

La serie "Borradores Semanales de Economía" es una publicación de la Subgerencia de Estudios Económicos del Banco de la República. Los Trabajos son de carácter provisional, las opiniones y posibles errores son responsabilidad exclusiva de los autores y sus contenidos no comprometen al Banco de la República ni a su Junta Directiva.

**GROWTH, WELFARE COSTS AND AGGREGATE  
FLUCTUATIONS IN ECONOMIES WITH MONETARY  
TAXATION**

**Por:  
Rodrigo Suescún M.**

**1995**

**No. 36**

Para comentarios favor dirigirse al autor:  
Fax: 2865936 - Teléfono 3421035.

**GROWTH, WELFARE COSTS AND AGGREGATE FLUCTUATIONS IN  
ECONOMIES WITH MONETARY TAXATION**

**Rodrigo Suescún M.\***

**Santafé de Bogotá, julio 1995**

---

\* Este artículo está basado en mi tesis doctoral en la Universidad de Pennsylvania. Agradezco a Alan Auerbach y Víctor Ríos-Rull por supervisar mi trabajo y por sus valiosas discusiones.

## **I. Introduction**

There is a large body of empirical literature devoted to study the relationship between inflation and long-run growth. Recently, Levine and Renelt (1992) encouraged by new developments in growth theory investigated, within a unified framework, the effect of a number of variables on per capita growth. The authors found that there was no robust relationship between the two variables. On the contrary, Fisher (1991, 1993) using the Levine and Renelt growth equation approach supports the conventional view that inflation is an important determinant of the rate of economic growth and that the effects of inflation are stronger at low and moderate inflation levels. Levine and Zervos (1992) included in the same framework an index of economic policy and concluded that growth and low inflation-low budget deficit are positively correlated. Additional evidence supporting a negative relationship between inflation and growth can also be found in De Long and Summers (1992) and De Gregorio (1993), among others.

The predominantly negative correlation between inflation and growth observed in the data has not been properly rationalized in models where identical agents behave rationally and where money has a significant impact on the evolution of real variables. In monetary versions of the neoclassical growth model the quantitative importance of money is quite modest inducing only small growth and welfare effects and playing almost no role in explaining the fluctuations of real variables. Because of the same reason, these models have not been successful at identifying a channel through which inflation plays a more meaningful role in the economy.

There are numerous plausible channels through which inflation may affect growth and welfare. However, the implications of many of them have not been fully explored or they simply have not been successful. Feasible channels are nominally denominated depreciation allowances, partially indexed tax bracketing, reserve requirement on bank deposits, investment purchases subject to cash-in-advance (CIA) constraint (Stockman, 1981), investment purchases and labor service payments subject to CIA constraint (Christiano, 1991), etc.

Nevertheless, as a result of this research program, the distorting effect of inflation on the labor-leisure choice has risen as the basic mechanism at work in monetary models. In models with no growth (Cooley and Hansen, 1989), inflation reduces labor effort through its effect on the return to working because part of the labor income has to be carried over, as cash balances, into the next period's cash-good trade. In models with endogenous growth (Gomme, 1993; Jones and Manuelli, 1993), inflation additionally affects the rate of utilization of human capital and thus, the rate of growth of the economy. Within the first type of models, the welfare cost of a 10% inflation rate was calculated in 0.4% of income; within the second, Gomme (1993) computes a welfare cost of less than 0.03% of income for a 8.5% inflation rate. This kind of evidence endorses the generally accepted conclusion that welfare costs of inflation are very small and that they are even smaller in models with endogenous growth<sup>1</sup>.

In this paper I explore one alternative avenue through which inflation can have real effects and estimate its quantitative importance. The assumption that taxes are directly collected in money is imposed to capture the real world feature that money is the required means of taxation payment. Most, if not all, of the literature has studied economies in which money exclusively has a private use (to buy goods or assets or factor payments) ignoring its public use in taxation and the fact that they are closely related in modern economic arrangements where the value of money is not tied down to gold or any other kind of backing. It has long been recognized that if the government "(...) declines to accept some kind of money in payment of obligations to itself, it is difficult to believe that it would retain much of its general acceptability. (...) Its general acceptability, which is its all-important attribute, stands or falls by its acceptability by the state" (Lerner, 1947). In

---

<sup>1</sup> A different strand of the literature -in economies where heterogeneous agents facing idiosyncratic risk (income variability) hold money to facilitate consumption smoothing- has found greater welfare costs. Imrohoroğlu (1992) estimates in 1.07% of total GNP the cost of a 10% inflation. In contrast, the paper presented adopts the transaction-based approach to motivate the demand for money in economies inhabited by identical agents.

consequence, it is natural to consider an economy in which money fulfills two functions: the government accepts money from households in the settlement of tax liabilities and money is used as a medium of exchange.

The paper is organized as follows. In sections 2 and 3 I study three model economies sharing the common features of steady state growth and tax payments explicitly modeled as a monetary obligation. I assume that taxes have to be paid with fiat money accumulated in advance. Welfare and growth effects of inflation are studied in an exogenous growth model, an AK model and an endogenous growth model with human capital accumulation. The principal finding is that the size of growth and welfare effects are higher than those found in comparable monetary models. In contrast to the existing literature, welfare costs are driven by the effect of inflation on the rate of growth instead of the effect on the labor-leisure choice. In an economy with monetary taxation, inflation strikes the growth rate directly through the after-tax real rate of return on investment. This is the same channel through which distortionary taxation has important real effects (Rebelo, 1991).

In section 4 a real business cycle model (RBC) with monetary taxation is parameterized, calibrated and simulated. I address the question of how the ability of the RBC model is affected when the tax payment technology is imposed. Section 5 extends the business cycle model to incorporate liquidity effects. The paper provides a "monetary" economy in which the observed labor market anomalies related to the correlation and relative volatility of hours worked and average productivity are not present. Section 6 presents a summary and conclusions.

## **II. The Basic Growth Model Economies**

In this section I characterize three monetary economies exhibiting growth in their deterministic steady states. Let us begin with the description of the general features shared by these economies. For the time being, assume that there is no uncertainty; agents are endowed with the natural gift of perfect foresight. Money is

valued in equilibrium because both the purchases of some goods -the so-called "cash goods" in the terminology of Lucas and Stokey (1983, 1987)- and tax payments are required to be carried out on a CIA basis. Each economy is composed of a government, a large number of firms and infinitely many homogeneous, infinite-lived households.

The government plays a trivial role in this setup. The tax policy is defined by a vector of three exogenous instruments  $(\bar{\mu}, \tau_H, \tau_K)$ , where  $\bar{\mu}$  is the gross rate of money growth,  $\tau_H$  is the tax rate on labor income while capital income is taxed at the rate  $\tau_K$ . Wealth effects and the role of government spending on both household's preferences and production possibilities are ignored by assuming that total revenues are rebated back to households in a lump-sum fashion. In equilibrium the corresponding lump-sum nominal transfer payments ( $v_t$ ) must satisfy the following government's budget constraint,

$$[1] V_t + T_t = (M_{t+1} - M_t) + \tau_K p_t (r_t - \delta^m) K_t^m + \tau_H p_t w_t N_t^m H_t$$

where  $p_t$  is the price level in period  $t$ ;  $r_t$  is the market return on capital;  $w_t$  is the real wage rate per efficiency unit of labor;  $K_t^m$ ,  $N_t^m$  and  $H_t$  are per capita, or aggregate, capital stock in the market sector which depreciates at the rate  $\delta^m$ , hours of work in that sector and human capital, respectively. Human capital is embodied in each worker and the endowment of time is normalized at one unit per worker, per period. The time endowment must be assigned to leisure ( $l_t$ ), to market work and - depending on the model economy analyzed- to the production of human capital.  $M_t$  is the per capita nominal stock of money carried over from the previous period to the beginning of the current one,  $t$ , and given  $M_0$ , its law of motion is expressed as follows:

$$[2] \cdot M_{t+1} = \bar{\mu} M_t = M_t + T_t$$

where newly created money is injected into the economy through lump-sum nominal transfers  $T_t$  per household at the beginning of the period.

Firms solve a standard profit maximization problem in which market consumption and investment goods are perfect substitutes in output. The production function is homogeneous of degree one in market capital and efficiency units of labor. In equilibrium profits are zero, factor prices are competitive and the number of firms is indeterminate so that it can be set to one without loss of generality.

### A. An Exogenous Growth Model with Monetary Taxation

This subsection presents a slightly modified version of the monetary model of Cooley and Hansen (1989) or its version with an additional fiscal sector (Cooley and Hansen 1991, 1992). The economy studied assumes divisible labor, steady state growth and, more importantly, monetary taxation. Cooley and Hansen (1991, 1992) include money and taxes in their model economies but implicitly assume that taxes can be paid with physical commodities; on the contrary, the transaction technology in this paper requires that previously accumulated money be used to settle tax liabilities.

The representative household maximizes its expected lifetime utility by choosing time paths for  $c_{1t}$ ,  $c_{2t}$ ,  $n_t^m$ ,  $x_t^m$ ,  $m_{t+1}$  and  $k_{t+1}^m$  subject to sequences of budget and liquidity constraints. Formally, the household's problem can be expressed as follows:

$$[P1] \text{ Max } \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, l_t)$$

Subject to:

$$[P1.1] \quad c_{1t} + \tau_K (r_t - \delta^m) k_t^m + \tau_H w_t n_t^m h_t \leq \frac{m_t + T_t}{p_t}$$

[P1.2]

$$c_{1t} + c_{2t} + x_t^m + \frac{m_{t+1}}{p_t} \leq (1 - \tau_K) r_t k_t^m + \tau_K \delta^m k_t^m + (1 - \tau_H) w_t n_t^m h_t + \frac{m_t + T_t + V_t}{p_t}$$

$$[P1.3] \quad 1 \geq l_t + n_t^m$$

$$[P1.4] \quad k_{t+1}^m \leq (1 - \delta^m) k_t^m + x_t^m$$

where  $c_{1t}$  and  $c_{2t}$  are the consumption of the cash and the credit-purchased goods;  $x_t^m$  is investment in physical capital. As usual, lower case letters, except for  $p_t$ ,  $w_t$  and  $r_t$ , stand for variables under the control of the household; the corresponding aggregates or economy-wide magnitudes are distinguished by capital letters. In equilibrium, aggregate consistency must be satisfied.

The first equation is the CIA constraint. Holdings of previously accumulated money balances and current lump-sum cash injections are required for purchases of cash goods and tax payments. This equation reflects the depreciation allowance built into the tax code; this treatment closes a potential route through which inflation may distort investment decisions when depreciation allowances are nominally denominated. The format of the liquidity constraint is based on the widely found idea in the CIA literature that current factor income earnings cannot be applied to current period consumption. This abstraction tries to capture real world circumstances where payments and receipts are not fully synchronized.



The second restriction is the household's budget constraint which specifies uses and sources of funds. Sources include after-tax labor and capital incomes, money balances carried over from the previous period and lump-sum money and tax transfer payments. Uses include purchases of the two consumption goods, expenditures on investment goods<sup>2</sup> and purchases of cash to be carried over into the next period. The third equation restricts the allocation of the time endowment. Equation [P1.4], given  $k_0^a$ , is the law of motion of physical capital; it takes one period to build productive capital from new investment.

Due to nonsatiation the household's budget constraint holds as equality while the CIA constraint binds if the nominal interest rate,  $R_t$ , is positive. An expression for the nominal interest rate can be obtained if a one-period nominal bond with return taxed at a rate  $\tau_k$  is introduced. From the efficiency conditions associated with the household's optimal decisions about how much to save in bonds and to consume of  $c_{1,t}$ , the following usual expression equating the costs and benefits of the saving decision can be obtained:

$$[3] \quad 1 + R_t(1 - \tau_k) = \frac{1}{\beta} \frac{p_{t+1}}{p_t} \frac{U_1(t)}{U_1(t+1)}$$

where  $U_1(t)$  is the derivative of the utility function with respect to its first argument; derivative evaluated at the period  $t$  optimal plan. The first order conditions (FOC's) associated with problem [P1] yield the following steady state relation among real returns on nominal and physical assets:

$$[4] \quad \frac{1 + R(1 - \tau_k)}{p/p_{-1}} = \frac{1 + r - \delta^m}{1 + (p/p_{-1})(r - \delta^m)\tau_k}$$

---

<sup>2</sup> Note that the investment good is a credit good.

where  $p/p_{-1}$  is the limiting gross rate of inflation.

In absence of monetary taxation, i.e., when [P1.1] in problem [P1] is rewritten to impose the CIA constraint only on consumption purchases, the Euler equations of the problem yield the following version of expression [4]:  $\frac{1 + R(1 - \tau_k)}{p/p_{-1}} = 1 + (1 - \tau_k)(r - \delta^m)$ . Then, the Fisher equation is satisfied in the sense that the nominal interest rate is made up of two elements: an inflation effect and a standard real return component. When the CIA constraint applies only to consumption purchases, as in Cooley and Hansen (1989, 1991, 1992) and Gomme (1993), the impact of inflation on the economy is propagated through its distorting effect on the labor-leisure choice, which in turn indirectly affects the marginal product of capital.

In the model with monetary taxation, inflation affects indirectly the real return on capital as well, but now, according to [4], there exists a direct channel through which the net-of-all-tax real rate of return on investment is reduced. Inflation represents an additional tax on capital income because agents are forced to hold money to settle tax liabilities.

Growth occurs at a exogenous rate  $\gamma$  given by the gross rate of growth of the stock of human capital,  $H_{t+1} = \gamma H_t$ . For the time being, I focus on steady state the implications of the models. King, Plosser and Rebelo (1988) specify restrictions on technologies and preferences in order to make steady state growth a feasible outcome. A Cobb-Douglas technology is assumed,  $F(K^m, HN^m) = K^\theta (HN^m)^{1-\theta}$ , and among the class of instantaneous utility functions that avoid leisure to growth along the growth path, adopt the following:

$$[5] U(c_1, c_2, l) = a \text{LOG}(c_1) + (1 - a) \text{LOG}(c_2) + B \text{LOG}(l)$$

To facilitate solving for an equilibrium and the comparison among economies exhibiting steady state growth, assume that household's human capital is along its

equilibrium path and transform variables to render the economy stationary. The symbol  $\hat{\cdot}$  denotes transformed variables. Let  $\hat{p}_t = \frac{p_t H_t}{M_{t-1}}$ ,  $\hat{m}_t = \frac{m_t}{M_t}$  and let the remaining nonstationary variables be expressed relative to  $H_t$ , the equilibrium stock of human capital; for example,  $\hat{c}_{1t} = \frac{c_{1t}}{H_t}$ . The transformed economy is the center of this part of the analysis.

Now focus on the firm's behavior. The firm seeks to maximize profit,  $\Pi_t = p_t F(K_t^m, H_t N_t^m) - p_t w_t H_t N_t^m - p_t r_t K_t^m$ , taking as given the wage rate and the rental rate on capital services. The FOC's for the transformed problem imply factor prices equal marginal products:

$$[6] \quad w_t = w(\hat{K}_t^m, N_t^m) = (1 - \theta) \left[ \frac{\hat{K}_t^m}{N_t^m} \right]^\theta$$

$$[7] \quad r_t = r(\hat{K}_t^m, N_t^m) = \theta \left[ \frac{N_t^m}{\hat{K}_t^m} \right]^{(1-\theta)}$$

Now formally define an equilibrium for the exogenous growth model economy with monetary taxation:

**Definition:** A Stationary Competitive Equilibrium is a sequence of prices and factor prices  $(\hat{p}_t, r_t, w_t)_{t=0}^\infty$ , individual household's real and monetary allocations  $(\hat{c}_{1t}, \hat{c}_{2t}, \hat{x}_t^m, \hat{k}_{t+1}^m, \hat{m}_{t+1}^m)_{t=0}^\infty$  and aggregate outcomes  $(\hat{c}_{1t}, \hat{c}_{2t}, \hat{x}_t^m, \hat{k}_{t+1}^m)_{t=0}^\infty$  such that:

- a) Given prices and factor prices, the sequence of household's allocations solve the consumer's maximization problem;
- b) Factor prices satisfy [6] and [7];

c) Aggregate consistency is satisfied:  $\hat{m}_{t+1} = 1$ ,  $\hat{c}_{1t} = \hat{c}_{1t}$ ,  $\hat{c}_{2t} = \hat{c}_{2t}$ ,  $\hat{x}_t = \hat{x}_t$  and  $\hat{k}_t^m = \hat{k}_t^m$ , for all  $t$ .

d) Government budget balance.

The steady state of the economy is derived from the necessary conditions of the household's problem that any interior equilibrium must satisfy:

$$[\text{SYS1.1}] \hat{C}_2 = \left( \frac{1 - a}{a} \right) \frac{\bar{\mu}}{\beta} \hat{C}_1$$

$$[\text{SYS1.2}] \frac{1}{\hat{p}} = \hat{C}_1 + \tau_K \hat{K}^m \left\{ \theta \left[ \frac{N^m}{\hat{K}^m} \right]^{1-\theta} - \delta^m \right\} + \tau_H N^m (1 - \theta) \left[ \frac{\hat{K}^m}{N^m} \right]^\theta$$

$$[\text{SYS1.3}] \hat{X}^m = \hat{K}^m (\gamma - 1 + \delta^m)$$

$$[\text{SYS1.4}] \hat{C}_1 = \frac{a}{B} \left( 1 - \frac{\bar{\mu}}{\beta} \tau_H \right) \frac{\beta}{\bar{\mu}} (1 - N^m) (1 - \theta) \left[ \frac{\hat{K}^m}{N^m} \right]^\theta$$

$$[\text{SYS1.5}] \frac{\gamma}{\beta} - 1 = \left( 1 - \frac{\bar{\mu}}{\beta} \tau_K \right) \left\{ \theta \left[ \frac{N^m}{\hat{K}^m} \right]^{1-\theta} - \delta^m \right\}$$

$$[\text{SYS1.6}] \hat{C}_1 \left[ 1 + \left( \frac{1 - a}{a} \right) \frac{\bar{\mu}}{\beta} \right] = [\hat{K}^m]^\theta [N^m]^{1-\theta} - \hat{X}^m$$

The system determines  $\hat{c}_1$ ,  $\hat{c}_2$ ,  $\hat{x}^m$ ,  $\hat{k}^m$ ,  $N^m$  and  $\hat{p}$  as functions of the parameters in the model and allows the study of the long-run effect of inflation-cum-money growth on the economy. The Friedman's optimal money rule is valid in this economy. When the limiting growth rate of money converges to  $\beta$ , it is possible to show that the cash-in-advance constraint is not binding and the equilibrium

allocations are identical to those obtained in an economy without money. However, the latter are not Pareto optimal allocations due to the presence of distortionary taxation.

## B. An AK Growth Model with Monetary Taxation

The welfare costs of inflation could be different in a growth model with endogenous growth. The intuition is that if monetary policy may affect growth, the welfare costs could be expected to be greater; however, as shown by Gomme (1993), this intuition is not necessarily true. The simplest endogenous growth model is the so-called AK model used by Rebelo (1991). It is a one-sector economy with a technology linear in the capital stock,  $F(K^m) = AK^m$ . The labor supply decision is ignored and, in contrast to the previous model, the distortion of the labor-leisure choice is now absent. The welfare effects of inflation are transmitted through the distorting impacts on the relative consumption of cash and credit goods and the growth rate.

The representative consumer in this economy with monetary taxation solves:

$$[P2] \text{ Max } \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})$$

Subject to:

$$[P2.1] c_{1t} + \tau_K(r_t - \delta^m)k_t^m \leq \frac{m_t + T_t}{p_t}$$

$$[P2.2] c_{1t} + c_{2t} + x_t^m + \frac{m_{t+1}}{p_t} \leq (1 - \tau_K) r_t k_t^m + \tau_K \delta^m k_t^m + \frac{m_t + T_t + V_t}{p_t}$$

$$[P2.3] k_{t+1}^m \leq (1 - \delta^m)k_t^m + x_t^m$$

where  $k_0^m$  and  $m_0$  are given.

The firm maximizes profits  $\pi_t = p_t (c_{1t} + c_{2t} + X_t^m) - p_t r_t K_t^m$  subject to  $c_{1t} + c_{2t} + X_t^m \leq A K_t^m$ . The first order condition for this problem yields:  $r_t = A$ , for all  $t$ .

Preferences are further specialized to be consistent with the absence of the labor-leisure choice.

$$[8] U(c_1, c_2) = a \text{ LOG}(c_1) + (1 - a) \text{ LOG}(c_2)$$

Equilibrium is defined in an analogous fashion as in the previous section. To facilitate solving for an equilibrium, assume that the household's capital is along the equilibrium path and nonstationary variables are expressed relative to  $K_t^m$ , to render the problem stationary. Transformed prices are obtained as follows,  $\bar{p}_t = \frac{p_t K_t^m}{M_{t,1}}$ .

Assuming that the cash-in-advance constraint is binding, the variables  $\hat{c}_1$ ,  $\hat{c}_2$ ,  $\hat{x}^m$ ,  $\gamma$  and  $\bar{p}$  must satisfy the following set of conditions in the steady state:

$$[\text{SYS2.1}] \hat{c}_2 = \left( \frac{1 - a}{a} \right) \frac{\bar{\mu}}{\beta} \hat{c}_1$$

$$[\text{SYS2.2}] \bar{p} = \left[ \hat{c}_1 + \tau_K (A - \delta^m) \right]^{-1}$$

$$[\text{SYS2.3}] \hat{x}^m = \gamma - 1 + \delta^m$$

$$[\text{SYS2.4}] \gamma = \beta \left[ 1 + \left( 1 - \frac{\bar{\mu}}{\beta} \tau_K \right) (A - \delta^m) \right]$$

$$[\text{SYS2.5}] \hat{c}_1 + \hat{c}_2 + \hat{x}^m = A$$

where  $\gamma$  is the limiting gross rate of growth of the economy. Equation [SYS2.1] shows the make-up of consumption as function of the limiting growth rate of money.

When  $\bar{\mu} = \beta$ , the cash-in-advance constraint is not binding and the ratio  $\bar{c}_2 / \bar{c}_1$  is determined by preferences: by the relative weights on credit and cash goods in the household's preferences<sup>3</sup>. When the nominal interest rate is positive,  $\bar{\mu} > \beta$ , a binding cash-in-advance constraint increases the relative consumption of the credit good over and above of what is dictated by preferences. Equation [SYS2.4] expresses the limiting gross rate of growth of the economy as function of the limiting rate of money growth, the marginal tax rate and parameters of taste and technology like the net marginal product of capital  $(A - \delta^m)$  and the discount factor.

If the liquidity constraint applies only to consumption purchases, as in its standard version, the limiting rate of growth is given by  $\gamma = \beta [1 + (1 - \tau_k)(A - \delta^m)]$ ; the asymptotic growth rate of the economy (and the real interest rate) is independent of the rate of growth of the money supply. Inflation may affect welfare through the composition of consumption expenditures but not the long-run growth rate. With monetary taxation, equation [SYS2.4], inflation increases the cost of taxation and reduces the return on investment and the rate of growth of the economy.

Equation [SYS2.2] is the CIA constraint; equation [SYS2.5] is the resource constraint and equation [SYS2.3] determines the steady state level of investment.

### **C. A Growth Model through Human Capital Accumulation**

Inflation distorts the labor-leisure choice. In the first model presented there is labor supply decision but growth is given exogenously. On the contrary, the second model allows for endogenous growth but there is no labor-leisure choice. A natural extension of the analysis is to study a model economy exhibiting both features. In such a model with human capital accumulation, that distortion can have growth effects by altering the rate of utilization of the stock of human capital. Adding

---

<sup>3</sup> See Ireland (1994) and Gillman (1993) for models in which the distinction between cash-purchased and credit-purchased goods is not tied down to preferences but determined endogenously.

monetary taxation growth effects are reinforced because inflation reduces both the real return on human and physical investment.

Consider a two-sector growth model of the class studied by Uzawa (1965), Lucas (1988) and Rebelo (1991). Consumption and investment goods are produced in the market sector while human capital production is modeled as a nontaxed household activity. Both production sectors use constant-return-to-scale technologies that combine physical capital and efficiency units of labor. Following Ben-Porath (1967), the production of human capital is assumed to require physical capital. Formally, the problem that the household must solve is:

$$[P3] \text{ Max } \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, l_t)$$

Subject to:

$$[P3.1] c_{1t} + \tau_K (r_t - \delta^m) k_t^m + \tau_H w_t n_t^m h_t \leq \frac{m_t + T_t}{p_t}$$

$$[P3.2] c_{1t} + c_{2t} + x_t^m + x_t^h + \frac{m_{t+1}}{p_t} \leq (1 - \tau_K) r_t k_t^m + \tau_K \delta^m k_t^m +$$

$$(1 - \tau_H) w_t n_t^m h_t + \frac{m_t + T_t + V_t}{p_t}$$

$$[P3.3] 1 \geq l_t + n_t^m + n_t^h$$

$$[P3.4] k_{t+1}^m \leq (1 - \delta^m) k_t^m + x_t^m$$

$$[P3.5] k_{t+1}^h \leq (1 - \delta^h) k_t^h + x_t^h$$

$$[P3.6] h_{t+1} \leq (1 - \delta) h_t + s [k_t^h]^\alpha [n_t^h h_t]^{1-\alpha}$$



where the notation and interpretation are similar to those in the first model.  $s$  is a scale parameter;  $k_t^m$  is the stock of physical capital in the market sector;  $k_t^h$  is the stock of physical capital allocated to the household activity and  $h_t$  is the household stock of human capital.  $\delta^m$ ,  $\delta^h$  and  $\delta$  are the corresponding depreciation rates and equations [P3.4], [P3.5] and [P3.6] are the corresponding laws of motion with  $x_t^h$  as physical investment in human capital.  $n_t^m$  is the fraction of the time endowment devoted to the market sector which combined with the stock of human capital,  $h_t$ , yields  $n_t^m h_t$  efficiency units of labor in that sector.  $n_t^h h_t$  efficiency units of labor are devoted to the production of human capital. The allocation of the household's time endowment determines the allocation of the stock of human capital, due to the embodiment assumption, and its rate of utilization. The last term on the right-hand side of equation [P3.6] can be interpreted as the human capital investment good.

The firm in the market sector maximizes profits, given by:  $\Pi_t = p_t (C_{1t} + C_{2t} + X_t^m + X_t^h) - p_t r_t K_t^m - p_t w_t N_t^m H_t$  subject to the constraint  $C_{1t} + C_{2t} + X_t^m + X_t^h \leq [K_t^m]^\theta [N_t^m H_t]^{1-\theta}$  and taking  $r_t$ ,  $w_t$  and  $p_t$  as given.

Preferences are again described by [5]. Preferences and technologies are consistent with steady state growth. To have a stationary representation, variables are transformed as in the first model. After manipulating the first order conditions, steady states values for  $\hat{C}_1$ ,  $\hat{C}_2$ ,  $\hat{X}^m$ ,  $\hat{X}^h$ ,  $N^m$ ,  $N^h$ ,  $\hat{K}^m$ ,  $\hat{K}^h$ ,  $\hat{P}$  and  $\gamma$  are obtained from the following nonlinear system of equations:

$$[\text{SYS3.1}] \hat{C}_2 = \left( \frac{1-a}{a} \right) \frac{\bar{\mu}}{\beta} \hat{C}_1$$

$$[\text{SYS3.2}] \frac{1}{\hat{P}} = \hat{C}_1 + \tau_K \hat{K}^m \left\{ \theta \left[ \frac{N^m}{\hat{K}^m} \right]^{1-\theta} - \delta^m \right\} + \tau_H N^m (1-\theta) \left[ \frac{\hat{K}^m}{N^m} \right]^\theta$$

$$[\text{SYS3.3}] \hat{X}^m = \hat{K}^m (\gamma - 1 + \delta^m)$$

$$[\text{SYS3.4}] \hat{X}^h = \hat{K}^h (\gamma - 1 + \delta^h)$$

$$[\text{SYS3.5}] \hat{C}_1 = \frac{a}{B} \left( 1 - \frac{\bar{\mu}}{\beta} \tau_H \right) \frac{\beta}{\bar{\mu}} (1 - N^m - N^h) (1 - \theta) \left[ \frac{\hat{K}^m}{N^m} \right]^\theta$$

$$[\text{SYS3.6}] \frac{\gamma}{\beta} - 1 = \left( 1 - \frac{\bar{\mu}}{\beta} \tau_K \right) \left\{ \theta \left[ \frac{N^m}{\hat{K}^m} \right]^{1-\theta} - \delta^m \right\}$$

$$[\text{SYS3.7}] \frac{\gamma}{\beta} - 1 + \delta^h = \left( 1 - \frac{\bar{\mu}}{\beta} \tau_H \right) (1 - \theta) \left[ \frac{\hat{K}^m}{N^m} \right]^\theta \left( \frac{\alpha}{1 - \alpha} \right) \left[ \frac{N^h}{\hat{K}^h} \right]$$

$$[\text{SYS3.8}] \frac{\gamma}{\beta} - 1 + \delta = s (N^m + N^h) (1 - \alpha) \left[ \frac{\hat{K}^h}{N^h} \right]^\alpha$$

$$[\text{SYS3.9}] \hat{C}_1 \left[ 1 + \left( \frac{1 - a}{a} \right) \frac{\bar{\mu}}{\beta} \right] + \hat{X}^m + \hat{X}^h = [\hat{K}^m]^\theta [N^m]^{1-\theta}$$

$$[\text{SYS3.10}] \gamma - 1 + \delta = s [\hat{K}^h]^\alpha [N^h]^{1-\alpha}$$

In this economy the CIA constraint is binding if the nominal interest rate is positive ( $\bar{\mu} > \beta$ ). When the growth rate of money converges to  $\beta$ , the liquidity constraint is not binding and allocations and the steady state growth rate are equal to those found in a nonmonetary economy.

### III. Welfare and Growth Implications of Inflation

In this section the experiment that I consider is the calculation of the steady state effects of inflation on growth rates and welfare for each of the three growth model economies. To compute the equilibrium and the quantitative effects under different growth rates of the money supply, it is required, first, to assign values to

the parameters included in each model and, second, to define a comparable measure of welfare costs.

### A. Calibrating the Three Model Economies

The three growth models are fully parameterized with the following set of 14 parameters:

Preferences:  $\beta, a, B$

Technology:  $\theta, \alpha, A, s$

Depreciation rates:  $\delta^m, \delta^h, \delta$

Tax rates:  $\tau_K, \tau_H$

Gross rates of growth of output and money:  $\gamma, \bar{\mu}$

It is standard in the literature on business cycles, since the paper of Kydland and Prescott (1982), to choose parameter values based on prior information and first moments of the data. The strategy followed here is to pick parameter values that are consistent with the model economies satisfying certain similar quantitative targets. By imposing these quantitative targets on the sets of FOC's [SYS1], [SYS2] and [SYS3] it is possible to assign values to the parameters included in each model. The following calibration targets are imposed:

- ◆ Gross rate of growth of per capita output,  $\gamma$ . It is set to 1.0035. This number is obtained using information from the NIPA and Survey of Current Business. GNP data were adjusted to include imputed services from both the stock of consumer durables and government capital and expressed in per capita terms by using the 16+ population<sup>4</sup>. This procedure yields an average quarterly growth rate of output of 0.35% for the 1954-1990 period. Note that the length of a period in the model economies has been made equal to a quarter.

---

<sup>4</sup> Civilian non-institutional population (16 years and over).

◆ Capital-output ratio. The average capital-output ratio is 3.1 according to the Economic Report of the President. This implies an approximate ratio of 12.4 if quarterly output is used in the computation.

◆ Consumption-output ratio. This ratio has been estimated in 0.728 in studies where output is supposed to be made up of consumption and investment components only. This corresponds to the ratio of consumption to the sum of consumption and investment.

◆ Money-output ratio. Money in the model economies resembles a monetary aggregate like M1. Using this definition of money, the average value of the ratio for the 1959:Q2 to 1990:Q4 period is 0.7584.

◆ Limiting gross rate of money growth,  $\bar{\mu}$ . This figure is 1.01, which roughly matches the observed quarterly growth rate of per capita M1 for the period from 1959:Q2 to 1990:Q4.

◆ Capital share in market output,  $\theta$ . By considering the imputed services from the stocks of consumer durables and government capital as payments to capital, and following the methodology described in Cooley and Prescott (1994) to distribute NIPA's proprietors income between labor and capital payments, an average value for  $\theta$  of 0.3715 was calculated for the 1954-1990 period. This figure is in between the 0.36 used, among others, by Kydland and Prescott (1982) and Cooley and Hansen (1991, 1992) and the 0.40 calculated by Cooley and Prescott (1994) and within the 0.25 to 0.43 plausible range computed by Christiano (1988).

◆ Marginal tax rates on capital and labor income,  $\tau_k$  and  $\tau_w$ . They can be calculated from Auerbach (1983), Joines (1981) and Barro and Sahasakul (1986) among others. The new time series recently constructed by Prakken, Varvares and Meyer (1991) is the source of the estimates used here.  $\tau_k$  set equal to 0.475 and  $\tau_w$  set equal to 0.28 correspond to the average effective marginal tax rates on corporate income and on wages and salaries, respectively, for the 1954-1988 period.

♦ Fraction of time devoted to market work,  $N^a$ . Greenwood and Hercowitz (1991) estimate in 0.24 the average ratio of total hours worked to total nonsleeping hours of the working age population.

♦ Fraction of time devoted to learning (education)  $N^h$ . This number has been approximately set to 0.10 based on the discussion and figures provided by Ríos-Rull (1993).

♦ After-tax real rate of return. I use the same number used by Fullerton and Rogers (1993). The annual rate is 4%. This target is employed in the calibration of the two models with endogenous growth; but it is not necessary in calibrate the exogenous one.

Finally, the rate of depreciation of physical capital devoted to the production of human capital, in the model with human capital accumulation,  $\delta^h$ , was assumed to be equal to  $\delta^a$ .

Table 1 summarizes the implicit or consistent parameter estimates grouped for each of the model economies considered.

## B. Measuring Welfare Costs

The welfare costs of inflation are calculated as the value of  $\lambda$  that solves the following nonlinear equation:

$$[9] \ 0 = \frac{1}{1 - \beta} \left[ a \left\{ \log(\hat{C}_1^a) - \log(\hat{C}_1^z + \lambda \hat{Y}^z) \right\} + (1 - a) \left\{ \log(\hat{C}_2^a) - \log(\hat{C}_2^z + \lambda \hat{Y}^z) \right\} \right] \\ + \frac{B}{1 - \beta} \left[ \log(L^a) - \log(L^z) \right] + \frac{\beta}{(1 - \beta)^2} \left[ \log(Y^a) - \log(Y^z) \right]$$

where  $\hat{Y}^z$  stands for the steady state level of market output.  $\lambda \hat{Y}^z$  is interpreted as the increase in consumption required to make the household as well off under the monetary policy "z" as under the Friedman (1969) optimum of deflation. The steady

state allocations under the first regime are distinguished by the superscript "z" while those under the optimal policy  $\bar{\mu} - \beta$  by the symbol "\*\*". The latter allocations are not Pareto optimal. To facilitate the comparison among model economies and with other results in the literature, the required increase in consumption is expressed as percentage of the steady state distorted output, as  $\lambda$ .

Note that equation [9] simply equals to zero the difference between the steady state lifetime utilities obtained from an economy under the Friedman's money rule and an economy with binding CIA constraint and compensated with a consumption gift ( $\lambda \hat{Y}^z$ ). The infinite sum involved in the computation of the lifetime utility converges when the variables defining the utility function are expressed in stationary form; when a steady state exists. The transformation of variables required to achieve a stationary representation gives rise to the term including the growth rate of the economy on the right side of [9].

With the help of equation [9], it is possible to express the total cost of inflation as the approximate sum of three forces: the consumption, leisure and the rate of growth effects. The consumption effect, for example, is calculated as the value of  $\lambda$  that solves the nonlinear equation [9] when the second and third terms in square brackets, on the right-hand side of [9], are set equal to zero. Thus, the consumption effect corresponds to the increase in consumption required to compensate the household only for the distortive effect on consumption of implementing the monetary policy "z" instead of the  $\bar{\mu} - \beta$  policy. Note that the decomposition of welfare costs is less accurate when the steady state of the alternative regime moves farther from that of the base optimal policy.

### **C. Implications of Inflation**

In this section, steady state growth and welfare implications of inflation are presented. Welfare costs, their decomposition, and limiting growth rates of output under alternative money growth rates are presented in table 2. Panel A shows the results for the exogenous growth model. Welfare costs are increasing in the rate of

inflation, as expected. The standard for comparisons in the literature is the welfare loss arising from a 10% inflation rate. A 10% inflation rate in the exogenous growth model with monetary taxation has a welfare cost equal to 0.51% of GNP. This number is not very different from previous findings. In comparable models, Cooley and Hansen (1989) found a cost of 0.38% and Cooley and Hansen (1991), in an economy with capital and labor income taxation, estimated the loss in 0.57% of GNP. On the other hand, within a partial equilibrium framework and assuming that money is superneutral, Fisher (1981) measured the Bailey (1957) area under the inverse demand-for-money schedule to calculate the deadweight loss of anticipated inflation in 0.38% of GNP; Lucas (1981) estimated it in 0.70%.

The driving force explaining this cost is the inefficiency of the substitution away from cash goods and work effort and toward credit goods, to avoid the inflation tax. The consumption effect costs 1.17% of GNP. In absence of a growth effect, the net effect of inflation depends on the interaction between the consumption and leisure effects. The inflation-induced substitution toward leisure and away from work effort and cash goods, allows for a welfare gain of 0.66% of GNP, that partially compensates the distorting effect on relative consumption expenditures.

In the results presented in panel B inflation is allowed to have growth effects but labor supply decisions are ignored. The AK model systematically exhibited greater welfare costs than the exogenous growth model. It suggests that once a growth rate effect is allowed, the magnitude of the costs of inflation turns out to become more meaningful. In this model, a 10% inflation rate yields a loss of 0.63% of income. It is worth noting that in contrast to the previous model, the consumption effect yields a benefit of 0.42% while the losses are explained by the effect of inflation on the rate of growth of the economy (1.04% of GNP). Welfare measures are sensitive to changes in the growth rate. Under the optimal money rule, the steady state rate of growth of the per capita output is 1.46% per year, while it decreases to 1.33% when the rate of inflation is 10%.

Panel C reports the results for the endogenous growth model through human capital accumulation. Now inflation is allowed to affect welfare and growth through the three dimensions mentioned earlier. The welfare loss of a 10% inflation rate is enormous -relative to existing evidence-, about 1.52% of GNP. The consumption effect does not make a quantitatively vital contribution to this result; it is driven by the effect of inflation on the limiting rate of growth of the economy which explains a loss of 3.18%. It is partially compensated by a gain of 1.63% coming from the leisure effect.

The cost of inflation is now 50 times bigger than that estimated in an comparable model. Gomme (1993) in a model with human capital accumulation and a standard cash-in-advance constraint estimated the welfare cost of a 8.5% inflation rate in less than 0.03% of income; the small size of this loss is explained in his model by a sizable gain from the leisure effect. When monetary taxation is introduced, welfare costs increase substantially and the force driving the result switches from the labor-leisure dimension to the rate of growth effect.

An interesting conclusion from Gomme's (1993) work is that it is not generally true the intuitive idea according to which larger welfare costs are expected to show up if public policies can affect the growth rate of the economy. The author found that the welfare costs of inflation in the endogenous growth model were lower than in its exogenous counterpart. He argues that this conclusion is not only robust to changes in parameters but to the form of the CIA constraint (p.71), as well. On the contrary, I found here that changing the form of the CIA constraint, to incorporate monetary taxation, the welfare costs of inflation in an endogenous growth model are no longer smaller.

#### **D. Real Effects and Welfare Costs of Taxation**

An important growth rate effect is expected to show up when considering the welfare costs of distortional taxation because it operates through the same channel



as inflation does in a model with monetary taxation: affecting the net-of-all-tax real rate of return on physical and human capital investment.

Table 3 presents the steady state growth rates of per capita output and welfare costs associated with alternative tax policies. In each panel welfare costs are calculated comparing economies under the same monetary regime, under the optimal rate of money growth, but they face different tax vectors. In the base economy there are no taxes. Obviously, the base of comparison are Pareto optimal allocations. The welfare cost of the tax policy  $\tau_K = 0.475$  and  $\tau_H = 0.28$  is 3.86% of GNP for the exogenous growth model, 14.45% for the AK model and 17.21% for the two-sector growth model. The result that public policies have greater effects in endogenous growth models is verified again. In the models with endogenous growth welfare costs are driven by the effect of anticipated inflation on the rate of growth of the economy.

The elimination of capital income taxation in the model with human capital accumulation while keeping constant the levy on labor income, increases the growth rate of the economy from 1.50% (baseline case) to 2.27% and reduces the costs of the resulting tax policy to 6.70% of GNP. On the other hand, if the tax rate on capital income is held constant while the tax rate on wage income is abolished, the effects on growth and welfare are even greater: welfare costs lower to 5.24% of GNP and the rate of growth increases to 3.57%.

#### **IV. A Real Business Cycle Model with Monetary Taxation**

The importance of monetary taxation as a channel through which inflation has growth and welfare effects could be inconsequential if the model loses its ability to mimic the volatility and co-movements among real and monetary variables that characterize the basic features of the US business cycle. In this section, a money-and-tax distorted RBC model -with monetary taxation- is formulated,

calibrated and solved numerically to evaluate if the second moments generated by the artificial economy match the corresponding moments from the US data.

### A. Specifying the Stochastic Model

From now on, I focus on the exogenous growth model described in section 2.A. To complete the description it only remains to specify the forcing processes. Agents face two independent sources of uncertainty about future productivity and money growth. The production function can be rewritten now as:  $F(\lambda_t^m, K_t^m, H_t N_t^m) = \lambda_t^m [K_t^m]^\theta [H_t N_t^m]^{(1-\theta)}$  where  $\lambda_t^m$ , at minor abuse of notation, stands for the productivity shock which is observed at the beginning of period  $t$  and is assumed to follow a first order linear Markov process,

$$[10] \text{Ln } \lambda_{t+1}^m = \rho^m \text{Ln } \lambda_t^m + \epsilon_{t+1}^m$$

$0 \leq \rho^m \leq 1$ , where  $\epsilon_{t+1}^m$  is an independently and normally distributed random variable with mean zero and variance  $\sigma_{\epsilon^m}^2$ .

On the other hand, the law of motion of the stock of money is now given by  $M_{t+1} = \mu_t M_t = M_t + T_t$  and the gross rate of money growth,  $\mu_t$ , is assumed to follow a first order autoregressive process,

$$[11] \text{Ln } \mu_{t+1} = (1 - \eta) \text{Ln } \bar{\mu} + \eta \text{Ln } \mu_t + \omega_{t+1}$$

$0 \leq \eta \leq 1$ , and  $\omega_t$  is white noise with mean zero and constant variance  $\sigma_{\omega}^2$ .

Finally, preferences are redefined over stochastic sequences of consumption and leisure chosen to maximize the expected discounted value of the utility stream,

$$[12] \max \mathcal{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, l_t) \right\}$$

where  $\mathcal{E}_0$  is the expectation operator conditional on information available at date 0, and the utility function  $U(\cdot, \cdot, \cdot)$  is parameterized as in [5]. The model is transformed into a stationary representation as indicated before (see section 2A).

The stochastic equilibrium for the sequence problem described so far cannot be computed analytically. There is available a number of techniques to find a numerical solution. The algorithm implemented here, and in the next section, is a straightforward variation of the method suggested by Kydland and Prescott (1982) and Cooley and Hansen (1989) which makes use of the notion of recursive equilibrium developed in Prescott and Mehra (1980). The information relevant for household decision making is characterized by a triple of aggregate state variables  $s = (L_n \lambda^m, L_n \mu, \hat{K}^m)$  and two individual state variables  $(\hat{k}^m, \hat{m})$ , where time subscripts have been dropped and a prime ( $'$ ) will denote the corresponding next-period values, adopting the standard convention. The individual household chooses the decision vector  $d = (\hat{c}_1, \hat{c}_2, n^m, \hat{x}^m, \hat{k}^{m'}, \hat{m}')$  taking as given the aggregate decision rules for  $N^m$  and  $\hat{x}^m$  and the price and rental prices  $\hat{p}$ ,  $w$  and  $r$  as functions of the aggregate state  $s$ . The household also takes as given the laws of motion for the individual capital stock, for the aggregate capital stock, for the technology shock, and for the monetary shock. The problem is a well-defined dynamic program which can be summarized recursively as follows:

$$[P4] v(S, \hat{k}^m, \hat{m}) = \text{Max}_d \left\{ U(\hat{c}_1, \hat{c}_2, 1 - n^m) + \beta \mathcal{E} [v(S', \hat{k}^{m'}, \hat{m}')] \right\}$$

Subject to:

$$[P4.1] \hat{c}_1 = \frac{\hat{m} + e^{L_n \mu} - 1}{\hat{p} e^{L_n \mu}} - \tau_K (r - \delta^m) \hat{k}^m - \tau_H w n^m$$

$$[P4.2] \hat{c}_2 = r(\hat{k}^m + \tau_K \hat{K}^m) - \tau_K \delta^m \hat{K}^m + w(n^m + \tau_H N^m) - \hat{x}^m - \frac{\hat{m}'}{\hat{p}}$$

$$[P4.3] \gamma \hat{k}^{m'} = (1 - \delta^m) \hat{k}^m + \hat{x}^m$$

$$[P4.4] \gamma \hat{K}^{m'} = (1 - \delta^m) \hat{K}^m + \hat{X}^m$$

$$[P4.5] Ln \lambda^{m'} = \rho^m Ln \lambda^m + e^{m'}$$

$$[P4.6] Ln \mu' = (1 - \eta) Ln \bar{\mu} + \eta Ln \mu + \omega'$$

$$\hat{p} = \hat{p}(S), \hat{X}^m = \hat{X}^m(S), N^m = N^m(S), r = r(S), w = w(S)$$

where  $v(S, \hat{k}^m, \hat{m})$  is the household's optimum value function. The solution to the problem yields stationary decision rules specifying allocation decisions as functions of the state variables which summarize the effect of past equilibrium decisions and new information. This leads us to the following definition:

**Definition:** A Recursive Competitive Equilibrium for the economy consists of a set of decision rules  $\hat{c}_1(S, \hat{k}^m, \hat{m})$ ,  $\hat{c}_2(S, \hat{k}^m, \hat{m})$ ,  $n^m(S, \hat{k}^m, \hat{m})$ ,  $\hat{x}^m(S, \hat{k}^m, \hat{m})$ ,  $\hat{k}^{m'}(S, \hat{k}^m, \hat{m})$  and  $\hat{m}'(S, \hat{k}^m, \hat{m})$ ; a set of aggregate decision rules  $N^m(S)$ ,  $\hat{X}^m(S)$ ,  $\hat{K}^{m'}(S)$ ; price functions  $\hat{p}(S)$ ,  $r(S)$  and  $w(S)$ ; and a value function  $v(S, \hat{k}^m, \hat{m})$  such that:

a) The functions  $v$ ,  $N^m$ ,  $\hat{X}^m$ ,  $\hat{K}^{m'}$ ,  $\hat{p}^m$ ,  $w$  and  $r$  satisfy [P4] and the allocations  $\hat{c}_1$ ,  $\hat{c}_2$ ,  $n^m$ ,  $\hat{x}^m$ ,  $\hat{k}^{m'}$  and  $\hat{m}'$  are the associated decision rules.

b) Profit maximization.

c) Individual decisions and aggregate outcomes are consistent:  
 $N^m(S) = n^m(S, \hat{K}^m, 1)$ ,  $\hat{X}^m(S) = \hat{x}^m(S, \hat{K}^m, 1)$ ,  $\hat{m}'(S, \hat{K}^m, 1) = 1$  and  
 $\hat{K}^{m'}(S) = \hat{k}^{m'}(S, \hat{K}^m, 1)$ .

d) Resource constraint is satisfied.

## B. Steady State, Calibration and Solution Method

The certainty version of this economy and the exogenous growth model described in section 2A share the same steady state, obviously, under the same benchmark parameterization and assuming that the corresponding CIA constraints are binding. Under these conditions, to simulate the stochastic model economy it only remains to assign values to the parameters defining the technology and monetary shock processes. For the technology process I pick the usual parameterization found in the literature. The persistence and size of the technology shock are set to  $\rho^a = 0.95$  and  $\sigma_{e^a} = 0.007$ . Both numbers are similar to those used by Prescott (1986) and others.

The parameter values included in the stochastic money growth process are estimated by fitting a first order autoregressive process to the per capita M1 aggregate over the 1959.Q2-1990.Q4 sample period. The estimates are  $\eta = 0.4508$  and  $\sigma_{\omega} = 0.009$ .

The solution method consists of substituting the constraints [P4.1], [P4.2] and [P4.3] into the return function to eliminate  $c_1$ ,  $c_2$  and  $x^m$ . The resulting nonlinear return function is then approximated, around the deterministic steady state, by a quadratic function. The problem is now transformed into a standard linear-quadratic programming problem in which the expectation operator has been dropped: the standard deviation of the shocks have been set to zero because in linear-quadratic problems the decision rules are independent of the covariance matrix of the shocks. Then, the method of successive approximations developed in Kydland and Prescott (1992) and Cooley and Hansen (1989) is used until a sequence of approximations to the value function obtained from the standard Bellman mapping converges to the optimal value function. Once with the value function it is possible to compute the aggregate decision rules that satisfy the recursive competitive equilibrium concept defined above. The following optimal linear feedback rules were calculated:

$$\begin{bmatrix} \nu^m \\ \hat{K}^{m'} \\ \hat{\rho} \end{bmatrix} = \begin{bmatrix} 0.2672 & 0.1877 & -0.0706 \\ 0.3156 & 1.2592 & 0.0641 \\ 1.6130 & -1.1995 & 0.5111 \end{bmatrix} \begin{bmatrix} 1 \\ \text{Ln } \lambda^m \\ \text{Ln } \mu \end{bmatrix} + \begin{bmatrix} -0.0020 \\ 0.9760 \\ -0.0286 \end{bmatrix} \hat{K}^m$$

### C. Simulation Results

The purpose of this section is to compare the cyclical behavior of the simulated economy with monetary taxation with that of the U.S. economy. The artificial economy is simulated with the help of the computed decision rules. The statistics reported in tables 4 and 5 provide information about three basic aspects of the aggregate cyclical behavior that have been highlighted in the literature (Kydland and Prescott, 1990): 1) the volatility of relevant variables; 2) their correlation with output and 3) the phase shift of a relevant variable relative to the output cycle. To compute statistics, all series were logged, except rates, and then filtered using the Hodrick-Prescott filter. The U.S. quarterly sample includes 127 periods from 1959:Q2 to 1990:Q4. Statistics for the simulated economies correspond to the sample means of statistics computed over 50 simulations of 227 periods long each, and where the first 100 periods have been discarded. The sample standard deviation of these moments are in parentheses.

Tables 4 and 5 show that the baseline economy with monetary taxation does not exhibit a poorer empirical performance than that of the basic RBC models. The model captures most of the basic broad features of aggregate fluctuations. The simulated economy predicts that output is more volatile than consumption and productivity, that investment is more volatile than output, and that consumption, investment, and hours are highly procyclical and tend to peak simultaneously with the (output) cycle. However, the model shares most of the anomalies exhibited by the rather simplest RBC model, the divisible labor model of Hansen (1985): consumption and hours are not as volatile and the correlation between hours and

productivity (0.82) is too far from zero, relative to the properties of the actual economy.

Now focus on the nominal properties of the cycle on which the RBC literature has been far less successful. In this regard the artificial economy shows mixed results. The model captures the countercyclical behavior of prices and the procyclical behavior of the nominal interest rate and income velocity of money as well as the volatility of prices, the inflation rate and the nominal interest rate. In general, the model is not good in capturing the phase shift patterns in the movement of output and the nominal variables (prices, monetary aggregates, income velocity, nominal interest rate, inflation rate and the rate of growth of per capita money balances). In addition, the model wrongly predicts a contemporaneous negative correlation between output and inflation and understates the volatility of the income velocity of money.

To sum, the introduction of monetary taxation causes no major changes in the cyclical behavior of real variables relative to the basic divisible labor model. Monetary taxation by itself does not have significant consequences at business cycle frequencies. In the next section I explore one avenue through which a monetary economy with monetary taxation affects the cyclical behavior of the economy.

## **V. Liquidity Effects in a Model with Monetary Taxation**

The RBC literature has devoted a number of papers to provide alternative solutions to the problems observed in the basic RBC models to account for two facts of the U.S. labor market<sup>5</sup>: the fact that total hours worked are much more volatile than productivity and the fact that these variables do not vary contemporaneously in a systematic way: the correlation in the data is close to zero.

---

<sup>5</sup> See Hansen and Wright (1992).

All the new refinements introduced into the basic framework directed at improving the model's predictions can be considered as "nonmonetary" explanations. Kydland and Prescott (1982) specified nonseparable preferences in leisure; Hansen (1985) assumed that labor is indivisible; Benhabib, et. al. (1991) incorporated a household production sector; Christiano and Eichenbaum (1992a) annexed stochastic real government spending; Bencivenga (1992) included stochastic shocks to preferences; Ambler and Paquet (1994) introduced stochastic depreciation shocks and Braun (1994) added tax rate disturbances. In general, these solutions operate through the labor supply curve; labor supply shifts decrease the strong positive correlation between hours and productivity that arises from technology shocks displacing the labor demand along a stable supply curve. In this section I propose an alternative solution to these labor market puzzles. In contrast to the existing literature, this alternative can be considered as "monetary". In this section I show that the mentioned labor market anomalies are not present in a RBC model with monetary taxation when a liquidity effect is included. In this paper, the liquidity effect alludes to the response of the nominal interest rate to an unexpected money growth shock; this type of model rationalizes the widely accepted idea that the nominal interest rate may be temporarily driven down by a money growth shock. Like most of the solutions proposed, this operates by shifting the labor supply curve, as well. In the presence of a liquidity effect, monetary injections temporarily affect the after-all-tax real interest rate which triggers intertemporal substitution effects on labor supply decisions.

### **A. Timing Conventions and the Model**

The household's behavior in the model analyzed so far (the business cycle model with exogenous growth of sections 2A and 4) is now slightly modified to



introduce the Lucas (1990) multiple-member household construct<sup>6</sup>. This methodology allows for heterogeneity but preserving the single agent artifice. I assume that the household is made up of three members: a shopper, a banker and a taxpayer. During a period, different family members use cash to transact in separated markets, thus, facing different trading opportunities, and then regroup to pool information, assets (into household's wealth) and goods (into household's consumption). The timing of events is very important in this setup. In contrast to Lucas (1990), the timing convention used here resembles that in Svensson (1985a, 1985b). The typical flow of events within a period is summarized as follows: *state of the economy - cash good market - IRS - factor, asset, and credit good markets*.

Now in detail. In a typical period each household enters period  $t$  holding wealth in the form of physical capital  $k_t^a$ , human capital  $h_t$ , and nominal cash balances  $m_t$ , all accumulated and carried over from the previous period. At the beginning of the period  $t$  the household splits up. Total money balances are allocated between the shopper ( $m_t - q_t \geq 0$ ) and the banker ( $q_t \geq 0$ ), who must pay market interest on the household's deposit, and each goes his own way. This portfolio decision is made under incomplete information in the sense that the current period money shock has not been learned<sup>7</sup>. Then, the money shock is announced and the current realization of the forcing processes on which the optimal feedback rules depend on, are fully known. The cash-good market opens and the shopper transacts obeying the liquidity constraint:

---

<sup>6</sup> This modeling strategy has been used by others in the liquidity-effect literature. See Fuerst (1992), Christiano (1991), and Christiano and Eichenbaum (1992b). Christiano (1991) presents the real business cycle implications of his models with liquidity effects (the Fuerst-Lucas and sluggish capital models). They do not clearly improve upon the performance of the basic cash-in-advance economy.

<sup>7</sup> To facilitate the experiment, I assume that the current technology shock is known at this stage. Note that multiple variants arise from different specifications for the flow of events within a period.

$$[13] p_t c_{1t} \leq m_t - q_t$$

In the meantime the bank receives the money transfer  $T_t$  and the taxpayer borrows from it the cash required to pay taxes. Clearing in the loan market requires:

$$[14] q_t + T_t = \tau_K p_t (r_t - \delta^m) k_t^m + \tau_H p_t w_t n_t^m h_t$$

Finally, the household reunites and the remaining decisions, payments and transactions are carried out. Labor, investment, and money holding decisions are materialized, as well as the purchases of credit goods and the tax rebate. Labor and capital services are remunerated. The taxpayer's debt with the bank is assumed by the household and it receives from the bank interests on the deposit and dividend payments. These operations are encompassed in the budget constraint [P1.2]<sup>8</sup>.

The model is transformed as usual into a stationary representation. Let  $\hat{q}_t = q_t / M_t$ . The household's problem belongs to the time invariant class of dynamic structures that satisfy the following Bellman's equation:

$$[P5] v(S, \hat{k}^m, \hat{m}) = \max_d \mathcal{E}_d \left\{ \max_{\hat{q}} \mathcal{E}_{\hat{q}} \left\{ U(\hat{c}_1, \hat{c}_2, 1 - n^m) + \beta v(S', \hat{k}^{m'}, \hat{m}') \right\} \right\}$$

Subject to:

$$[P5.1] \hat{c}_1 = \frac{\hat{m} - \hat{q}}{\hat{p} e^{Ln \mu}}$$

---

<sup>8</sup> Under the timing conventions, it is possible to have nonbinding liquidity constraints (equations [13] and [14]). The simulation experiments deal only with the case of binding constraints. It was possible to make an ex-post verification of the assumption.

$$[P5.2] \frac{\hat{q} + e^{Ln \mu} - 1}{\hat{p} e^{Ln \mu}} = \tau_K (r - \delta^m) \hat{k}^m + \tau_H w n^m$$

$$[P5.3] \hat{c}_2 = r(\hat{k}^m + \tau_K \hat{K}^m) - \tau_K \delta^m \hat{K}^m + w(n^m + \tau_H N^m) - \hat{x}^m - \frac{\hat{m}'}{\hat{p}}$$

$$[P5.4] \gamma \hat{k}^{m'} = (1 - \delta^m) \hat{k}^m + \hat{x}^m$$

$$[P5.5] \gamma \hat{K}^{m'} = (1 - \delta^m) \hat{K}^m + \hat{X}^m$$

$$[P5.6] Ln \lambda^{m'} = \rho^m Ln \lambda^m + \epsilon^{m'}$$

$$[P5.7] Ln \mu' = (1 - \eta) Ln \bar{\mu} + \eta Ln \mu + \omega'$$

$$\hat{p} = \hat{p}(S), \hat{X}^m = \hat{X}^m(S), N^m = N^m(S), r = r(S), w = w(S)$$

where the state  $s$  is defined as above, and  $\mathcal{E}_t$  and  $\mathcal{E}_d$  are the expectation operators conditional on the information available at the moment of making the decision on how much money to lend to the bank,  $\hat{q}$ , and the remaining decisions  $d$ ,  $d = (\hat{c}_1, \hat{c}_2, n^m, \hat{x}^m, \hat{k}^{m'}, \hat{m}')$ , respectively. The definition of a recursive competitive equilibrium for this economy is a straightforward generalization of the definition given for the economy without a liquidity effect.

## B. Calibration and Solution Method

Note that the specification of the model with liquidity effect does not involve the introduction of new parameters. In addition, it is possible to show, assuming binding liquidity constraints, that the economies with and without liquidity effect share the same deterministic steady state.

To simulate the economy numerically, I introduce a simple variant to the methodology used in the previous section. Constraint [P5.2] can be seen as an expression defining  $n^m$  and whose aggregate version gives an expression for  $N^m$ . Substituting constraints [P5.1], [P5.2], and [P5.3] into the return function and then

obtaining a quadratic approximation to the resulting function, I end up with a standard linear-quadratic problem. The following are the optimal linear feedback rules computed:

$$\begin{bmatrix} \hat{Q} \\ \hat{K}^{ml} \\ \hat{\rho} \end{bmatrix} = \begin{bmatrix} 0.4222 & 0.3867 & -0.6382 \\ 0.3139 & 1.2611 & 0.0641 \\ 1.6126 & -1.1991 & 0.5111 \end{bmatrix} \begin{bmatrix} 1 \\ \text{Ln } \lambda^m \\ \text{Ln } \mu \end{bmatrix} + \begin{bmatrix} -0.0077 \\ 0.9761 \\ -0.0286 \end{bmatrix} \hat{K}^m$$

The computed decision rule for the equilibrium fraction of money holdings lent to the bank,  $\hat{\rho}$ , depends on the aggregate state  $s$ ,  $s = (\text{Ln } \lambda^m, \text{Ln } \mu, \hat{K}^m)$ ; since this portfolio decision is assumed to be made without observing the money shock, the appropriate decision rule is simply the conditional expectation of the rule where the conditioning set is  $s \setminus \omega$ .

### C. Findings

The results from simulating the artificial economy displaying the liquidity effect appear in tables 4 and 5 labeled as "Liquidity I", which corresponds to the model where only the portfolio decision is made under incomplete information, i.e. before the money shock is revealed to everybody. Before discussing the business cycle properties of the model, let us study the liquidity effect of an unexpected money shock.

The baseline RBC model with monetary taxation (model presented in section 4) is incapable of generating a dominant liquidity effect. Figure 1 illustrates the response of the quarterly nominal interest rate to a one-standard-deviation money growth shock. The first order effect of a money shock is to increase the nominal interest rate through an expected-inflation mechanism even though the Fisher relation does not hold exactly in this context; thereafter, the nominal interest rate remains above its steady state value as long as the money growth rate remains

above its steady state rate. Figure 1 also depicts the response of the nominal interest rate in the model with liquidity effect (Liquidity I). Note that after the third period both responses are almost identical; all the action occurs within the first two quarters following the shock. The Liquidity I model accounts for the idea that an unexpected money shock drives the nominal interest rate down. In the period of the shock, a short-lived liquidity effect dominates the expected inflation effect; but soon vanishes. In the next period the interest rate catches up and its behavior is dominated by the expected inflation effect thereafter.

It is interesting to review the response of total hours worked to the same type of shock. For the baseline economy (described in section 4), figure 2 shows that one period after the shock the level of work effort lowers relative to its steady state value. In this model, the money shock temporarily shifts left the labor supply curve; this is because two reasons. First, the real interest rate is falling though the nominal interest rate is increasing; this induces an intertemporal substitution effect which reduces today's work effort. Second, since part of the current wage is used to acquire money balances to be carried over into the next period, higher inflation makes the current real wage worth less what reduces the labor supply. However, the displacement in the labor supply curve exhibited by the baseline model is not large enough to reduce the positive correlation between hours worked and average productivity, as shown before in section 4.

The impulse response of labor supply in the Liquidity I model shows a jump in the period after the shock associated with the hike in nominal and real interest rates after the liquidity effect dissipates. When the after-all-tax nominal and real interest rates overtake, the level of labor supply increases in response to a intertemporal substitution effect. Then, the liquidity model may potentially solve the labor market anomalies. But, is the displacement in the labor supply curve large enough and the overall results of the liquidity model consistent with the U.S. data?

Tables 4 and 5 show that the liquidity model is very successful at predicting the real properties of the U.S. business cycles. The predicted volatility of output

matches the observed GNP standard deviation. The volatility of consumption increases, but tends to overstate the corresponding moment of the U.S. data. More importantly, the labor market anomalies are not longer present. Hours worked fluctuate considerable more than productivity. The model predicts a ratio between their standard deviations of 2, while in the data it is 1.66. However, the former number is in the 1.37-2.15 range of plausible values for this ratio calculated by Hansen and Wright (1992) using different series and sample periods. The model captures the absolute volatility of average productivity and tends to slightly overstate the volatility of hours worked. On the other hand, Hansen and Wright (1992) argue that the correlation between hours and productivity may plausibly range between -0.35 and 0.10. The liquidity model predicts that such a correlation is -0.157 (0.11).

Tables 4 and 5 also report the results of an experiment in which both the portfolio decision and the physical capital investment decision are made before the money shock is revealed. Now, the appropriate decision rule for the next period capital stock (second linear feedback rule) is simply the conditional expectation of the rule where the conditioning set is again  $s \setminus \omega$ . The simulation results for this artificial economy, the "Liquidity II", are not qualitatively different from those obtained with the Liquidity I model.

## **VI. Concluding Remarks**

This paper argues and empirically supports the idea that incorporating the real world fact that money is the required means of taxation payment, money has far greater effects on growth and welfare than those found in existing monetary models. Based on three model economies exhibiting steady state growth, the evidence found seems to dispute the conclusion that independently of the form of the CIA constraint, the welfare costs of inflation are smaller in an endogenous growth model than in its exogenous counterpart (Gomme, 1993).

At business cycle frequencies, the introduction of monetary taxation does not deteriorate the ability of the model to mimic key aspects of the U.S. aggregate fluctuations, but it exhibits the same type of anomalies presented by the simplest business cycle model, the divisible labor model of Hansen (1985). This paper provides a solution to the labor market anomalies but, in contrast to the existing solutions, this is "monetary" in nature. When a liquidity effect is introduced in the model with monetary taxation, the predicted correlation between hours worked and average productivity as well as its relative volatility, roughly match the U.S. data. This evidence seems to dispute the generally accepted conclusion that monetary shocks do not contribute to explain the cyclical behavior of U.S. real variables (Cooley and Hansen, 1989).

Future work should be directed at enhancing the predictions of the model regarding the nominal features of the business cycle as well as some countefactual real side predictions. Among the several counterfactual implications, it is worth noting that the model tends to overstate the volatility of the nominal interest rates and consumption. One possible avenue is to smooth the interest rate series modeling a liquidity effect that persist beyond the period of the shock.

## References

- Ambler, S. and A. Paquet (1994): "Stochastic Depreciation and the Business Cycle," **International Economic Review**, 35, 101-116.
- Auerbach, A. (1983): "Corporate Taxation in The United States," **Brookings Papers on Economic Activity**, 2, 451-505.
- Bailey, M. (1956): "The Welfare Costs of Inflationary Finance," **Journal of Political Economy**, 64, 93-110.
- Barro, R. and C. Sahasakul (1986): "Average Marginal Tax Rates from Social Security and Individual Income Tax," **Journal of Business**, 1986, 555-566.
- Bencivenga, V. (1992): "An Econometric Study of Hours and Output Variation with Preference Shocks," **International Economic Review**, 33, 449-471.
- Benhabib, J., R. Rogerson and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," **Journal of Political Economy**, 99, 1166-1187.
- Ben-Porath, Y. (1967): "The Production of Human Capital and the Life Cycle of Earnings," **Journal of Political Economy**, 75, 352-365.
- Braun, A. (1994): "Tax Disturbances and Real Economic Activity," **Journal of Monetary Economics**, 33, 441-461.
- Christiano, L. (1988): "Why does Inventory Investment Fluctuate so Much," **Journal of Monetary Economics**, 21, 247-280.
- Christiano, L. (1991): "Modeling the Liquidity Effect of a Money Shock," **Federal Reserve Bank of Minneapolis Quarterly Review**, Winter, 3-34.
- Christiano, L. and M. Eichenbaum (1992a): "Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," **American Economic Review**, 82, 430-450.
- Christiano, L. and M. Eichenbaum (1992b): "Liquidity Effects and the Monetary Transmission Mechanism," **American Economic Review**, 82, 346-353.
- Cooley, T. and G. Hansen (1989): "The Inflation Tax in a Real Business Cycle Model," **American Economic Review**, 79, 733-748.



- Cooley, T. and G. Hansen (1991): "The Welfare Costs of Moderate Inflation," **Journal of Money, Credit and Banking**, 23, 483-503.
- Cooley, T. and G. Hansen (1992): "Tax Distortions in a Neoclassical Monetary Economy," **Journal of Economic Theory**, 58, 290-316.
- Cooley, T. and G. Hansen (1994): "Money and the Business Cycle," in T. Cooley (ed.), **Frontiers of Business Cycle Research**, (Princeton: Princeton University Press), forthcoming, .
- Cooley, T. and E. Prescott (1994): "Economic Growth and Business Cycles," in T. Cooley (ed.), **Frontiers of Business Cycle Research**, (Princeton: Princeton University Press), forthcoming.
- De Gregorio, J. (1993): "Inflation, Taxation, and Long-Run Growth," **Journal of Monetary Economics**, 31, 271-298.
- De Long, B. and L. Summers (1992): "Macroeconomic Policy and Long-Run Growth," **Federal Reserve Bank of Kansas City Economic Review**, 77, No. 4, 5-30.
- Fisher, S. (1981): "Towards an Understanding of the Welfare Costs of Inflation: II," **Carnegie-Rochester Conference on Public Policy**, 15, 5-42.
- Fisher, S. (1991): "Macroeconomics, Development, and Growth," **NBER Macroeconomics Annual 1991**, 329-364.
- Fisher, S. (1993): "The Role of Macroeconomic Factors in Growth," **Journal of Monetary Economics**, 32, 485-512.
- Friedman, M. (1969): **The Optimum Quantity of Money and Other Essays**, (Chicago: Aldine).
- Fuerst, T. (1992): "Liquidity, Loanable Funds and Real Activity," **Journal of Monetary Economics**, 29, 3-24.
- Fullerton, D. and D. Rogers (1993): **Who Bears the Lifetime Tax Burden?**, (Washington, D.C.: The Brookings Institution).
- Gillman, M. (1993): "The Welfare Cost of Inflation in a Cash-in-Advance Economy with Costly Credit," **Journal of Monetary Economics**, 31, 97-115.

- Gomme, P. (1993):"Money and Growth Revisited. Measuring the Costs of Inflation in an Endogenous Growth Model," **Journal of Monetary Economics**, 32,51-77.
- Greenwood, J. and Z. Hercowitz (1991):"The Allocation of Capital and Time over the Business Cycle", **Journal of Political Economy**, 99, 1188-1214.
- Hansen, G. (1985):"Indivisible Labor and the Business Cycle," **Journal of Monetary Economics**, 16, 309-327.
- Hansen, G and E. Prescott (1994):"Recursive Methods for Computing Equilibria of Business Cycle Models," in T. Cooley (ed.), **Frontiers of Business Cycle Research**, (Princeton: Princeton University Press), forthcoming.
- Hansen, G and R. Wright (1992):"The Labor Market in Real Business Cycle Theory," **Federal Reserve Bank of Minneapolis Quarterly Review**, Spring, 1-12.
- Imrohorglu, A. (1992):"The Welfare Cost of Inflation under Imperfect Insurance," **Journal of Economic Dynamics and Control**, 16, 79-91.
- Ireland, P. (1994):"Money and Growth: An Alternative Approach," **American Economic Review**, 84, 47-65.
- Joines, D. (1981):"Estimates of Effective Marginal Tax Rates on Factor Incomes," **Journal of Business**, 54, 191-226.
- Jones L. and R. Manuelli (1992):"Growth and the Effects of Inflation," mimeo, NBER Working Paper 4523.
- King, R., C. Plosser and S. Rebelo (1988):"Production, Growth and Business Cycles: I. The Basic Neoclassical Model," **Journal of Monetary Economics**, 21,195-232.
- Kydland, F. and E. Prescott (1982):"Time to Build and Aggregate Fluctuations," **Econometrica**, 50, 1345-1370.
- Kydland, F. and E. Prescott (1982):"Business Cycles: Real Facts and a Monetary Myth," **Federal Reserve Bank of Minneapolis Quarterly Review**, 14, Spring, 1-18.
- Lerner, A. (1947):"Money as a Creature of the State," **American Economic Review**, Papers and Proceedings, 37, 312-317.

- Levine, R. and D. Renelt (1992): "A Sensitivity Analysis of Cross-Country Growth Regressions," **American Economic Review**, 82, 942-963.
- Levine, R. and S. Zervos (1992): "Looking at the Facts: What We Know about Policy and Growth from Cross-Country Analysis," mimeo, October, (Washington, D.C.: World Bank).
- Lucas, R.E., Jr. (1981): "Discussion of: Stanley Fisher, Towards an Understanding of the Costs of Inflation: II," **Carnegie-Rochester Conference on Public Policy**, 15, 43-52.
- Lucas, R.E., Jr. (1988): "On The Mechanics of Economic Development," **Journal of Monetary Economics**, 22, 3-42.
- Lucas, R.E., Jr. (1990): "Liquidity and Interest Rates," **Journal of Economic Theory**, 50, 237-264.
- Lucas, R.E., Jr. and N.L. Stokey (1983): "Optimal Fiscal and Monetary Policy in an Economy without Capital," **Journal of Monetary Economics**, 12, 55-93.
- Lucas, R.E., Jr. and N.L. Stokey (1987): "Money and Interest in a Cash-in-Advance Economy," **Econometrica**, 55, 491-513.
- Prakken, J., C. Varvares and L. Meyer (1991): "Tax Reform and Potential Output: A Retrospective Analysis of The Tax Reform Act of 1986," **Carnegie-Rochester Conference Series in Public Policy**, 113-180.
- Prescott, E. (1986): "Theory Ahead of Business Cycle Measurement," **Federal Reserve Bank of Minneapolis Quarterly Review**, 10, Fall, 9-22.
- Prescott, E. and R. Mehra (1980): "Recursive Competitive Equilibrium: The Case of Homogeneous Households," **Econometrica**, 48, 1365-1379.
- Rebelo, S. (1991): "Long-Run Policy Analysis and Long-Run Growth," **Journal of Political Economy**, 31, 500-521.
- Ríos-Rull, J. (1993): "Working in Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach," **American Economic Review**, 83, 893-907.
- Stockman, A. (1981): "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy," **Journal of Monetary Economics**, 8, 387-393.

Svensson, L. (1985a): "Currency Prices, Terms of Trade, and Interest Rates: A General Equilibrium Asset Pricing, Cash-in-Advance Approach," **Journal of International Economics**, 18, 17-41.

Svensson, L. (1985b): "Money and Asset Prices in a Cash-in-Advance Economy," **Journal of Political Economy**, 93, 919-944.

Uzawa, H. (1965): "Optimum Technical Change in an Aggregative Model of Economic Growth," **International Economic Review**, 6, 18-31.

Table 1

**Parameter Values**

Parameter	Exogenous Growth Model	AK Growth Model	Growth through Human Capital Accumulation
<b>PREFERENCES</b>			
$\beta$	0.99753	0.99371	0.99371
$a$	0.70936	0.54248	0.68040
$B$	1.94166	----	1.67985
<b>TECHNOLOGY</b>			
$\theta$	0.3715	idem	idem
$\alpha$	----	----	0.27314
$A$	----	0.03748	----
$s$	----	----	0.01830
<b>DEPRECIATION RATES</b>			
$\delta^m$	0.01844	0.01844	0.01844
$\delta^h$	----	----	0.01844
$\delta$	----	----	0.00082
<b>TAX RATES</b>			
$\tau_k$	0.475	idem	idem
$\tau_h$	0.28	----	idem
<b>GROSS RATES OF GROWTH</b>			
$\gamma$	1.0035	idem	idem
$\bar{\mu}$	1.01	idem	idem

**Table 2**  
**Steady State Welfare Costs of Alternative Money Growth Rates**  
 (% of GNP)

	Welfare Cost:	Approximate Contribution to Welfare Costs				Annual Growth Rate Per Capita GNP (%)
		Consumption Effect	Leisure Effect	Growth Effect	Rest	
<b>A. EXOGENOUS GROWTH MODEL</b>						
Optimal Money Rule	0.00	0.00	0.00	0.00	0.00	1.40
Average Money Growth	0.20	0.46	-0.26	0.00	0.00	1.40
Annual Inflation Rate						
5%	0.30	0.69	-0.39	0.00	0.00	1.40
10%	0.51	1.17	-0.66	0.00	0.00	1.40
15%	0.73	1.67	-0.92	0.00	-0.02	1.40
20%	0.96	2.18	-1.19	0.00	-0.03	1.40
30%	1.45	3.23	-1.73	0.00	-0.05	1.40
40%	1.97	4.33	-2.27	0.00	-0.09	1.40
50%	2.53	5.49	-2.83	0.00	-0.13	1.40
100%	5.98	12.17	-5.75	0.00	-0.44	1.40
<b>B. AK GROWTH MODEL</b>						
Optimal Money Rule	0.00	0.00	0.00	0.00	0.00	1.46
Average Money Growth	0.29	-0.20	0.00	0.48	0.01	1.40
Annual Inflation Rate						
5%	0.39	-0.27	0.00	0.66	0.00	1.38
10%	0.63	-0.42	0.00	1.04	0.01	1.33
15%	0.86	-0.56	0.00	1.43	-0.01	1.29
20%	1.11	-0.71	0.00	1.82	0.00	1.24
30%	1.63	-1.00	0.00	2.62	0.01	1.15
40%	2.18	-1.29	0.00	3.46	0.01	1.06
50%	2.76	-1.58	0.00	4.32	0.02	0.97
100%	6.19	-2.96	0.00	9.14	0.01	0.52
<b>C. ENDOGENOUS GROWTH MODEL WITH HUMAN CAPITAL ACCUMULATION</b>						
Optimal Money Rule	0.00	0.00	0.00	0.00	0.00	1.50
Average Money Growth	0.70	0.05	-0.78	1.46	-0.03	1.40
Annual Inflation Rate						
5%	0.96	0.08	-1.06	2.01	-0.07	1.36
10%	1.52	0.12	-1.63	3.18	-0.15	1.29
15%	2.11	0.17	-2.18	4.40	-0.28	1.22
20%	2.72	0.22	-2.73	5.67	-0.44	1.14
30%	4.01	0.34	-3.77	8.35	-0.91	1.00
40%	5.38	0.46	-4.78	11.21	-1.51	0.85
50%	6.85	0.60	-5.74	14.28	-2.29	0.71
100%	15.54	1.41	-9.98	32.79	-8.68	0.03

Table 3

**Steady State Welfare Costs of Alternative Fiscal Policies  
(% of GNP)**

	Welfare Cost:	Approximate Contribution to Welfare Costs				Annual Growth Rate Per Capita GNP (%)
		Consumption Effect	Leisure Effect	Growth Effect	Rest	
<b>A. EXOGENOUS GROWTH MODEL (under optimal money rule)</b>						
$\tau_K = 0.475 ; \tau_H = 0.28$	3.86	14.57	-10.04	0.00	-0.67	1.40
$\tau_K = 0.475 ; \tau_H = 0$	1.38	3.48	-2.07	0.00	-0.03	1.40
$\tau_K = 0 ; \tau_H = 0.28$	1.58	8.91	-6.99	0.00	-0.34	1.40
$\tau_K = 0.575 ; \tau_H = 0.28$	5.25	17.46	-11.39	0.00	-0.82	1.40
$\tau_K = 0.475 ; \tau_H = 0.38$	5.92	21.08	-13.96	0.00	-1.20	1.40
<b>B. AK GROWTH MODEL (under optimal money rule)</b>						
$\tau_K = 0.475$	14.45	-11.93	0.00	26.49	-0.11	1.46
$\tau_K = 0$	0.00	0.00	0.00	0.00	0.00	5.05
$\tau_K = 0.575$	24.09	-14.42	0.00	38.67	-0.16	0.70
<b>C. ENDOGENOUS GROWTH MODEL WITH HUMAN CAPITAL ACCUMULATION (under optimal money rule)</b>						
$\tau_K = 0.475 ; \tau_H = 0.28$	17.21	-3.29	-15.86	83.02	-46.66	1.50
$\tau_K = 0.475 ; \tau_H = 0$	5.24	0.06	-7.21	16.67	-4.28	3.57
$\tau_K = 0 ; \tau_H = 0.28$	6.70	-3.85	-11.61	43.41	-21.25	2.27

Table 4

**Volatility in the U.S. Data and Artificial Model Economies**  
(standard deviations in percent)

	Quarterly	Simulated Economies with Monetary Taxation			
	U.S. Data 1990:Q4	1959:Q2-	Baseline	Liquidity I	Liquidity II
Real Output	1.64		1.34 (0.15)	1.65 (0.13)	1.70 (0.14)
Consumption Non-Durables	0.86		0.50 (0.06)	1.39 (0.09)	1.46 (0.08)
Investment			3.96 (0.49)	3.82 (0.51)	4.02 (0.50)
Fixed Private Investment	7.72				
Total Investment	6.22				
Hours	1.50		0.76 (0.08)	1.76 (0.11)	1.78 (0.10)
Productivity	0.90		0.64 (0.07)	0.86 (0.07)	0.88 (0.08)
Monetary Aggregates			1.69 (0.26)	1.77 (0.25)	1.77 (0.27)
M1	1.62				
M2	1.43				
MB	0.82				
Nominal Interest Rate			1.98 (0.12)	3.89 (0.26)	3.87 (0.22)
Federal Funds Rate	1.82				
Treasury Bills Rate	1.39				
Price Level			1.87 (0.26)	1.91 (0.26)	1.91 (0.28)
Price Deflator	1.12				
CPI	1.49				
Income Velocity of Money			0.51 (0.06)	1.43 (0.09)	1.43 (0.08)
M1 Velocity	2.18				
M2 Velocity	1.99				
MB Velocity	1.59				
Inflation Rate			1.00 (0.08)	1.00 (0.09)	1.00 (0.08)
Price Deflator	1.07				
CPI	0.54				
Money Growth Rate			0.91 (0.06)	0.92 (0.07)	0.91 (0.06)
M1	0.83				
M2	0.70				
MB	0.43				

The U.S. data, except price indexes, rates and velocities, have been expressed in per-capita terms by using the 16+ population. Actual hours and productivity statistics are taken from Benhabib, et. al. (1991); productivity is defined by the ratio output to hours. All variables, except rates, have been logged; and all detrended using the Hodrick-Prescott filter. Then, standard deviations are computed. Standard deviations for the simulated economies correspond to the sample means of statistics computed over 50 simulations of 127 periods long each. The sample standard deviation of these moments are in parentheses.



**Table 5**

**Dynamic Comovements in the U.S. Data and Artificial Model Economies**  
(cross-correlations of output and other variables)

**Part I: Real Variables**

Variable	Source	J=	Cross-Correlation of Output in t+J and variable at t						
			-3	-2	-1	0	1	2	3
<b>REAL OUTPUT</b>									
	U.S. Data		0.42	0.65	0.85	1.00	0.85	0.65	0.42
	Baseline		0.22 (0.12)	0.37 (0.10)	0.66 (0.06)	1.00 (0.00)	0.66 (0.06)	0.37 (0.10)	0.22 (0.12)
	Liquidity I		0.16 (0.10)	0.32 (0.09)	0.59 (0.10)	1.00 (0.00)	0.59 (0.10)	0.32 (0.09)	0.16 (0.10)
	Liquidity II		0.25 (0.11)	0.40 (0.11)	0.61 (0.08)	1.00 (0.00)	0.61 (0.08)	0.40 (0.10)	0.25 (0.11)
<b>CONSUMPTION</b>									
	U.S. Data		0.34	0.53	0.70	0.79	0.78	0.68	0.51
	Baseline		0.33 (0.13)	0.44 (0.11)	0.57 (0.09)	0.67 (0.06)	0.26 (0.09)	-0.01 (0.11)	-0.12 (0.10)
	Liquidity I		0.01 (0.09)	0.02 (0.08)	0.21 (0.09)	0.72 (0.04)	0.18 (0.08)	0.02 (0.07)	-0.03 (0.09)
	Liquidity II		0.12 (0.10)	0.25 (0.09)	0.38 (0.09)	0.81 (0.04)	0.28 (0.08)	0.12 (0.08)	0.05 (0.08)
<b>PRIVATE INVESTMENT</b>									
	U.S. Data		0.27	0.51	0.74	0.90	0.77	0.60	0.42
	Baseline		0.16 (0.12)	0.31 (0.10)	0.61 (0.06)	0.98 (0.01)	0.70 (0.07)	0.43 (0.11)	0.28 (0.13)
	Liquidity I		0.22 (0.10)	0.42 (0.09)	0.66 (0.08)	0.86 (0.04)	0.68 (0.09)	0.42 (0.10)	0.24 (0.11)
	Liquidity II		0.30 (0.10)	0.41 (0.09)	0.62 (0.08)	0.89 (0.04)	0.71 (0.08)	0.51 (0.11)	0.34 (0.11)
<b>HOURS</b>									
	U.S. Data		0.30	0.53	0.74	0.86	0.82	0.69	0.52
	Baseline		0.14 (0.12)	0.31 (0.10)	0.61 (0.07)	0.96 (0.01)	0.61 (0.07)	0.32 (0.10)	0.20 (0.11)
	Liquidity I		-0.04 (0.08)	0.04 (0.08)	0.30 (0.09)	0.84 (0.02)	0.33 (0.09)	0.15 (0.08)	0.08 (0.09)
	Liquidity II		0.10 (0.10)	0.26 (0.09)	0.43 (0.08)	0.89 (0.02)	0.40 (0.08)	0.24 (0.09)	0.16 (0.10)

(continued on the following page)

Table 5 (Continued)

Part II: Nominal Variables

Variable Source	J=	Cross-Correlation of Output in t+J and variable at t						
		-3	-2	-1	0	1	2	3
<b>PRICE LEVEL</b>								
U.S. Data	Deflator	-0.14	-0.26	-0.28	-0.36	-0.46	-0.54	-0.50
	CPI	0.01	-0.17	-0.34	-0.51	-0.66	-0.75	-0.76
	Baseline	-0.25 (0.18)	-0.29 (0.16)	-0.39 (0.15)	-0.51 (0.16)	-0.41 (0.16)	-0.38 (0.17)	-0.42 (0.18)
	Liquidity I	-0.08 (0.16)	-0.12 (0.15)	-0.27 (0.13)	-0.49 (0.11)	-0.44 (0.11)	-0.25 (0.12)	-0.12 (0.12)
	Liquidity II	0.09 (0.16)	-0.04 (0.16)	-0.29 (0.15)	-0.62 (0.13)	-0.62 (0.13)	-0.51 (0.15)	-0.40 (0.16)
<b>MONETARY AGGREGATES</b>								
U.S. Data	M1	0.09	0.14	0.21	0.32	0.35	0.36	0.27
	M2	-0.21	-0.01	0.20	0.42	0.58	0.68	0.67
	MB	0.25	0.31	0.34	0.37	0.34	0.28	0.18
	Baseline	-0.16 (0.18)	-0.12 (0.18)	-0.08 (0.18)	-0.05 (0.18)	-0.14 (0.19)	-0.28 (0.19)	-0.40 (0.20)
	Liquidity I	0.13 (0.15)	0.19 (0.14)	0.09 (0.14)	-0.09 (0.14)	-0.06 (0.14)	-0.01 (0.13)	0.00 (0.14)
	Liquidity II	0.29 (0.14)	0.16 (0.15)	-0.04 (0.16)	-0.31 (0.16)	-0.34 (0.17)	-0.31 (0.18)	-0.29 (0.18)
<b>INCOME VELOCITY</b>								
U.S. Data	M1	0.18	0.25	0.34	0.33	0.14	-0.05	-0.15
	M2	0.41	0.39	0.40	0.32	0.02	-0.26	-0.42
	MB	0.21	0.34	0.51	0.59	0.38	0.15	-0.02
	Baseline	0.16 (0.11)	0.28 (0.09)	0.56 (0.07)	0.89 (0.03)	0.68 (0.08)	0.45 (0.12)	0.31 (0.14)
	Liquidity I	-0.06 (0.09)	0.04 (0.08)	0.31 (0.09)	0.79 (0.03)	0.27 (0.09)	0.11 (0.08)	0.05 (0.10)
	Liquidity II	0.12 (0.10)	0.29 (0.09)	0.46 (0.09)	0.85 (0.03)	0.33 (0.08)	0.18 (0.09)	0.11 (0.10)
<b>NOMINAL INTEREST RATE</b>								
U.S. Data	Fed. Funds	0.48	0.48	0.45	0.37	0.13	-0.19	-0.39
	T. Bills	0.41	0.42	0.39	0.36	0.15	-0.14	-0.33
	Baseline	0.01 (0.12)	0.00 (0.11)	0.07 (0.12)	0.12 (0.11)	0.04 (0.11)	0.11 (0.11)	0.14 (0.10)
	Liquidity I	-0.07 (0.08)	0.03 (0.08)	0.25 (0.08)	0.47 (0.06)	-0.29 (0.06)	-0.15 (0.08)	-0.06 (0.08)
	Liquidity II	0.12 (0.08)	0.24 (0.07)	0.32 (0.09)	0.49 (0.06)	-0.26 (0.06)	-0.13 (0.08)	-0.09 (0.07)
<b>INFLATION RATE</b>								
U.S. Data	Deflator	0.12	0.02	0.09	0.12	0.09	-0.03	0.00
	CPI	0.51	0.45	0.45	0.39	0.24	0.03	-0.11
	Baseline	0.08 (0.11)	0.18 (0.12)	0.23 (0.12)	-0.20 (0.10)	-0.05 (0.11)	0.09 (0.10)	-0.03 (0.09)
	Liquidity I	0.08 (0.10)	0.28 (0.10)	0.43 (0.08)	-0.08 (0.09)	-0.35 (0.09)	-0.24 (0.10)	-0.06 (0.10)
	Liquidity II	0.27 (0.10)	0.46 (0.09)	0.60 (0.08)	-0.02 (0.08)	-0.21 (0.08)	-0.22 (0.09)	-0.18 (0.08)
<b>MONEY GROWTH RATE</b>								
U.S. Data	M1	-0.09	-0.11	-0.17	-0.08	-0.02	0.15	0.16
	M2	-0.37	-0.40	-0.43	-0.35	-0.21	-0.01	0.11
	MB	-0.10	-0.08	-0.07	0.00	0.10	-0.14	0.17
	Baseline	-0.07 (0.12)	-0.07 (0.13)	-0.05 (0.12)	0.16 (0.12)	0.23 (0.12)	0.21 (0.11)	-0.02 (0.11)
	Liquidity I	-0.09 (0.11)	0.17 (0.10)	0.40 (0.09)	-0.05 (0.10)	-0.08 (0.11)	-0.02 (0.11)	0.04 (0.12)
	Liquidity II	0.29 (0.10)	0.42 (0.12)	0.53 (0.09)	0.02 (0.10)	-0.06 (0.10)	-0.07 (0.10)	-0.08 (0.10)

Actual U.S. dynamic comovement pattern for Hours is taken from Cooley and Hansen (1994). Sample standard deviations in parentheses.

Figure 1

**Response of the Nominal Interest Rate to a One-Standard-Deviation Money Growth Shock in Period 1**  
(Deviation from steady state rate X 100)

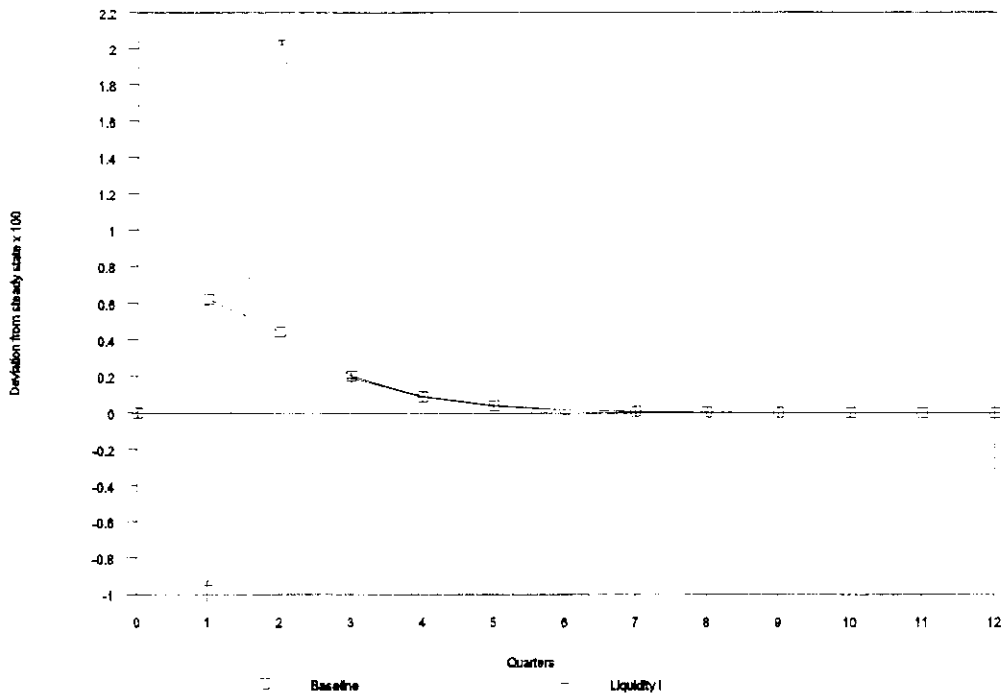


Figure 2

Response of Hours Worked to a One-Standard-Deviation Money Growth Shock in Period 1

