

# Seigniorage and the Welfare Cost of Inflation in Colombia

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## ABSTRACT

We compute both seigniorage rate and welfare cost of inflation rate in Colombia using a Sidrauski-type model in which preferences are separable functions of the service flows of non-durable goods and money holdings. The set of the estimated parameters imply sizeable welfare cost of inflation and seigniorage rates. However, eventhough for low inflation rates seigniorage rate markedly increases with the rate of inflation, for very high inflation rates it reaches an asymptote.

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## I. Introduction

Reduction of welfare cost of inflation has been one of the goals of monetary policy in Colombia. However, estimates assessing quantitatively the welfare losses associated with different rates of inflation have shown quite different results, which means that further research in this sense would be useful.

Among the first approximations to this measure for Colombia was the paper by Carrasquilla-Galindo-Patrón (1994). Their estimates of welfare loss for an increase in inflation rate from 5 per cent to 20 percent reach the sizable figure of 7 percent of the GDP. On the other hand, the estimates made by Posada (1995) and Riascos (1997) for an inflation rate of 20 percent are around 3.9 percent and 1.5 percent of GDP, respectively. The paper by Carrasquilla-Galindo-Patrón has more information about the Colombian economy and a richer econometric framework than the one used by Riascos (1997) and Posada (1995) (which is a calibration exercise), but the estimates of their model are quite large in comparison with previous results for countries with inflation rates higher or lower than the Colombian, such as Israel or the United States, respectively.

Carrasquilla-Galindo-Patrón used an approach similar to the one used by Eckstein and Leiderman (1992), but did not take into account the service flow of purchases of goods for more than one period, even though the consumption variable used by them included consumption of durable goods. Besides, the monetary aggregate used was M2, which would not be a good aggregate to analyze the effects of the inflation rate on welfare.

In this paper, we try one approach closer to the one used by Eckstein and Leiderman than the one used by Carrasquilla-Galindo-Patrón. The first part of the paper deals with estimation - on quarterly time series for Colombia- of the parameters of a model that treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in the modern monetary theory (Sidrausky, 1967).

After obtaining estimates for the key parameters, the second and main part of our work consist of comparing steady states of the model assuming different rates of inflation to determine both, the welfare loss associated with different steady states rates of inflation and the

relationship between inflation rate and seigniorage revenue predicted by the model. We calculate that the steady state welfare cost of a moderate inflation of 10 percent per year at 1.37 percent of GNP, quite similar to the one of Israel, 1.4 per cent and more than twice as big as the available estimates for the United States - given the same inflation rate. The welfare cost of an inflation rate around 20 percent per year is about 2.4 per cent of GDP according to our estimates, which are in between the estimates by Posada and Riascos.

The remainder of this paper is organized as follows. In section 2 we describe the model and discusses some steady state implications of the model. In section 3 we describe the data and econometric methodology used in estimation. Section 4 presents the parameter estimates and tests of the over-identifying restrictions. In section 5, we use parameter estimates and auxiliary assumptions about hypothetical steady state are used to determine the model's quantitative implications for the relation between seigniorage and the rate of inflation and for the welfare cost of inflation. Concluding remarks are presented in section 6.

## 2. The Model

This model treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in modern monetary theory (see e.g. Eichenbaum, Hansen and Singleton (1988) for similar specifications)

Suppose that consumers rank alternative sequences of consumption of services from goods using the utility functional:

$$(1) \quad E_0 = \sum_{t=0}^{\infty} \beta^t U(m_t, c_t^*),$$

In (1),  $c_t^*$  is the consumption percapita of services from goods at date t,  $m_t$  denotes real money balances per capita,  $\beta \in (0,1)$  is a subjective discount factor, and the period utility function is of the constant relative risk-averse form,

$$(2) \quad U(m_t, c_t^*) = \frac{[m_t^\gamma c_t^{*1-\gamma}]^\theta - 1}{\theta} \quad 0 < \gamma < 1 \quad \theta < 1$$

where  $\gamma$  and  $\theta$  are preferences parameters, with coefficient of relative risk aversion  $(1-\theta)$ . The parameter  $\theta$  must be less than unity in order to obtain a concave utility. The lower  $\theta$  the higher the relative risk aversion coefficient and the lower the intertemporal elasticity of substitution.

The consumption of services from goods is not measured and, therefore, it is necessary to specify a technology for transforming goods into services in order to proceed with the empirical analysis. Following Telser and Graves (1972), all consumers are assumed to have access to linear technologies that transform consumption goods purchased today into services flows in the future. The service flow  $c_t^*$  is assumed to be given by

$$(3) \quad c_t^* = \delta_0 c_t + \delta_1 c_{t-1} + \delta_2 c_{t-2} + \dots + \delta_m c_{t-m}, \quad m < \infty$$

where  $c_t$  denotes actual purchases of consumer goods,  $\delta_0 = 1$  and in this paper  $m = 1$ <sup>1</sup>. Thus consumption purchases at time  $t$  directly affect consumption services in both  $t$  and  $t + 1$ .

Each household's budget constraint, in per capita real units is given by

$$(4) \quad b_t = b_{t-1} \frac{(1+r_{t-1})}{(1+n_t)} + \frac{m_{t-1}}{(1+n_t)(1+\pi_t)} + y_t - m_t - c_t$$

where  $b_t$ ,  $m_t$ , and  $c_t$  are, respectively, the real per capita values of one-period financial assets, money balances, and consumption.  $N_T$  and  $\pi_t$  denote population growth and the rate of inflation, respectively, and the real interest factor  $(1+r_{t-1})$  is equal to  $(1+R_{t-1})/(1+\pi_t)$ , where  $R_{t-1}$  denotes the nominal return on assets.  $y_t$  is real per capita income from other sources.

Substituting the specification about the relation between consumption services and purchases into (1) we have to solve the problem

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<sup>1</sup> For non-durable goods, it is common in the literature to use  $m=1$ , however, for durable goods, see Dunn and Singleton (1985) for an alternative specification that take into account a different technology for transforming goods into services.

$$(5) \quad \underset{b_t, m_t}{Max} \quad E_0 = \sum_{t=0}^{\infty} \beta^t U(m_t, (c_t + \delta c_{t-1})),$$

and from the budget constraint (4),

$$(6) \quad c_t = b_{t-1} \frac{(1+r_{t-1})}{(1+n_t)} + \frac{m_{t-1}}{(1+n_t)(1+\pi)} + y_t - m_t - b_t,$$

$$(7) \quad c_{t+1} = b_t \frac{(1+r_t)}{(1+n_{t+1})} + \frac{m_t}{(1+n_{t+1})(1+\pi_{t+1})} + y_{t+1} - m_{t+1} - b_{t+1}$$

Therefore, differentiating with respect to  $b_t$  and  $m_t$  and dividing by  $U_{c^*}(t)$  we obtain the Euler equations,

$$(8) \quad \beta E_t \left[ \frac{U_{c^*}(t+1)}{U_{c^*}(t)} \left[ \frac{(1+r_t)}{(1+n_{t+1})} - \delta \right] \right] + \beta^2 \delta E_t \left[ \frac{U_{c^*}(t+2)}{U_{c^*}(t)} \frac{(1+r_t)}{(1+n_{t+1})} \right] - 1 = 0$$

$$(9) \quad \frac{U_{m_t}(t)}{U_{c^*}(t)} \beta E_t \left[ \frac{U_{c^*}(t+1)}{U_{c^*}(t)} \left[ \frac{1}{(1+n_{t+1})(1+\pi_{t+1})} - \delta \right] \right] + \beta^2 \delta E_t \left[ \frac{U_{c^*}(t+2)}{U_{c^*}(t)} \frac{1}{(1+n_{t+1})(1+\pi_{t+1})} \right] - 1 = 0$$

Euler eq.(8) relates the disutility of giving up one unit of consumption at date  $t$  to the present value of the utility from shifting that unit of consumption in the next period. Euler eq.(9) relates the expected utility cost of giving up one unit of consumption at date  $t$  to the expected benefits from allocating in money holdings the foregone consumption during one period. This Euler equations can be used to construct orthogonality conditions for use in estimation and inference.

The marginal utilities with respect to  $m_t$  and  $c_t^*$  implied by (2) are given by

$$(10) \quad U_{m_t}(t) = \gamma (m_t)^{\gamma\theta-1} (c_t + \delta c_{t-1})^{\theta(1-\gamma)}$$

$$(11) \quad U_{c^*}(t) = (1-\gamma) (m_t)^{\gamma\theta} (c_t + \delta c_{t-1})^{\theta(1-\gamma)-1}$$

## 2.1. Implications of the model for seigniorage and welfare cost of inflation

The implications of the model for seigniorage and welfare cost of inflation are derived by comparing steady states of the model assuming different rates of inflation.

It is assumed that percapita consumption and real money balances grow in steady states at a constant rate  $\Phi$ , that population grows at a constant rate  $n$ , and that all real variables do not change with respect to steady state changes in the rate of inflation.

With this assumptions and rearranging Euler equation (9), we obtain a steady state ‘demand for money’ which depends on explicit preference parameters,

$$(12) \quad m = \frac{\left( \frac{\gamma}{(1-\gamma)} \right) \left( 1 + \frac{\delta}{(1+\Phi)} \right)^* c}{1 + \alpha_1 - \frac{\alpha_2}{(1+\pi)}}$$

where  $c$  and  $\pi$  denotes the steady state values of consumption per capita and rate of inflation and,

$$\alpha_1 = \beta\delta(1+\Phi)^{\theta-1}$$

$$\alpha_2 = (1+n)^{-1} (1+\alpha_1)\beta(1+\Phi)^{\theta-1}$$

Welfare cost of various steady state levels of inflation are calculated by substituting eq(12) into (2) and compute the percentage decrease in consumption per capita that would generate the same welfare loss as that from moving from  $\pi=0$  to  $\pi>0$ . This welfare loss is expressed as a percentage of GNP and given by,

$$(13) \quad WL = \left( \left( \frac{1 + \alpha_1 - \alpha_2(1+\pi)^{-1}}{1 + \alpha_1 - \alpha_2} \right)^\gamma - 1 \right) \Psi$$

Where  $\Psi$  is the ratio of consumption to GDP<sup>2</sup>.

<sup>2</sup> Den Haan (1990) shows that a welfare measure based on an expression such as eq.(13) leads to very similar answers as the measure that calculates the area under the steady-state money demand function of the structural model” (Eckstein and Leiderman, 1992).

Seigniorage per capita is given by

$$(14) \quad S_t = \left(1 - \frac{H_{t-1}}{H_t}\right) h_t$$

where  $H$  is the monetary base and  $h$  denotes the monetary base in real per capita units. In steady state equilibrium the gross rate of change of the monetary base ( $H_t/H_{t-1}$ ) is equal to  $(1+n)(1+\Phi)(1+\pi)$ . Substituting for  $h_t$  the derived demand for real monetary base from eq.(12) and dividing by GDP per capita we obtain the ratio of seigniorage to GDP in steady state :

$$(15) \quad SR = \left[1 - \frac{1}{(1+n)(1+\Phi)(1+\pi)}\right] \frac{\left(\frac{\gamma}{(1-\gamma)}\right) \left(1 + \frac{\delta}{(1+\Phi)}\right)^* \Psi \kappa}{1 + \alpha_1 - \frac{\alpha_2}{(1+\pi)}}$$

Where  $\Psi$  is the ratio of consumption to GDP and  $\kappa$  is the inverse of the money supply multiplier. As in the standard literature about seigniorage, there are two components of SR: the inflation-tax rate, that increases when inflation accelerates, and the tax base which is the demand for real balances and it decreases when inflation accelerates.

If  $[1-\beta(1+\Phi)^\theta] > 0$ , SR would be increasing with respect to  $\pi$ . This would be the case if  $\beta < 1$ ,  $\Phi \geq 0$ , and  $\theta \leq 0$ . In this case SR would not exhibit a Laffer curve that arises from a model based on a Cagan-type money demand.

### 3. The Econometric model and the data

Consider the first-order condition (8) together with (9), from the agent's intertemporal optimum problem, let

$$(16) \quad u_{1t+2} \equiv \beta E_t \left[ \frac{U_c^*(t+1)}{U_c^*(t)} \left[ \frac{(1+r_t)}{(1+m_{t+1})} - \delta \right] \right] + \beta^2 \delta E_t \left[ \frac{U_c^*(t+2)}{U_c^*(t)} \frac{(1+r_t)}{(1+m_{t+1})} \right] - 1$$

$$(17) \quad u_{2t+2} \equiv \frac{U_m(t)}{U_c^*(t)} \beta E_t \left[ \frac{U_c^*(t+1)}{U_c^*(t)} \left[ \frac{1}{(1+m_{t+1})(1+\pi+1)} - \delta \right] \right] + \beta^2 \delta E_t \left[ \frac{U_c^*(t+2)}{U_c^*(t)} \frac{1}{(1+m_{t+1})(1+\pi+1)} \right] - 1$$

Where

$$\frac{Um(t)}{U_{c^*}(t)} = \left( \frac{\gamma}{1-\gamma} \right) \left( \frac{c_t + \delta c_t}{m_t} \right)$$

$$\frac{U_{c^*}(t+1)}{U_{c^*}(t)} = \left( \frac{m_{t+1}}{m_t} \right)^{\theta\gamma} \left( \frac{c_{t+1} + \delta c_t}{c_t + \delta c_{t-1}} \right)^{\theta(1-\gamma)-1}$$

$$\frac{U_{c^*}(t+2)}{U_{c^*}(t)} = \left( \frac{m_{t+2}}{m_t} \right)^{\theta\gamma} \left( \frac{c_{t+2} + \delta c_{t+1}}{c_t + \delta c_{t-1}} \right)^{\theta(1-\gamma)-1}$$

Accordingly, we interpret the  $u_{kt}$  as the disturbances in our econometric analysis. Eq.(16) and (17) are the two-equation system to be estimated whose parameter vector is  $\sigma = (\beta, \delta, \gamma, \theta)$ .

Hansen's (1982) Generalized Method of Moments (GMM) provides a convenient framework for estimating nonlinear system of simultaneous equations. Suppose that the goal is to estimate a system of n nonlinear equations of the form

$$y_t = f(\sigma, x_t) + u_t$$

For  $x_t$  a  $(k \times 1)$  vector of explanatory variables and  $\sigma$  a  $(p \times 1)$  vector of unknown parameters. Let  $z_{it}$  denote a vector of instruments that are uncorrelated with the  $i$ th element of  $u_t$ . The q orthogonality conditions for this model are

$$f(\sigma, w_t) = \begin{bmatrix} [y_{1t} - f_1(\sigma, x_t)]z_{1t} \\ [y_{2t} - f_2(\sigma, x_t)]z_{2t} \\ \vdots \\ \vdots \\ [y_{nt} - f_n(\sigma, x_t)]z_{nt} \end{bmatrix}.$$

Hansen(1982<sup>a</sup>) shows that the estimator of  $\sigma_0$  with the smallest asymptotic covariance matrix given our choice of instruments is obtained by minimizing the criterion function

$$(16) \quad J_T(\sigma) = g_T(\sigma)' S_T^{-1} g_T(\sigma)$$

where  $g_T(\sigma) = T^{-1} \sum Z_t u_t(\sigma)$ , and  $S_T^{-1}$  is a consistent estimator of the population weighting matrix. The weighting matrix is positive definite and, for this model, due to the presence of a two-period-ahead forecast error in the Euler equations, the type of covariance matrix estimate that would be more suitable would be the Heteroscedasticity Autocorrelation Consistency Covariance Matrix.

Identification requires an order condition ( $q \geq p$ ) and that the columns of  $(\partial f(\sigma, w_t) / \partial \sigma')$  be linearly independent.

The expressions in (16) and (17) are scaled by  $U_c^*(t) = [(1-\gamma) (m_t)^\theta (c_t + \delta c_{t-1})^{\theta(1-\gamma)-1}]$

in order that the disturbances will be strictly stationary processes in the presence of certain types of real growth in purchases of goods and money balances.

The sample period for the empirical analysis is 1977.2 through 1997.4. Quarterly data on total private consumption were obtained from National Planning Department. We also used a measure for real purchases of non-durable plus services based on the classification made by Alejandro López (1996). Money is defined as the standard M1. The nominal interest rate is the quarterly lending rate charged by banks (the appendix presents estimates with the banks deposits rate). The inflation rate is measured by the percentage change in the GDP price deflator.

#### 4. Parameter estimates and test results

The results from estimating the model are displayed in table 1. We report two set of estimates corresponding to two alternative definitions of consumption (total private consumption and consumption of non-durable plus services).

In addition to the parameter estimates and estimates of their respective standard errors, we report a statistic for testing the validity of the over-identifying restrictions implied by the model, the  $J_T$  statistics. The instrument vector,  $z_t$ , associated with the disturbance  $u_{1t}$  and  $u_{2t}$  included the constant unity, the first lagged value of the growth rates of consumption and real money balances per capita, the inflation rate and the real interest factor. With these five instruments and two equations there are ten orthogonality conditions,  $q=10$ . Since there are four parameters to be estimated,  $p=4$ , there are six overidentifying restrictions.

The parameter estimates displayed in table 1 are qualitatively similar for the two alternative definitions of consumption. The point estimates of the concavity parameter,  $\theta$ , are lower than zero, which means a relatively high risk aversion coefficient and a low intertemporal elasticity of substitution. Estimates of  $\gamma$  and  $\beta$  are between zero and one as expected and large relative to their estimated standard errors. The estimates of the lag for consumption of non-durable,  $\delta$ , are economically plausible and similar to the estimates for Israel (between 0.3-0.6) and the United States (0.6 according to Dunn and Singleton estimates)

The test statistic,  $J_T(\sigma)$ , is equal to 8.08 when total consumption is used and to 8.25 when the proxy for non-durable and services variable is used. The null hypothesis is that the model is correctly specified, and one compares the test statistic to a  $\chi^2_{q-p}$ . In this case, the critical  $\chi^2_6$  is 12.6 at 5 percent significance level. So we do not reject the null hypothesis and assume that the model is correctly specified.

Parameters	C	CN
$\beta$	0.959 (235.67)	0.959 (245.5)
$\gamma$	0.052 (36.29)	0.055 (35.46)
$\theta$	-1.310 (-3.04)	-1.102 (-2.79)
$\delta$	0.717 (1.71)	0.697 (2.66)
	$J(\sigma)= 8.08$	$J(\sigma)= 8.25$

##### 5. Implications for seigniorage and welfare cost of inflation.

Based on the parameter estimates obtained in the previous section, and based on eq. (13) and (15), we made some estimates from the seigniorage as a percentage of GDP and from the welfare cost of inflation for Colombia. Table 2 report the results for seigniorage as a percentage of GDP and

for the welfare cost of inflation for Colombia and also for Israel according to the estimates obtained by Eckstein and Leiderman (1992). The parameter values used are:

	Israel	Colombia
$\beta$	0.980	0.960
$\gamma$	0.050	0.060
$\theta$	-1.500	-1.100
$\delta$	0.300	0.600
$\Psi$	0.610	0.700
$n$	0.580	0.510
$\phi$	0.008	0.005

where the values for  $\Psi$ ,  $n$ ,  $\phi$  correspond to the quarterly sample means of the share of consumption in GDP, the rate of change of population, and the rate of change of consumption percapita, respectively.

The results for Seigniorage rate, SR, in table 2 show that Seigniorage rate is an increasing function of the rate of inflation. That is, government can raise more revenue by increasing monetary base growth and inflation. For low rate of inflation SR markedly increases with increases in  $\pi$ , but then SR reaches an asymptote. Besides SR is higher in Colombia than in Israel. Finally, the result suggest that an inflation rate of 20 percent per year would result in a seigniorage rate of about 3.1 percent of GDP<sup>3</sup>.

In order to compute the decrease in per capita consumption (expressed as percent of GNP) that would generate the same welfare loss as that from increasing inflation from zero to a given rate, we use eq. (13). Welfare cost of inflation depends on the degree of risk aversion. The higher the degree of risk aversion, the lower is the welfare cost of inflation. From table 2, we see that a

<sup>3</sup> Inflation rate average in the period 1977-1997 was about 23 percent and the observed seigniorage rate was 2.9 percent of GNP according to document of the department of Monetary and Reservs from the Central Bank of Colombia (1995).

shift from zero inflation to an annual rate of inflation of 10 percent (i.e., 2.41 per quarter) results in a loss in utility equivalent to about 1.3 percent of GDP in Israel and Colombia. This estimate is much lower than the estimate for Colombia obtained by Carraquilla-Galindo-Patron (1994), where a decrease of the inflation rate from 20 to 5 percent represents decrease in the welfare cost of inflation of about that was about 7 percent of GDP. Moreover, our estimates for the welfare loss due to an increase in the inflation rate from 10% to 20% are equivalent to about 1.0 per cent of GDP, similar to the estimate of 1.2 per cent of GDP found by Posada (1995)<sup>4</sup>.

Comparing the estimate of welfare loss of around 1.3 percent of GDP when inflation increases from zero to 10 percent in Colombia and Israel, to the estimates found for the United States, it is more than twice as big as the welfare loss for the United States. For example, for the same inflation rate the estimated welfare loss for the United States are 0.28% of GDP according to McCallum (1989), 0.3% of GDP according to Fisher(1981), and 0.39 percent of GDP computed by Cooley and Hansen(1989).

Table 3: Seignorage ratio and Welfare cost of inflation for Israel and Colombia  
percentage of GDP

$\pi$ (quarterly)	SR Israel	SR Colombia	WL Israel	WL Colombia
0,0	1,2	1,0	0,0	0,0
1,2	1,7	1,8	0,8	0,7
2,4	2,0	2,4	1,4	1,3
5,0	2,3	3,1	2,5	2,3
10,0	2,7	3,9	3,8	3,7
15,0	2,8	4,3	4,6	4,6
20,0	2,9	4,5	5,3	5,3
28,0	3,0	4,8	6,0	6,2
32,0	3,0	4,8	6,3	6,6
50,0	3,1	5,1	7,2	7,7
70,0	3,1	5,2	7,9	8,5
90,0	3,1	5,2	8,3	9,0
100,0	3,2	5,3	8,9	9,3
120,0	3,2	5,3	8,8	9,6
130,0	3,2	5,3	8,9	9,7

<sup>4</sup> The welfare cost calculations of Posada (1995) allow the possibility of endogenous production and capital acumulation. However, he based his estimation on parameter values estimated by Carrasquilla-Galindo and Patrón.

## 6. Summary

In this paper we have presented estimates from the parameters of a model that treats consumption and money demand behavior as jointly arising from a single optimizing framework of a representative agent, as in the modern monetary theory ( Sidrausky, 1967).

The point estimates of the concavity parameter,  $\theta$  is lower than zero and has the expected sign, which means a relatively high risk aversion coefficient and low intertemporal elasticity of substitution . The discount factor estimated was around 0.96, the preference parameter  $\gamma$  is around 0.05 (much lower than the one estimated by Carrasquilla-Patron-Galindo (1994). Finally, the parameter that captures the service flow of the consumption goods,  $\delta$ , is around 0.7, a little higher than the estimates found in previous studies for the U.S.A. and Israel which are around 0.6

The second part of this paper consisted of comparing steady states of the model assuming different rates of inflation to determine both, the welfare loss associated with different steady states of inflation and the relationship between inflation rate and the seigniorage revenue. The results show that the welfare loss due to an increase in the inflation from 5% to 20% is no higher than 2.3% of the GDP, again much lower estimates than the estimates from Carrasquilla-Patron – Galindo , around 7% of GDP.

On the other hand, our estimates for the welfare loss due to an increase in the inflation rate from 10% to 20% are equivalent to about 1% of the GDP, similar to the 1.2% of GDP found by Posada (1995).

Finally, the results on seigniorage rates shows that seigniorage rate is an increasing function of inflation rate, but it reaches an asymptote. It does not have the shape of the Laffer curve. Besides, seigniorage rate is higher in Colombia than in Israel; an inflation rate of 20% per year in Colombia would result in a seigniorage rate of about 3.1% of GDP.

## Appendix

Table : Estimates under alternative asset return Banks Deposit Rate	
Parameter estimates and t-values	
Parameters	CN
$\beta$	0.959 (245.5)
$\gamma$	0.055 (35.46)
$\theta$	-1.102 (-2.79)
$\delta$	0.697 (2.66)
	J( $\sigma$ )= 8.25

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