Some Univariate Time Series Properties of Output

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This paper deals with the size of the random walk property of Colombia´s output in two periods 1925-1994 and 1950-1994. GDP and GDPPC were both found to be integrated of order one a result which is very well known. The sequences are highly persistent, specially in the period 1950-1994. The forecast error when an innovation of 1 percent enters into the economy is about 1.5 percent in the very long run, when GDP is considered. The response is about 1.3 percent in the case of GDPPC, which seems to give support to the idea that population growth is a source of nonstationarity in some macroeconomic aggregates. For the larger sample (1925 - 1994) persistence is less. This result could cast some doubt on the method of estimation of GDP for the period 1925-1950. Finally, evidence of nonlinearity is found only in Hodrick-Prescott filtered variables dated between 1925 and 1994. This leaves open the question about whether the HP filter introduces nonlinearity in the high frequency variable that it generates.

Keywords: unit roots, persistence, nonlinearities, logistic function, ESTAR and, LSTAR .models.

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1. Introduction

The object of this work is to estimate the effects of an innovation on the behaviour of Colombia's output, as measured by real *GDP* and real *GDP* per capita (*GDPPC*). We deal with a number of questions: is there any reaction in output when a shock occurs? Is such a reaction permanent or temporary? How large is the reaction in output? Do the reactions of *GDP* and *GDPPC* have the same statistical content? Is there any important difference in the answer when the sample size is extended from the post second world war to the pre war period? Are the series linear?

The definition of the time series properties of any macroeconomic process highly depends on whether the reactions caused by innovations or unforecastable shocks are *permanent* or *temporary*. We associate innovations to that part of the current value of any variable which past values fail to predict. Their importance is central to the descriptive view of economic fluctuations of this chapter, as in most of the works on business cycles since Slutzky [(1927), 1937] and Frisch [(1933), 1965]. The interpretation of output fluctuations as the summation of random causes has been an important argument in the business cycle theory since the experiment of Slutzky who took a series of random numbers (based on the numbers drawn in a lottery), to generate cyclical (or ondulatory) processes which matched the behaviour of output. These fluctuations could, additionally, be represented by stable, low-order, stochastic difference equations. Frisch observed the distinction between random shocks and their propagation mechanism. He was able to show how, under a set of exact mathematical conditions, a dynamic system produced damped cyclical (wave-shaped) movements. This description of the time behaviour of output has been labelled as 'pendulum dynamics'.

The distinction between random shocks and their propagation mechanism was later considered by Adelman and Adelman [1959], who introduced innovations into the Klein-Goldberg model of the US economy. According to Adelman and Adelman, the linear growth of the variables in such a model could not explain the persistent oscillatory process undergone by aggregate economic activity. To remove the excess of stability in the economy described by the model, they included random shocks in the fitted equations. This procedure produced better results than plugging the innovations in the exogenous variables of the model.

Lucas [1977] pointed to the shocks as the cause of co-movements -in deviations from the trend- in different aggregate time series. Moreover, according to Lucas, these business cycles seem alike in qualitative terms. First, prices, short-term and also longer- term interest rates, monetary aggregates, velocity measures, and business profits, were procyclical; second, production of durables was more volatile than output and less procyclical than the previous aggregates; and, finally, there were harmonic movements of output across sectors. The Real Business Cycle theory, a more recent approach to the study of fluctuations, first developed by Kydland and Prescott [1982], uses technological-driven economies to explain the business cycles phenomena: technological shocks are posed as the first cause of economic fluctuations, which are propagated across the economy due to the intertemporal substitutability of leisure.

The empirical analysis of the cyclical behaviour of economic activity in Colombia has utilised some of the above ideas. As a result, the statistical characterisation of the evolution of *GDP* has benefited, among others, from the work of Carrasquilla and Uribe [1991] who estimated the measures of persistence developed by Campbell and Mankiw [1987a,b] and Cochrane [1988]; and also from the work of Gaviria and Uribe [1994], who showed the structural changes which have produced permanent movements in aggregate *GDP*.

In this work we apply various techniques which may be useful in the characterisation of the main features of the evolution of output in a univariate framework assuming that the initial impulse received by the economy is random. First, we test for the existence of unit roots by using the procedure of Dickey and Fuller [1979]. Second, we deal with the "size" of the random walk component of output by using the concepts of persistence of Campbell and Mankiw [1987a,b] and Cochrane [1988]. Finally, following Terasvirta [1994], Terasvirta and Anderson [1992] and Granger and Terasvirta [1993], we present the results of the linearity tests.

2. Unit Roots

The order of integration of a variable (i.e. the number of times that it needs to be differenced before becoming covariance stationary *[I~(0)]*) is a basic time series property of any variable in the context of business cycles [Nelson and Plosser, 1982]. Furthermore, the use of standard asymptotic theory requires stationarity [see Granger and Newbold, 1986]. Nelson and Plosser [1982], show that a *nonstationary* process, Y_t, can be represented by two different mechanisms: *trend-stationary* and *difference-stationary*. The former incorporates a *deterministic (possibly) linear time trend* plus a *stationary* and *invertible* autoregressive moving average (*ARMA*) stochastic process e_t; that is,

$$Y_t = a + bt + e_t \qquad (1.1)$$

where *t* is time and a and b are fixed parameters. The latter mechanism, used to represent *changing trends*, involves a *stochastic trend* (usually a *random walk* component) plus a *stationary* and *invertible ARMA* stochastic process u_t; that is:

$$\Delta Y_t = a + u_t \qquad (1.2)$$

where $\Delta Y_t = Y_t - Y_{t-1}$.

The traditional representation of the time behaviour of economic variables through (1.1) was first questioned by Nelson and Plosser [1982], who presented statistical evidence about the existence of a stochastic trend in eleven, out of fourteen, aggregate variables of the US economy[‡]. The analysis here is focused on output which is represented by the logarithm of real *GDP* and real *GDP* per capita in two periods: 1925-1994 and 1950-1994 (see figures 1.1 and 1.2 at the end of this work)[§].

[‡] *Nelson and Plosser [1982] concluded that real shocks dominate as a source of output fluctuations. That is, fluctuations driven by aggregate demand (monetary shocks) are not a satisfactory explanation of output fluctuations.*

[§] *Source of data: GDP in real terms (1975=100) from "Principales Indicadores Económicos. 1923-1992. Banco de la República. Bogotá", for period 1950-1990 and from Revista Banco de la República, different*

To test the null hypothesis that the processes were better described by (1.2) against the alternative of (1.1), Nelson and Plosser used both the procedure of Dickey and Fuller [1979] and the correlogram. We first consider the Dickey-Fuller (DF) test but instead of using the correlogram we present, in the next section, further evidence about the results obtained here.

Consider an unrestricted version of (1.2) such as:

$$Y_t = a + r Y_{t-1} + u_t \qquad (1.3)$$

where r is a parameter. The null hypothesis in the DF test is that of nonstationarity, which in a parameterisation such as:

$$\Delta Y_t = a + l Y_{t-1} + u_t \qquad (1.4)$$

corresponds to H_O: $l = 0$, where $l = r - 1$. The alternative hypothesis is H_1: $l < 0$. Errors are assumed to be independent and with finite variance. The test can also be based on the following regression:

$$\Delta Y_t = a + b t + l Y_{t-1} + u_t \qquad (1.5)$$

which nests (1.1) and (1.2). The use of (1.4) or (1.5) depends on the possible presence of a deterministic trend which can be determined by inspection. The augmented version of the DF test, labelled ADF, incorporates *k*-additional terms in order to rule out possible serial correlation in the error term. Thus, we have:

$$\Delta Y_t = a + l Y_{t-1} + \sum_{i=1}^{k} d_i \Delta Y_{t-i} + u_t \qquad (1.6)$$

and,

$$\Delta Y_t = a + b t + l Y_{t-1} + \sum_{i=1}^{k} d_i \Delta Y_{t-i} + u_t \qquad (1.7)$$

where d's are constant parameters. However, the larger the value of *k* the less the power of the test due to the loss of degrees of freedom produced by the estimation of additional parameters. To determine the order of *k*, Campbell and Perron [1990] suggest to start by estimating an autoregression including some upper bound of *k;* if the lag is found to be significantly different from zero, using the standard normal asymptotic distribution, then select that *k*. If it is not different from zero, the process continues by estimating a new regression with *k-1* lags[**].

The results of table 1.1, at the end of this paper, show that the DF test fails to reject the null of nonstationarity for *GDP* and *GDPPC* in levels for the two periods considered. Once differenced, however, all the sequences are stationary. With these results, we may expect that the variance of the long-term forecast error of

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issues for 1991-1994. GDP (1925-1994) from Easterly [1994] and Cuddington and Urzúa [1989] for period 1930-1949 and from the two former sources for the remainder as well as Population.

output will increase without bound, because of the random walk component in the time behaviour of output. Put another way, since output can be represented by (1.2), the second mechanism above, the effect of any innovation will never die out: any shock will have effects on the evolution of the variable which are permanent.

Gaviria and Uribe [1994] describe some features of the permanent changes in the behaviour of aggregate GDP which also relate to the results obtained here[††]. They question whether it is sensible to consider, as it is implicit in Nelson and Plosser [1982], that all random shocks have permanent effects on the sequence of output. To test for nonstationarity, Gaviria and Uribe [1994] use the *variable trend* procedure, suggested by Perron [1989, 1990]. They pick up six exogenous shocks and introduce the same number of possible changes in the intercept of the trend, in the slope or in both. The changes are regarded as structural only if they are able to subtract the unit root of the sequence, otherwise more structural shifts are needed. Thus, they consider as changes *potentially* structural: the second world war; the coffee bonanzas in the fifties and seventies; the institutional changes in 1967; the recession of early eighties together with the collapse of the coffee prices and the debt crisis; and finally the economic openness of Colombia at the beginning of nineties.

Individually considered, the second world war and the institutional changes of 1967 introduced significant changes in the slope of the trend while the recession of eighties modified significantly not only the slope but also its intercept. In addition, to be able of rejecting the null of a nonstationary process of output, any combination of the six shocks must include those three shocks already mentioned. That is, only those three facts, out of the six, have had a permanent effect on the sequence of output. In other words, not all shocks have had a permanent effect on output which denies the hypothesis of Nelson-Plosser.

If we take into account that those events traced by Gaviria and Uribe [1994] as causing structural -permanent- movements in output are spread through the sample period[‡‡], it is not very difficult to accept the evidence of output having a random walk component. It may be noted that Gaviria and Uribe [1994], as Nelson and Plosser [1982], link the relevant events with the supply side: the first with protectionism (second world war), the second with modifications on the exchange rate determination (institutional changes of 1967), and the third with the deterioration of the terms of trade and the debt crisis (recession of eighties) [§§]. With respect to this, Plosser [1991, p. 257] writes:

> *..Variations in real opportunities can arise from many sources including changes in tax rates; real government spending; changes in terms of trade brought about through tariffs or import-exports restrictions; changes in regulations, in addition to more general changes in productivity or preferences, just to name a few. Of course this is part of theory's strength*

[**] *There are other methods to select k. Campbell and Perron [1991] also propose the use of the information criterion or a joint-F test of significance on additional lags.*

[††] *Their result in applying the Dickey-Fuller test to the series 1936-1991 of aggregate GDP is similar to that obtained here (see Gaviria and Uribe [1994], page 5, footnote 3).*

[‡‡] *The events were about 1945, 1967 and 1981.*

[§§] *Recall, however, that Nelson and Plosser explicitly refer to shocks having such a characteristic of remaining forever in the sequence of output as supply (technological) shocks.*

and weakness. Since there is no single, always easily observable impulse that initiates the cycle, systematic empirical investigations are difficult to conduct.

Therefore, to a great extent, the view of Nelson and Plosser [1982] is applicable to Colombia's output. However, to gather more features about output fluctuations, we next deal with the issue of persistence.

3. Persistence

With the suggestion of the previous section about a nonstationary evolution of GDP and GDPPC, we can examine the relative size of the random walk or, in other words, the relative importance of the permanent component (the stochastic trend) in the evolution of output. Assume that Y_t is nonstationary, so that ΔY_t can be represented as:

$$\Delta Y_t = a + y(L)e_t = a + \sum_{i=0}^{\infty} y_i e_{t-i} \qquad (1.8)$$

where y_k measures the impact produced on ΔY_t, *k*-periods ahead, by an innovation in period *t*, denoted by e_t. By the same token, $\sum_{i=0}^{k} y_i$ measures the effect of e_t on *Y*, *k*-periods ahead. When $k=\infty$, the sum of the moving average coefficients gives the ultimate effect of e_t on *Y*, which can be written as $y(1)=\sum_{i=0}^{\infty} y_i$. Thus, for a stationary sequence $y(1)=0$, while for a random walk $y(1)=1$, since $y_i=0$ for $i>0$ in a moving average representation. Estimating a factor which involves a sum of infinite terms as $y(1)=\sum_{i=0}^{\infty} y_i$ introduces some difficulties, however. At least two approaches about persistence have been proposed recently, each with an alternative measure of $y(1)$: the ARMA approach with the *impulse response measure* and the non-parametric approach with the *variance ratio measure.*

The ARMA approach associates the concept of persistence with the duration of the effect of any unforecastable shock to the economy. Thus, a time series is more persistent than another when the effect of a shock on it lasts for a longer period. This concept is linked not only with the presence of unit roots in the sequence of output but also with the economic dynamics [Campbell and Mankiw, 1987a,b]. The non-parametric approach, on the other hand, argues that an appropriate measure of persistence is not related to the presence of unit roots in output. In fact, the measure of persistence, put forward by Cochrane [1988], allows a stationary variable to exhibit much more persistence than one with unit roots (see Cochrane [1991, p. 207]).

Campbell and Mankiw [1987a,b] derive their parametric measure of persistence approximating $y(L)$ by a ratio of finite order of polynomials. In fact, they compute $y(1)$ from the MA representation of a set of parsimonious ARMA models (up to order three for both *p* and *q*, in the case that they analyse) for the first difference of GDP:

$$f(L)\Delta Y_t = q_0 + q(L)e_t \qquad (1.9)$$

where $f(L)=1-f_1 L-...-f_p L^p$ and $q(L)=1-q_1 L-...-q_q L^q$. Solving for ΔY_t, gives the moving average representation or impulse response function of ΔY_t:

$$\Delta Y_t = f(L)^{-1} q_0 + f(L)^{-1} q(L)e_t = a + y(L)e_t \qquad (1.10)$$

as in (1.8). The corresponding expression for Y_t is obtained as:

$$Y_t = a + (1-L)^{-1} y(L)e_t \qquad (1.11)$$

where, as before, y_k is the impact of the innovation on ΔY in period $t+k$ while $1+y_1+....+y_k$ is the impact of the shock on the level of output in period $t+k$.

Following Campbell and Mankiw[***], we have estimated ARMA models for the first difference of *GDP* and *GDPPC* during 1925-1994 and 1950-1994, setting the maximum order for both the AR and the MA components equal to two (see table 1.2). We assume that for annual data as in our case, models nested in an ARMA (2,2) will suffice to capture all the dynamics of output[†††]. The models in table 1.2 are the result of considering the fulfilment of stationarity and invertibility conditions, sensible values for $y(L)$, and convergence of the estimation procedure[‡‡‡]. In table 1.2 an ARMA(3,0) is included out of curiosity since it is the only one of order three in *p* and/or *q*, surpassing the bound we use by invoking parsimony, which accomplishes the above conditions.

Figures 1.3 and 1.4[§§§] show the impulse response functions implied by the different ARIMA models estimated. The responses have been obtained by recursive substitution assuming a (positive) shock of *1* percent in period *1*. In the case of Δ *GDP* between 1950 and 1994 (figure 1.3), the mean reversion property of the stationary sequence appears after four periods if the ARMA(0,2) is used or after about eight periods if the ARMA(1,0) is used. The response to the impulse under the ARMA(3,0) disappears after about twelve periods. This specification reports much richer and complicated dynamics for the Colombian output than the former two models defined under the parsimony principle. For Δ *GDPPC* the effect of any innovation persists for about six-seven periods. In the period 1925-1994, the same variables revert to the mean after approximately five periods (see figure 1.4).

[***] *Krishnan and Sen [1995] replicate the exercise of Campbell and Mankiw [1987b] to the case of India.*

[†††] *The estimation method we use, exact maximum likelihood estimation, explicitly recognizes that the starting values of the disturbances are random (see Harvey [1993], Doan [1992]).*

[‡‡‡] *Building parsimonious ARIMA models for the GDP of Colombia has been troublesome. Moreover, if we had adopted the Box and Jenkins [1970] procedure of selecting the ARIMA models by making subjective judgements based on autocorrelation functions (ACF) and partial autocorrelation functions (PACF), the situation would not have been made easier. The pictures of the ACF for the sequences in levels and first differences (not shown here) are not straightforward.. Cuddington and Urzua [1989], for example, estimated DGDP:* $0.044+(1+0.336L-0.368L^4-0.284L^5)e_t$. *Clavijo [1992] reports a specification which is similar to Cuddington and Urzuas' for the sample period 1930-1985.*

[§§§] *In the figures, the suffixes S (for short period) and L (for long period) identify the sample between 1950-1994 and 1925-1994, respectively.*

Table 1.3 presents the accumulated value of the responses. Between 1950 and 1994 any shock produced a reaction on *GDP* (computed as $y\ (1) = \sum_{i=0}^{30} y_i$) between 1.3% and 1.8% after four periods depending upon the mechanism chosen to represent such a process. The accumulated response is about 1.3% after four periods for *GDPPC* in the same period. When this is extended to the pre second world war period, the accumulated responses for both definitions of output are 1.2%. These estimates confirm that an innovation of 1 percent in real *GDP* and *GDP* per capita will increase the forecast of those time series by more than 1 percent. This result is further evidence of a random walk component on output.

If the impulse response measures of persistence were applied to ARMA models (3) and (5) estimated by Clavijo [1992, p.374] for Δ *GDP*[****], the change in the forecast one, five and ten periods ahead, after a shock of one percent, would be 2.17%, 1.85%, and 1.56% for the first model and 2.15%, 1.82%, and 1.55% for the second model. These values describe an aggregate *GDP* process more persistent in the short run than that described above but the accumulated responses are similar in longer periods. The sample period as well as the model specification possibly explain the differences.

Carrasquilla and Uribe [1991] also applied the parametric ARMA approach but used the Beveridge and Nelson [1981] decomposition, instead of the implied impulse response functions, to estimate the effects of an innovation on *GDP* in the long run[††††]. The results obtained by Carrasquilla and Uribe [1991] are very different from those we find here. However, it is important to point out that they use an estimation method which sets $e_0=0$ and allows for p and q greater than two. Only in the case of their model (8), which is an ARIMA (1,1,1), is the level of persistence estimated similar to that obtained here: about 1.42%. Other estimates of persistence reported by them vary between 0.56% and 0.87%.

Cochrane's concept of persistence is different from Campbell and Mankiw's. Instead of observing the number of periods that the effects of the shock last, Cochrane [1991, p. 207-8] observes the magnitude of the response, which can be large even if the sequence is stationary[‡‡‡‡]. The nonparametric measure of persistence proposed by Cochrane [1988], known as the *variance ratio*, relates the variance of *k*-differences of the sequence of output to the variance of its first differences, $V_k = s_k^2 / s_1^2$. Explicitly, the variance ratio can be written as:

$$V_k = k^{-1} \frac{var(Y_t - Y_{t-k})}{var(Y_t - Y_{t-1})} \qquad (1.12)$$

If the series of output is a random walk, the variance ratio will tend to one ($V_k \rightarrow 1$) as k increases since the variance of its *k*-differences will increase linearly with k; if the series is trend stationary, the variance ratio will

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[****] *The corresponding models to periods 1930-1985 and 1930-1987, respectively are* $DY_t=0.0429+(1+0.174L-0.320L^4-0.295L^6)e_t$ *and* $DY_t=0.0434+(1+0.152L-0.331L^4-0.276L^6)e_t$. *L is the lag operator.*

[††††] *For implementing the Beveridge and Nelson decomposition, Carrasquilla and Uribe use the linear approximation suggested by Cuddington and Urzua [1989].*

tend to zero ($V_k \longrightarrow 0$) as *k* increases. Cochrane [1988] introduces two corrections for the same number of sources of small-sample bias of the estimator of s_k^2. As a result, the estimator of s_k^2 is unbiased when computed from a pure random walk with drift. First, Cochrane uses the sample mean of the first differences to estimate the drift term at all *k* rather than estimate a distinct drift term at each *k* from the mean of the *k*-differences. Second, Cochrane uses the factor *T/(T-k-1)* to make a correction for degrees of freedom; without multiplying by this factor, *1/k* times the variance of *k*-differences will tend to zero as $k \longrightarrow T$ for any process because of the shortage of available data points.

In practice, the variance ratio can be computed as:

$$V_k = 1 + 2\sum_{j=0}^{k}\left(1 - \frac{j}{k+1}\right)\left[\frac{\left(\frac{T}{T-j-1}\right)\sum_{t=j+1}^{T}\Delta y_t\,\Delta y_{t-j}}{\sum_{t=1}^{T}(\Delta y_t)^2}\right] \qquad (1.13)$$

where the term in square brackets is the *j*-th autocorrelation coefficient for *DY*. Consequently, the "triangular" pattern pictured by (1.13) gives linearly declining weights to the higher-order autocorrelations, out to the *k*-th autocorrelation. As written in (1.13), the non-parametric measure of persistence is construed by Cochrane, in terms of frequency domain, as the Bartlett estimator of the spectral density at frequency zero[§§§§]. Such a frequency is equivalent, in time domain terms, to considering an infinite sum of the MA coefficients as in the term $y\ (1)$ above.

Campbell and Mankiw [1987a,b] relate (1.13) to the measure $y\ (1)$ obtained through the ARMA representation of Δ *GDP* and Δ *GDPPC* by the following approximation:

$$y\ (1)^k = \sqrt{\frac{V_k}{1-R^2}} \qquad (1.14)$$

where $R^2 = 1 - s_e^2 / s_{\Delta Y}^2$, is the fraction of the variance in ΔY_t that is explained by its lagged values. For computational purposes R^2 is substituted with the square of the first-order (sample) autocorrelation r_1^2 of ΔY_t. Cochrane [1988] has criticised the use of the impulse response functions based on ARIMA models to measure persistence since those models have been designed to capture short-run dynamics rather than long-run correlations. The non-parametric measure, however, provides only an 'approximate' estimate of $y\ (1)$. It has large standard errors and the *window size, k*, can be difficult to determine [Mills, 1993].

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‡‡‡‡ *Pischke [1991] presents some explanations about the discrepancies between the Cochrane and Campbell-Mankiw statistics of persistence. See also Mills [1993].*

§§§§ *In other words, it is an estimate of the mass spectrum (the normalized spectral density) at frequency zero which uses a Bartlett window: the smoothing factor (1-j/k+1) in (1.13).*

The high value of the estimators of V_k (see table 1.4 and figures 1.5 and 1.6[*****]) suggests that the permanent component of the growth rates of *GDP* is large or, put another way, the innovation variance of the random walk component is very high. This result is more evident with *GDP* and *GDPPC* after 1950 than in the complete period. In no case, however, are the estimators of the variance ratio significant after 10 years when their values are greater than one. Hence, we could point out that the effect of any (past) innovation has been part of the trend of output for at least ten years (see table 1.4). After ten years, the standard errors of the estimates are relatively large[†††††]. Cochrane [1988] points out the growth of population as a source of nonstationarity in macroeconomic aggregates. Thus, to rule out such a possible nuisance, Cochrane recommends using *GDPPC* instead of *GDP*. Here, we use both and find that the sequence of aggregate *GDP* presents more persistence than the sequence of *GDPPC* for both sample periods. So, it may give some support to the conclusion of Cochrane.

Table 1.4 also contains the results of the non-parametric measure of persistence of Campbell and Mankiw; the y^k estimates of persistence are qualitatively the same as those of V_k. Our estimates of persistence of *GDP* between 1925 and 1994 are also similar, at least for k=10, to those computed by Carrasquilla and Uribe [1991] under both non-parametric methods.

Since *GDP* and *GDPPC* are less persistent for the period 1925-1994 than between 1950-1994, for all *k*, we could infer that after 1950 the behaviour of *GDP* starts "to fit" much better to a stochastic trend. There could be two possible explanations. First, and more plausible, that the results are being affected by a smooth retropolation procedure used to estimate output (or population) before 1950, and second, that stabilisation policy was more effective in the period before fifties. However, the link between stabilisation policy and persistence is not straightforward. To see this, in the companion table we list the standard deviation of the temporary component of the logarithm of output obtained by using the Hodrick-Prescott [1980] filter:

Temporary Component of :	*1925-1950*	*1951-1994*	*1925-1994*
GDP	*0.033*	*0.021*	*0.026*
GDPPC	*0.033*	*0.023*	*0.027*

The fluctuations of the sequences are sharper between 1925 and 1950, which seems to be the case in other countries[‡‡‡‡‡]. These results could suggest that fluctuations, between 1951-94, have been dampened by stabilisation policy contrary to what we just said above. Nevertheless, note that the measures of persistence are different; for instance the Cochrane statistic is a *ratio of variances* while the above values are *absolute*

[*****] *The suffix AK in the keylabels of those figures identifies the nonparametric estimates of Campbell and Mankiw that we label* y^k *in the text.*

[†††††] *Campbell and Mankiw [1987b, p. 873] argue that the usefulness of the standard errors is unclear.*

[‡‡‡‡‡] *A comparison of the severity of the business cycles is carried out by Sheffrin [1988], who concludes that, with the only exception of Sweden out of six European countries, there was no substantial reduction in*

estimates of variability. Instead, these changes in the deviations could suggest that some sort of non-linear behaviour is present in the sequence of output, an issue that we explore next.

4. Testing Linearities

Testing for linearities is a recent development in the characterisation of the time series properties of any process. However, nonlinearity is an issue far from new in the context of output fluctuations[§§§§§], which are inherently non-linear. Knowing about its presence can improve the forecasts generated by linear models (such as the ARMA models we used for computing persistence) which are capable only of generating symmetric cyclical fluctuations[******].

The asymmetry of the business cycle has been an issue of extreme importance in macroeconomics. Fluctuations of output (business cycles) are said to be asymmetric when the distance from trough to peak is different from the distance from peak to trough [Granger and Terasvirta, 1993][††††††]. This characteristic cannot be accounted for by linear univariate models. Consider, for instance, the ARMA(*p*,*q*) model:

$$f(L)\Delta Y_t = q_0 + q(L)e_t \qquad (1.15)$$

where e_t is white noise and *f(L)* and *q(L)* are polynomials in the lag operator ($L^d = X_{t-d}$). However, the representation in (1.15) is not appropriate when the true underlying structural process generating ΔY_t is non-linear in parameters.

When *f(L)* is invertible, the ARMA representation (1.15) also has the MA(∞) representation $\Delta Y_t = y_t = a + f^{-1}(L)q(L)e_t$, in which linearity holds as long as e_t is *i.i.d.* Thus, apart from requiring that the disturbances are white noise in a well specified ARMA process, linearity further requires independence of the disturbances [Peel and Speight, 1995a]. Therefore, specifications such as the Autoregressive Conditional Heteroscedastic (ARCH), Bilinear, Threshold Autoregressive (TAR), or Smooth Transition Autoregressive (STAR) models which are capable of generating asymmetric cycles ought to be considered. Here we shall focus on STAR models because of the small sample size of our data sets. We will briefly review such non-linear models.

4.1. Some Nonlinear Representations

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the severity of the business cycles between 1951 - 1984 in comparison with those undergone between 1871 and 1914. Greater severity of the busines cycle is found, without exception, in the interwar period.

§§§§§ *Early references on this are Mitchell [1927] and Keynes[1936].*

****** *Moreover, the methods currently used for solving general equilibrium stochastic models of business cycles rely on the fact that nonlinearities are not the dominant characteristic of the macroeconomics aggregates in order to approximate nonlinear models by using the first or second order Taylor series expansion .*

†††††† *Zarnowitz [1992, chapter 8], documents the existence of asymmetries in some US indexes of business activity between 1875 and 1933.*

First, the ARCH characterisation [Engle, 1982] accounts for persistence and clustering in conditional variance. Thus, for the error term in (1.15), e_t, we can write a *q*th order ARCH(*q*) model in multiplicative form as:

$$e_t = e_t h_t ; \quad h_t^2 = j_0 + j(L)e_t^2 = j_0 + \sum_{i=1}^{q} j_i e_{t-i}^2 \tag{1.16}$$

where $j_O > 0$, $j_i \geq 0$, and $\sum_q j_i < 1$ for $i > 0$, and the $\{e_t\}$ is *i.i.d*; e_t is white noise process such that Var(e_t)=1 and E(e_t)=0, and independent of e_{t-i}. Extensions of the original ARCH model include Bollerslev [1986], where the conditional variance is allowed to follow an ARMA process.

To show the second form, the Bilinear representation, we can write first the moving average representation of (1.15) as:

$$y_t = f^{-1}(L)q_0 + f^{-1}(L)\, q(L)e_t = a + y(L)e_t = a + \sum_{j=0}^{\infty} y_j e_{t-j} \tag{1.17}$$

where $DY_t = y_t$.

Taking the Volterra series expansion involving quadratic, cubic and higher order components yields the non-linear expression[‡‡‡‡‡‡]:

$$y_t = a + \sum_{j=0}^{\infty} y_j e_{t-j} + \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} y_{jk} e_{t-j} e_{t-k} + \sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty} y_{jkl} e_{t-j} e_{t-k} e_{t-l} + \ldots \tag{1.18}$$

The obvious difficulty of estimating an infinite number of parameters in the non-linear representation (1.18) has been overcome by approximating them by the bilinear model. A general form of it is:

$$y_t = d + \sum_{j=1}^{p} d_j y_{t-j} + e_t + \sum_{j=1}^{q} k_j e_{t-j} + \sum_{i=1}^{P}\sum_{j=1}^{Q} f_{ji} y_{t-i} e_{t-j} \tag{1.19}$$

which is a sum of an ARMA(p,q) process and bilinear terms involving products of lagged values of y_t and e_t. This model implies the estimation of $p+q+PQ$ coefficients, plus the variance of e [Granger and Terasvirta, 1993].

Third, the two-regime threshold autoregressive (TAR) model of order one and delay parameter equal to two, can be written as:

$$\begin{aligned} y_t &= b_1 y_{t-1} + m_t \quad if \quad y_{t-2} > 0; \\ y_t &= b_2 y_{t-1} + m_t \quad if \quad y_{t-2} \leq 0 \end{aligned} \tag{1.20}$$

where $\beta_1 \neq \beta_2$, so that the parameters of the autoregression vary according to the switching rule [see Tong, 1990].

Finally, the smooth transition autoregressive (STAR) model which we express as:

[‡‡‡‡‡‡] *The Volterra series expansion is a nonlinear generalization of the Wold representation.*

$$y_t = b_0 + \sum_{j=1}^{p} b_j y_{t-j} + (b_0^* + \sum_{j=1}^{p} b_j^* y_{t-j}) F(y_{t-d}) + e_t \qquad (1.21)$$

where y_t is stationary and e_t is an *i.i.d.* process with zero mean and finite variance. *F* is a transition function bounded by zero and one. In our testing strategy we will focus on two transition functions: The logistic function:

$$F(y_{t-d}) = (1 + \exp\{-g(y_{t-d} - c)\})^{-1}, \qquad g > 0 \qquad (1.22)$$

in which case (1.21) is called the logistic STAR (LSTAR) model, and the exponential function[§§§§§§]:

$$F(y_{t-d}) = 1 - \exp(-g(y_{t-d} - c)^2), \qquad g > 0 \qquad (1.23)$$

in which case (1.21) is called the exponential STAR (ESTAR) model.

Notice the monotonic change produced by y_{t-d} in the parameters of (1.21). Note also that when *g*→¥ in (1.22) and y_{t-d} >*c* then *F*=1, but when $c \geq y_{t-d}$, *F=0,* so that (1.21) collapses into a TAR model of order *p*. When *g*→*0* in (1.22), (1.21) becomes an AR(*p*) model. The LSTAR model can describe one type of dynamics for booming phases of an economy and another for slow-down ones. It can generate asymmetric realisations On the other hand, note that the ESTAR model becomes linear both when *g*→*0* and when *g*→¥ in (1.23). This model implies that contraction and expansion have similar dynamics [Terasvirta and Anderson, 1992].

Recent investigations show that nonlinearities are stronger in industrial production than in *GDP* [Granger and Terasvirta, 1993]. Peel and Speight [1995b], consider the simultaneous presence of nonlinearity in the conditional mean and the conditional variance of international industrial production in Germany, US, United Kingdom, Italy and Japan, as well as in sectoral production of the United Kingdom and US. They report strong evidence of joint-nonlinearity in the case of Italian and US industrial production, in US durables production and UK manufacturing and consumer goods and evidence of nonlinearity in conditional variance in UK industrial production and US manufacturing and non-durable production.

4.2. Testing Strategy

Since our aim here is to construct a STAR model, the strategy involves three steps[*******] which we describe next.

1. Carry out the complete specification of a linear AR(p) model. The maximum value of the lag *p* has to be determined from the data if the economic theory is not explicit about it. Michael, et al. [1996] use the partial autocorrelation function (PACF), but other techniques such as the information criterion can be employed. If the true model is non-linear, it is possible that the value selected for *p* is greater than the maximum in the non-linear model. This could reduce the power of the test compared to the case where the maximum lag is known. On the other hand,

[§§§§§§] *See Terasvirta [1994].*

if the selected value for p is too low, the estimated AR could have autocorrelated residuals. In this case, the test is biased against rejecting the non-linear model when the true model is linear [Terasvirta and Anderson, 1992].

2. Test linearity for different values of the delay parameter d. If linearity is rejected for more than one value of d, choose the one for which the P-value of the test is the lowest. Note that testing H_o: g= 0 in (1.21) - with either (1.22) or (1.23) -, assuming that y_t is stationary and ergodic[†††††††] under H_o, is a non-standard testing problem since (1.21) is only identified under the alternative H_1:γ ≠ 0. This problem is overcome by estimating the artificial regression:

$$y_t = p_{00} + \sum_{j=1}^{p}(p_{0j}y_{t-j} + p_{1j}y_{t-j}y_{t-d} + p_{2j}y_{t-j}y_{t-d}^2 + p_{3j}y_{t-j}y_{t-d}^3) + e_t \qquad (1.24)$$

and then testing the null H_O : $p_{1j}=p_{2j}=p_{3j}$ =0, (j=1,...,p), against the alternative that H_O is not valid. In practice the Lagrange multiplier (LM) test of linearity is replaced by an ordinary F-test in order to improve the size and power of the test[‡‡‡‡‡‡‡].

3. Treat the value of d as given and choose between ESTAR and LSTAR models. This is done by a sequence of tests nested in (1.24). Such a sequence is:

$$H_{O3} : p_{3j} = 0, \qquad j=1,..., p. \qquad (1.25)$$

$$H_{O2} : p_{2j} = 0| \; p_{3j} = 0 , j=1,..., p. \qquad (1.26)$$

$$H_{O1} : p_{1j} = 0| \; p_{2j} = p_{3j} = 0 , \quad j=1,..., p. \qquad (1.27)$$

and is based on the relationship between the parameters in (1.24) and (1.21) with either (1.22) or (1.23). For the ESTAR model p_{3j} = 0, j = 1,...., p, but p_{2j} ≠ 0 for at least one j if b_j^*≠0. For the LSTAR model p_{1j} ≠ 0 for at least one j if b_j^*≠0. If H_{O3} is rejected, a LSTAR model is selected. If H_{O3} is accepted and H_{O2} is rejected then an ESTAR model is selected. If H_{O3} and H_{O2} are accepted but H_{O1} is rejected a LSTAR model is selected. The only inconclusive case is when H_{O2} and H_{O1} are rejected. In this case we test:

$$H'_{O2} : p_{2j} = 0 \,|\, p_{1j} = p_{3j} = 0, \; j=1,..., p \; (1.28)$$

If H_{O2} is rejected then H'_{O2} should be rejected even more strongly. In any case, the decision is based on whether H_{O3}, H_{O2} or H_{O1} is rejected more strongly. Terasvirta [1994] found that the selection procedure works

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******* *These steps are explained in Terasvirta [1994]; Granger, Terasvirta and Anderson [1993] and elsewhere.*

††††††† *For satisfying this property new observations added to the sample bring useful information to the time average of a process (say x_t) since the values distant enough are almost uncorrelated. Thus, the time average $\bar{x}_n$ =1/n $\sum_t x_t$ is an unbiased and consistent estimate of the population mean m so that the var($\bar{x}_n$)¯0 as n® ¥ and E($\bar{x}$)=m, all n [Granger and Newbold, 1986, page 4-5].*

very well when the true model is LSTAR or ESTAR but in the latter case the observations have to be symmetrically distributed around *c*. When this is not the case, the ESTAR model can be approximated by a LSTAR model. However, another explanation for rejecting the ESTAR model more frequently is that the testing strategy could be biased against it by design. As a check for this possibility, Michael et al. [1996] add another *F*-test:

$$H_{OO}: p_{1j} = p_{3j} = 0, \qquad j = 1,..., p \qquad (1.29)$$

which they apply when modelling nonlinearities in deviations from PPP.

4.3. Results

We test for linearities in *GDP* and *GDPPC* in the two periods we have considered so far: 1925-1994 and 1950-1994. In addition, since applying the procedures requires stationary variables, we use two standard methods on the natural logs of output: first differences and the Hodrick-Prescott filter (HP). However, notice that only the AR(1) model of *GDP* between 1950 and 1994 presents a coefficient that is significant when the variables are first differenced (see table 1.5).

Here we consider a maximum delay of three periods. Evidence of nonlinearities is found only in *GPD* and *GDPPC* for the longer period when the variables are HP filtered[§§§§§§§]: they present the smallest *P*-value, for the *F*-test corresponding to testing the null $H_O : p_{1j} = p_{2j} = p_{3j} = 0$, (*j=1,...,p*), in (1.24). Moreover, from table 1.6 we can point out that the nonlinearity can be parameterized through a LSTAR model. In fact, the procedure fails to reject H_{O3} and H_{O2} but H_{01} is rejected. Furthermore, this selection seems adequate if we attend the test suggested by Michael et al. [1996], labelled H_{OO} following their notation. The null H_{OO} is rejected. The models estimated are:

$$y_t = 0.932\, y_{t-1} - (0.706\, y_{t-2}) * (1 + exp\{-1.035 * (y_{t-3})\})^{-1} + \hat{e}_t$$

(8.625) *(-3.286)* *(-1.198)*

se = 0.016 *DW = 1.977*

for *GDP*, and:

$$y_t = 0.917\, y_{t-1} - (0.698\, y_{t-2}) * (1 + exp\{-37.987 * (y_{t-3})\})^{-1} + \hat{e}_t$$

(8.215) *(-3.096)* *(-1.194)*

se = 0.017 *DW = 1.995*

[‡‡‡‡‡‡‡] *Recall that LM-type test is an asymptotic one which has better performance when the sample size is large.*

[§§§§§§§] *This gives rise to an issue to be investigated in the future: Does the HP filter introduce nonlinearities (asymmetries) to the variables? Considering this is extremely important due to the widespread use of the HP filter into the modern business cycle research.*

for *GDPPC*. The numbers in parenthesis are *t*-statistics, whereas *se* is standard error of estimate and *DW* is the Durbin-Watson statistic. The models produce a smaller standard error than the corresponding AR models. In both cases, the value of the ratio of the *se* corresponding to the non-linear model to the *se* corresponding to the linear one is 0.94. However, both the value of $\hat{g}$ and its *t*-statistic are rather low which could indicate that the nonlinearity is not strong.

5. Conclusions

In this paper we have considered the behaviour of output in two periods 1925-1994 and 1950-1994. *GDP* and *GDPPC* were both found to be integrated of order one. The sequences are highly persistent, specially in the period 1950-1994. The forecast error when an innovation of 1 percent enters into the economy is about 1.5 percent in the very long run, when we consider *GDP*. However, the response is about 1.3 percent when *GDPPC* is considered, which seems to give support to the idea that population growth is a source of nonstationarity in some macroeconomic aggregates.

However, for the larger sample (1925 - 1994) persistence is less. This result could cast some doubt on the method of estimation of *GDP* for the period 1925-1950. Finally, evidence of nonlinearity is found only in Hodrick-Prescott filtered variables dated between 1925 and 1994. This leaves open the question, in which the author is currently working, about whether the HP filter introduces nonlinearity in the high frequency variable that it generates. The type of asymmetric dynamics implied by the models we have fitted (LSTAR), suggests that the motion of Colombian output is different for booming and slow-down phases.

Table 1.1 Dickey-Fuller Test for Unit Roots

	Levels				*First Differences*		
	k	*a*	*b*	*l*	*k*	*a*	*l*
GDPPCS	1	2.11	2.15	-2.09	0	3.94	-5.07***
GDPPCL	1	2.80	2.79	-2.70	4	4.94	-5.39***
GDPS	3	1.13	1.50	-0.86	2	3.75	-3.91***
GDPL	4	2.50	2.32	-2.27	2	5.12	-5.30***

*NOTE: The values correspond to the t-statistics for a, b, and l in the ADF autoregression, $D Y_t = a + bt + l Y_{t-1} + \sum_{i=1}^{k} d \, D Y_{t-i} + u_t$. GDPPCL and GDPL correspond to 1925-1994, while GDPPCS and GDPS correspond to 1950-1994. *, **, and *** mean significantly different from zero with 90%, 95%, and 99% probability, respectively.*

Table 1.2 ARMA Models for DGDP and DGDPPC

Variable	*AR1*	*AR2*	*AR3*	*MA1*	*MA2*	*S E*	*Q[P]*
GDPS				0.31 (2.29)	0.53 (3.78)	0.015	8.76[0.46]
	0.35 (2.42)					0.015	10.66[0.30]
	0.36 (2.32)	0.22 (1.40)	-0.33 (-2.19)			0.015	5.43[0.60]
GDPPCS	0.231 (1.52)					0.018	6.03[0.74]
GDPL	0.156 (1.50)					0.020	9.77[0.87]
	-0.21 (-1.51)			0.49 (3.09)		0.020	12.62[0.63]
GDPPCL	-0.19 (-1.21)			0.42 (2.34)		0.020	11.15[0.74]

NOTE: GDPPCL and GDPL correspond to 1925-1994, while GDPPCS and GDPS correspond to 1950-1994; t-statistics in parenthesis.; SE is the standard error of the estimate. Q is the statistic of Ljung-Box, based on 10 lags, accompanied with the P-value in brackets.

Table 1.3 Accumulated Impulse Response of GDP and GDPPC

Variable	ARIMA Model	After 1 Period	After 2 Periods	After 3 Periods	After 4 Periods	After 5 Periods	After 10 Periods	After 20 Periods
GDPS	(0,1,2)	1.310	1.840	1.840	1.840	1.840	1.840	1.840
	(1,1,0)	1.352	1.476	1.520	1.543	1.543	1.543	1.543
	(3,1,0)	1.359	1.712	1.589	1.361	1.350	1.339	1.338
GDPPCS	(1,1,0)	1.231	1.285	1.297	1.300	1.301	1.301	1.301
GDPL	(1,1,0)	1.157	1.181	1.185	1.186	1.186	1.186	1.186
	(1,1,1)	1.256	1.194	1.209	1.207	1.207	1.207	1.207
GDPPCL	(1,1,1)	1.230	1.186	1.194	1.193	1.193	1.193	1.193

NOTE: GDPPCL and GDPL correspond to 1925-1994, while GDPPCS and GDPS correspond to 1950-1994.

Table 1.4 Non-parametric Measures of Persistence

	k-Years	2	3	5	10	20	30
GDPL	$Y(1)^k$	1.051	1.083	1.062	0.967	0.826	0.669
	V_k	1.079 (0.30)	1.145 (0.35)	1.100 (0.40)	0.912 (0.44)	0.665 (0.43)	0.437 (0.34)
GDPS	$Y(1)^k$	1.158	1.251	1.281	1.351	1.121	0.806
	V_k	1.178 (0.41)	1.376 (0.53)	1.441 (0.66)	1.689 (1.01)	1.105 (0.90)	0.570 (0.56)
GDPPCL	$Y(1)^k$	1.046	1.077	1.056	0.961	0.821	0.666
	V_k	1.059 (0.29)	1.097 (0.34)	1.018 (0.37)	0.702 (0.33)	0.387 (0.25)	0.366 (0.28)
GDPPCS	$Y(1)^k$	1.114	1.204	1.232	1.333	1.079	0.775
	V_k	1.115 (0.38)	1.218 (0.47)	1.179 (0.54)	1.133 (0.68)	0.487 (0.39)	0.346 (0.34)

NOTE: The suffixes L and S in GDP and GDPPC corresponds to the sample periods 1925-1994 and 1950-1994, respectively. Standard Error computed as V_k ´$[(0.75$ ´$(k+1)T]^{-1/2}$ [see Cochrane , 1988].

Table1.5 LinearityTest: P-values and Coefficients of AR Models

	1925 - 1994				*1950 - 1994*			
	GDP		*GDPPC*		*GDP*		*GDPPC*	
Delay	*D*	*HP*	*D*	*HP*	*D*	*HP*	*D*	*HP*
1	0.408	0.649	0.803	0.950	0.469	0.744	0.816	0.700
2	0.594	0.056	0.404	0.078	0.781	0.556	0.552	0.751
3	0.813	0.007	0.959	0.018	0.377	0.232	0.812	0.501
Order of AR model	1	2	1	2	1	2	1	2
Coefficients								
AR1	0.156 (1.50)	0.842 (7.97)	0.117 (1.11)	0.837 (7.75)	0.352 (2.42)	1.078 (7.14)	0.231 (1.52)	0.963 (6.40)
AR2		-0.264 (-2.69)		-0.249 (-2.46)		-0.379 (-2.58)		-0.328 (-2.18)
S E	0.020	0.017	0.021	0.018	0.015	0.013	0.015	0.015
D W	1.781	1.949	1.850	1.958	1.97	2.090	1.970	2.059

NOTE: D and HP represent first-differenced and Hodrick-Prescott filtered variables.

Table1.6. Test Selection of Non-linear Models

	1925 - 1994	
	GDP - HP	*GDPPC - HP*
Null Hypothesis	*d=3 p=2*	*d=3 p=2*
H_{03}	*0.229*	*0.353*
H_{02}	*0.132*	*0.231*
H_{01}	*0.004*	*0.005*
H_{00}	*0.001*	*0.004*
Suggested Model	*LSTAR*	*LSTAR*

NOTE: The table presents P-values of the F-tests. HP stands for Hodrick-Prescott Filtered variables.

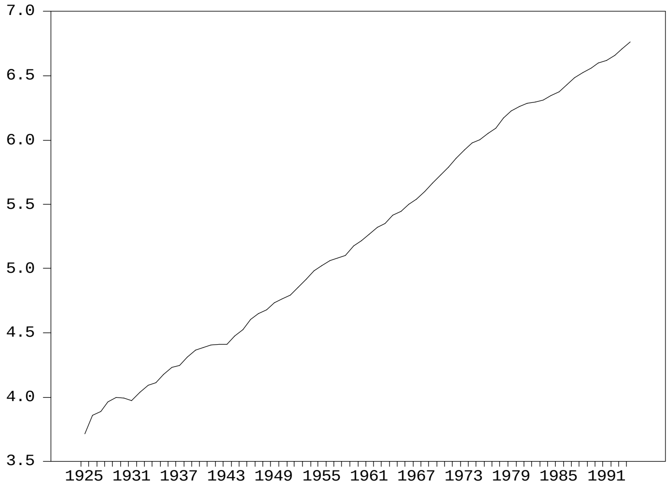

Figure 1.1. Logarithm of GDP: 1925 - 1994

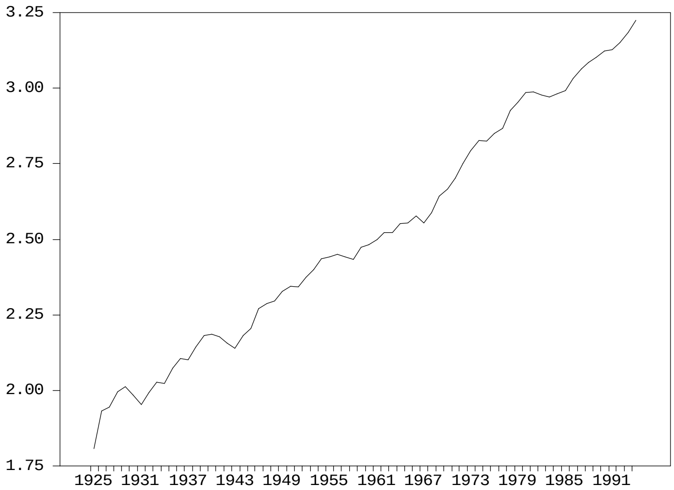

Figure 1.2. Logarithm of GDPPC: 1925 - 1994

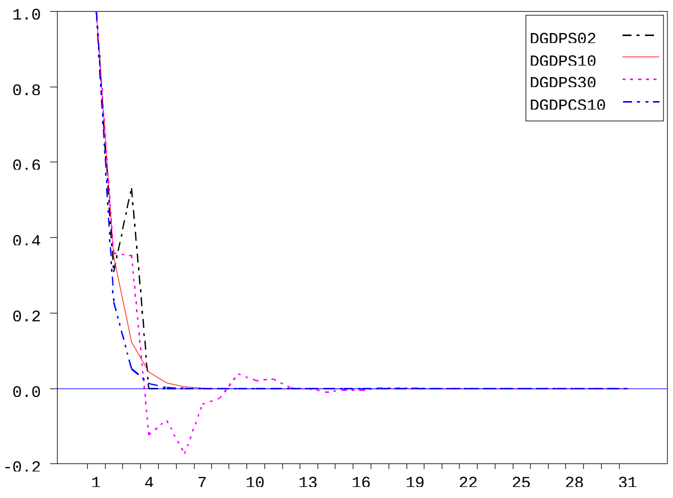

Figure 1.3 Impulse Response of DGDP and D GDPPC. 1950 - 1994

Note: DGDPS02 identifies the response computed from the ARIMA(0,1,2) specification of GDP, while DGDPS10 and DGDPS30 identify the responses implied by the ARIMA(1,1,0) and ARIMA(3,1,0) of the same variable. DGDPCS10 identifies the response computed from the ARIMA(1,1,0) for GDPPC.

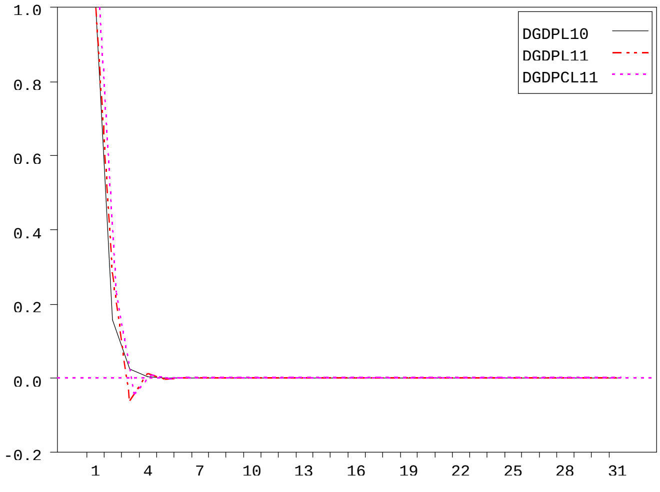

Figure 1.4 Impulse Response of DGDP and D GDPPC. 1925 - 1994

Note: DGDPL10 identifies the response computed from the ARIMA(1,1,0) specification of GDP, while DGDPCL11 identifies the responses computed from the ARIMA(1,1,1). DGDPCL11 identifies the response computed from the ARIMA(1,1,1) for GDPPC

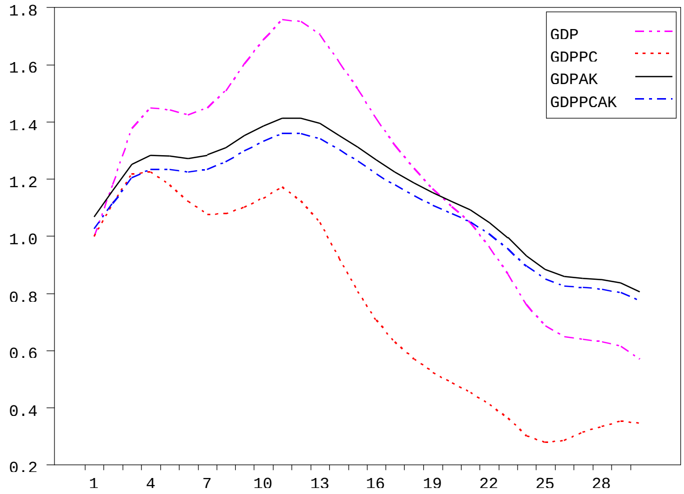

Figure 1.5 Persistence of GDP and GDPPC: 1950 - 1994

NOTE: The suffix AK in the keylabels in the figure identifies the nonparametric estimates of Campbell and Mankiw that we label y^k in the text. Thus GDPAK shows the behaviour of Campbell and Mankiw's measure of persistence for GDP.

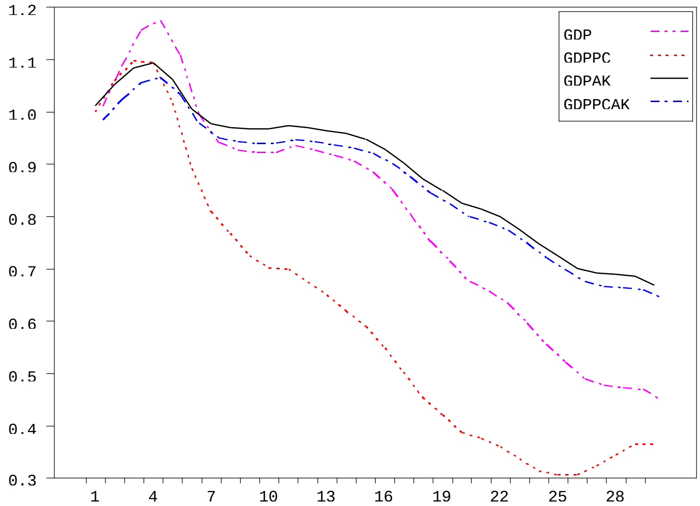

Figure 1.6 Persistence of GDP and GDPPC: 1925 - 1994

NOTE: The suffix AK in the keylabels in the figure identifies the nonparametric estimates of Campbell and Mankiw that we label y^k in the text. Thus GDPAK shows the behaviour of Campbell and Mankiw's measure of persistence for GDP.

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