individual accounts with solidarity system, however, a fixed percentage of the monthly individual savings goes to an *individual account*, and the pension depends on the balance of this account at the age of retirement. Therefore, in the individual accounts with solidarity system the pension depends critically on the Pension Funds' investment performance.

The contract signed between Colombian citizens, represented by the Central Government, and Pension Funds, to manage the individual accounts in the individual accounts with solidarity system may be understood as a *principal-agent problem with minimum performance insurance*. The *principal* (Colombian citizens represented by the government), assigns a *task* (the management of the mandatory individual savings account) to an *agent* (Pension Fund firms), which, after its execution, produces *revenue* to the principal (the returns on the fund's investments). However, the revenue produced by the agent is not completely under her control (the investment returns depend on market conditions also). The principal can not observe the level of *effort* (the Pension Funds' ability to identify "good investments opportunities") applied by the agent to perform the task. The principal's *wealth* is the revenue produced by the agent net of the *reward* (the administration fee Pension Funds receive) for performing the task. The minimum performance *insurance* protects the principal against low performances (individuals are entitled to a minimum return on their balance). See [16], [9],

[8], [14], [15], [11], [2] and [17].

Since the action chosen by the agent is not observable, a *moral hazard* problem arises. The agent chooses the action that maximizes her utility regardless of the principal's expected wealth. Thus, the agent's choice may adversely affect the principal's wealth.

A widely known result in Principal-Agent theory establishes that an optimal risk-sharing reward maximizes the principal's expected wealth. Proposals to apply this result to pension management contracts are already well known. See [12], [18], [13], [5], [7] and [1].

Caution should be taken when implementing this kind of optimal solutions as it may induce short run risky behavior on Pension Funds' managers, a culprit in the current financial crisis. Optimal risk sharing and risk adjusted rewards would do the for the present case. See [3] and the shape of the optimal solution in [6].

However, the current Pension Funds' contract in Colombia establishes an administration fee of 10% of deposits regardless of the Pension Funds' investment performance, and establishes a minimum return insurance on individual accounts balances. Therefore, Colombian Pension Funds lack contract incentives to apply effort in managing the pension accounts. In fact, Pension Funds' optimal behavior is to enroll high income individuals, and/or increase the number of enrollees, at the expense of pension accounts returns.

Moreover, the efficiency of having a minimum return insurance has not been studied and is recurrent not only in Colombia but also in other emerging countries. This note tries to fill this gap.

The rest of this note is organized as follows. In section two we set up the two models we deal with in this note, the plain-vanilla principal-agent model and the principal-agent model with minimum performance insurance. In section three we solve these problems when the principal induces the lowest effort and prove the insurance related inefficiency for this particular case. This result shows a second source of inefficiency in the current Pension Funds' contract in Colombia. In the fourth, we solve these problems when the principal induces the highest effort and prove the insurance related inefficiency for this particular case. This result points to the optimal solution to be implemented in the Pension Funds' contract. In the appendix we prove the insurance related inefficiency in the general setting where there is a continuum of actions to chose from.

2 Setting Up The Problem

In this section we set up the two models we deal with in this note. In the first subsection we review the plain-vanilla two-action principal-agent model. In the second we bring the minimum performance insurance into the previous problem. In addition, we formalize the first source of inefficiency in the Colombian Pension Funds' contract.

2.1 The Principal Agent Problem

At the beginning of the period the *principal*, a *risk neutral* individual or firm with bargaining power, offers a *contract* to an *agent*, a *risk averse* individual or firm, to perform a *task* that produces revenue x to the principal at the end of the period. The contract determines the *reward* function, $s()$, the agent receives for performing the task.

The agent is free to accept the contract and then, autonomously, choose the action that maximizes her utility, or give it up to earn a reservation utility U_R. The principal, however, does not take any further action besides offering the contract.

Each of the actions the agent may choose from is mapped into a unique level of *effort* she applies in performing the task, $a = 0 =$ low effort, and $a = 1 =$ high effort. Since revenue is not completely under the agent's control, conditional on the level of effort it has a distribution $(X/A = a) \sim F_{X/A=a}(x)$ on $\mathcal{X}$, the set of all possible revenue realizations for $a \in \{0, 1\}$. These distributions are such that the higher the effort, the higher the frequency of better results. Thus, the expected revenue function is strictly increasing on effort, $E[X/A = 0] < E[X/A = 1]$. Moreover, the agent's cost function is strictly increasing on effort, $C(0) < C(1)$.

However, the action the agent chooses is *hidden* to the principal. The level of effort applied by the agent is not observable and can not be inferred with certainty from the resulting revenue.

The agent's utility function is separable in money and effort, $U(s(), x, a) = u(s()) - C(a)$, where $u()$ is strictly increasing and concave, and continuously differentiable. The conditional distribution functions are defined on $\mathcal{X} = \mathbb{R}$, have the monotone likelihood ratio property and do not have a shifting support. See [12], [18] and [4].

For a realized revenue x, the ex-post wealth of the principal is the revenue net of the reward $W(x, s()) = x - s()$, and, for a given level of effort a, the ex-post utility of the agent is $U(s(), x, a) = u(s()) - C(a)$.

The principal chooses the reward function $s()$ that maximizes his expected wealth in a two step procedure. In the first step he determines the optimal reward for each level of effort, $s_0()$ and $s_1()$ respectively, and in the second he chooses the action that maximizes his expected wealth. In this way, the principal determines the effort he wants to induce the agent to.

Ex ante, given that the principal wants to induce the agent to take action

a, he chooses the reward $s()$ that maximizes his expected wealth

$$\max_{s()} E[X - s()/A = a] \tag{1}$$

However, the agent accepts the contract only if her expected utility net of costs is higher than her reservation utility, U_R. Otherwise, she refuses the contract and takes an alternative activity earning U_R. If the principal wants the agent to accept the contract, the reward is constrained to

$$E[u(s())/A = a] - C(a) \geq U_R \tag{2}$$

which is known as the participation constraint.

If the reward satisfies the participation constraint, the agent signs the contract and chooses the effort she will apply according to

$$\max_{a\in\{0,1\}} \quad E[u(s())/A = a] - C(a) \tag{3}$$

producing, at the end of the period, an observed level of revenue x from $F_{X/A=a}(x)$.

However, if the agent chooses $1-a$ instead of a, the reward should make a more appealing to the agent than $1-a$, so $s()$ might also be constrained to

$$E[u(s())/A = a] - C(a) > E[u(s)/A = 1 - a] - C(1 - a) \tag{4}$$

which is known as the incentive compatibility constraint.

Provided the principal induces the agent to take action a, the optimal solution to the principal-agent problem is denoted as $s_a()$ for $a \in \{0, 1\}$.

Now we are able to set the following results:

Result 1. *If the marginal expected revenue is higher than the marginal expected optimal reward,*

$$E[X/A = 1] - E[X/A = 0] > E[s_1()/A = 1] - E[s_0()/A = 0]$$

the principal increases his expected wealth by inducing the agent to apply the highest effort. If the marginal expected revenue is lower than the marginal expected optimal reward, the principal's expected wealth is higher under the lowest effort and he induces the agent to it.

In fact, if $E[X/A = 1] - E[X/A = 0] > E[s_1()/A = 1] - E[s_0()/A = 0]$, the principal's expected wealth under action $a = 1$, $E[X/A = 1] - E[s_1()/A = 1]$ is strictly higher than his expected wealth under $a = 0$, $E[X/A = 0] - E[s_1()/A = 0]$.

Result 2. *A constant reward satisfying the participation constraint induces the agent to choose the lowest effort, $a = 0$.*

Since the reward is constant, $s() = \overline{s}$, the agent's expected utility does not depend on effort, $E[u(\overline{s})/A = a] = u(\overline{s})$, then, the smaller the cost of the action, the higher the agent's expected utility, $0 = \arg\max_a [E[u(\overline{s})/A = a] - C(a)]$ since $C(0) = \min_a C(a)$. Then the agent's optimal choice is to apply the lowest effort $a = 0$.

The current Pension Funds' contract in Colombia establishes an administration fee of 10% of deposits regardless of both, the funds' investment performance and the unobserved action. Therefore, the current Colombian Pension Funds contract induces them to apply the lowest effort in managing the individual pension accounts.

The previous exposition (and the two step procedure), leads to the the first two problems we deal with in the following two sections:

Problem 1 (**The principal induces the lowest level of effort**). *The principal chooses the reward $s()$ that maximizes his expected wealth subject to the participation constraint, given $a = 0$. Solves 1 subject to 2 for $a = 0$.*

Because of result two above, only the participation constraint is required under a constant reward.

Problem 2 (The principal induces the highest level of effort). *The principal chooses the reward $s()$ that maximizes his expected wealth subject to the participation constraint and the incentive compatibility constraint, given $a = 1$. Solves 1 subject to 2 and 4 for $a = 1$.*

2.2 The Minimum Performance Insurance

The agent warrants a minimum level of revenue θ to the principal. If the realized revenue x is at least θ, the agent receives a reward $s()$, otherwise she pays back $\theta - x$ to honor the insurance. Therefore, the reward function, $S(x, \theta, s())$, splits into an undetermined component, $s()$ when $x \geq \theta$, and a completely determined and contingent component, $-(\theta - x)$ when $x < \theta$, as follows

$$S(x, \theta, s()) = s()I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x) \tag{5}$$

where $I_A(x)$ is the indicator function defined on a set A, $I_A(x) = 1$ if $x \in A$ and $I_A(x) = 0$ otherwise.

For a realized revenue x, the ex-post wealth of the principal is the revenue net of the reward, $W(x, \theta, S) = x - S(x, \theta, s())$, and for a given level of effort a, the ex-post utility of the agent becomes $U(S, x, a, \theta) = u(S(x, \theta, s())) - C(a)$.

Ex ante, the principal determines the optimal undetermined component of the contract in a two step procedure. Provided the principal wants to induce the agent to a, he chooses the undetermined component of the reward,

$s()$, that maximizes his expected wealth

$$\begin{aligned}\max_{s()} \quad & E[W(x,\theta,S)/A=a] \Leftrightarrow \\ \max_{s()} \quad & E[(X-s())/X\geq\theta, A=a]P[X\geq\theta/A=a]+ \\ & +\theta P[X<\theta/A=a] \end{aligned} \tag{6}$$

subject to the participation constraint,

$$\begin{aligned}& E[u(S(x,\theta,s()))/A=a]-C(a)\geq U_R \Leftrightarrow \\ & E[u(s())/X\geq\theta, A=a]P[X\geq\theta/A=a]+ \\ & +E[u(-(\theta-X))/X<\theta, A=a]P[X<\theta/A=a]\geq U_R+C(a) \end{aligned} \tag{7}$$

and the incentive compatibility constraint

$$\begin{aligned}& E[u(S(x,\theta,s()))/A=a]-C(a)>E[u(S(x,\theta,s()))/A=1-a]-C(1-a) \Leftrightarrow \\ & E[u(s())/X\geq\theta, A=a]P[X\geq\theta/A=a]+ \\ & +E[u(-(\theta-X))/X<\theta, A=a]P[X<\theta/A=a]-C(a)> \\ & E[u(s())/X\geq\theta, A=1-a]P[X\geq\theta/A=1-a]+ \\ & +E[u(-(\theta-X))/X<\theta, A=1-a]P[X<\theta/A=1-a]-C(1-a) \end{aligned} \tag{8}$$

if required.

Parallel to the two problems above, we establish the problems the principal may face in this case:

Problem 3 (**The principal induces the lowest level of effort**). *The principal chooses the undetermined component of the reward, $s()$, that maximizes his expected wealth subject to the participation constraint, given $a=0$. Solves 6 subject to 7 for $a=0$. Because of result two above only the participation constraint is required under a constant reward.*

Problem 4 (The principal induces the highest level of effort). *The principal chooses the undetermined component of the reward, $s()$, that maximizes his expected wealth subject to the participation constraint and the incentive compatibility constraint, given $a = 1$. Solves 6 subject to 7 and 8 for $a = 1$.*

3 The Principal Induces the Lowest Effort

The marginal expected revenue is lower than the marginal expected optimal reward. Thus the principal increases his expected wealth by inducing the agent to choose the lowest effort.

Under the assumption that the principal wants to induce the lowest effort, we find the optimal solution of the plain-vanilla principal-agent problem and the principal-agent problem with minimum performance insurance. Based on the result that a constant reward induces the action of lower effort, we find, in sub sections one and two, the optimal *constant* reward for problems one and three above. The insurance related inefficiency is shown in sub section three where we show that the optimal solution of problems one and three are constant and unique, and show that the insurance reduces the principal's expected wealth. *This is the second source inefficiency in the Pension Funds contract in Colombia.*

3.1 The Optimal Constant Solution to the Principal-Agent Model

The principal solves problem one above under the assumption that the reward is constant. If the principal wants to induce the action of lowest effort, he might rely on the previous result to set a constant reward[1]. The less ex-

[1]Another approach to see constancy in this case is to resort on the Khun-Tucker first order condition, $\frac{1}{u'(s())} = \lambda$, $\forall x \in \mathcal{X}$, where $\lambda > 0$ since the participation constraint

pensive *constant* reward satisfying the participation constraint in problem one is obtained from this constraint with equality

$$E[u(\bar{s}_0)/A = 0] - C(0) = U_R \Leftrightarrow$$
$$u(\bar{s}_0) - C(0) = U_R \Leftrightarrow$$

which yields

$$\bar{s}_0 = u^{-1}(U_R + C(0)) \tag{9}$$

The reward 9 has the agent sign the contract and apply the action of lowest effort $a = 0$. The agent's expected utility becomes U_R, and the principal's expected wealth becomes

$$E[W(X, \bar{s}_0)/A = 0] = E[X/A = 0] - u^{-1}(U_R + C(0)) \tag{10}$$

3.2 The Optimal Constant Solution to The Principal Agent Problem with Insurance

The principal solves problem three above under the assumption that the undetermined component of the reward is constant. As the argument above shows, a *constant* reward satisfying the participation constraint induces the agent to choose $a = 0$. The optimal constant undetermined component of 5 satisfying the participation constraint with equality is

$$E[u(S(x, \theta, s()))/A = 0] - C(0) = U_R \Leftrightarrow$$
$$\bar{s}_0^i =$$
$$u^{-1}\left(\frac{C(0) + U_R - E[u(-(\theta - X))/X < \theta, A = 0]P[X < \theta/A = 0]}{P[X \geq \theta/A = 0]}\right)$$
$$\forall x \geq \theta \tag{11}$$

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is binding. Moreover, since u' is decreasing, $s()$ and λ are related directly, then $s()$ is constant.

The reward 5 with $s() = \overline{s}_0^i$ as in 11 has the agent sign the contract and choose $a = 0$. The agent's expected utility becomes U_R and the principal's expected wealth under 11 becomes

$$\begin{aligned} E[W(x,\theta,S)/A=0] &= E[X/A=0] - E[\overline{s}_0^i I_{[\theta,\infty)}(X)/A=0] + \\ &+ E[(\theta - X)I_{(-\infty,\theta)}(X)/A=0] \\ &= \{E[X/X \geq \theta, A=0] - \overline{s}_0^i\}P[X \geq \theta/A=0] + \\ &+ \theta P[X < \theta/A=0] \end{aligned} \quad (12)$$

Result 3. *The optimal constant undetermined component of the reward in the principal-agent problem with insurance is given by 5 and the principal's expected wealth 12 where s() is given by 11.*

3.3 Insurance Related Inefficiency when the Principal Induces the Lowest Effort: The Case of the Colombian Private Pension Funds

The case just considered describes the current Pension Funds' contract in Colombia. The reward (10% of deposits), does not depend on the Pension Funds' investment performance and there is a minimum return warranted by law on the account balance. Therefore, Pension Funds increase their utility by applying the lowest effort in managing the individual pension accounts.

The Colombian case differs slightly from 5 since the constant reward is charged beforehand. By transforming the warranted return to $\theta' = \theta - \overline{s}$ we get to 5 and our analysis holds.

The minimum performance insurance is inefficient because it reduces the principal's expected wealth. To prove this statement we have to prove that the optimal solution to problems one and three above are constant and unique. Then we are able to prove the statement in this particular case.

Problem 1 is equivalent to minimizing $E[s()/A = 0]$ subject to the participation constraint,

$$\min_s E[s()/A = 0]$$
$$\text{subject to} \qquad E[u(s())/A = 0] \geq U_R + C(0)$$

The participation constraint is binding. If it were not, $E[u(s())/A = 0] > U_R + C(0)$, and since u^{-1} is strictly increasing, $u^{-1}(E[u(s())/A = 0]) > u^{-1}(U_R + C(0))$, and since u^{-1} is strictly convex, because of Jensen's inequality, $E[s()/A = 0] = E[u^{-1}(u(s()))/A = 0] > u^{-1}(U_R + C(0))$. However, the constant reward $\overline{s}_0 = u^{-1}(U_R + C(0))$ satisfies the participation constraint and reduces the expected reward with respect to $s()$, $\overline{s}_0 = E[\overline{s}_0/A = 0] < E[s()/A = 0]$. Then, if the participation constraint is not binding $s()$ is not optimal. Therefore the participation constraint is binding.

Infinitely many reward functions satisfy $E[u(s())/A = 0] = U_R + C(0)$. We argue that the optimal solution is constant and unique, $s() = \overline{s}_0$. In fact, if it were not, since it satisfies the participation constraint and u^{-1} is increasing,

$$\begin{aligned} E[u(s())/A = 0] &= U_R + C(0) \Leftrightarrow \\ u^{-1}(E[u(s())/A = 0]) &= u^{-1}(U_R + C(0)) = \overline{s}_0 \end{aligned}$$

and since u^{-1} is convex, from Jensen's inequality we obtain

$$u^{-1}(U_R + C(0)) = u^{-1}(E[u(s()/A = 0]) \leq E[u^{-1}(u(s()))/A = 0] = E[s()/A = 0]$$

then

$$\overline{s}_0 = u^{-1}(U_R + C(0)) \leq E[s()/A = 0]$$

In Jensen's inequality equality holds if and only if for every line $a + bY$ tangent to $u^{-1}(y)$ at $y = E[Y/a = 0]$, $P[u^{-1}(Y) = a + bY/A = 0] = 1$.

Since u^{-1} is strictly convex, the only way that $P[u^{-1}(Y) = a + bY/A = 0] = 1$ for every $a + bY$ tangent to $u^{-1}(y)$ at $y = E[Y/A = 0]$, is that $P[Y = E[Y/A = 0]/A = 0] = 1$. That is, $P[s() = E[s()/A = 0]/A = 0] = 1$. Then the optimal reward is constant and unique, given by $s_0() = \overline{s}_0 = u^{-1}(U_R + C(0))$.

Result 4. *The optimal solution to the principal-agent problem where the principal induces the lowest effort is constant and unique, given by 9.*

Following the same argument it can be shown that the optimal undetermined component of the reward for the the principal-agent problem with minimum performance insurance, where the principal induces the lower effort, satisfies the participation constraint with equality

$$E[u(s_0^i()I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x))/A = 0] = U_R + C(0) \tag{13}$$

which leads to 11.

Therefore, the optimal reward under insurance becomes

$$S(x, \theta, \overline{s}_0^i) = \overline{s}_0^i I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x)$$

with $\overline{s}_0^i = u^{-1}(\frac{C(0)+U_R-E[u(-(\theta-X))/X<\theta,A=0]P[X<\theta/A=0]}{P[X\geq\theta/A=0]})$.

Result 5. *The optimal solution to the principal-agent problem with insurance where the agent induces the lowest effort is constant and unique, given by 5 where s is given by 11.*

From Jensen's inequality and strict convexity, the second inefficiency follows from 13

$$E[S()/A = 0] = E[u^{-1}(u(\overline{s}_i I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x)))/A = 0] >$$
$$u^{-1}(E[u(\overline{s}_i I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x))/A = 0]) = u^{-1}(U_R + C(0)) = \overline{s}_0$$

Therefore, the insurance reduces the expected wealth of the principal.

Result 6. *The principal's expected wealth under insurance is **strictly** lower than his expected wealth without insurance when the principal induces the lowest effort.*

4 The Principal Induces the Highest Effort

The marginal expected revenue is higher than the marginal expected optimal reward. Thus the principal increases his expected wealth by inducing the agent to choose the highest effort.

4.1 The Optimal Solution to the Principal-Agent Model

The principal solves problem two above. A constant reward will not do since the principal wants to induce $a = 1$, which is achieved only through the incentive compatibility constraint.

The necessary and sufficient condition for the optimal solution is the Khun-Tucker first order condition

$$\frac{1}{u'(s_1(x))} = \lambda + \mu[1 - \frac{f_{X/A=0}(x)}{f_{X/A=1}(x)}] \tag{14}$$

$\forall x \in \mathcal{X}$, where λ and μ are the Lagrange multipliers of the participation and incentive compatibility constraints respectively, and $f_{X/A=a}(x) = \frac{dF_{X/A=a}(x)}{dx}$. See [12] and [18].

Both, the participation and incentive compatibility constraints are binding, that is, $\mu > 0$ and $\lambda > 0$. Since $\mu = 0$ is consistent with a constant reward, $\mu > 0$. And $\lambda > 0$ since otherwise we could find a better deal still satisfying the participation constraint. See [12] and [18].

Given that u is strictly concave, u' is decreasing, then $s_1()$ and $\lambda + \mu[1 - \frac{f_{X/A=0}(x)}{f_{X/A=1}(x)}]$ are related inversely, and $s_1()$ and $\frac{f_{X/A=1}(x)}{f_{X/A=0}(x)}$ directly. Thus, the greater the likelihood ratio $\frac{f_{X/A=1}(x)}{f_{X/A=0}(x)}$ that the action chosen was $a = 1$, the greater the reward. In other words, the optimal solution entails risk sharing, that is, the reward is contingent on revenue, x.

4.2 The Optimal Solution to the Principal-Agent Model with Insurance

The principal solves problem four above. The reward function is determined by 5, and the principal determines the optimal undetermined component of the reward, $s()$. Since the principal wants to induce the highest effort, the incentive compatibility constraint is in place.

The Kuhn-Tucker first order condition is the same as before

$$\frac{1}{u'(s_1^i())} = \lambda + \mu[1 - \frac{f_{X/A=0}(x)}{f_{X/A=1}(x)}]$$

only for $x \geq \theta$, where $\lambda > 0$ and $\mu > 0$ are the Lagrange multipliers of the participation and incentive compatibility constraints respectively.

Therefore, the optimal solution entails risk sharing $\forall x \geq \theta$, and the reward becomes

$$S(x, \theta, s) = s_1^i(x) I_{[\theta,\infty)}(x) - (\theta - x) I_{(-\infty,\theta)}(x)$$

where $s_1^i(x)$ satisfies the first order condition $\forall x \geq \theta$. Otherwise $S(x, \theta, s)$ is completely determined and contingent on x.

4.3 Insurance Related Inefficiency when the Principal Induces the Highest Effort

The efficiency of contracting the insurance is determined by comparing the expected principal's wealth with and without the minimum performance insurance.

Let us rewrite the maximization problem of the principal-agent with minimum performance insurance in a more convenient way. The principal maximizes his wealth

$$\max_{S()} E[X - S()/A = 1]$$

subject to the participation constraint

$$E[u(S())/A = 1] - C(1) \geq U_R$$

the incentive compatibility constraint

$$E[u(S())/A = 1] - C(1) > E[u(S())/A = 0] - C(0)$$

and the insurance constraint

$$S() = s()I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x)$$

By writing the problem this way we have proved that the maximization set under the minimum performance insurance

$$\begin{aligned} \mathcal{S}_w = \{S() \quad : \quad & E[u(S())/A = 1] - C(1) \geq U_R, \\ & E[u(S())/A = 1] - C(1) > E[u(S())/A = 0] - C(0), \text{ and} \\ & S() = s()I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x)\} \end{aligned}$$

is *strictly* contained in the plain-vanilla principal-agent maximization set

$$\begin{aligned} \mathcal{S} = \{S() \quad : \quad & E[u(S())/A = 1] - C(1) \geq U_R, \text{ and} \\ & E[u(S())/A = 1] - C(1) > E[u(S())/A = 0] - C(0)\} \end{aligned}$$

Therefore, since the minimum performance insurance imposes a particular behavior to the reward function $S()$ for $x < \theta$, the expected principal's wealth with insurance is lower than or equal than without it,

$$E[W(x, \theta, S())/A = 1] \leq E[W(x, s())/A = 1]$$

Equality of the maximized expected wealth with and without minimum performance insurance is possible under strong restrictions. For instance, [6] propose an optimal linear risk sharing reward function of the form $s(x) = \delta + \gamma x$ for the principal-agent problem. However, for the minimum performance insurance to be optimal under this linear reward schedule, $\delta = -\theta$ and $\gamma = 1$ for $x < \theta$, a strong constraint even under this kind of reward.

However, an unconstrained linear reward schedule would increase the principal's expected wealth with respect to the current Colombian contract.

Result 7. *The principal's expected wealth under insurance is lower than or equal than his expected wealth without insurance when the principal induces the highest effort.*

5 Conclusions and Policy Implications

5.1 Conclusion

A minimum return insurance reduces the principal's expected wealth and a constant reward induces the agent to the lowest effort. Because of these inefficiencies, Colombian pension savers have been cumulating a wealth loss since the introduction of the Individual Accounts with Solidarity system in Colombia in 1993.

An optimal linear risk sharing reward schedule like [6] would improve the Pension Funds' contract in Colombia. However, caution should be taken in applying these kind of reward schedules as they may induce short run risky behavior on Pension Funds managers. A linear risk adjusted reward schedule may be more appropriate under these circumstances, and would agree with the current proposal of introducing risk varying funds. See [15].

5.2 Policy Implications

- The minimum return insurance in the Pension Funds' contract should be eliminated as it is wealth reducing to the principal.

- A risk-sharing reward program to induce the conformance of Pension Funds' actions to pension savers' interests should be established.

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Appendix

A Insurance Related Inefficiency in the General Setting

Consider now the case in which there is a continuum of possible actions the agent may choose from, $a \in \mathcal{A}$, which correspond to a continuum of possible levels of effort, where $\mathcal{A}$ is a compact subset of $\mathbb{R}$. The optimal reward schedule in the plain-vanilla principal-agent problem is obtained by choosing simultaneously the reward and action that maximize the principal's expected wealth,

$$\max_{a,S()} E[X - S()/A = a]$$

subject to the incentive compatibility constraint

$$a \in \arg\max_{a} E[u(S())/A = a] - C(a)$$

and the participation constraint

$$E[u(S())/A = a] - C(a) \geq U_R$$

which has an optimization set

$$\begin{aligned} \mathcal{S} = \{(S(), a) \quad : \quad & a \in \mathcal{A}, \;\; E[u(S())/A = a] - C(a) \geq U_R, \text{ and} \\ & a \in \arg\max_{a} E[u(S())/A = a] - C(a)\} \end{aligned}$$

However, the optimization set under the insurance is

$$\begin{aligned} \mathcal{S}_w = \{(S(), a) \quad : \quad & a \in \mathcal{A}, \;\; E[u(S())A = a] - C(a) \geq U_R, \text{ and} \\ & a \in \arg\max_{a} E[u(S())/A = a] - C(a), \\ & S() = s()I_{[\theta,\infty)}(x) - (\theta - x)I_{(-\infty,\theta)}(x)\} \end{aligned}$$

which is *strictly* contained in $\mathcal{S}$.

Therefore, since the insurance imposes a particular behavior to the reward function $S()$ for $x < \theta$, the principal's expected wealth under the insurance is lower than or equal than under the plain vanilla principal-Agent problem.

Moreover, under the assumptions

1. $\int_{-\infty}^{y} F_{X/A=a}(x)dx$ is non decreasing convex in a $\forall y \in \mathcal{X}$.

2. $E[X/A = a]$ is non decreasing concave in a.

3. $\frac{df_{X/A=a}(x)/da}{f_{X/A=a}(x)}$ is non decreasing convex in x $\forall a$

4. $u(u'^{-1}(1/z))$ is concave $\forall z > 0$

the first order conditions approach to the maximization problem above holds. In this case the Khun-Tucker first order condition is

$$\frac{1}{u'(s())} = \lambda + \mu \frac{df_{X/A=a}(x)/da}{f_{X/A=a}(x)} \tag{15}$$

Therefore, under these assumptions a reward schedule satisfying 15 is optimal, and the insurance is wealth reducing to the principal. See [10].

However, the first order conditions approach as described above *does not work* under arbitrary distribution functions $F_{X/A=a}(x)$. A class of distribution functions that satisfy the fist two assumptions above are the exponential family in an appropriate parametrization

$$f_{X/A=a} = \theta(x)\psi(a)e^{\alpha(a)\beta(x)}$$

where α and β are non decreasing, $\beta(x)$ is concave, and the expected revenue function is concave. See [10].