

# Colombian Purchasing Power Parity Analysed Using a Framework of Multivariate Cointegration

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## Abstract

This paper tests for purchasing power parity (PPP) between Colombia and its main trading partners using the Johansen framework of multivariate cointegration. The tests shows that PPP does not hold in the strong sense, but a clear purchasing power relationship is, nevertheless, shown to exist. The model is, furthermore, shown to have significant forecasting power. It outperforms a random walk in out-of-sample forecasting on the 12 and 24-month horizon but not on the 3 and 6-month horizon.

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# 1 Introduction

“Purchasing power parity (PPP) is the disarmingly simple empirical proposition that, once converted to a common currency, national price levels should be equal. The basic idea is that if goods market arbitrage enforces broad parity in prices across a sufficient range of individual goods (the law of one price), then there should also be a high correlation in aggregate price levels.”<sup>1</sup> However, few economists would accept PPP as a short-term proposition, even if most instinctively believe that some form of purchasing power parity exists in the long run.

This paper tests the PPP hypothesis adjusted for the Balassa-Samuelson effect on Colombian data for the last 23 years. Quarterly data is used from the first quarter 1980 to the third quarter 2002, for the nominal effective exchange rate, together with domestic and external prices. The Johansen framework of multivariate cointegration is used for the empirical analysis, to allow for short-run dynamics. A valid cointegrating relationship is found to exist, but this is relatively far from the parity relationship stated by the PPP hypothesis. Nevertheless, the estimated model seems to have considerable forecasting power. It outperforms a random walk model in out-of-sample forecasting at the 12 and 24-month horizon, but not at the 3 and 6-month horizon. That the model does relatively bad at short-term forecasts is, however, not that surprising. While long-term exchange rates tend to be governed by fundamentals, short-term rates are to a large extent influenced by speculation.

Chapter 2 of the paper tells the history behind the PPP theory, as well as how this can be adjusted for the Balassa-Samuelson effect. The chapter also briefly reviews the literature in the area. Chapter 3 contains the empirical analysis. The data set is discussed, the Johansen framework is introduced, and the analysis and the results are presented. In chapter 4 the forecasting power of the model is analysed and discussed, and chapter 5 concludes the paper.

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<sup>1</sup> Rogoff (1996), p. 647.

## 2 Purchasing Power Parity: Theory and Empirical Findings

PPP is a classical theory in international economics. The first section of this chapter introduces the PPP hypothesis. However, if two economies have different rates of labour productivity growth, they will normally have different rates of inflation, and PPP in its classical form might, therefore, not hold. The second section shows how the PPP hypothesis can be adjusted to account any such difference in labour productivity growth rates. In the last section, the literature in the area is briefly reviewed.

### 2.1 The Purchasing Power Parity Hypothesis

“Purchasing power parity (PPP) asserts (in the most common form) that the exchange rate change between two currencies is determined by the change in the two countries’ relative price levels.”<sup>2</sup> The theory is one of the classical concepts in international economics.<sup>3</sup> The idea that exchange rates are related to national price levels has been traced back to the sixteenth century and the School of Salamanca in Spain,<sup>4</sup> but it was not until 1918 that the Swedish economist Gustav Cassel coined the term *purchasing power parity*. In an analysis of the exchange rate changes during World War I, Cassel wrote:

The general inflation which has taken place during the war has lowered this purchasing power in all countries, though in a different degree, and the rates of exchange should accordingly be expected to deviate from their old parities in proportion to the inflation of each country. At every moment the real parity is represented by this quotient between the purchasing power of the money in the one country and the other. I propose to call this parity “purchasing power parity”. As long as anything like free movement of merchandise and a somewhat comprehensive trade between the two countries takes place, the actual rate of exchange cannot deviate very much from this purchasing power parity.<sup>5</sup>

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<sup>2</sup> Dornbusch (1992), p. 236.

<sup>3</sup> For a definition and discussion on PPP, see, for example, Dornbusch (1992), Isard (1995), and Rogoff (1996).

<sup>4</sup> Officer (1982), who cites Grice-Huchinson (1952).

<sup>5</sup> Cassel (1918), p. 423.

Two alternative forms of PPP have evolved over time, *absolute* PPP and *relative* PPP. The absolute PPP hypothesis states that the exchange rate between the currencies of two countries should equal the ratio of the price levels of the two countries, i.e.

$$S = \frac{P}{P^*} \quad (2.1)$$

where  $S$  is the nominal exchange rate measured in units of domestic currency per unit of foreign currency,  $P$  is the domestic price level, and  $P^*$  is the foreign price level. The relative PPP hypothesis, on the other hand, states that the exchange rate should be proportionate to the ratio of the price level, which is expressed as

$$S = k \frac{P}{P^*} \quad (2.2)$$

where  $k$  is a constant parameter. Since information on national price levels normally is available in the form of price indices rather than absolute price levels, absolute PPP may be difficult to test empirically. We will of this reason use relative PPP for the study in this paper, in line with earlier empirical studies.

Finally, the PPP hypothesis does not make any general assertion about the direction of causality between the variables. It only states the relationship. Causality between prices and the exchange rate might very well run in both directions. The exchange rate may respond to a change in the ratio of the national price levels, while an exchange rate depreciation might feed inflation.

## **2.2 Purchasing Power Parity and the Balassa-Samuelson Effect**

As explained by the structural models of inflation, two economies with different productivity growth rates will normally enjoy different inflation rates even if the

exchange rate does not change.<sup>6</sup> In such a case, the classical PPP hypothesis does not hold, but has to be adjusted for the different rates of labour productivity growth.

The structural models divide the economy into two sectors, of which one is producing tradables, exposed to foreign competition, and the other is producing non-tradables, sheltered from foreign competition. We assume output in the two sectors to be defined by Cobb-Douglas production functions,<sup>7</sup>

$$Y_T = A_T L_T^\theta K_T^{1-\theta} \quad (2.3)$$

$$Y_N = A_N L_N^\delta K_N^{1-\delta} \quad (2.4)$$

where  $Y$  is the output,  $L$  is labour,  $K$  is capital,  $A$  represents the effectiveness of labour,<sup>8</sup> and  $\theta$  and  $\delta$  represent the labour intensity of production in each of the sectors. Labour is assumed to be perfectly mobile between the sectors, which implies nominal wage equalisation,

$$W_T = W_N \quad (2.5)$$

The profit margin in the two sectors is assumed to be constant, and workers are paid the value of their marginal product, which is expressed as

$$\frac{\partial Y_j}{\partial L_j} = \frac{W_j}{P_j}; \quad j = T, N \quad (2.6)$$

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<sup>6</sup> See Rowland (2003b) for a discussion of the Scandinavian model of inflation. See also, for example, Lindbeck (1979), and Maynard and van Rijkeghem (1976).

<sup>7</sup> The tradable and non-tradable sectors are indicated by subscripts.

<sup>8</sup> This is sometimes also said to represent knowledge. See Romer (1996) for a further discussion on the Cobb-Douglas production function.

It can easily be shown that the ratio of marginal productivities is proportional to the ratio of average productivities under Cobb-Douglas production technology, i.e.

$$\frac{\partial Y_T / \partial L_T}{\partial Y_N / \partial L_N} = \frac{\theta Y_T / L_T}{\delta Y_N / L_N} \quad (2.7)$$

Inserting (2.5) and (2.6) into (2.7) yields

$$\frac{P_N}{P_T} = \frac{\theta Y_T / L_T}{\delta Y_N / L_N} = \frac{\theta Q_T}{\delta Q_N} \quad (2.8)$$

where labour productivity  $Q$  is defined as output  $Y$  divided by labour  $L$ . Assuming that labour intensity is equal in the two sectors, i.e. that  $\theta$  equals  $\delta$ , and expressing equation (2.8) in logs, we have

$$p_N - p_T = q_T - q_N \quad (2.9)$$

where lower-case letters represent the logarithm of a variable. In line with the structural models, we assume the price level in the economy to be equal to the weighted average of the price levels in the two sectors, that is

$$p = a p_N + (1 - a) p_T \quad (2.10)$$

where  $a$  is the weight of non-tradables in the consumer price index. For the foreign economy this equation will be

$$p^* = a p_N^* + (1 - a) p_T^* \quad (2.11)$$

if we assume that the weight of non-tradables  $a$  is the same as in the domestic economy. In line with the structural models, we, furthermore, assume PPP between prices in the tradable sectors of the two economies, which is stated as

$$s = k + p_T - p_T^* \quad (2.12)$$

Equation (2.12) can together with equation (2.10) and (2.11) be rewritten as

$$s = k + p - p^* - a p_{NT} \quad (2.13)$$

where

$$p_{NT} = (p_N - p_T) - (p_N^* - p_T^*) \quad (2.14)$$

is called the Balassa-Samuelson effect after two seminal papers by Balassa (1964) and Samuelson (1964), which laid the ground for the structural models of inflation. If equation (2.9) is inserted in equation (2.14) the Balassa-Samuelson effect can also be expressed in terms of labour productivity differentials, i.e.

$$p_{NT} = (q_T - q_N) - (q_T^* - q_N^*) \quad (2.15)$$

The advantage of expressing PPP as equation (2.13) rather than (2.12) is that if it is used to forecast the exchange rate, it is normally easier to forecast consumer prices and the Balassa-Samuelson effect than forecasting prices of tradable goods. Forecasting the latter generally boils down to forecasting the exchange rate itself.

### 2.3 Review of the Literature

Relative PPP has been tested in a large number of studies, and empirical evidence strongly confirms that PPP is not a valid hypothesis about the relationship between nominal exchange rates and national price levels in the short term.<sup>9</sup> Empirical testing has, nevertheless, shown that the PPP hypothesis may, even in the short term, have

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<sup>9</sup> See, for example, Isard (1995) for a discussion.



considerable validity during hyperinflations or other periods of very large changes in price levels.<sup>10</sup>

The main theoretical argument against the validity of long-term PPP comes from the structural models of inflation.<sup>11</sup> Balassa (1964) made an important contribution to the development of these set of arguments.

In the long term, PPP has, nevertheless, received considerable empirical support. Flood and Taylor (1996) shows that cross-sectional data yields very high correlations between changes in nominal exchange rates and relative national price levels over 10- or 20-year horizons. A number of studies from the mid 1980s and onwards have also tested if divergence from PPP between national price levels can be explained in terms of the Balassa-Samuelson effect.<sup>12</sup> The literature does, however, not provide a unanimous agreement on how to interpret the evidence. Froot and Rogoff (1995) argue that the Balassa-Samuelson effect may be relevant in the medium term, but that the spreading of knowledge, together with the mobility of physical as well as human capital, generates a tendency toward absolute PPP over the very long run. It is, nevertheless, worth noting that empirical support for long-term PPP is particularly weak for countries, such as Japan and Argentina, where real output has undergone sharp changes relative to real output of the rest of the world.<sup>13</sup>

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<sup>10</sup> See Frenkel (1976).

<sup>11</sup> See Rowland (2003b) for a discussion.

<sup>12</sup> See, for example, Edison and Klovland (1987), and Marston (1987). See also Froot and Rogoff (1995) for an extensive discussion.

<sup>13</sup> Froot and Rogoff (1995).

### **3 Empirical Analysis**

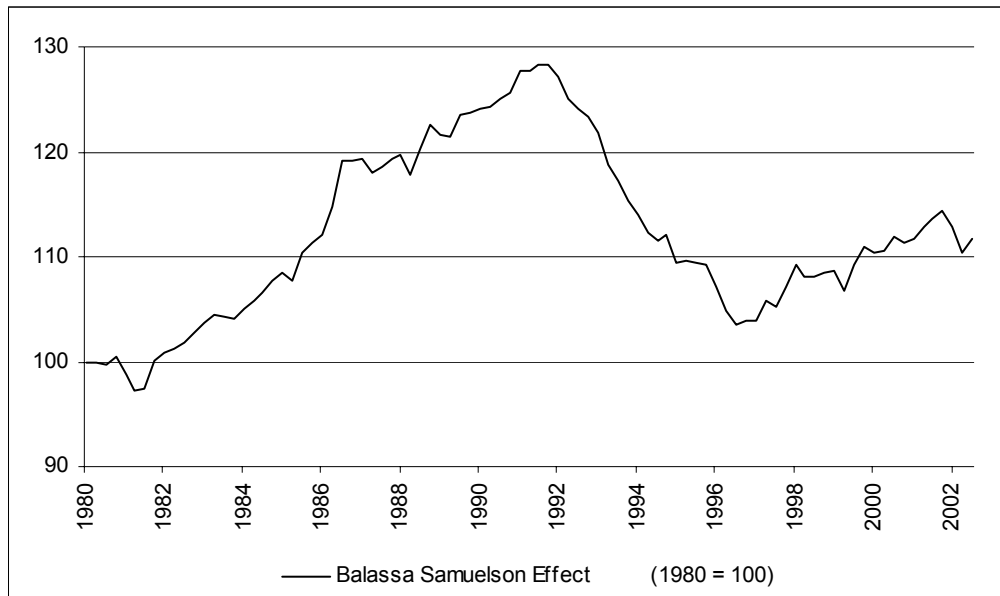
This chapter presents the tests of the long-run PPP relationship between Colombia and its trading partners. The data is discussed in the first section. The second section discusses the different exchange rate regimes in Colombia during the period analysed, and what implication they have on the analysis. The third section introduces the Johansen framework, and in the last section the econometric analysis and the results are presented.

#### **3.1 The Data Set**

To test the PPP hypothesis we use 23 years of quarterly data from the first quarter of 1980 to the third quarter of 2002. For the exchange rate we use a nominal effective exchange rate (NEER) index, which represents an index of the period-average exchange rate of the Colombian peso to a weighted geometric average of exchange rates for the currencies of Colombia's main trading partners. The Balassa-Samuelson effect is derived from equation (2.14) using the consumer price index as a proxy for the non-tradable goods price index and the producer price index as a proxy for the tradable goods price index. The external price indices are calculated as weighted geometric averages of the price indices of the main Colombian trading partners. The source of the data used is Banco de la República.

The Balassa-Samuelson effect is shown in figure 3.1. It might seem strange that this has not increased more, since Colombia is a developing country and should have had a higher growth rate during the period. However, between end-1979 and end-2001 Colombian real gross domestic product grew by 85.7 percent compared to 90.0 percent, which was the growth rate of the United States, Colombia's main trading partner.

**Figure 3.1.** The Balassa-Samuelson effect



*Source:* Banco de la República.

### **3.2 The Different Exchange Rate Regimes in Colombia**

From 1967 and up until 1991, the exchange rate regime in Colombia was defined by a crawling peg. The Colombian peso was pegged to the US dollar at a pre-specified exchange rate and was not allowed to depart significantly from this rate. This exchange rate was, furthermore, devalued daily at a pre-determined and continuous devaluation rate. The exchange rate regime was combined with a system of thorough capital controls, where all foreign exchange transactions had to be made through the Banco de la República.<sup>14</sup>

The crawling peg regime was abolished in June 1991, following a sharp fall in international coffee prices and a deterioration in the trade balance. A market for foreign

<sup>14</sup> For a thorough discussion on the Colombian exchange rate regimes, see Villar and Rincón (2000), as well as Cárdenas (1997). The discussion here draws heavily from Villar and Rincón (2000), as well as from Rowland (2003a).

exchange was created, where the exchange rate was freely determined.<sup>15</sup> However, the Banco de la República continued to intervene in the market, and in practice the new exchange rate regime was a managed floating regime with many similarities to a crawling exchange rate band.

In January 1994, the central bank introduced an official crawling band regime. This was to regain control over monetary variables, after a period of very low real interest rates in combination with very large capital inflows. The exchange rate was allowed to fluctuate around a pre-determined central rate, which initially was to be continuously devalued at an annual rate of 11 percent. The actual exchange rate could depart with as much as 7 percent from the central rate. As shown by figure 3.2, the regime, in fact, very much resembled a managed float, since the limits of the band were shifted several times, and since the band was relatively wide. “In this sense, the currency band was not supposed to create obstacles in the process of adjustment of the exchange rate but to guarantee a more orderly and gradual adjustment when such a process was grounded in fundamental macroeconomic changes”.<sup>16</sup>

In September 1999, the exchange rate band was dismantled, and the exchange rate was allowed to float freely. This followed a period of economic difficulties. Colombia was in a recession, the government was running a large fiscal deficit, and the credibility of the currency band system had rapidly been deteriorating. The floating regime, which has been in place since then, is close to a free float. The central bank can only intervene to reduce short-term exchange rate volatility, and has not done so until earlier this year.<sup>17</sup>

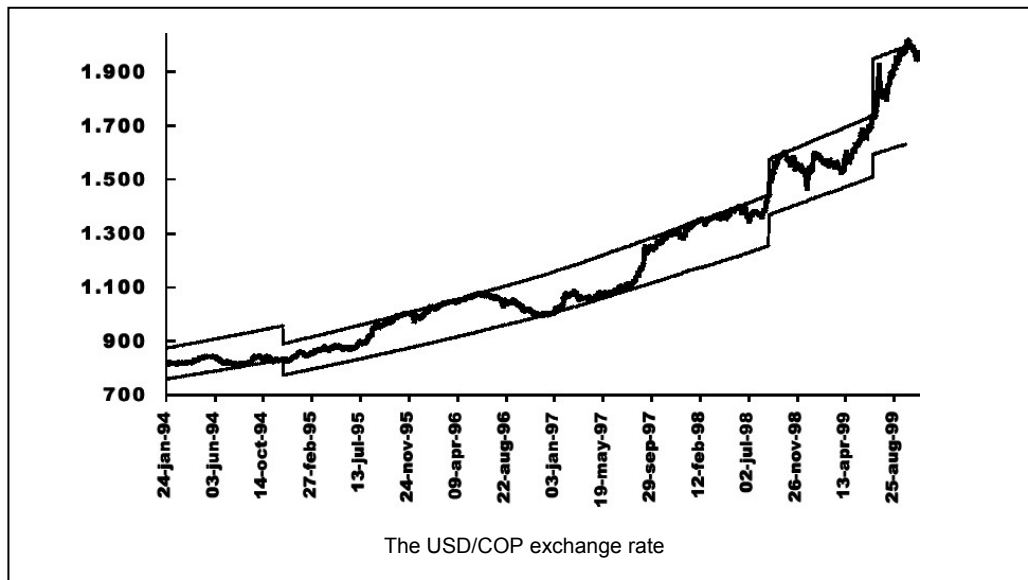
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<sup>15</sup> The market traded Exchange Rate Certificates (Certificados de Cambio) which were US dollar denominated interest bearing papers issued by the Banco de la República. See Villar and Rincón (2000), pp 27ff.

<sup>16</sup> Villar and Rincón (2000), p. 30.

<sup>17</sup> The central bank can only intervene if the average exchange rate of a given day deviates more than 4 percent from its 20-day moving average.

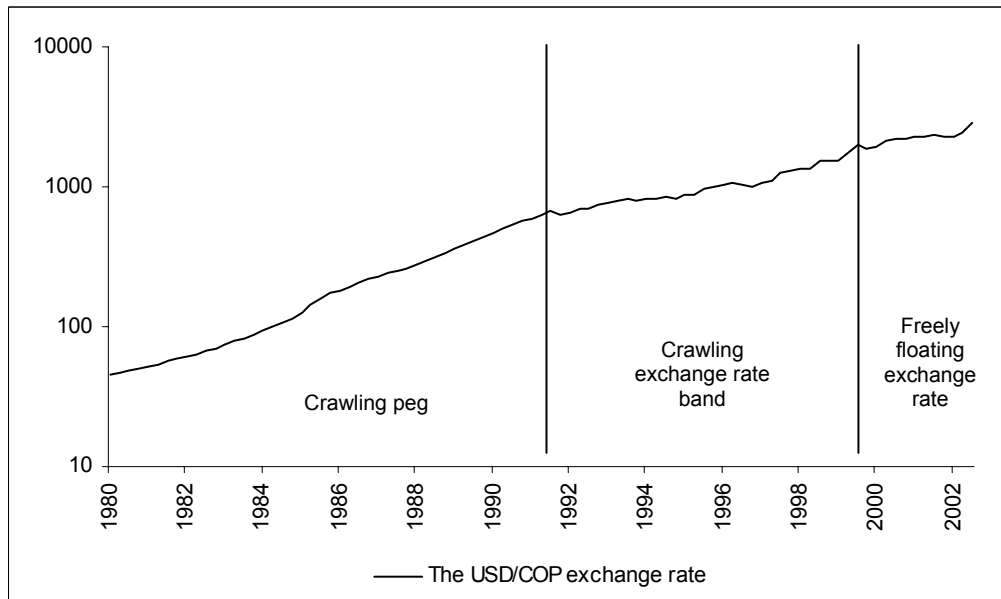
**Figure 3.2.** The Colombian exchange rate band 1994 – 1999



*Source:* Villar and Rincón (2000), p. 31.

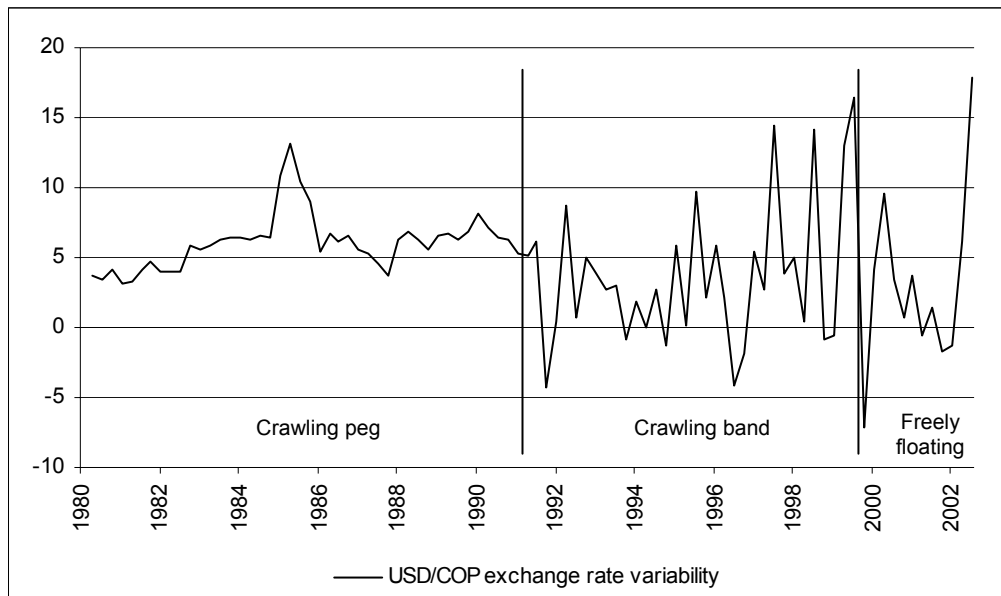
Figure 3.3 shows the exchange rate development since 1970, and figure 3.4 shows the exchange rate variability. It is apparent from figure 3.3 that the exchange rate left its path of a long-term stable depreciation rate in 1991, when the crawling peg was abandoned. As expected, the short-term variability of the exchange rate also increased significantly, as shown by figure 3.4. However, there was no significant change in exchange rate variability between the crawling band regime and the floating regime, which was introduced in 1999. If we calculate the average absolute weekly change for the periods January 1994 to September 1999 and October 1999 to August 2002 we receive values of 0.72 percent and 0.68 percent respectively.

**Figure 3.3.** The USD/COP exchange rate under the different regimes (logarithmic scale)



Source: Banco de la República.

**Figure 3.4.** Short-term variability of the USD/COP exchange rate, expressed as percentage change from previous quarter



Source: Banco de la República.

In our analysis we are using dummy variables to represent the different exchange rate regimes. We are using one dummy variable to represent the exchange rate band regime, running from the third quarter 1991 to the third quarter 1999, and another dummy variable to represent the floating rate regime from the fourth quarter 1999 and onwards.<sup>18</sup>

### 3.3 The Johansen Framework of Multivariate Cointegration

The statistical framework that we use for the analysis of multivariate cointegration is the one originally developed by Johansen (1988).<sup>19</sup> This estimation test procedure, which by now is well known, is a method for estimating the cointegrating relationships that exist between a set of variables as well as testing these relationships. The application of this framework on the PPP relationship with the Balassa-Samuelson effect, as stated by equation (2.13), can very briefly be introduced as follows. First, a vector autoregressive model with a maximum distributed lag length of  $k$  is defined,

$$\mathbf{X}_t = \mathbf{r}_1 \mathbf{X}_{t-1} + \dots + \mathbf{r}_k \mathbf{X}_{t-k} + \mathbf{e}_t, \quad t = 1, \dots, T \quad (3.1)$$

where  $\mathbf{X}_t = (s_t, p_t, p_t^*, p_{NT,t})^T$ ,  $\mathbf{r}_i$  are 4x4 coefficient matrices and  $\mathbf{e}_t$  is a 4x1 vector of independent and identically distributed error terms.<sup>20</sup> The distributed lag length  $k$  should be specified long enough for the residuals not to be serially correlated. The cointegrating matrix  $\mathbf{r}$ , which defines the long-term solution of the system, is defined as

$$\mathbf{r} = -\mathbf{I} + \mathbf{r}_1 + \dots + \mathbf{r}_k \quad (3.2)$$

where  $\mathbf{I}$  is the 4x4 identity matrix. The Johansen procedure now continues with decomposing the matrix  $\mathbf{r}$  into two  $N \times r$  matrices  $\boldsymbol{\alpha}$  and  $\boldsymbol{\beta}$ ,

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<sup>18</sup> Since the change from the floating band regime to the floating regime did not seem to represent a significant structural break, we could, alternatively, have chosen to use one dummy variable to represent the whole period from the third quarter 1991 and onwards.

<sup>19</sup> See also Johansen (1990, 1991, 1995).

<sup>20</sup> A 'T' in superscript behind a vector or matrix indicates that this vector or matrix is to be transposed.

$$r = \alpha \beta^T \quad (3.3)$$

The rows of the matrix  $\beta$  now define the cointegrating relationships among the four variables in the vector  $X$ , and the rows of the matrix  $\alpha$  show how these cointegrating vectors are loaded into each equation in the system. Johansen, furthermore suggests a maximum likelihood estimation procedure to estimate the two matrices  $\alpha$  and  $\beta$  together with test procedures to test the number of distinct cointegrating vectors. Linear parameter restrictions on the data can, furthermore, be tested by testing the matrix  $\beta$ , and the direction of causality within the system can be tested by testing the matrix  $\alpha$ .

### 3.4 Likelihood Estimation and Results

In this section we present the results of the Johansen cointegration test procedure, used to test the PPP hypothesis. As explained in the previous section, we have a set of variables in logarithms,  $(s, p, p^*, p_{NT})$ , where  $s$  stands for the nominal effective exchange rate,  $p$  and  $p^*$  are the domestic and external consumer price indices, and  $p_{NT}$  is the Balassa-Samuelson effect.

The system is tested using the following scheme: Tests for exclusion, stationarity and weak-exogeneity are used jointly with tests associated with the behaviour of the residuals of the model at different distributed lag lengths. The maximum lag length is chosen using an information criteria,<sup>21</sup> the levels and the signs of the parameters of the cointegrating vector together with the performance of the model in terms of the normality assumption.

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<sup>21</sup> In this case we use the Akaike information criteria.



**Table 3.1.** Testing for exclusion, stationarity and weak exogeneity  
(using quarterly data from Q1 1980 to Q3 2002)

Test	Critical Value	$s$	$p$	$p^*$	$p_{NT}$
<i>Testing under 1 cointegrating vector</i>					
Exclusion	3.84	6.14	13.35	12.85	5.03
Stationarity	7.81	28.07	27.97	29.39	19.52
Weak exogeneity	3.84	2.50	10.47	1.07	4.60

**Table 3.2.** Estimation of the model  
(using quarterly data from Q1 1980 to Q3 2002)

<i>Model</i>	VAR(5): Drift
<i>Variables</i>	$(s, p, p^*, p_{NT})$
<i>Restriction test</i> $(\beta_{12} = -\beta_{13}; \alpha_{13} = 0)$	$\chi^2(2) = 1.15$ P-value: 0.56
<i>Cointegrating vector</i> $\beta' = (\beta_{11} \ \beta_{12} \ \beta_{13} \ \beta_{14})$	$\beta' = (1.000 \ -1.809 \ 1.809 \ -1.830)$
<i>Speed of adjustment</i> $\alpha' = (\alpha_{11} \ \alpha_{12} \ \alpha_{13} \ \alpha_{14})$	$\alpha' = (-0.11 \ 0.09 \ 0.00 \ -0.05)$

**Table 3.3.** Residual tests  
(using quarterly data from Q1 1980 to Q3 2002)

Test	Test Statistic	P-value
<i>Multivariate Normality</i>		
Lütkepohl test	$\chi^2(6) = 12.5$	0.13
<i>Autocorrelation</i>		
Portmanteau test	Portmanteau(12) = 113.9	0.10
LM test	LM(1) = 13.4	0.65

Table 3.1 presents the initial diagnostic tests under one cointegrating vector. All the variables of the system are non-stationary, and none of them are excluded from the cointegrating vector. As suggested by economic theory, evidence of weak-exogeneity is most likely to be found in the external consumer price index. Table 3.2 reports the results of the Johansen estimation of the model for the sample of the analysis, and table 3.3 shows the residual tests.

We can draw several conclusions from the results presented in table 3.2. The cointegrating vector shows that a valid purchasing power relationship does, indeed, exist. However, the parameter estimates of the cointegrating vector are relatively far from the values of (1, -1, 1,  $a$ ) suggested by the PPP hypothesis expressed by equation (2.13). Note that  $a$  is the weight of non-tradables in the consumer price index, and should consequently be somewhere between zero and one. The parameter estimates for domestic and external prices, -1.809 and 1.809, are relatively far from their parity values of minus one and one. The homogeneity restrictions of minus one and one are, furthermore, rejected by a high chi square test statistic,  $\chi^2(3)$ , of 23.39. The parameter estimate of the Balassa-Samuelson effect is not only out of its range of between zero and one, but also of the wrong sign. This might, nevertheless, be explained by the fact that the United States, which is Colombia's main trading partner, grew faster than Colombia during the period

analysed, while the Balassa-Samuelson effect, as stated by equation (2.14), increased by 11.6 percent.<sup>22</sup>

The restrictions imposed on the model, i.e. that the external consumer price index is exogenous and that the parameter estimates for domestic and external prices are of the same value but of opposite sign, are shown to be valid by a low chi square test statistic,  $\chi^2(2)$ , of 1.15.

The estimate of the vector  $\alpha$ , furthermore, yields that the disequilibrium level of the exchange rate is adjusted by 11 percent in one quarter. This is a considerably higher speed of adjustment than consensus estimates of roughly 15 percent per year, suggested by other studies.<sup>23</sup>

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<sup>22</sup> It might also be explained by the fact that we are using a measure of the Balassa-Samuelson effect based on the differentials between consumer prices and producer prices as a proxy for the differentials between the prices of non-traded and traded goods. Using another measure might have produced a better result.

<sup>23</sup> Rogoff (1996), p. 647.

## 4 Using the Model for Exchange Rate Forecasting

To test the forecasting power of the model, we compare forecasts of the model with those of a random walk, in line with Meese and Rogoff (1983). The model is shown to outperform a random walk on the 12 and 24-months forecast horizons, but not on the 3 and 6-months horizons. Section 4.1 presents the methodology for evaluating the forecasting power of the model, and in section 4.2 the actual results are presented

### 4.1 The Methodology for Evaluation of Out-of-Sample Forecasts

The data set used is quarterly data from the first quarter 1980 to the third quarter 2002, as discussed earlier. The model is initially estimated using data from the first quarter 1980 until the last quarter 1996. Forecasts are then generated at horizons of 3, 6, 12 and 24 months.<sup>24</sup> The data for the first quarter 1997 is then added to the sample, and the model is re-estimated, and new forecasts generated. These rolling re-estimations are continued until the third quarter 2002 is included in the sample.

We have chosen to begin our forecast period in the first quarter 1997 to give the model enough time to adjust to the crawling band that was introduced in June 1991. As shown earlier by figure 3.4, this change of exchange rate regime implied a considerable structural break in the time series data of the nominal USD/COP rate of exchange, which, since the United States is Colombia's largest trading partner, weighs heavy in the nominal effective exchange rate index. The following changes of exchange rate regime in January 1994 and in September 1999 did however not produce such large structural breaks.

In the model, only the external price level  $p^*$  is exogenous, and the model does, therefore, neither allow us to use our own forecasts nor, as in Meese and Rogoff (1983), actual data for the other variables when forecasting the exchange rate. We are, consequently, letting

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<sup>24</sup> This corresponds to 1, 2, 4 and 8 quarters using the quarterly data.

the model forecast all the variables. Since the external price level is, indeed, exogenous, we could have chosen to feed the model with actual data of this price level to forecast the other variables. However, we decided not to give the model this advantage, but instead to let it forecast the whole set of variables.

During every forecast iteration, we impose the restriction that the parameter estimates of domestic and external price levels are the same but of opposite sign.<sup>25</sup> The restriction that the external price level is exogenous is also imposed.<sup>26</sup> These two restrictions pass the test of validity in each of the forecast iterations.

The out-of-sample accuracy of the forecasts is measured by two statistics, root mean square error (RMSE) and mean absolute error (MAE). These are defined as follows:

$$RMSE = \left( \sum_{s=0}^{N_k-1} \frac{(F(t+s+k) - A(t+s+k))^2}{N_k} \right)^{1/2} \quad (4.1)$$

$$MAE = \sum_{s=0}^{N_k-1} \frac{|F(t+s+k) - A(t+s+k)|}{N_k} \quad (4.2)$$

where  $k = 1, 2, 4, 8$  denotes the forecast steps in quarters,  $N_k$  the total number of forecast iterations in the projection period for which the actual value of the exchange rate  $A(t)$  is known, and  $F(t)$  the forecasted value of the exchange rate. The forecast starts in period  $t$ .

## 4.2 The Results

Table 4.1 shows the statistics of the root mean square error and the mean absolute error for the forecasts of the model as well as for the spot rate (the forecast of a random walk model) for the different forecast horizons.

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<sup>25</sup> That is  $\beta_{12} = -\beta_{13}$

<sup>26</sup> I.e.  $\alpha_{13} = 0$

**Table 4.1.** Statistics for the forecast errors

Horizon	Model Forecasts		Random Walk Forecasts	
	RMSE	MAE	RMSE	MAE
3 months	5.15	4.46	5.24	3.91
6 months	8.23	7.39	7.84	6.17
12 months	12.16	9.88	12.54	11.09
24 months	22.43	17.57	23.08	20.44

*Note:* All values are in approximate percentage terms (the difference between two logarithmic values is approximately the same as the relative difference between the variables).

It is apparent from the values in the table that the model outperforms a random walk on the 12 and 24-month forecasting horizon. For the 3-month horizon the model outperforms the random walk according to the RMSE statistic but not according to the MAE statistic.

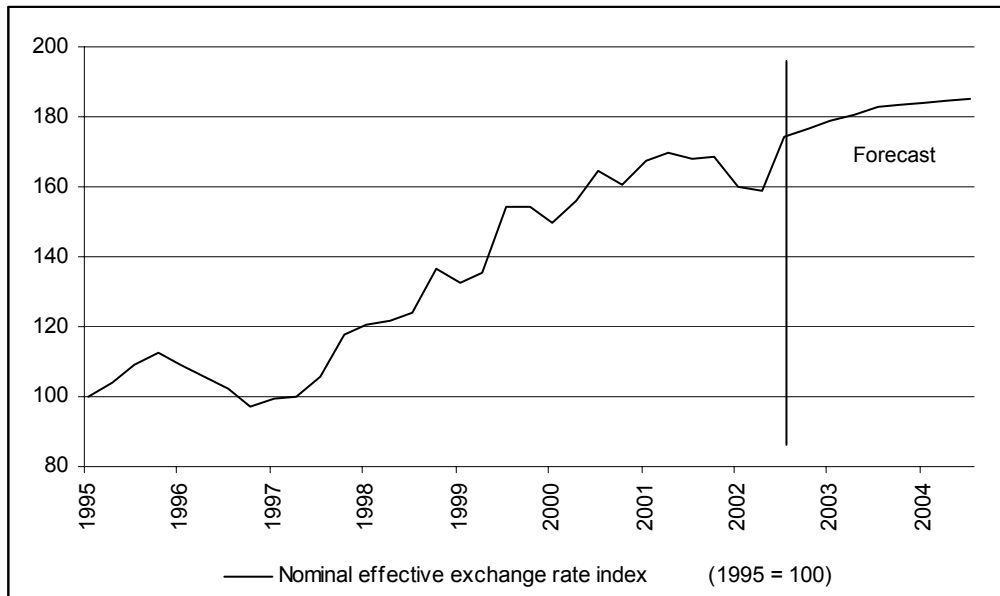
To forecast exchange rates in the short term using fundamentals should also be more difficult than to forecast in the medium and long term. A number of studies have shown that, due to incomplete information in the short term, the behaviour of foreign exchange market participants is to a large extent based on *technical analysis* of short-term trends or other patterns in the observed behaviour of the exchange rate.<sup>27</sup> In support of such behaviour, simulations have shown short-term trading strategies based on technical analysis to generate significant profits.<sup>28</sup> The long-term behaviour of exchange rates is, on the other hand, much more governed by fundamentals. This also implies that short-term exchange rates will vary much more widely than is justified by changes in fundamentals.<sup>29</sup>

<sup>27</sup> See, for example, Taylor and Allen (1992).

<sup>28</sup> See Cumby and Modest (1987), Dooley and Shafer (1983), and Sweeney (1986).

<sup>29</sup> Models have been developed where *feedback traders* coexist with *fundamentalists* as market participants. The former base their trading strategies on the recent history of exchange rates, while the latter base their strategies on analysis of economic fundamentals. In these types of models, the fundamentalists have the predominant influence of exchange rates in the long term. However, risk aversion together with substantial uncertainties regarding news and new information, leads to feedback traders dominating the market in the short term. See Lyons (1993), Kyle (1985), Frankel and Froot (1990), and Cutler, Poterba and Summers (1990).

**Figure 4.1.** The nominal effective exchange rate index of Colombia



*Source:* Banco de la República, and own forecast

If we use the model to forecast the nominal effective exchange rate index of Colombia for the next 12 and 24 months starting from end of September 2002 we receive the results presented in figure 4.1. From September 2002 to September 2003 the model predicts a depreciation of the nominal effective exchange rate of 4.8 percent, and from September 2002 to September 2004 a depreciation of 6.0 percent.<sup>30</sup>

<sup>30</sup> We define the depreciation as the percentage change of the inverted nominal effective exchange rate index. We can alternatively say that the nominal effective exchange rate index will increase by 5.1 and 6.1 percent from September 2002 to September 2003 and September 2004 respectively.

## 5 Conclusion

In this study we have examined the purchasing power parity hypothesis for Colombia, using the nominal effective exchange rate together with domestic and external prices in a model that takes into account the Balassa-Samuelson effect. A number of interesting findings were reported. In particular, we demonstrated that a purchasing power relationship existed during the period, even if this was not the parity relationship stipulated by the PPP hypothesis. In addition, we demonstrated that the estimated model has some forecasting power, outperforming a random walk on both the 12- and 24-month forecasting horizon.

However, the research also concluded that the relationship found was relatively far away from the parity relationship stated by the PPP hypothesis. This can possibly be explained by the relatively short period of data. The analysis was done using quarterly data for the past 23 years, which when investigating long-run cointegrating relationships, is a relatively short time indeed.<sup>31</sup>

Overall, the study suggests that the PPP hypothesis, with allowance made for complex short-run dynamics, might be usefully applied. Particularly, the out-of-sample forecasting performance of the model demands further research.

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<sup>31</sup> See Juselius (1999) for a discussion on the length of the time span of the data set, and long-term cointegrating relationships.



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