An Estimation of the Nonlinear Phillips Curve in Colombia

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Abstract

As originally drawn and estimated by professor Phillips, the Phillips curve is a curve indeed, not a straight line as often thought. Following Laxton, et. al. (1999) we estimate a convex Phillips curve and model the NAIRU as a variable that is unobserved. Using Colombian data, we provide confidence bands for the NAIRU and report estimated sacrifice ratios. Using the unobserved components methodology along with the Kalman filter, we find evidence in favor of a nonlinear Phillips curve and no evidence against a NAIRU that is constant. This latter finding is explained by the high level of uncertainty in the estimation of the NAIRU. Nonlinearity implies that the sacrifice ratio increases with unemployment, in other words, the cost of decreasing inflation is higher the higher the unemployment rate.

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1 Introduction

As originally drawn and estimated by professor Phillips, the Phillips curve is a curve, not a straight line. If the Phillips curve were linear, the cost of decreasing inflation would be the same in a recession or in a boom. It would not mater whether the economy was in a profound recession or in the midst of a boom, employment losses would decrease inflation by exactly the same amount. As the Phillips curve is convex, the cost of disinflation is not a constant. When the economy is in a recession, further increases in unemployment do not "produce" much disinflation. When the economy is overheated an increase in unemployment produces faster disinflation.

Non linearity in the Phillips curve has an important policy implication for Colombia: when the unemployment rate is high, the sacrifice ratio is also high.

Even though the original work of Phillips was based on a non linear specification for the curve, Laxton (1999) finds that there are at least two reasons why it has been difficult to identify nonlinearity in more recent specifications. The first reason is that the NAIRU is generally estimated according to a linear specification of the Phillips Curve, the second one is that an economic authority that successfully smooths out the economic cycle provides sample information lying on the center of the Phillips curve. It leaves both ends of the curve empty. The ends are required to identify non linearity.

In this paper we estimate a convex Phillips curve for Colombia in which the NAIRU is modeled as an unobserved component. Our sample spans over the nineties in order to take advantage of the more recent sample data that lies on the right end of the Phillips curve, that is, higher unemployment and lower inflation than historically observed. We provide an estimate of the NAIRU that is consistent with the specified non linearity, present the confidence bands for the estimated NAIRU as an indicator of its estimation uncertainty, and report the implied sacrifice ratios.

The paper has seven sections including this introduction. In section two we summarize the literature on the estimation of the Phillips curve in Colombia. In section three and four we present the model. In section four we give an intuitive interpretation of the Phillips curve in Colombia and the role of supply shocks, expectations and the pass-through. In sections five and six we present the data and the estimation results. In section eight we conclude.

y_t	$=$	3.66	$+$	0.01	t	$+$	0.15	Δx_t	$+$	0.70	y_{t-1}	$+$	ε_t
		(3.43)		(3.11)			(2.18)			(8.20)			
R^2	$=$	0.99											

Table 1: Review of the Literature on the Phillips Curve in Colombia: Birchenall (1999)

Dependet Variable: Inflation gap.	
Lagged inflation gap $(t-1)$	0.821
	(0.118)
Lagged inflation gap $(t-2)$	-0.414
	(0.150)
Lagged inflation gap $(t-3)$	0.259
	(0.152)
Lagged inflation gap $(t-4)$	-0.476
	(0.111)
Output gap $(t-8)$	0.343
	(0.160)

Table 2: Review of the Literature on the Phillips Curve in Colombia: Julio and Gómez (1999)

2 Previous Estimations of the Phillips Curve in Colombia

Birchenall (1999) estimates the Phillips curve that we present in Table 1, where y_t is log output, and Δx_t is the change in log nominal income.

Following the model of Lucas (1972) and using the Kalman filter, Birchenall estimates a time varying slope for the Phillips curve where the coefficient on nominal income decreases from 0.25 at the beginning of the seventies to 0.1 in the late eighties. As reported in Table 1, the average slope of the Phillips curve is 0.15.

Julio and Gómez (1999) estimate the Phillips curve that we here present in Table 2. Following Smets (1998), they estimate the output gap and the Phillips curve with the Kalman Filter. As their Phillips curve is linear, their implied sacrifice ratio is a constant: $1/0.343 = 2.9$. They provide a confidence interval for the output gap, but their sample does not cover the important events of the 1999 recession.

Dependent Variable: Inflation (t)	
Dummy (t)	0.025
	(4.34)
Dummy ($t-1$)	0.013
	(2.78)
Dummy ($t-2$)	-0.008
	(-1.86)
Output Gap ($t-1$)	0.189
	(2.19)
Inflation of imports ($t-4$)	0.212
	(2.42)
Lagged inflation ($t-4$)	0.324
	(2.89)
Dummy 1986	-0.070
	(-5.32)
Probability regime switch	0.033
	(4.46)
Probability regime switch	0.023
	(3.52)

Table 3: Review of the Literature on the Phillips Curve in Colombia: Misas and López (1999)

Misas and López (1999) estimate the Phillips curve presented in Table 3. They follow Fillion and Léonard (1997) in introducing a new variable in the determination of inflation, the probability of a switch to a regime of lower and more stable inflation. The switch probably took place by 1990. They estimate the output gap with a structural VAR, they estimate the probability if a switch in regime with Hamilton's switching procedure and the Phillips curve with OLS. The implied sacrifice ratio of their Phillips curve is $1/0.189 = 5.3$.

Uribe, Gómez, and Vargas (1999) estimate the Phillips curve in Table 4 where π_t^N is nontraded goods' inflation gap, y_t the output inflation gap, and q_t real exchange rate. They follow Svensson (1998). With quarterly data, output is significant in their Phillips curve with one, two and three lags. The cumulative effect of the gap on inflation is 7/10 and the implied sacrifice ratio is 10/7. They estimated the output gap with the Hodrick Prescott filter and the Phillips curve with GMM-IV. The implied sacrifice ratio of their Phillips curve is $1/(0.243 + 0.238 + 0.214) = 1.4$.

$\pi_t^{N_4}$	$=$	α_π^1	$\pi_{t-1}^{N_4}$	$+$	α_π^2	$\pi_{t-4}^{N_4}$	$+$	α_y^1	y_{t-1}
		0.602			-0.487			0.243	
		(8.657)			(1.802)			(1.802)	
	$+$	α_y^2	y_{t-2}	$+$	α_y^3	y_{t-3}	$+$	α_q^1	q_{t-4}
		0.238			0.214			0.046	
		(2.545)			(2.518)			(0.791)	
	$+$	α_q^2	q_{t-8}	$+$	c	$+$	ε_t^π		
		0.214			-0.103				
		(1.406)			-0.449				
Sample	1990:1	1999:2							
R^2	$=$	0.664							
Q	$=$	8.530	Signif	0.482					

Table 4: Review of the Literature on the Phillips Curve in Colombia: Uribe, Gómez, and Vargas (1999)

All the currently estimated Phillips curves in Colombia are linear except for the one of Birchenall whose Phillips curve has a time varying slope. In this paper we estimate a nonlinear Phillips curve.

3 The Model

Following the lines of Laxton, Rose and Tambakis (1998) we study the following convex Phillips curve:

$$\pi_t = \pi_t^c + \gamma \left(\frac{u_t^* - u_t}{u_t} \right) + \varepsilon_t^\pi \tag{1}$$

$$\pi_t^c = -\gamma + \widehat{\theta}_1 \pi_{t-1} + \widehat{\theta}_2 \pi_{t-2} + \widehat{\delta}_0 s_t + \widehat{\delta} 1 s_{t-1} + \widehat{\delta}_2 s_{t-2} + \eta \pi_t^M \tag{2}$$

$$u_{t+1}^* = u_t^* + \varepsilon_t^u \tag{3}$$

where π_t is CPI inflation, π_t^c is core inflation or a measure of expected inflation, u_t^* is the unobserved deterministic NAIRU that by Eq. (3), follows a random walk, u_t is the unemployment rate, s_t is an indicator of supply shocks measured as suggested by King and Watson (1994, footnote 18), and π_t^M is the inflation of imports. The variance of ε_t^π is σ_π^2, the variance of ε_t^u is σ_u^2, and the two residual terms are independent.

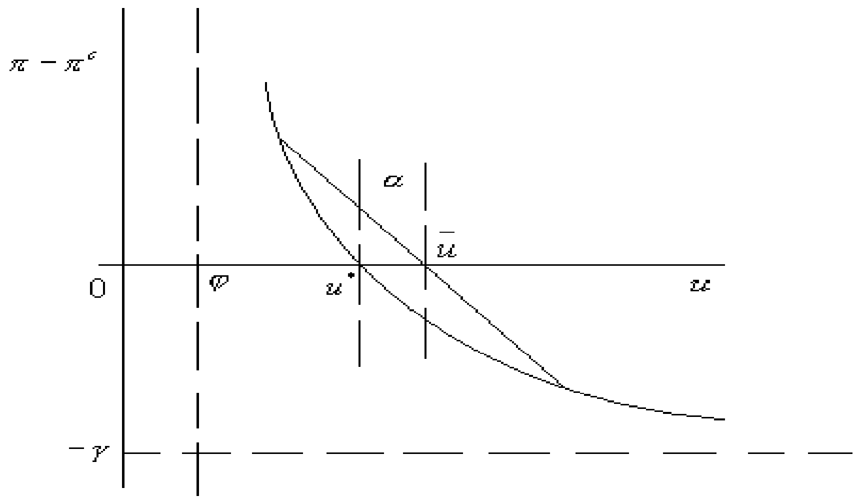

Figure 1: The Convex Phillips Curve

The restriction $\theta_1 + \theta_2 = 1$ is the natural rate hypothesis. If this restriction holds, when unemployment is at the NAIRU inflation is constant, when unemployment is above the NAIRU inflation is decreasing, and when unemployment is below the NAIRU inflation is increasing.

The restriction $\delta_0 + \delta_1 + \delta_2 = 0$ implies that supply shocks have no long run effect on inflation.

Figure 1 depicts the convex Phillips curve. Convexity implies that decreasing inflation has an increasing cost in terms of increases in unemployment, that is, the sacrifice ratio is increasing.

In Figure 1, u^* is the NAIRU in the absence of stochastic shocks, and $\bar{u}$ is the expected NAIRU when there are stochastic shocks. Thus, the expected NAIRU $\bar{u} = u^* + \alpha$ where $\alpha > 0$ is an increasing function of both the convexity of the Phillips curve, and the variance of the deviation of inflation from the core. In other words, the deterministic NAIRU, u^*, is the unemployment rate at which $\pi = \pi^c$ in the absence of stochastic shocks. If there are shocks and unemployment is in the deterministic NAIRU, u^*, inflation is increasing, hence, the NAIRU must be $\bar{u}$. To illustrate the difference between $\bar{u}$ and u^*, if $\pi - \pi^c$ is uniformly distributed between -1 and 1, the expected NAIRU is $\bar{u} = \frac{u_1+u_2}{2} < u^*$.

Eq. (1) may be written as

$$\pi_t - \pi_t^c = \gamma \left(\frac{u_t^* - u_t}{u_t} \right) + \varepsilon_t^\pi$$

spliting the terms in parenthesis, we get

$$\pi_t - \pi_t^c = -\gamma + a_t z_t + \varepsilon_t^\pi \tag{4}$$

where $z_t = \frac{1}{u_t}$, $u_t^* = \frac{a_t}{\gamma}$.

Using Eqs. (4), (2) and (3) we can write the model in a state space representation with transition equation

$$a_{t+1} = a_t + \varepsilon_t^a \tag{5}$$

and state equation

$$\pi_t = z_t a_t + \begin{bmatrix} \theta_1 & \theta_2 & \delta_0 & \delta_1 & \delta_2 & \eta & -\gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi_t \\ s_t \\ s_{t-1} \\ s_{t-2} \\ \pi_t^M \\ 1 \end{bmatrix} + \varepsilon_t^\pi \tag{6}$$

where $\varepsilon_t^\alpha = \gamma \varepsilon_t^u$.

The transition equation is clearly nonstationary and the state equation presents a time varying coefficient, z_t. If the variance σ_a^2 is zero, u^* is a constant. In this case the model may be estimated by OLS[1]:

$$\pi_t = -\gamma + a z_t + \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \delta_0 s_t + \delta_1 s_t + \delta_2 s_t + \eta \pi_t^M + \varepsilon_t^\pi$$

where $u^* = \frac{\alpha}{\gamma}$.

4 On the Phillips Curve in Colombia

Figure 2 presents data on inflation and unemployment in Colomba for the nineties. Although the relationship between inflation and unemployment is inverse and appears convex, inflation data has not yet been adjusted for expectations nor supply shocks.

4.1 The Role of Supply Shocks

Since supply shocks may change inflation without changing unemployment, we introduce a measure of supply shocks in the estimation of the Phillips curve. Our indicator is defined as

$$s_t = 100 * (\log P_l^A - \log P_{t-4}^A) - 100 * (\log P_l - \log P_{t-4})$$

where P_t is the CPI, and P_t^A is the price of food.

Figure 3 presents our measure of supply shocks. There are positive supply shocks March 1991, September 1992, and June 1998, and negative shocks in September 1993, June 1996, and June-September 1999.

[1]See for instance Staiger, Stock, and Watson (1995).

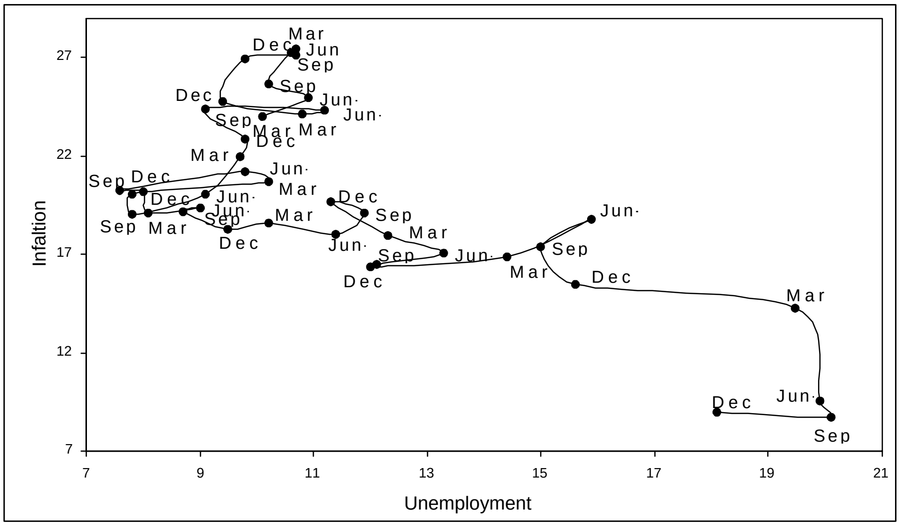

Figure 2: The Phillips Curve in Colombia 1990:1-1999:4.

The supply shocks of Figure 3 correspond to outliers in Figure 2. The adjustment of Figure 2 for supply shocks appears to improve the negative and convex relationship between inflation and unemployment.

4.2 The Role of Expectations

Data on forward looking inflation expectations is not available in Colombia now. Hence, our measure of inflation expectations is adaptive; a backward looking function of inflation:

$$\pi_t^e = \widehat{\theta}_1 \pi_{t-1} + \widehat{\theta}_2 \pi_{t-2}$$

A permanent increase in inflation shifts the Phillips curve upwards. The permanent increase in inflation that took place in Colombia by 1972 is a shift of this kind. When inflation permanently increased in Colombia by 1972 output grew strongly. When inflation decreased to a single digit in 1999 the recession was the biggest in more that 50 years. The convexity of the Phillips curve has an important policy implication. Stimulating the economy has a large cost in terms of inflation. Once the Phillips curve has shifted upwards because of the increase in inflation expectations, stabilizing inflation has a cost in terms of unemployment that is higher than the former gains in employment.

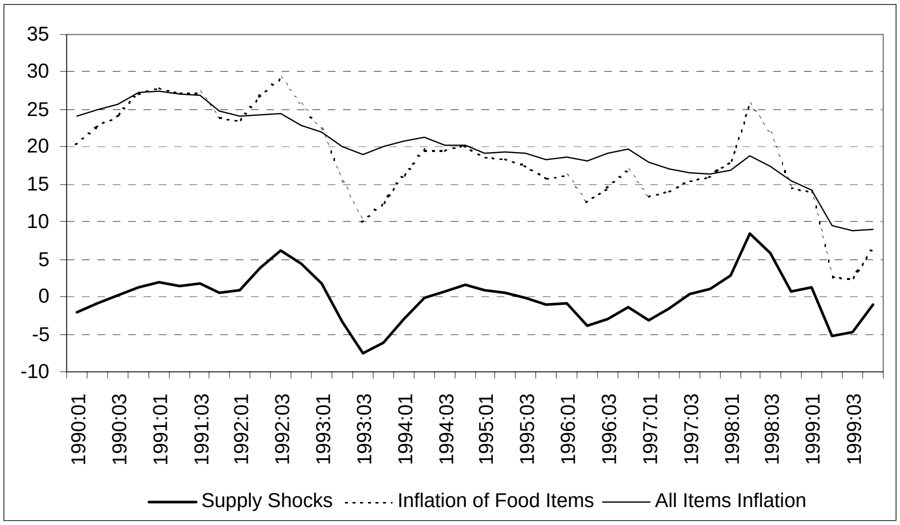

Figure 3: Inflation and Supply Shocks

5 The Data

Data are quarterly running from 1990:1 to 1999:4. Inflation is measured as $100 * (\log P_t - \log P_{t-4})$ where P_t is the CPI. Supply shocks are measured as described above. Unemployment figures correspond to urban unemployment in the main 7 cities.

6 Results

We report results obtained with two methodologies, the Kalman filter and OLS. Table 5 shows the estimation results with the Kalman filter. Figure 4 shows the deterministic NAIRU or u^* along with a one standard deviation confidence band. The confidence interval is approximately 6.0% to 11.0% and the middle point is an unemployment rate of about 8.5%.

An important result of Table 5 is that the variance of a, σ_a^2, is zero. As a is a multiple of u^*, the result that $\sigma_a^2 = 0$ implies that u^* is not statistically different from a constant. This enable us to estimate the model along with the NAIRU, u^*, by OLS.

Table 6 shows the estimation results with OLS. All the estimates are significant. The estimated deterministic NAIRU, u^*, is 8.4%. The OLS point estimate of u^* is close to the estimate provided by the Kalman Filter. Figure 6 shows the unemployment rate along with u^* calculated by OLS.

Table 7 and Figure 7 present the four quarter ahead inflation forecast. As in the Phillips curve, the relationship between the inflation forecast and

Parameter	Estimate	Standard Error	T-Statistic
γ	1.296	0.249	5.189
θ_1	1.305	0.126	10.298
θ_2	−0.305	0.127	−2.393
δ_0	0.384	0.039	9.624
δ_1	−0.544	0.074	−7.346
δ_2	0.159	0.068	2.329
η	0.057	0.016	3.536
σ_π^2	0.260	0.060	4.282
σ_a^2	0.000	0.001	–
Normality T* Statistic		4.396	
Normality P-Value		0.111	
Ljung-Box Statistic(9)		5.491	
Ljung-Box P-Value		0.482	

Table 5: Estimation Results with the Kalman Filter

Parameter	Estimate	Standard Error	T-Statistic
γ	−1.314	0.441	−2.983
θ_1	1.303	0.133	9.768
θ_2	−0.303	0.133	−2.275
δ_0	0.384	0.036	10.799
δ_1	−0.543	0.076	−7.136
δ_2	0.159	0.063	2.490
η	0.057	0.018	3.171
α	11.093	4.431	2.504
σ_π^2	0.299	–	–
R^2		0.988	
Significance of Q		0.857	
DNAIRU		8.439	
NAIRU		12.043	

Table 6: Estimation Results with Ordinary Least Squares

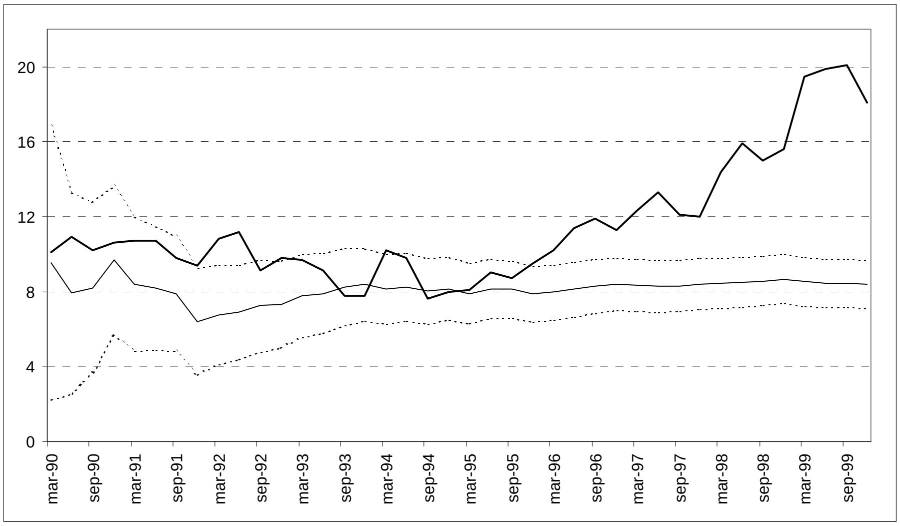

Figure 4: The Unemployment Rate, the Deterministic NAIRU, and its Confidence Band Estimated with the Kalman Filter

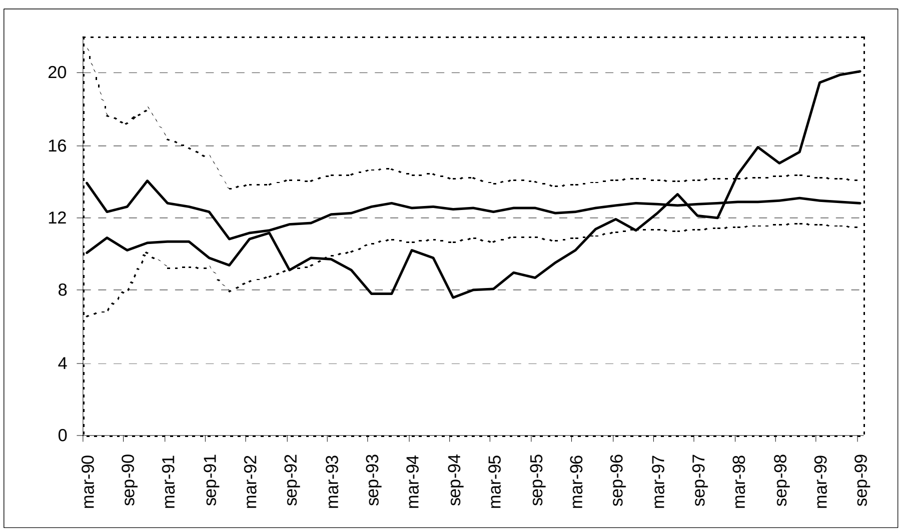

Figure 5: The Unemployment Rate, the Expected NAIRU and its Confidence Band Estimated with the Kalman Filter.

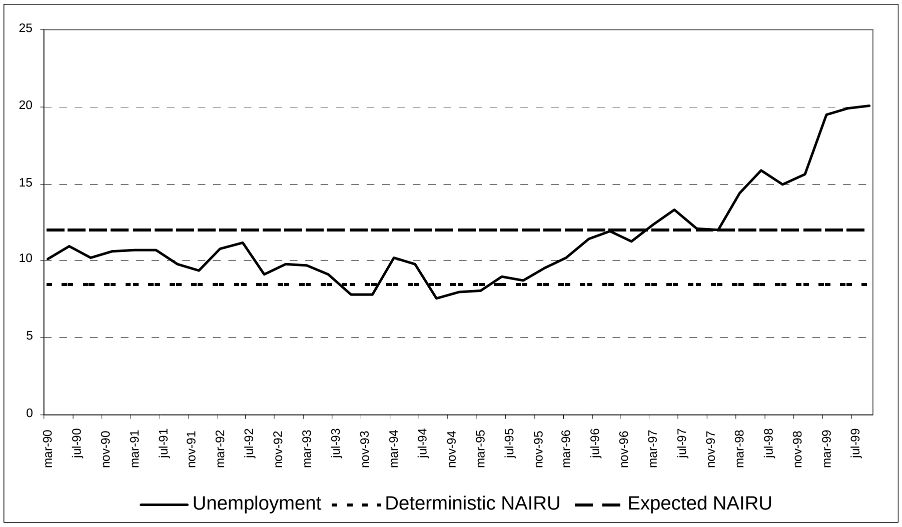

Figure 6: The Unemployment Rate, the Deterministic NAIRU, and the Expected NAIRU Estimated by OLS.

unemployment is convex.

As shown in Table 7, a decreasing of percentage point in the unemployment rate decreases inflation by an increasing amount, hence the sacrifice ratio, that is, the number of points of unemployment that has to be paid to decrease inflation by one percentage point, is increasing in the unemployment rate. Column three in Table 7 and Figure 8 present the sacrifice ratio.
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7 Conclusions

Using Colombian data, we have found evidence of a Phillips curve that is not linear. This finding implies that the sacrifice ratio increases with unemployment. An important policy implication of convexity in the Phillips curve is that the right moment to stabilize inflation is when the economy is in a boom, and the worst, when the economy is in a recession. According to Laxton et.al. (1998) the non linearity of the Phillips curve points to a gradual approach to disinflation.

In 2000, inflation has decreased, in part because of a favorable supply shock, in part because of an increase in unemployment. What the non linearity of the Phillips curve implies is that the sacrifice ratio in the recession is higher than before the recession, hence, if the Phillips curve is fixed because expectations do not change, a cold shower approach to disinflation may be very costly.

Inflation Forecast	Unemployment	Sacrifice Ratio
5.1	20	7.4
5.3	19	6.7
5.4	18	6.0
5.6	17	5.4
5.8	16	4.8
6.1	15	4.2
6.3	14	3.7
6.7	13	3.2
7.0	12	2.7
7.5	11	2.3
8.0	10	1.9
8.6	9	1.6
9.4	8	1.3
10.4	7	1.0
11.8	6	0.7
13.7	5	0.5

Table 7: Inflation Forecast and Sacrifice Ratio

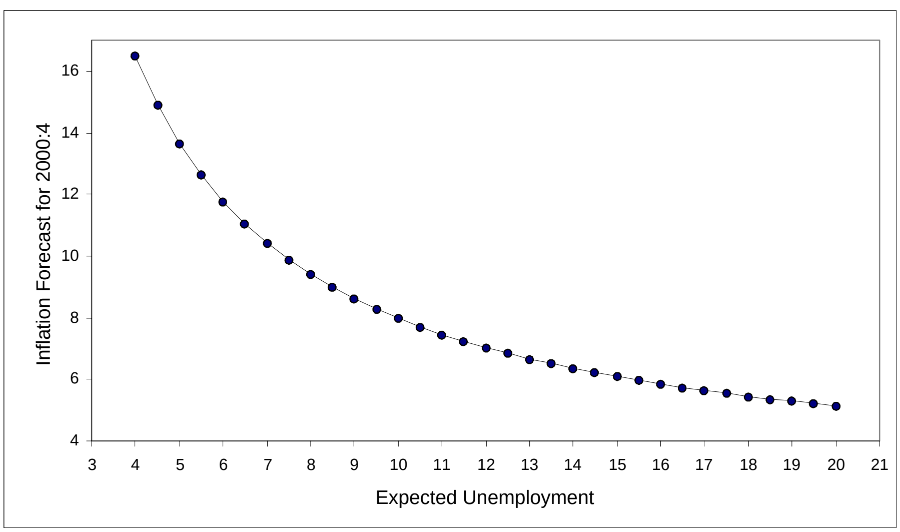

Figure 7: Inflation Forecast for 2000:4 for Different Rates of Expected Unemployment.

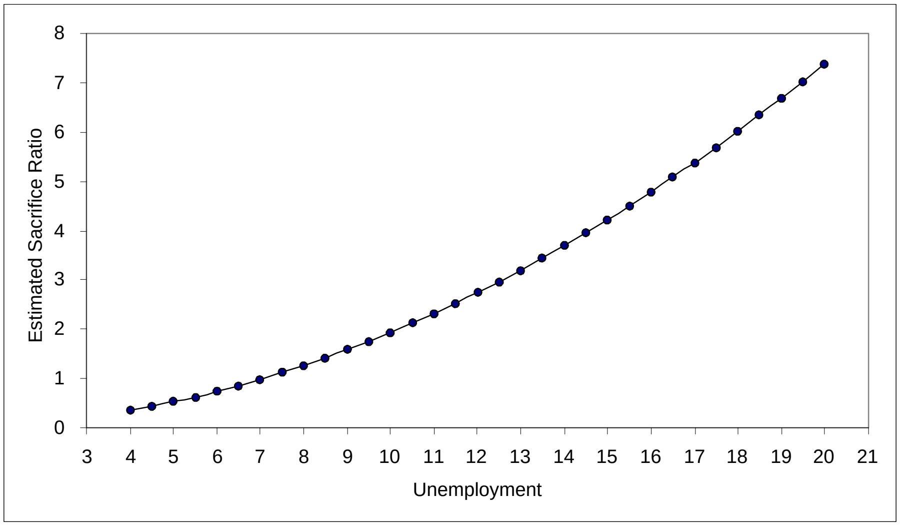

Figure 8: Sacrifice Ratio.

The decreasing marginal effect of unemployment on inflation points to an opportunistic approach to disinflation (See Orphanides (1996) and Isard and Laxton (1996)). According to the opportunistic approach to disinflation, the monetary authorities should wait until supply shocks or unforeseen recessions decrease inflation. Once inflation has decreased, they should avoid U turns in inflation by the means of contractionary aggregate demand policy, otherwise the boom may imply further and permanent additional losses in employment.

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