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Abstract

In this paper we set up a small open economy model with financial frictions, following Curdia and Woodford (2010)’s model. Unlike other results in the literature such as Curdia and Woodford (2010), McCulley and Ramin (2008) and Taylor (2008), we find that optimal monetary policy should not respond to changes in domestic interest rate spreads when the source of fluctuations are exogenous financial shocks. A novel result here is that the optimal size of policy responses to changes in the credit spread is large when the disturbance source are shocks to the foreign interest rate. Our results suggest that such a response is welfare enhancing.

JEL Classification: E44, E50, E52, E58, F41

Keywords: financial frictions, optimal interest rate rules, interest rate spreads, welfare, small open economy, second order approximation

1 Introduction

The 2007 global financial crisis and its aftermath have brought to the forefront of macroeconomic policy research the need to incorporate the role of financial intermediaries in the understanding of aggregate macroeconomic fluctuations and its implications for the conduct of stabilization policies. In the field of monetary policy, the use of quantitative measures and the deployment of non-traditional instruments by the central banks of major advanced economies to provide monetary accommodation, as the zero lower bound on nominal interest rates was reached, has sparked renewed interest on the rationale and effects of unconventional monetary policy. Also, the expanded mandates for central banks around the world to formally incorporate financial stability objectives have translated into new avenues for research in the design of optimal monetary policy. Contributions in this direction include Christiano et al. (2008), who argue that monetary policy should take into account financial variables such as...

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credit dynamics to mitigate financial frictions and to reduce the likelihood that monetary policy inadvertently contributes to boom-bust episodes. However, the optimality criterion used to gauge the policy response in terms of welfare is not clear. Curdia and Woodford (2010), on the other hand, evaluate the optimality of the policy response in terms of microfounded welfare measures. They find that there are no significant gains in terms of welfare from including credit fluctuations in the monetary policy rule and that central banks should react instead to movements in the spread between borrowing and lending interest rates. However, both analyses are framed in a closed economy model. Here, we extend on that literature by investigating whether monetary policy should respond either to credit spreads or to private debt in the context of a small open economy.

For that purpose, we extend Curdia and Woodford (2010) to a small open economy version. Even though we maintain several basic features of Curdia and Woodford (2010), such as the introduction of heterogeneous agents (borrowers and lenders) we introduce several additional features to better capture the characteristics of a small open economy. In particular, we allow financial intermediaries fund their activities both with domestic and foreign liabilities, which permits us to capture the effect of cross-border banking flows in the transmission of shocks (in a very stylized manner). Also, we allow consumers to consume three different baskets of goods: domestic tradable goods, domestic non-tradable goods and foreign tradable goods. However, the introduction of these features makes it difficult to derive a closed-form expression for a micro-founded loss function. Hence, we use the numerical welfare analysis proposed by Schmitt-Grohe and Uribe (2004a) to find the optimal policy rule that maximizes the expected welfare of this model economy.

We find that under the usual disturbance sources such as productivity shocks, Curdia and Woodford (2010)’s conclusions does hold. That is, optimal monetary policy should not react neither to private debt nor credit spread fluctuations in the case of productivity shocks. Furthermore, we find a novel result which states that there are substantial welfare gains from responding to credit spreads in the face of foreign interest rate shocks. In particular, we find that the response coefficient to credit spread in the Taylor rule is larger than 1, which is equivalent to have the borrowing interest rate in the Taylor rule instead of the lending rate. This finding is very important for monetary policy analysis in small open economies, particularly in the context of policies to manage capital flows when this are driven by global liquidity conditions.

This recent conclusion can be explained as follows. Suppose, there is an exogenous, unanticipated and temporary increase in the credit spread, which raises the borrowing interest rate, which in turn reduces the households consumption. Then, if the central bank wants to offset to this shock, it has to cut the lending rate until let the borrowing rate reach its before-shock’s level. Since private banks can easily substitute between domestic and foreign funding, the loans-supply is not going to be affected. Therefore, the exogenous credit spread distortion could not be offset by monetary policy and there will not be welfare gains.
Unlike the exogenous credit spread shock, foreign interest rate shock effect on credit spread can be offset easily by the central bank. Suppose, there is an exogenous, unanticipated and temporary increase in the foreign interest rate, which raises the real exchange rate, inflation, private banks’ domestic funding but reduces their foreign funding. However, since the surge in the real exchange rate is larger than the reduction on the foreign funding, then the real foreign funding, deflated in units of domestic good, will increase and therefore private banks will have more available resources for lending to households. This raise in domestic loans increases the credit spread. Thus, for the central bank it would be easy to offset this shock as long as it can raise the domestic lending interest rate to offset the above real exchange rate boost, and therefore reduce the real foreign funding, deflated in units of domestic good, until its before-shocks level.

The paper is organized as follows. In Section 2, we review the structure of the model, emphasizing the role of agent heterogeneity and imperfect financial intermediation, the introduction of the small open economy assumptions, and the discussion of its numerical calibration. In Section 3 we derive the optimal policy rules and the methodology used to evaluate the welfare implications of each one of these. Finally, in Section 4 we summarize our findings and present some conclusions.

2 A Small Open Economy Model with Financial Frictions

2.1 Model Description

2.1.1 Households

The following section presents the model that extends Curdia and Woodford (2010)’s model to a small open economy version. Similar to C&W, we depart from the assumption of a representative household and suppose a continuum of households with different preferences that in any given period can be either borrowers (b) or savers (s). Each household i seeks to maximize a discounted intertemporal value function given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u^{\tau_t}(c_t(i); \xi_t) - \int_0^1 v^{\tau_t}(h_t(j, i); \xi_t) dj \right],$$

(1)

where $\tau_t(i) \in \{b, s\}$ indicates household i’s type in period $t$ (borrower or saver), with the utility from consumption taking the form:

$$u^{\tau_t}(c_t(i); \xi_t) \equiv \frac{[c_t(i)]^{1-\frac{1}{\sigma}} (\bar{C}^\tau)^{\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}}$$

(2)

and the disutility of labor given by:

\footnote{For succinctness we will refer to Curdia and Woodford (2010)’s model as C&W going forward.}
\[ v^\tau(h_t(j,i); \xi_t) = \frac{\psi_{\tau}}{1 + \nu} [h_t(j,i)]^{1+\nu} \hat{H}_t^{-\nu}, \]  

(3)

where \( \xi_t \) is a vector of aggregate taste shocks, \( \sigma_{\tau} \) is the intertemporal elasticity of substitution of type \( \tau \)'s household, \( \psi_{\tau} \) is a scalar factor for each household's type \( \tau \), \( \nu \) is the inverse of the Frisch elasticity of labor supply. As in C&W we assume a continuum of differentiated goods, each produced by a monopolistic competitive supplier, where \( c_t(i) \) is a Dixit-Stiglitz aggregator of household \( i \)'s purchases of differentiated goods indexed by \( j \). Each household \( i \) supplies a continuum of different types of specialized labor, also indexed by \( j \), which are hired by the \( j-th \) firm in the non-tradable sector of the economy, which subsequently we will refer to domestic economy.

Unlike C&W, we allow the existence of three baskets of goods: two baskets of domestically produced goods (tradable and non-tradable), and one basket of imported goods. The tradable domestic good is consumed by both domestic and foreign households, while the non-tradable domestic good is consumed by domestic households, the government and financial intermediaries. This assumption allows us to introduce monopolistic competition and sticky price formation in the supply sector. Accordingly, both types of households face the following consumption baskets:

\[ c^\tau_t(j) = \left[ (\gamma)^{\frac{1}{\rho}} (c^\tau_{h,t}(j))^{\frac{1}{\rho}} + (1 - \gamma)^{\frac{1}{\rho}} (c^\tau_{f,t}(j))^{\frac{1}{\rho}} \right]^{\rho \gamma / (\gamma + \rho)} \]  

(4)

\[ c^\tau_{h,t}(j) = \left[ (\gamma_{hn})^{\frac{1}{\rho_{nx}}} (c^\tau_{hn,t}(j))^{\frac{1}{\rho_{nx}}} + (1 - \gamma_{hn})^{\frac{1}{\rho_{nx}}} (c^\tau_{h,t}(j))^{\frac{1}{\rho_{nx}}} \right]^{\rho_{nx} \gamma / (\rho_{nx} + \gamma)} \]  

(5)

where \( c^\tau_{h,t} \) denotes the consumption basket of domestically produced goods, with \( c^\tau_{hn,t} \) and \( c^\tau_{xn,t} \) denoting non-tradable and tradable goods, respectively. \( c^\tau_{f,t} \) denotes the consumption basket of imported goods, \( \gamma \) is the parameter controlling the participation of expenditure in domestic goods’ consumption on total consumption’s expenditure, \( \rho \) denotes the intratemporal elasticity of substitution between imported and home goods consumption, \( \gamma_{hn} \) is a parameter controlling the participation of expenditure in domestic non-traded goods’ consumption on domestic-goods consumption’s expenditure and \( \rho_{nx} \) denotes the intratemporal elasticity of substitution between non-traded and traded home-produced goods consumption.

Each agent’s type, \( \tau_t(i) \), evolves as an independent two-state Markov chain. In each period a household’s type remains the same with probability \( 0 < \delta < 1 \), and with probability \( 1 - \delta \) a new type is drawn. When a new type is drawn, household \( i \) will be type \( b \) with probability \( \pi_b \) and type \( s \) with probability \( \pi_s \), where \( 0 < \pi_b, 0 < \pi_s, \) and \( \pi_b + \pi_s = 1 \). These constant probabilities imply that, in the aggregate, probability \( \pi_t \) can be interpreted as the fraction of households who are of type \( \tau(i) \).

We also assume that borrowers are more impatient than savers so that \( u^b(c_t(i)) > u^s(c_t(i)) \) for all levels of \( c \), and that marginal utility for type \( b \)
households varies less with the current level of consumption than for type $s$ households, resulting in a greater degree of intertemporal substitution of type $b$'s expenditures in response to interest rate changes and therefore in lower relative risk aversion. The difference in households’ risk aversion coefficients implies assorted marginal utilities of income. Therefore, it is necessary to assume different marginal disutility of working a given number of hours for each type of household in order for both household types to choose to work the same number of hours in the steady state. For simplicity, we also assume that the elasticities of labor supply of the two types are the same. As pointed by C&W the coexistence of the two types of households with differing impatience to consume creates a social function for financial intermediation. Likewise, and as it is explained in detail by C&W, type $b$ households will always choose to borrow from financial intermediaries, while type $s$ households will deposit their savings with them.

Since households face random shocks associated with their type, one way to facilitate their aggregation is to allow them to sign state-contingent contracts with one another so that optimal consumer decisions are not affected by their type’s uncertainty. To give a significant role to the financial intermediary sector it is assumed, as in C&W, that these claims are traded through an insurance agency which only transfers money to households when each one of them receives a draw with a new type. This implies that during the periods in which households’ type remains constant they will not accrue transfers from the insurance agency, so they have to engage one-period contracts with financial intermediaries.

Given that our model’s households’ intertemporal optimization problem is the same as in C&W, so is the set of Euler equations that characterize household’s optimal consumption choices:

$$
\lambda^b_t = (1 + \rho^b_t)\beta E_t[(\delta + (1 - \delta)\pi_b)\frac{\lambda^b_{t+1}}{\pi_{t+1}} + (1 - \delta)\pi_s\frac{\lambda^s_{t+1}}{\pi_{t+1}}] \tag{6}
$$

$$
\lambda^s_t = (1 + \rho^s_t)\beta E_t[(1 - \delta)\pi_b\frac{\lambda^b_{t+1}}{\pi_{t+1}} + (\delta + (1 - \delta)\pi_s)\frac{\lambda^s_{t+1}}{\pi_{t+1}}] \tag{7}
$$

where $\rho^b_t$ is the nominal interest rate that type $s$ households receive from their deposits in the financial sector, $\rho^s_t$ is the nominal interest rate that type $b$ households pay to financial intermediaries by their loans, $\pi_{t+1} = \frac{P_t}{P_{t-1}}$ denotes the gross inflation rate, $P_t$ is the consumer price index in period $t$ of the home country, $\lambda^s_t$ and $\lambda^b_t$ denote the marginal utility of income for types $s$ and $b$, respectively. These first order conditions also imply:

$$
\lambda^s_t = \left(\frac{\rho^s_t}{\rho^s_t} \right)^{-\sigma^{-1}} \tag{8}
$$

where $\tilde{C}^s_t$ denotes a preference shock with different unconditional means for each household’s type, representing changes in their spending opportunities.
The optimal supply of labor of each type of household is given by:

\[ \mu_t w^{\nu_t} (h(j)^{\tau}; \xi_t) = \lambda_t \frac{W(j)_t}{P_t} \]  

(9)

where \( W(j)_t \) is the nominal wage for labor of type \( j \), and the exogenous factor \( \mu_t^{\nu} \) represents a possible "wage markup".

Since consumer preferences are separable along time, households' intertemporal optimization problem can be solved separately from the intratemporal optimization one. Therefore, each household chooses the optimal basket of home and foreign goods by taking the optimal total consumption basket \( c^{\tau}_t(j) \) as given.² These optimal baskets are similar to those of the solution of standard small open economy models.

2.1.2 Firms

The production technology for each differentiated non-tradable good produced by firm \( j \) is described by the following decreasing returns to scale function:

\[ y_{n,t}(i) = Z_t h_t(i)^{\phi} \]  

(10)

where \( h_t(i) \) is hours worked (a non-tradable input), \( Z_t \) is an aggregate exogenous productivity process and \( \phi \geq 1 \). Prices for the different non-tradable varieties are sticky and follow a Calvo pricing process with constant probability \( \alpha \) of keeping them unchanged. Demand for non-tradable goods can be derived as a function of relative prices with a constant price elasticity (given the differentiation assumption).

Imported goods are homogenous on the border but are differentiated at the retail stage in a monopolistic competition setting. Prices in local currency of imported goods also follow a Calvo pricing process.

 Tradable goods are assumed to be commodity goods which sell at prices determined in international markets. Their production technology \( (y_{x,t}) \) and prices \( (p_{x,t}) \) are described by exogenous stochastic processes (see Appendix).

2.1.3 Resources Constraint

Physical resource constraints are thus given by:

\[ y_{n,t} \geq c_{hn,t} + g_t + \Xi(b_t) \]  

(11)

\[ y_{x,t} \geq c_{hx,t} + c_{hx,t}^* \]  

(12)

where \( g_t \) is the government expenditure, \( \Xi(b_t) \) represents the resources consumed by the intermediary sector (we describe this function’s properties below), \( b_t \) is domestic loans that financial intermediaries lend to type-b households \( c_{hx,t}^* \) is the foreign consumption of tradable home good (exports).

²We present the intratemporal optimization problems and optimal basket choices in the Appendix.
2.1.4 Financial Intermediaries

Following C&W, we assume that there are two frictions associated with financial intermediation: the first represents a screening cost that banks face when they are lending resources, $\Xi(b_t)$, which is a non-decreasing and weakly convex function on the amount domestic real credit, $b_t$; the second one is the presence of an exogenous non-negative loss rate, $\chi_t$, that represents the proportion of loans which are not repaid. Additionally, we allow financial intermediaries to fund their operations with both domestic and foreign savings. Therefore, the representative bank’s optimization problem is represented by:

$$\max_{(b_t, b^*_t)} \frac{D^\text{int}_{t+1}}{P_{t+1}}$$

subject to

$$b_t + \Xi(b_t) + \chi_t b_t = x_t + q_t b^*_t$$

where $\frac{D^\text{int}_{t+1}}{P_{t+1}}$ denotes one-period-ahead real dividends, $b_t$ real domestic credit, $b^*_t$ real foreign debt, $x_t$ denotes the real quantity of deposits, $q_t$ the real exchange rate,

$$\frac{D^\text{int}_{t+1}}{P_{t+1}} = \frac{b_{t-1}(1 + \psi_{t-1})}{\pi_t} - \frac{(1 + \psi_{t-1})x_{t-1}}{\pi_t}$$

$$- \frac{(1 + \psi^d_{t-1})(1 + \varphi((q_{t-1}b^*_t - 1))(b^*_t - 1))}{\pi_t}$$

and the intermediation technology

$$\Xi(b) = \Xi(b)^\eta$$

where $\Xi_t$ is an exogenous scale factor process, $\eta$ denotes the inverse of elasticity of intermediation production respect to domestic debt $\varphi((2b^*_t - b^*_t - 1))$ is the risk premium, a non-decreasing and convex function of aggregate external debt to product ratio’s deviations from its steady state. Here, we use the same functional form for $\psi((2b^*_t - b^*_t - 1))$ as Schmitt-Grohe and Uribe (2003). That is:

$$\varphi_t = \psi_1 \left( e^{\frac{\eta b^*_t}{\eta b^*_t - \eta b^*_t}} - 1 \right)$$

The introduction of financial frictions implies that the interest rate faced by depositors ($i^d_t$) differs from the one charged to borrowers ($i^b_t$). The solution to the optimization problem of the banks yields two results: First, the interest rate spread will be a non-decreasing function of real loans that can shift over

---

3 As in C&K, we treat opportunities for fraudulent borrowing as being equally distributed across all households.
time by exogenous changes either in the cost function, $\Xi$ or in the loss rate, $\chi_t$ (see equation 18). Second, financial intermediaries will demand foreign real resources up to the point where the marginal cost of domestic funding equals the exchange rate-adjusted marginal cost of foreign borrowing (see equation 19).

$$\frac{(1 + \bar{i}_t^f)}{\pi_{t+1}} = \frac{(1 + \bar{i}_t^d)(1 + \Xi' b_t + \chi_t)}{\pi_{t+1}}$$  \hspace{1cm} (18)$$

$$\frac{(1 + \bar{i}_t^d q_t)}{\pi_{t+1}} \leq \frac{(1 + \bar{i}_t^d)(1 + \varphi_t)q_{t+1}}{\pi_{t+1}}$$  \hspace{1cm} (19)$$

where,

$$\omega_t = \Xi' b_t + \chi_t$$  \hspace{1cm} (20)$$

2.1.5 Government

Government purchases, $G_t$, and real public debt $b^g_t$ are represented by exogenous autoregressive stochastic processes. The public debt process is a reduced-form representation of the fiscal sector, which has a government budget constraint and a fiscal policy rule responding to government-debt deviations from its steady state value. In addition, we allow for exogenous income tax rate, $\tau_t$, shocks which are equally distributed to consumers in form of lump-sum subsidies.

Finally, monetary policy is represented by a Taylor rule, in which the central bank provides real domestic funds to financial intermediaries at a rate $i_t^d$, which we will describe in section 3.

2.2 Calibration

In this paper we provide various calibration exercises for this model to check our outcomes’ robustness for small open economies. The first calibration exercise consists in checking how our C&W’s properties (like impulse response functions and optimal policy rules) change when this is generalized to a small open economy version. To do this, we need to keep the same calibrated parameters of C&W’s. However, when we extend the model to a small open economy, we require more parameters that are not present in Curdia and Woodford (2010). Therefore, we need to calibrate a new set of parameters. Since this is a theoretical exercise, we calibrated them by using some values obtained from small open economy literature, this calibration exercise is fully described in the appendix.

The second calibration exercise consists in a robustness check of our novel results. Non-arbitrage equilibrium condition implies that this expression holds with inequality and therefore financial intermediaries could choose positive balances of foreign debt.

Some readers of this draft have pointed out that the C&W calibrated parameters are different than those provided in the small open economy literature. However, for this first exercise, we have decided to keep the C&W’s calibrated parameters because we want to show that our novel result is not a consequence of different parameters values but of the fact of allowing banks to borrow abroad.
result for a set of different countries. In this working paper, we only present the first calibration exercise. The second one will be available soon.

3 Welfare Analysis

3.1 Methodology

We assume a Central Planner whose objective is to average household expected welfare, $W_t$:

$$W_t = \bar{U}_t + \beta E_t W_{t+1}$$

First we compute the optimal parameters, of three different policy rules (we will show them further), that maximize the following measure of welfare.

$$\bar{U}_t (c^b_t, c^s_t, Y_{n,t}, \xi_t) = \pi_b \left( \frac{(c_b^t)^{1-\sigma_b^{-1}} (C_b^t)^{\sigma_b^{-1}}}{1 - \sigma_b^{-1}} \right) + (1 - \pi_b) \left( \frac{(c_s^t)^{1-\sigma_s^{-1}} (C_s^t)^{\sigma_s^{-1}}}{1 - \sigma_s^{-1}} \right)$$

$$- \frac{\psi}{1 + \nu} \left( \frac{\Delta h_t}{\lambda_t} \right)^{1+\omega} H_t^{-\nu} \left( \frac{\psi_b}{\lambda_b^t} \right)^{1+\omega}$$

$$\Delta h_{n,t} = \alpha \Delta h_{n,t-1} (\pi_{n,t+1})^{\psi_b h_t (1+\omega)} + (1 - \alpha) \left( \frac{K_{n,t}^h}{F_{n,t}^h} \right)^{-\theta h (1+\omega)}$$

$$\left( \frac{K_{n,t}^h}{F_{n,t}^h} \right)^{-\theta h (1+\omega)}$$

denotes the ratio of the optimizing-firm’s price to the industry price level in the domestic non-tradable sector (More details are provided in the Appendix).

$$\Lambda_t^{1+\omega} = \psi \left[ \pi_b \psi_b^{-\frac{1}{\omega}} (\lambda_t^h)^{1+\omega} (1 - \pi_b) \psi_s^{-\frac{1}{\omega}} (\lambda_t^s)^{1+\omega} \right]$$

$$\tilde{\lambda}_t = \psi \left[ \pi_b \left( \frac{\lambda_t^b}{\psi_b^b} \right)^{\frac{1}{\omega}} (1 - \pi_b) \left( \frac{\lambda_t^s}{\psi_s^s} \right)^{\frac{1}{\omega}} \right]^\nu$$

Unlike Curdia and Woodford (2010), Svensson and Woodford (2003), Gertler et al. (1999), Gali and Monacelli (2005), along with others compute optimal policy rules by using conventional loss functions, as the following form, as a measure of welfare.

$$L = E_0 \sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi)^2 + \theta x_t^2]$$
These loss functions are useful and popular because of their algebraic tractability and simplicity for welfare evaluation. However, they can lead to biased outcomes since most of them, like those used by Svensson and Woodford (2003) and optimal monetary policy literature, lack of deep parameters and omit other variables than inflation and output. However, if those loss functions were derived from a specific welfare function, as Curdia and Woodford (2010) did, they would lead to trustworthy outcomes but they are difficult to algebraically derive for large-scale models.

Therefore, following Schmitt-Grohe and Uribe (2004c) we evaluate the second order approximation of expected welfare function, \( W_t \), with the first and second moments of model’s policy functions approximated up to second order, to compute optimal policy rules and to make welfare comparisons among them. By using this computational approach, we can avoid spurious welfare rankings as Kim and Kim (2003) showed.\(^6\)

The following are the monetary policy rules that we consider in this paper:

1. **Standard Taylor Rule:**
   \[
   i_t^d = i_{t}^{n.d} + \phi_\pi^* (\pi_t - \pi) + \phi_y^* (\hat{\text{gdp}}_t) \tag{27}
   \]

2. **Standard Taylor Rule plus Credit Spread:**
   \[
   i_t^d = i_{t}^{n.d} + \phi_\pi^* (\pi_t - \pi) + \phi_y^* (\hat{\text{gdp}}_t) + \phi_w^{so} (\omega_t - \omega) \tag{28}
   \]
   Where, \( \phi_w^{so} \) is the optimal parameter that maximize (21) given \( \phi_\pi \) and \( \phi_y \) whose values are the same as those set in the standard Taylor rule.

3. **Optimal Interest Rate Rule:**
   \[
   i_t^d = i_{t}^{n.d} + \phi_\pi^* (\pi_t - \pi) + \phi_y^* (\hat{\text{gdp}}_t) \tag{29}
   \]

4. **Optimal Interest Rate Rule with Credit Spread**
   \[
   i_t^d = i_{t}^{n.d} + \phi_\pi^* (\pi_t - \pi) + \phi_y^* (\hat{\text{gdp}}_t) + \phi_w^* (\omega_t - \omega) \tag{30}
   \]
   where,
   \[
   \hat{\text{gdp}}_t = \text{gdp}_t - \text{gdp}^n_t \tag{31}
   \]
   \( \text{gdp}_t \) is the output gap, \( i_{t}^{n.d} \) is the natural interest rate, \( \pi \) and \( \omega \) are the steady-state values of inflation and credit spread respectively.\(^7\)

\(^6\)We can provide the matlab codes that show how our approach can replicate the Kim and Kim (2003) solution to spurious welfare evaluations.

\(^7\)Our model solution provides the dynamics and moments for natural variables. Here we assume that natural rates are those belonging to the flexible prices and constant inflation version of this model.
\( \phi_*^t, \phi_*^y \) and \( \phi_*^w \) are the optimal parameters that maximize (21). Following Schmitt-Grohe and Uribe (2004a) we restrict each one of these parameters to take values between 0 and 3 and consider a grid with a step side of 0.05 to find the optimal values that maximize (21) conditional to each one of the three previous policy rules.

How significant are those welfare differences? following Schmitt-Grohe and Uribe (2004a)'s methodology to calculate how much percentage of every period consumption for type, \( \Gamma \), should compensate agents from changing from interest rate rules which react to inflation and output fluctuations to those of which react to inflation, output and credit spread fluctuations. To compute \( \Gamma \), we have to solve the following expression.

\[
\mathcal{W}_t^a \left[ c_t^a, c_t^b, Y_t^a, \xi_t \right] - \mathcal{W}_t^r \left[ (1 - \Gamma) c_t^r, (1 - \Gamma) c_t^b, Y_t^r, \xi_t \right] = 0 \quad (32)
\]

where, \( a \) denotes the alternative policy regime and \( r \) the reference policy regime, in this case the interest rate rules which react to inflation and output.

Since (32) is a nonlinear function in \( \Gamma \), we solve these equations through numerical methods.8

4 Results

In this section we provide, first, the performance in terms of welfare of above policy rules conditional to each one of the shocks that this small open economy faces. These shocks are foreign interest rate shock, credit spread shock and productivity shock. Second, we present this model’s dynamics (through generalized impulse-response functions) for each one of the previous mentioned shocks and compare the model dynamics for each one of the previous policy rules.

Welfare performance of these policy rules is described in Tables 1 to 3. In first column, we denote the model’s number. That is, Model 1 denotes the model with the standard Taylor Rule parameters, Model 2 denotes the standard Taylor Rule parameters plus the optimal credit spread parameter. Model 3 denotes the model with the optimal flexible inflation targeting rule (i.e. the arguments are only inflation and output), and Model 4 denotes the model with optimal interest rate rule with credit spread (i.e. the arguments are only inflation, output and credit spread). In second to fourth columns we provide the parameters value associated to each one of the previous rules. In the fifth column, we provide the expected welfare measure, \( \mathcal{W}_t \) associated to its respective rule. Finally, in the sixth column we provide the welfare losses(gains), \( \Gamma \), measured in units of consumption, of moving away from interest rates which react only to inflation and output fluctuations.

Model dynamics comparison among these policy rules is summarized in figures (1) to (12).

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8This solution is written in matlab and the codes are available upon request.
4.1 Foreign Interest Rate Shock

From Table 1 we learn that optimal policy rule should react to credit spread. What is new here is that we do not need to exogenously shock the credit spread function, as Curdia and Woodford (2010) did to generate this result, but a foreign interest rate shock is sufficient to get this new result. Moreover this is more realistic and likely observable shock than the exogenous credit spread shock designed by Curdia and Woodford (2010).

Second we learn that moving away from the standard Taylor Rules to alternative optimal policy rules produces significantly huge welfare gains. Even though welfare is just an ordinal measure, we found that units of consumption, required to get the same welfare, of moving away from the standard Taylor to alternative optimal policy rules, such as (29), or (30), are positive, since $\lambda$ is a cost measure.9

From Figures 1 to 4 we learn that model dynamics with standard Taylor and optimal policy rules (29) and (30) are quite similar. In addition, these dynamics are consistent with those of standard small open economy models, see Schmitt-Grohe and Uribe (2003).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\phi_\pi$</th>
<th>$\phi_y$</th>
<th>$\phi_\omega$</th>
<th>$W_t$</th>
<th>$\Gamma$</th>
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<td>1.50</td>
<td>0.13</td>
<td>0.00</td>
<td>6.59</td>
<td>0.00</td>
</tr>
<tr>
<td>2.00</td>
<td>1.50</td>
<td>0.13</td>
<td>1.00</td>
<td>14.06</td>
<td>-0.07</td>
</tr>
<tr>
<td>3.00</td>
<td>1.00</td>
<td>1.05</td>
<td>0.00</td>
<td>1287.26</td>
<td>0.00</td>
</tr>
<tr>
<td>4.00</td>
<td>1.10</td>
<td>2.30</td>
<td>1.00</td>
<td>1498.65</td>
<td>-2.20</td>
</tr>
</tbody>
</table>

Table 1: Welfare Comparison - 1% Foreign Interest Rate as a Uncertainty Source

4.2 Credit Spread Shock

From Table 2 we learn first that optimal policy rules should react only to inflation fluctuations around their target and to output gap. Unlike Curdia and Woodford (2010) and McCulley and Ramin (2008), we found that optimal policy rule should not react to credit spread, a possible explanation of this finding is that we allow banks to easily substitute between domestic and foreign funding, then monetary policy will not have a considerable effect on loans and therefore on the credit spread. Second we learn that moving away from standard Taylor Rule to optimal policy rules generates large welfare gains.

From Figures 5 to 8 we learn that model dynamics with standard Taylor and optimal policy rules are slightly different, except to the Non-Traded goods inflation. This result is predictable since the standard Taylor rule is more aggressive than the optimal rules provided in this paper.

9If we compare our welfare gains values with those calculated in the literature of welfare evaluation, for example Schmitt-Grohe and Uribe (2004b), ours are quite larger even in the cases where welfare differences are small. The reason of this result comes from the risk aversion coefficient calibration, $\sigma^{-1}$. Our calibration implies a smaller risk aversion coefficient value.
4.3 Productivity Shock

From Table 3 we learn first that optimal policy rules (29) to (30) are the same. Since rules are the same, therefore expected welfare does. This result is consistent with that of Curdia and Woodford (2010). That is, there are not welfare gains of reacting to credit spreads or private debt fluctuations when disturbance sources come from productivity shocks.

Second we learn that moving away from standard Taylor Rules to optimal policy rules generates less gains in welfare under productivity shocks than under credit spread or foreign interest rate shocks.

From Figures 9 to 12 we learn that model dynamics with Standard Taylor and optimal policy rules (29) and (30) are slightly different except when we compare their output gap dynamics. The main reason is that optimal policy rules (29) to (30) do not respond aggressively to inflation deviations from its target while the Taylor Rule does.

5 Final Comments

We extend the closed economy version of C%W’s model with financial frictions to a small open economy version. Here, we found that optimal monetary policy should react to credit spread fluctuations when the only disturbance sources are the foreign interest rate shocks. However, we found that optimal policy rules that react to inflation fluctuations and output gap are sufficient to enhance large welfare gains when disturbances sources came from credit spread shocks and productivity shocks. Even though, all previous results were obtained

than that of Schmitt-Grohe and Uribe (2004a).
by numerical methods, we cannot generalize this conclusion to any small open economy. Therefore, one future extension of this paper is to calibrate and estimate this model for different countries, like Colombia, Chile, Mexico, Canada, South Africa, and so on in order to evaluate how sensitive are our conclusions, provided here, to different parameter values.

Appendix

Since this is a small open economy model, preferences, budget constraints, and resources constraints are different from our Benchmark model. These differences imply a new set of intratemporal first order conditions for consumers because now we allow agents to choice among different goods: tradable and non-tradable. However, consumer intertemporal conditions remain the same as Curdia and Woodford (2010) because, here we assume that only financial intermediaries have access foreign assets (or debt) market.

A Consumers Intratemporal Optimization Problem

Since consumer’s preferences are separable across time, we can solve the intratemporal optimization problem independently of the intertemporal optimization problem. The first intratemporal optimization problem describes a consumer who seeks to minimize his expenditure between home goods and foreign goods given his total consumption basket $c_t$. That is:

$$[c_{h,t}(j), c_{f,t}(j)] = \arg \min p_{h,t} c_{h,t}(j) + p_{f,t} c_{f,t}(j)$$  \hspace{1cm} (A.1)

subject to:

$$c_{\tau,t}(j) = \left[ (\gamma) \frac{\rho_{hf}}{\rho_{hf} - 1} (c_{h,t}(j))^{\rho_{hf} - 1} + (1 - \gamma) \frac{1}{\rho_{hf}} (c_{f,t}(j))^{\rho_{hf} - 1} \right]^{\frac{1}{\rho_{hf}}}$$  \hspace{1cm} (A.2)

From this optimization problem, we have the conditional demands for home goods, imported good and the Consumer Price Index (CPI). That is:

$$c_{h,t}(j) = \gamma c_{h,t}^* (\tilde{p}_{h,t})^{-\rho_{hf}}$$  \hspace{1cm} (A.3)

$$c_{f,t}(j) = (1 - \gamma) c_{f,t}^* (\tilde{p}_{f,t})^{-\rho_{hf}}$$  \hspace{1cm} (A.4)

$$p_t = \left[ \gamma p_{h,t}^{1-\rho_{hf}} + (1 - \gamma) p_{f,t}^{1-\rho_{hf}} \right]^{\frac{1}{1-\rho_{hf}}}$$  \hspace{1cm} (A.5)

where $p_t$ is the CPI, $\tilde{p}_{h,t} = \frac{p_{ht}}{p_t}$ is the relative price of home goods and $\tilde{p}_{f,t} = \frac{p_{ft}}{p_t}$ is the relative price of imported goods.
The second intratemporal optimization problem describes a consumer who seeks to minimize his expenditure between non tradable home goods and tradable home goods given his consumption basket of home goods, chosen above \( c_{h,t} \). That is:

\[
\begin{align*}
\left[ c_{h,n,t}(j), c_{h,x,t}(j) \right] = & \arg \min p_{n,t}^{h} c_{n,t}(j) + p_{x,t}^{h} c_{x,t}(j) \\
\text{subject to:}
\end{align*}
\]

\[
c_{h,t}(j) = \left[ (\gamma_{h})^{\frac{1}{\rho_{nx}}} \left( c_{n,t}^{h}(j) \right)^{\frac{\rho_{nx}}{1-\rho_{nx}}} + (1-\gamma_{h})^{\frac{1}{\rho_{nx}}} \left( c_{x,t}^{h}(j) \right)^{\frac{\rho_{nx}}{1-\rho_{nx}}} \right]^{\frac{1}{\rho_{nx}}}
\]

From this optimization problem, we have the conditional demands for home non tradable goods, home tradable goods and the Price Index for Home produced goods. That is:

\[
\begin{align*}
\hat{c}_{h,n,t}(j) = & \gamma_{h} c_{h,t}(j) \left( \frac{p_{n,t}^{h}}{p_{n,t}} \right)^{-\rho_{nx}} \\
\hat{c}_{h,x,t}(j) = & (1 - \gamma_{h}) c_{h,t}(j) \left( \frac{p_{x,t}^{h}}{p_{x,t}} \right)^{-\rho_{nx}} \\
p_{h,t} = & \left[ \gamma \left( \frac{p_{n,t}^{h}}{p_{n,t}} \right)^{1-\rho_{nx}} + (1 - \gamma) \left( \frac{p_{x,t}^{h}}{p_{x,t}} \right)^{1-\rho_{nx}} \right] \frac{1}{\rho_{nx}}
\end{align*}
\]

where \( p_{h,t} \) is the Price Index for Home produced goods, \( \frac{p_{n,t}^{h}}{p_{n,t}} \) is the relative price of non-traded home goods and \( \frac{p_{x,t}^{h}}{p_{x,t}} \) is the relative price of traded home goods.

The third optimization problem describes a consumer who seeks to minimize his expenditure among a set of differentiated no traded goods \( c_{n,t}^{h}(i) \), that is:

\[
\left[ c_{n,t}^{h}(j,i) \right] = \arg \min \int_{0}^{1} p_{n,t}^{h}(i) c_{n,t}^{h}(j,i) di
\]

subject to:

\[
c_{n,t}^{h}(j) = \int_{0}^{1} \left[ \left( c_{n,t}^{h}(j,i) \right)^{\frac{\rho_{nx}}{1-\rho_{nx}}} di \right] \frac{1}{\rho_{nx}}
\]

From this optimization problem, we have the conditional demands for each variety \( i \) of home non tradable good and its respective price index, that is:

\[
\begin{align*}
c_{n,t}^{h}(j,i) = & \left( \frac{p_{n,t}^{h}(i)}{p_{n,t}^{h}} \right)^{-\theta_{h}} c_{n,t}^{h}(j) \\
p_{n,t}^{h} = & \left[ \left( \frac{p_{n,t}^{h}(i)}{p_{n,t}^{h}} \right)^{1-\theta_{h}} di \right] \frac{1}{\theta_{nx}}
\end{align*}
\]
The fourth optimization problem describes a consumer who seeks to minimize his expenditure among a set of differentiated imported goods \( c_{f,t}^h(i) \):

\[
[c_{f,t}(j, i)] = \arg \min \int_0^1 p_{f,t}(i) c_{f,t}(j, i) \, di \tag{A.15}
\]

subject to:

\[
c_{f,t}(j) = \int_0^1 \left( c_{f,t}(j, i) \right)^{\theta_f - 1} \frac{\theta_f}{\theta_f - 1} \, di \tag{A.16}
\]

From this optimization problem, we have the conditional demands for each variety \( i \) of the imported good and its respective price index, that is:

\[
c_{f,t}(j, i) = \left( \frac{p_{f,t}(i)}{p_{f,t}} \right)^{\theta_f} c_{f,t}(j) \tag{A.17}
\]

\[
p_{f,t} = \left[ \left( p_{f,t}(i) \right)^{1-\theta_f} \, di \right]^{\frac{1}{1-\theta_f}} \tag{A.18}
\]

### B  Firms’ price setting problem

In this model we have nominal rigidities in both non-tradable home goods and imported goods. Since most of the tradable goods exported by small open economies are commodities, we assume that the tradable goods market in our model is perfectly competitive and therefore home-tradable goods price is given.

#### B.1 Domestic Non-Traded Good: \( y_{h,n,t}^h \)

Each domestic non traded good producer \( i \) has monopolistic power in its variety, then he seeks a nominal price \( p_{n,t}^h(i) \) such that maximize the expected profits. That is:

Firm \( i \) seeks to:

\[
\overline{p_{n,t}^h(i)} = \arg \max E_t \sum_{T=t}^\infty (\alpha)^{T-t} Q_t \hat{T}_{h,n}(i) \hat{y}_{h,n,t}^h(i) (1 - \tau_{h,n,t}) \hat{y}_{h,n,t}^h - \hat{y}_{h,n,t}^{inc}(i) \tag{B.1}
\]

subject to:

\[
y_{n,t}^h(i) = \hat{z}_t h(i)^{\frac{1}{\tau}} \tag{B.2}
\]

\[
y_{n,h}(i) = \left( \frac{p_{n,t}^h(i)}{p_{n,t}^h} \right)^{\theta_h} y_{n,t}^h \tag{B.3}
\]

as long as one of the resource constraints states that \( c_{h,n,t}^h = y_{n,t}^h \).
\[ \omega^c_t = \psi \mu^w_t \left( \frac{h_t(i)}{H_t} \right)^\nu \hat{\lambda}_t^{-1} \]  

(B.4)

where,

\[ Q_{t,T} = \beta^{T-t} \frac{L_t}{L_T} \frac{p_t}{p_T} \]  

(B.5)

\[ \Lambda_t = \pi_b \lambda^b_t + (1 - \pi_b) \lambda^s_t \]  

(B.6)

\[ \hat{\lambda}_t \equiv \psi \left[ \pi_b \left( \frac{\lambda^b_t}{\psi_b} \right)^{\frac{1}{\nu}} + \pi_s \left( \frac{\lambda^s_t}{\psi_s} \right)^{\frac{1}{\nu}} \right]^{\nu} \]  

(B.7)

\[ L_{inc}^c(i) \equiv \omega^c h_c(i) \]  

(B.8)

By using (B.2) to (B.8), we get:

\[ L_{inc}^c(i) = \psi \mu^w_T \hat{\lambda}_t^{-1} \left( \frac{y_{T,T}^h}{z_T} \right)^{(1+\nu)} \left( \frac{p_{n,t}(i)}{p_{n,T}^h} \right)^{-\theta_h} p_{n,T}^h \]  

(B.9)

Replacing (B.2), (B.3) and (B.9) into (B.1), we simplify the above maximization problem into:

\[ \frac{p_{n,t}^h(i)}{p_{n,T}^h} = \arg \max E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{L_t}{L_T} \frac{p_t}{p_T} [p_{n,t}(i)(1 - \tau_{n,T}^h) \left( \frac{p_{n,t}(i)}{p_{n,T}^h} \right)^{\theta_h} y_{n,T}^h ... \left( \frac{y_{T,T}^h}{z_T} \right)^{(1+\omega_y)} \left( \frac{p_{n,t}(i)}{p_{n,T}^h} \right)^{-\theta_h(1+\omega_y)} p_{n,T}^h]^{(1+\omega_y)} \]  

(B.10)

F.O.C

\[ E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{L_t}{L_T} \frac{p_t}{p_T} [(1-\theta_h)(1-\tau_{n,T}^h) \left( \frac{p_{n,t}(j)}{p_{n,T}^h} \right)^{-\theta_h} y_{n,T}^h + \psi \mu^w_T \hat{\lambda}_t^{-1} \left( \frac{y_{T,T}^h}{z_T} \right)^{(1+\omega_y)} \left( \frac{p_{n,t}(i)}{p_{n,T}^h} \right)^{-\theta_h(1+\omega_y)} p_{n,T}^h]^{(1+\omega_y)} \]  

(B.11)

From this first order condition, we can solve for \( p_{n,t}^h(j)^{1+\theta_h \omega_y} \), that is:

\[ p_{n,t}^h(j)^{1+\theta_h \omega_y} = \frac{E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{L_t}{L_T} \left[ \alpha \mu^p \mu_{ph} \left( \frac{y_{n,T}^h}{z_T} \right)^{(1+\omega_y)} \left( \frac{p_{n,t}(j)}{p_{n,T}^h} \right)^{(1+\omega_y)} \left( \frac{1}{p_{n,T}^h} \right)^{-\theta_h(1+\omega_y)-1} \right] }{E_t \sum_{T=t}^{\infty} (\alpha \beta)^{T-t} \frac{L_t}{L_T} \left[ (1-\tau_{n,T}^h) y_{n,T}^h \left( \frac{1}{p_{n,T}^h} \right)^{-\theta_h} \right] } \]  

(B.12)
Dividing by \((p_{h,n,t}^{i})^{1+\theta_h\omega_y}\) we have:

\[
\frac{p_{h,n,t}^{i}}{p_{h,n,t}^{i}}^{1+\theta_h\omega_y} = \frac{e^T_{t} \sum_{i=1}^{\infty}(\alpha\beta)^{T-i} \Delta T \left[ \phi_{h}^{\prime}(1+\omega_y) \right]^{1+\omega_y} \theta_h(1+\omega_y)+1}{e^T_{t} \sum_{i=1}^{\infty}(\alpha\beta)^{T-i} \Delta T \left[ \phi_{h}^{\prime}(1+\omega_y) \right]^{1+\omega_y} \theta_h(1+\omega_y)+1}
\]

(R.13)

Rearranging some terms, we have:

\[
\frac{p_{h,n,t}^{i}}{p_{h,n,t}^{i}}^{1+\theta_h\omega_y} = \frac{e^T_{t} \sum_{i=1}^{\infty}(\alpha\beta)^{T-i} \Delta T \left[ \phi_{h}^{\prime}(1+\omega_y) \right]^{1+\omega_y} \theta_h(1+\omega_y)+1}{e^T_{t} \sum_{i=1}^{\infty}(\alpha\beta)^{T-i} \Delta T \left[ \phi_{h}^{\prime}(1+\omega_y) \right]^{1+\omega_y} \theta_h(1+\omega_y)+1}
\]

(B.14)

\[
K_{h,n,t} = \Lambda(\psi_{h,T}^{\prime} \psi_{h,T})^{1+\omega_y} (1+\omega_y) p_{h,n,t}^{i} + \alpha\beta E_t[K_{h,n,t+1}(\pi_{h,n,t+1})^{\theta_h(1+\omega_y)}]
\]

(B.15)

\[
P_{h,n,t} = \Lambda(1-\pi_{h,n,t+1}) \theta_{h,n,t} p_{h,n,t}^{i} + \alpha\beta E_t[P_{h,n,t+1}(\pi_{h,n,t+1})^{\theta_h-1}]
\]

(B.16)

\[
p_{h,n,t}^{i} = \left[(\alpha(\pi_{h,n,t}^{i})^{1-\theta_h} + (1-\alpha)(p_{h,n,t}^{i}(i))^{1-\theta_h}) \right]^{1-\theta_h}
\]

(B.17)

Since all non-tradable domestic good producers are alike, then \(p_{h,n,t}^{i}(k) = p_{h,n,t}^{i}\), we have:

\[
\frac{p_{h,n,t}^{i}}{p_{h,n,t}^{i}(k)}^{1+\theta_h\omega_y} = K_{h,n,t}^{i}
\]

(B.19)

\[
1 = \left[\alpha(\pi_{h,n,t}^{i})^{1-\theta_h} + (1-\alpha)(\frac{p_{h,n,t}^{i}(k)}{p_{h,n,t}^{i}})^{1-\theta_h}\right]^{1-\theta_h}
\]

(B.20)

\[\text{B.2 Foreign Traded Good: } y_{f,t} - \text{Incomplete E.R Pass-through}\]

Foreign traded good is imported by a continuum of retailers, each one make a small variation over the homogenous good and sell to consumer a differentiated good. Since they have monopolistic power, the can set their sale price. This
price setting power is characterized by Calvo Staggered price setting strategy, which implies a imperfect pass-through.

Consider the Local Retailer \( f \) optimization problem in which seek a price \( p_{f,t}(f) \) such that maximize its expected discounted profit, that is:

\[
\overline{p}_{f,t}(f) = \arg \max E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} p_{f,t}(f) (1 - \tau_{f,T}) c_{f,t}(f) - s_t p^*_f(f) c_{f,t}(f)
\]

subject to:

\[
c_{f,t}(f) = \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^{-\theta_f} c_{f,t}
\]

Replacing (B.22) in (B.21), we simplify the above maximization problem into:

\[
\overline{p}_{f,t}(f) = \arg \max E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \left\{ \frac{p_{f,t}(f)}{p_{f,T}} \right\}^{-\theta_f} \left[ \frac{1}{p_{f,T}} \right]^{-\theta_f} (1 - \tau_{f,T}) c_{f,T} + \theta_f s_t p^*_f(c_{f,T})^\theta_f c_{f,T}
\]

F.O.C

\[
E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^{-\theta_f} \left[ \frac{1}{p_{f,T}} \right]^{-\theta_f} (1 - \tau_{f,T}) c_{f,T} + \theta_f s_t p^*_f(c_{f,T})^\theta_f c_{f,T} = 0
\]

solving for \( p_{f,t}(f) \)

\[
\overline{p}_{f,t}(f) = \frac{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^{-\theta_f} \left[ \frac{1}{p_{f,T}} \right]^{-\theta_f} (1 - \tau_{f,T}) c_{f,T}}{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} Q_{t,T} \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^\theta_f (1 - \tau_{f,T}) c_{f,T}}
\]

Then we have:

\[
\overline{p}_{f,t}(f) = \frac{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} \Lambda_T \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^\theta_f (1 - \tau_{f,T}) c_{f,T}}{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} \Lambda_T \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^\theta_f (1 - \tau_{f,T}) c_{f,T}}
\]

Dividing by \( p_{f,t} \) on both sides of (B.26), we have:

\[
\frac{\overline{p}_{f,t}(f)}{p_{f,t}} = \frac{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} \Lambda_T \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^\theta_f (1 - \tau_{f,T}) c_{f,T}}{E_t \sum_{T=t}^{\infty} (\alpha)^{T-t} \Lambda_T \left[ \frac{p_{f,t}(f)}{p_{f,T}} \right]^\theta_f (1 - \tau_{f,T}) c_{f,T}}
\]
Let,

\[ K_{f,t} = \Lambda_t \mu p_f q_t \tilde{P}_{f,t} + (\alpha \beta) E_t K_{f,t+1} \pi_{f,t+1} \theta_f \]  

(B.28)

\[ F_{f,t} = \Lambda_t (1 - \tau_{f,t}) c_{f,t} \tilde{p}_{f,t} + (\alpha \beta) E_t F_{f,t+1} \pi_{f,t+1} \theta_{f-1} \]  

(B.29)

Since all imported goods retailers are alike, then \( \bar{p}_{f,t} = \bar{p}_f = \bar{p}_{f,t} \), we have:

\[ \left( \frac{p_{f,t}}{p_f} \right) = \frac{K_{f,t}}{F_{f,t}} \]  

(B.30)

If we assume that \( i = f \) and by (A.18), we have:

\[ p_{f,t} = \left[ \int_0^1 (p_{f,t}(f))^{1-\theta_f} df \right]^{1/\theta_f} \]  

(B.31)

and therefore,

\[ 1 = \alpha \pi_{f,t} \theta_{f-1} + (1 - \alpha) \left( \frac{p_{f,t}}{p_f} \right)^{1-\theta_f} \]  

(B.32)

C Income Distribution

Wage income shares for each type of consumer are derived as follows:

From households first order condition, we have:

\[ \lambda_t \omega(j) = \psi_t h_t(j)^{1+\nu} \Pi_t^{-\nu} \mu_t^\nu \]  

(C.1)

which is the labor supply function.

\[ \Rightarrow h_t(j) = \left( \frac{\lambda_t \omega(j)}{\psi_t \mu_t^\nu} \right)^{\frac{1}{\nu}} \Pi_t \]  

(C.2)

\[ \forall \tau = b, s \]

Since,

\[ h_t(j) = \pi_b h^b_t(j) + (1 - \pi_b) h^s_t(j) \]  

(C.3)

And by using (C.2) and (C.3), we have:

\[ \omega_t(j) = \psi \mu_t^\nu \left( \frac{h_t(j)}{H_t} \right)^\nu \lambda_t^{-1} \]  

(C.4)

Let’s assume that the \( j^t h \) household has the skill required by \( i^t h \)’s firm, then:

\[ y^h_{m,t}(j) = z_t h_t(j)^{\frac{1}{\beta}} \]  

(C.5)
By (C.4) and (C.5), we have:

$$
\omega_t(j) = \psi \mu_t^w \lambda_t^{-1} \left[ \frac{y_{n,t}^h(j)}{z_t} \right] \frac{1}{\bar{H}_t} \nu
$$  \hspace{1cm} (C.6)

From B.3 we know that:

$$
y_{n,t}^h(j) = \left( \frac{p_{n,t}^h(j)}{p_{n,t}^h} \right)^{-\theta_h} y_{n,t}^h
$$  \hspace{1cm} (C.7)

By replacing (C.7) into (C.6), we get:

$$
\omega_t(j) h_t(j) = \psi \mu_t^w \lambda_t^{-1} \left( \frac{p_{n,t}^h(j)}{p_{n,t}^h} \right)^{-\theta_h(1+\omega_y)} \left[ \frac{y_{n,t}^h}{z_t} \right] \nu
$$  \hspace{1cm} (C.8)

Then integrating over \( j \), we have the aggregate labor income of this economy:

$$
\int_0^1 \omega_t(j) h_t(j) dj = \psi \mu_t^w \lambda_t^{-1} \Delta_{n,t}^h \left[ \frac{1}{\bar{H}_t} \right]^{-\nu} \left( \frac{y_{n,t}^h}{z_t} \right)^{1+\omega_y}
$$  \hspace{1cm} (C.9)

Where: \( \Delta_{n,t}^h = \int_0^1 \frac{p_{n,t}^h(j)}{p_{n,t}^h} dj \)

Let, \( \omega_t^\tau = \frac{\omega_t(j) h_t(j)}{y_{n,t}^h(j)} \) denotes the labor income share for each consumer’s type, \( \tau \) and consider equations (C.2) and (C.3), then

$$
\omega_t^\tau = \frac{\int_0^1 \omega_t(j) h_t(j) \psi \mu_t^w \lambda_t^{-1} \left( \frac{p_{n,t}^h(j)}{p_{n,t}^h} \right)^{-\theta_h(1+\omega_y)} \left[ \frac{y_{n,t}^h}{z_t} \right] \nu \psi \mu_t^w \lambda_t^{-1} \Delta_{n,t}^h \left[ \frac{1}{\bar{H}_t} \right]^{-\nu} \left( \frac{y_{n,t}^h}{z_t} \right)^{1+\omega_y} dj}{\int_0^1 \omega_t(j) \psi \mu_t^w \lambda_t^{-1} \left( \frac{p_{n,t}^h(j)}{p_{n,t}^h} \right)^{\frac{1}{\nu}} \omega_t(j) \psi \mu_t^w \lambda_t^{-1} \left[ \frac{1}{\bar{H}_t} \right] \psi \mu_t^w \lambda_t^{-1} \Delta_{n,t}^h \left[ \frac{1}{\bar{H}_t} \right]^{-\nu} \left( \frac{y_{n,t}^h}{z_t} \right)^{1+\omega_y} dj}
$$  \hspace{1cm} (C.10)

This expression can be simplified as follows:

$$
\omega_t^\tau = \left( \frac{\lambda_t^\psi}{\lambda_t^\psi_{\psi_t^w}} \right)^{\frac{1}{\nu}} \psi \mu_t^w \lambda_t^{-1} \Delta_{n,t}^h \left[ \frac{1}{\bar{H}_t} \right]^{-\nu} \left( \frac{y_{n,t}^h}{z_t} \right)^{1+\omega_y}
$$  \hspace{1cm} (C.11)

Therefore, the labor income for each type of household is computed as:

$$
W_t^\tau = \omega_t^\tau \int_0^1 \omega_t(j) h_t(j) dj
$$  \hspace{1cm} (C.12)

Finally,

$$
W_t^\tau = \left( \frac{\lambda_t^\psi}{\lambda_t^\psi_{\psi_t^w}} \right)^{\frac{1}{\nu}} \psi \mu_t^w \lambda_t^{-1} \Delta_{n,t}^h \left[ \frac{1}{\bar{H}_t} \right]^{-\nu} \left( \frac{y_{n,t}^h}{z_t} \right)^{1+\omega_y}
$$  \hspace{1cm} (C.13)
$W_t^* = \left( \frac{\lambda^*}{\lambda} \frac{\psi}{\psi_a} \right)^{\frac{1}{\nu}} \psi \mu_t \tilde{\lambda}^{-1} \Delta_{\tau(t)}^{-\nu} \left( \frac{y^h_{t-1} \varphi_t}{z_t} \right)^{(1+\omega)}$

(C.14)

D Balance of Payments - (Model Consistent)

Following Curdia and Woodford (2010), the aggregate net end-of-period net financial wealth for borrowers at $t, B_t$, we have:

$$B_t = -\int_{B_t} A_t(i) + \pi_b R_t^b$$

(D.1)

In the same way, the aggregate net end-of-period net financial wealth for lenders at $t, X_t$ is defined as follows:

$$X_t + B_t^d \int_{A_t} A_t(i) - \pi_s R_t^s$$

(D.2)

Where:

$B_t$ is the set of households $i$ for which $A_t(i) < 0$, being $A_t(i)$ the household $i$’s beginning-of-period nominal financial wealth, $A_t$ is the set of households $i$ for which $A_t(i) > 0$, being $A_t(i)$ the household $i$’s beginning-of-period nominal financial wealth.

$$\int_{B_t} A_t(i) di = (1 - \delta) \pi_b A_t - \delta B_{t-1} (1 + i^b_{t-1}) + \delta \pi_b D_{t}^{int}$$

(D.3)

$$\int_{A_t} A_t(i) di = (1 - \delta) \pi_s A_t + \delta (X_{t-1} + B_{t-1}^d) (1 + i^d_{t-1}) + \delta \pi_s D_{t}^{int}$$

(D.4)

$$D_{t}^{int} = B_{t-1} (1 + i^b_{t-1}) - X_{t-1} (1 + i^d_{t-1})$$

$$-s_{t-1} B_{t-1}^r (1 + i^d_{t-1}) (1 + \varphi_{t-1})$$

(D.5)

$$R_{t}^{\tau} = c_t^\tau - W_t^\tau - p_{x,t} y_{x,t}^h - D_{n,t}^h - D_{x,t}^b - D_{f,t}^b - T_{t}^{\tau}$$

(D.6)

for all $\tau = b, s$

$T_{t}^{\tau}$ is a lump sum transfer that government provide uniformly to each consumer $D_{n,t}^h$ are the dividends of home non trade good firms, $D_{x,t}^b$ are the dividends of home trade good firms and $D_{f,t}^b$ are the dividends of foreign goods importers.
By using (D.1), (D.2), and (D.4), we have:

\[ B_t - X_t = -(1 - \delta)(\pi_b + \pi_s)A_t + \delta[B_{t-1}(1 + \delta t_{t-1}) - X_{t-1}(1 + \delta t_{t-1})] \]
\[-\delta(\pi_b + \pi_s)D_{t}^{int} + \pi_b R_t^b + \pi_s R_t^s + B_t^q - \delta B_{t-1}^g(1 + \delta t_{t-1}) \]  
(D.7)

Since \(\pi_b + \pi_s = \pi_b + (1 + \pi_b) = 1\) and using (D.5), we have:

\[ B_t - X_t = -(1 - \delta)A_t + \delta(1 + \varphi_{t-1})(1 + \delta t_{t-1})s_tB_{t-1}^r \]
\[+\pi_b R_t^b + \pi_s R_t^s + B_t^q - \delta B_{t-1}^g(1 + \delta t_{t-1}) \]  
(D.8)

Consider the aggregate beginning-of-period assets \(A_t\) of all households:

\[ A_t = \int_{B_t} A_t(i)\,di + \int_{A_s} A_t(i)\,di \]  
(D.9)

by plugging (D.9) on (D.3) and (D.4), and solving for \(A_t\), we obtain:

\[ A_t = B_t^p(1 + \dot{t}_{t-1}) - D_t^{int} - (1 + \varphi_{t-1})(1 + \delta t_{t-1})s_tB_{t-1}^r + D_t^{int} \]  
(D.10)

By replacing (D.10) into (D.8), we have:

\[ B_t - X_t = -(1 - \delta)B_t^p(1 + \dot{t}_{t-1}) + (1 + \varphi_{t-1})(1 + \delta t_{t-1})s_tB_{t-1}^r \]
\[+\pi_b R_t^b + \pi_s R_t^s + B_t^q - \delta B_{t-1}^g(1 + \delta t_{t-1}) \]  
(D.11)

By substituting (D.6) into (D.11) and assuming perfect competition in the home traded god market (i.e. \(D_{t,x,t}^h = 0\)), we get:

\[ B_t - X_t = -(1 - \delta)B_t^p(1 + \dot{t}_{t-1}) + (1 + \varphi_{t-1})(1 + \delta t_{t-1})s_tB_{t-1}^r \]
\[-\delta B_{t-1}^g(1 + \delta t_{t-1}) + \pi_b p_t e_t^b - \pi_b W_t^b - \pi_b p_x y_{x,t}^b - \pi_b D_{f,t} - \pi_b D_t^h \]
\[+\pi_s p_t e_t^s - \pi_s W_t^s - \pi_s p_x y_{x,t}^s - \pi_s D_{f,t} - \pi_s D_t^h + B_t^q - T^g \]  
(D.12)

Since \(\pi_s + \pi_b = 1\), then

\[ B_t - X_t = -(1 - \delta)B_t^p(1 + \dot{t}_{t-1}) + (1 + \varphi_{t-1})(1 + \delta t_{t-1})s_t \]
\[+\pi_t c_t - W_t - D_{f,t} - D_t^h \]
\[+B_t^q - T^g - p_{x,t} y_{x,t}^b \]  
(D.13)
Where: \( W_t = \pi_s W_t^s + \pi_b W_t^b \)

Then using firms current profit definitions \( D_n^h \) and \( D_f^t \), and using both (12) and the government budget constraint, we get:

\[
B_t - X_t = -(1 - \delta)B_t^q(1 + \varphi_{t-1}) + (1 + \varphi_{t-1})(1 + \varphi_{t-1})s_tB_{t-1}^s + p_t c_t - W_t - p_{f,t} c_{f,t} + p_{j,t} s_{f,t} + p_{h,t} y_{h,t} - p_{x,t} y_{x,t} - p_{x,t} c_{x,t} - p_{h,t} y_{h,t} + p_{x,t} e_{x,t} - p_{h,t} e_{h,t}
\]

(D.14)

Since \( p_t c_t = p_{n,t} c_{n,t} + p_{x,t} e_{x,t} + p_{f,t} c_{f,t} \) and simplifying the above expression,

\[
B_t - X_t = -(1 - \delta)B_t^q(1 + \varphi_{t-1}) + (1 + \varphi_{t-1})(1 + \varphi_{t-1})s_tB_{t-1}^s + p_{n,t} c_{n,t} + p_{f,t} c_{f,t} + p_{x,t} e_{x,t} - p_{n,t} y_{n,t} - p_{n,t} y_{h,t} - p_{x,t} y_{x,t} + B_{t-1}^s(1 + \varphi_{t-1}) + p_{n,t} g_t + p_{x,t} g_{x,t} - p_{x,t} e_{x,t} - p_{h,t} e_{h,t}
\]

(D.15)

By substituting (11) in D.15 and simplifying, we have:

\[
B_t - X_t = -(1 - \delta)B_t^q(1 + \varphi_{t-1}) + (1 + \varphi_{t-1})(1 + \varphi_{t-1})s_tB_{t-1}^s + B_{t-1}^s(1 + \varphi_{t-1}) + B_{t-1}^s(1 + \varphi_{t-1}) + p_{n,t} y_{h,t} + s_t p_{f,t} c_{f,t} + p_{n,t} y_{n,t} + p_{x,t} y_{x,t} - p_{x,t} e_{x,t} - p_{h,t} e_{h,t}
\]

(D.16)

Let define the imports value, \( IM_t = s_t p_{f,t} c_{f,t} \), as the value in domestic currency of foreign good consumption by domestic households and the exports value, \( EX_t = p_{x,t} e_{x,t} \), as the value in domestic currency of traded home good consumption by foreigners.

\[
B_t - X_t = -(1 - \delta)B_t^q(1 + \varphi_{t-1}) + (1 + \varphi_{t-1})(1 + \varphi_{t-1})s_tB_{t-1}^s + B_{t-1}^s(1 + \varphi_{t-1}) + B_{t-1}^s(1 + \varphi_{t-1}) + p_{n,t} y_{h,t} + s_t p_{f,t} c_{f,t} + p_{n,t} y_{n,t} + p_{x,t} y_{x,t} - p_{x,t} e_{x,t} - p_{h,t} e_{h,t} + IM_t - EX_t
\]

(D.17)

Simplifying the above expression, we get

\[
B_t - X_t = (1 + \varphi_{t-1})(1 + \varphi_{t-1})s_tB_{t-1}^s + B_{t-1}^s(1 + \varphi_{t-1}) + IM_t - EX_t
\]

(D.18)
In as much as, the financial intermediaries funding constraint is:

\[ B_t - X_t = B^*_{t-1} s_t - p^b_{t,t} \Xi b_t \]  

(D.19)

Then plugging (D.19) in (D.18), we obtain:

\[ s_t B^*_{t} = s_t B^*_{t-1} (1 + \varphi_{t-1})(1 + i^{d*}_{t-1}) + IM_t - EX_t \]  

(D.20)

Rearranging terms in the above equation, we get our model-derived Balance of payments.

\[ IM_t - EX_t + s_t B^*_{t-1} [(1 + \varphi_{t-1})(1 + i^{d*}_{t-1}) - 1] = (B^*_t - B^*_{t-1}) s_t \]  

(D.21)

Where the left hand side term is the current account deficit and the right hand side is the capital account.

By the UIP condition, equation (19), we have:

\[ IM_t - EX_t + B^*_{t-1} s_t i^{d*}_{t-1} = (s_t B^*_t - s_{t-1} B^*_{t-1}) \]  

(D.22)

In order to include the balance payments in our model, we can express it in real terms as follows:

\[ \tilde{p}_{f,t} c_{f,t} - \tilde{p}_{x,t} c_{x,t} + \frac{q_{t-1} b^*_{t-1}(1 + i^{d*}_{t-1})}{\pi_t} = q_t b^*_t \]  

(D.23)

E Calibration

E.1 Set Values

In Table 4, we provide the parameters’ calibrated values and the sources where they were taken. These values are set in the model and all of them are taken from close related literature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>Probability that household i’s type remain the same as in the previous period.</td>
<td>0.9750</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>( \pi_b )</td>
<td>Probability that household i be a type b once a new type is drawn.</td>
<td>0.5</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>( \pi_s )</td>
<td>Probability that household i be a type s once a new type is drawn.</td>
<td>0.5</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor.</td>
<td>0.9874</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Inverse of the Frisch elasticity of labor supply.</td>
<td>0.1048</td>
<td>Curdia and Woodford (2010)</td>
</tr>
</tbody>
</table>

Continue on Next Page...
Table 4 – Continued

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>Inverse of constant firm $i$'s labor elasticity of output.</td>
<td>1.3333</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Probability that firm keep its price unchanged from one period to the next.</td>
<td>0.66</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Elasticity of substitution between varieties for the non-trade good produced at home.</td>
<td>7.65</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>Labor Markup.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Inverse of elasticity of intermediation production respect to domestic debt.</td>
<td>5.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>Elasticity of intertemporal substitution for type $b$ agents.</td>
<td>13.8</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Elasticity of intertemporal substitution for type $s$ agents.</td>
<td>2.76</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Aggregate welfare’s scale parameter.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Steady state’s credit spread.</td>
<td>$(1.02^{\dagger}) - 1$</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Persistence parameter for shocks persistence.</td>
<td>0.9</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$s_c$</td>
<td>Private expenditure share of GDP.</td>
<td>0.7</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$b$</td>
<td>Private debt to GDP ratio.</td>
<td>3.2</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\tau_{bn}$</td>
<td>Firms’ income tax rate.</td>
<td>0.2</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\bar{H}$</td>
<td>Steady state value for the exogenous labor-supply disturbance process.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\bar{Z}$</td>
<td>Steady state value for the exogenous labor-productivity disturbance process.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\bar{\chi}$</td>
<td>Steady state value for the exogenous loss rate of not repaid loans.</td>
<td>0.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$b_g$</td>
<td>Steady state value for the exogenous government debt.</td>
<td>0.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Total inflation’s steady state value.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$gd$</td>
<td>Gross domestic product steady state value.</td>
<td>1.00</td>
<td>Curdia and Woodford (2010)</td>
</tr>
<tr>
<td>$\psi_1$</td>
<td>Risk premium sensitivity parameter to changes in net borrowing.</td>
<td>0.000742</td>
<td>Schmitt-Grohe and Uribe (2003)</td>
</tr>
<tr>
<td>$b^*$</td>
<td>Steady state foreign debt.</td>
<td>0.7442</td>
<td>Schmitt-Grohe and Uribe (2003)</td>
</tr>
<tr>
<td>$\rho_{bf}$</td>
<td>Elasticity of substitution between home and foreign goods.</td>
<td>1.00</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\rho_{nx}$</td>
<td>Elasticity of substitution between non-traded and traded goods produced at home.</td>
<td>1.00</td>
<td>Gali and Monacelli (2005)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Parameter that controls the participation of expenditure in domestic goods’ consumption on total consumption’s expenditure.</td>
<td>0.000742</td>
<td>Liu (2010)</td>
</tr>
<tr>
<td>$q$</td>
<td>Steady state real exchange rate.</td>
<td>1.00</td>
<td>We impose this value to guarantee PPP in the long run.</td>
</tr>
</tbody>
</table>

**NOTE:** Private debt to annual GDP is 80%. However, our model’s calibration is quarterly, then $\frac{b}{y^{\text{annual}}} = \frac{b^*}{4 \times y^{\text{quarterly}}}$. Therefore, $\frac{b}{y^{\text{annual}}} = 4 \times \frac{b^*}{y^{\text{quarterly}}} = 3.2$

E.2 Definitions

The following are some definitions that we have in our model. They are useful for calibrating the rest of our model parameters.
\[ gdp_t \equiv \tilde{p}_{hn,t}c_{hn,t} + \tilde{p}_{hx,t}c_{hx,t} + \tilde{p}_b^e c_{hx,t} + \tilde{p}_{hn,t}g + \tilde{p}_{hn,t}\Xi(b_t) \]  
(E.1)

where, this equation is the demand-side GDP.

\[ gdp_t = \tilde{p}_{hn,t}y_{hn,t} + \tilde{p}_{hx,t}y_{hx,t} \]  
(E.2)

where, this equation is the supply-side GDP.

\[ 1 + \omega_y \equiv \phi(1 + \nu) \]  
(E.3)

\[ H_{pf} \equiv \frac{\theta_f}{(\theta_f - 1)} \]  
(E.4)

\[ H_{ph} \equiv \frac{\theta_h}{(\theta_h - 1)} \]  
(E.5)

\[ \psi \equiv \left[ \pi_b \psi_b^{-1} + \pi_s \psi_s^{-1} \right]^{-1} \]  
(E.6)

\[ \tilde{\gamma} \equiv \gamma(1 - \gamma)^{1-\gamma} \]  
(E.7)

\[ \tilde{\gamma}_{hn} \equiv \gamma_{hn}^{\gamma_{hn}} (1 - \gamma_{hn})^{1-\gamma_{hn}} \]  
(E.8)

\[ \Omega \equiv \frac{\lambda_b}{\lambda_s} \]  
(E.9)

**E.3 Steady State**

The following are some steady state relations among this model variables. They are useful for calibrating the rest of our model parameters.

By solving for \( i^d \) from (6) and (7), we have:

\[ i^d = \beta^{-1} \left[ (\delta + 1) + (\omega - 1)(\delta + (1 - \delta)\pi_s) - \sqrt{[(\delta + 1) + (\omega - 1)(\delta + (1 - \delta)\pi_s)]^2 - 4\delta \omega} \right] \]  
(E.10)

By solving for \( \Omega \) from (6) and (7), we have:

\[ \Omega = \frac{1 - i^d \beta(\delta + (1 - \delta)\pi_s)}{i^d \beta(1 - \delta)\pi_b} \]  
(E.11)

By using (C.2) and the assumption of same worked hours at the steady state for both consumers’ types, we have:

\[ \psi_s = \psi \left( \pi_b \Omega^{-1} + \pi_s \right)^\nu \]  
(E.12)
\[ \psi_b = \psi_s \Omega \quad (E.13) \]

By equations (16) and (20), we have

\[ \frac{\omega}{(\gamma b^{\alpha - 1})} \quad (E.14) \]

**E.3.1 Calibration for \( \tilde{\lambda} \)**

From intratemporal households' optimization problem and using the consumption aggregator among households' types, which is \( c_t = \pi_b c^b_t + \pi_s c^s_t \), we have:

\[ \tilde{p}_{hn} c_{hn} = \gamma_{hn} \gamma s_c \quad (E.15) \]

\[ \tilde{p}_{hx} c_{hx} = (1 - \gamma_{hn}) \gamma s_c \quad (E.16) \]

By solving for exports value from the balance of payments equation, derived above, we have:

\[ \tilde{p}^*_t c^*_{hx,t} = (1 - \gamma) \frac{1 - \tau_{f,t}}{\mu_{pf}} s_c + b^* \left( \frac{1 + i_d}{1 + \pi} - 1 \right) \quad (E.17) \]

By plugging (E.15), (E.16) and (E.17) into (E.1), and solving for \( \tilde{p}_{hn} \), we have:

\[ \tilde{p}_{hn} = \frac{gdp - \gamma_{hn} \gamma s_c - (1 - \gamma_{hn}) \gamma s_c - (1 - \gamma) \frac{1 - \tau_{f,t}}{\mu_{pf}} s_c + b^* \left( \frac{1 + i_d}{1 + \pi} - 1 \right)}{g + E(b)} \quad (E.18) \]

By using the real resources for domestically produced traded goods, equation (12), (E.16) and the balance of payments, we have:

\[ \tilde{p}_{hx} y_{hx} = (1 - \gamma_{hn}) \gamma s_c + (1 - \gamma) \frac{1 - \tau_{f,t}}{\mu_{pf}} s_c + b^* \left( \frac{1 + i_d}{1 + \pi} - 1 \right) \quad (E.19) \]

By using (E.2),(E.19) and (E.18), we have:

\[ y_{hn} = \frac{gdp - (1 - \gamma_{hn}) \gamma s_c - (1 - \gamma) \frac{1 - \tau_{f,t}}{\mu_{pf}} s_c + b^* \left( \frac{1 + i_d}{1 + \pi} - 1 \right)}{\tilde{p}_{hn}} \quad (E.20) \]

Finally, by using equation (B14), (B15), (B18) and (E.20 ), and assuming \( \pi = 0 \), we have:

\[ \tilde{\lambda} = \frac{\psi_{mu} \psi_{ph} \tilde{H}^{-\nu} y_{hn}^\omega_s (1 + \omega_y)}{1 - \tau_{hn}} \quad (E.21) \]
E.3.2 Calibration for $\lambda_s$ and $\lambda_b$

By using (B.7), and (E.21), we have:

$$\lambda_s = \frac{\tilde{\lambda}}{\psi \left[ \pi_b \Omega^{1/2} \psi_b^{1/2} + \pi_s \psi_s^{1/2} \right]} \quad (E.22)$$

By using (E.11), we obtain:

$$\lambda_b = \Omega \lambda_s \quad (E.23)$$

E.3.3 Calibration for $\bar{c}_b$ and $\bar{c}_s$

From (8), we have:

$$c_b = \bar{C}_b (\lambda_b)^{-\sigma_b} \quad (E.24)$$

$$c_s = \bar{C}_s (\lambda_s)^{-\sigma_s} \quad (E.25)$$

Since $\pi_s$ and $\pi_b$ are constant, we can aggregate consumption as follows:

$$c = \pi_s c_s + \pi_b c_b \quad (E.26)$$

Since we normalize GDP to 1 at steady state, we have

$$s_c = \pi_s c_s + \pi_b c_b \quad (E.27)$$

Also we know that

$$c_b - c_s = \vartheta \quad (E.28)$$

where, $\vartheta$ is the difference between two types households’ consumptions. $\vartheta$ value is calibrated such that private debt dynamics holds at steady state.

Finally, by plugging (E.25) and (E.24) into (E.26) and (E.28), we have:

$$\begin{pmatrix} \bar{C}_b \\ \bar{C}_s \end{pmatrix} = \begin{pmatrix} (\lambda_b)^{-\sigma_b} & 0 \\ 0 & (\lambda_s)^{-\sigma_s} \end{pmatrix} \begin{pmatrix} \pi_b & \pi_s \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} s_c \\ \vartheta \end{pmatrix} \quad (E.29)$$

E.3.4 Exogenous External Variables Calibration

By assuming PPP and zero risk premium in the steady state, we have

$$r^* = r^d \quad (E.30)$$

By using E.18 and consumer prices indexes implied by this model, we have:

$$p_h \bar{x} = (\tilde{\gamma}^{-1} \tilde{\gamma}_h \tilde{\gamma}_n \rho \tilde{n}^{1/2} \gamma_h n \gamma)(\mu_{pf} / (1 - \tau_f))^{1-\gamma - \frac{1}{1-\gamma_h \mu_{pf} / (1 - \tau_f) \gamma_h n \gamma}} \quad (E.31)$$
By assuming Law of One price at steady state, we have:
\[ \tilde{p}_t = \tilde{p}_h x \] (E.32)

By using the model-derived’s balance of payments and E.31, we have:
\[ c_{hx}^* = \frac{(1 - \gamma)(1 - \tau_f) s_c + b^* r^*}{\tilde{p}_h x} \] (E.33)

Finally, by using the domestically-produced traded good’s resource constraint (or equation (11)), we obtain:
\[ y_{hx} = \frac{(1 - \gamma_{hn}) \gamma s_c + \tilde{p}_h x c_{hx}^*}{\tilde{p}_h x} \] (E.34)

### E.4 Additional Calibration Procedures

#### E.4.1 Calibration for \( \gamma_{hn} \)

Since we do not have a calibrated or estimated value for \( \gamma_{hn} \), we provide the following way to get a broad calculation of \( \gamma_{hn} \) ’s value.

From Balsam and Eckstein (2001) we know that \( VN = 1.36 \) and \( VT = 1 \), where \( VN = \frac{2 \gamma_N}{\gamma_T} \). Since in Balsam and Eckstein (2001) ’s model \( \gamma_N + \gamma_T = 1 \), then \( \frac{\gamma_N}{\gamma_T} + 1 = \frac{1}{\gamma_T} \). Therefore, we have \( \gamma_T = 0.4237 \) and \( \gamma_N = 0.5763 \). However, our model is more complex because we have three different goods: Non Traded, Traded produced at home and Traded imported. We can decompose tradable goods between home produced and produced abroad. By Liu (2010), and Medina and Soto (2006), we know that 30% of goods are imported and 70% of them are home-produced. Also we can decompose \( \gamma_T \) such that \( \gamma_T = \gamma_{HT} + \gamma_{FT} \). Hence, \( \gamma_{HT} = (0.7)(0.4237) = 0.2965 \) and \( \gamma_N + \gamma_{HT} = 0.5763 + 0.2965 = 0.8728 \).

Define \( \gamma_{hn} = \frac{\gamma_N}{\gamma_N + \gamma_{HT}} \) and \( \gamma_{hx} = \frac{\gamma_{HT}}{\gamma_N + \gamma_{HT}} \) to normalize \( \gamma_{hn} + \gamma_{hx} = 1 \). Finally, we obtain \( \gamma_{hn} = \frac{0.5763}{0.8728} = 0.6602 \).

#### E.5 Model Implied Values

In Table 5, we described the calibrated value for parameters and long run means implied by this model’s steady state solution. These values come after evaluating Table 4’s values into the definitions and steady state equations described in the previous subsections.
Table 5: Model Implied Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_{hn} )</td>
<td>A parameter controlling the participation of expenditure in domestic non-traded goods’ consumption on domestic-goods consumption’s expenditure.</td>
<td>0.6600</td>
<td>Previous subsection</td>
</tr>
<tr>
<td>( 1 + \omega_y )</td>
<td>Homogeneity degree of Non-traded good’s cost function.</td>
<td>1.4730</td>
<td>Equation E.3</td>
</tr>
<tr>
<td>( \mu_{p}^{h} )</td>
<td>Non-traded good firm’s Markup.</td>
<td>1.1504</td>
<td>Equation E.5</td>
</tr>
<tr>
<td>( \mu_{p}^{f} )</td>
<td>Distributed imported good firm’s Markup.</td>
<td>1.1504</td>
<td>Equation E.4</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>Type b consumer to type s marginal utilities of consumption’s steady state’s ratio.</td>
<td>1.2175</td>
<td>Equation E.9</td>
</tr>
<tr>
<td>( \psi_{s} )</td>
<td>Type s households’ scale factor.</td>
<td>0.9439</td>
<td>Equation E.12</td>
</tr>
<tr>
<td>( \psi_{b} )</td>
<td>Type b households’ scale factor.</td>
<td>1.1492</td>
<td>Equation E.13</td>
</tr>
<tr>
<td>( \bar{C}^{b} )</td>
<td>Long run mean of type b households’ preferences shock.</td>
<td>7973.10</td>
<td>Equation E.29</td>
</tr>
<tr>
<td>( \bar{C}^{s} )</td>
<td>Long run mean of type s households’ preferences shock.</td>
<td>2.4691</td>
<td>Equation E.29</td>
</tr>
<tr>
<td>( r^{*} )</td>
<td>Real foreign interest rate (risk free rate).</td>
<td>0.01</td>
<td>Equation E.30</td>
</tr>
<tr>
<td>( p^{*} )</td>
<td>Relative steady state foreign imports price.</td>
<td>0.1310</td>
<td>Equation E.32</td>
</tr>
<tr>
<td>( c^{*} )</td>
<td>The foreign consumption of home traded good.</td>
<td>1.6206</td>
<td>Equation E.33</td>
</tr>
<tr>
<td>( y_{hx} )</td>
<td>Traded good steady state output.</td>
<td>2.8909</td>
<td>Equation E.34</td>
</tr>
</tbody>
</table>

**F Welfare function**

When considering optimal policy we assume that the central planner maximizes the average household expected welfare, where each individual’s utility is

\[
U^{\tau(j)}(i) = u^{\tau(j)}(c_{i}(j); \xi_{i}) - \int_{0}^{1} v^{\tau(j)}(h_{t}(i, j); \xi_{t})di \tag{F.1}
\]

In equilibrium each household \( j \) works \( h_{t}(j, i) \) hours at firm \( i \), see equation (C.2).

\[
h_{t}(j, i) = \left( \frac{\lambda_{t}^{\tau(i)}}{\psi_{\tau(i)}(\lambda_{t})} \right) \frac{1}{\nu} \Pi_{t}(j) \tag{F.2}
\]

Therefore,

\[
\int_{0}^{1} v^{\tau(i)}(h_{t}(j, i); \xi_{t})dj = \frac{\pi_{h}(\lambda_{h}^{\nu})^{\frac{1+N}{1+N}} + (1 - \pi_{b})(\lambda_{b}^{\nu})^{\frac{1+N}{1+N}}}{1 + \nu} \left( \frac{\psi_{b}}{\lambda_{b}} \right)^{\frac{1}{1+N}} \bar{H}_{t}^{-\nu} \int_{0}^{1} \tilde{H}_{t}(j)^{1+\nu} dj \tag{F.3}
\]
\[ \int_{0}^{1} v^{\tau}(h(x,j,i) ; \xi_{t}) \frac{1}{1 + \nu} R_{t}^{-\nu} \left( \frac{Y_{n,t}^{h}}{Z_{t}} \right)^{1+\omega_{\nu}} \Delta_{n,t}^{h} \]  

(H.4)

Hence, the welfare function at \( t \) is

\[ \tilde{U}_{t} = \int_{0}^{1} U_{1}(j) dj \]  

(H.5)

\[ \tilde{U}_{t} = \pi_{b} \frac{(\psi^{b})^{1-\sigma_{b}^{-1}}(C_{t}^{b})^{\sigma_{b}^{-1}}}{1 - \sigma_{b}^{-1}} + (1 - \pi_{b}) \frac{\psi^{s} (C_{t}^{s})^{1-\sigma_{s}^{-1}}}{1 - \sigma_{s}^{-1}} \]

\[ - \frac{\psi}{1 + \nu} \left( \frac{\tilde{A}_{t}}{\lambda_{t}} \right)^{1+\nu} \bar{H}_{t}^{-\nu} \left( \frac{Y_{n,t}^{h}}{Z_{t}} \right)^{1+\omega_{\nu}} \Delta_{n,t}^{h} \]  

(H.6)

with

\[ \tilde{A}_{t}^{1+\nu} \equiv \psi^{b} \left[ \pi_{b} \psi_{b}^{1-\sigma_{b}^{-1}} (\lambda_{t}^{b})^{1+\nu} + (1 - \pi_{b}) \psi_{s}^{1-\sigma_{s}^{-1}} (\lambda_{t}^{s})^{1+\nu} \right] \]  

(H.7)

By replacing (H.7) in (H.6)

\[ \tilde{U}_{t} = \pi_{b} \frac{(\lambda_{t}^{b})^{1-\sigma_{b}^{-1}}(C_{t}^{b})}{1 - \sigma_{b}^{-1}} + (1 - \pi_{b}) \frac{(\lambda_{t}^{s})^{1-\sigma_{s}^{-1}}(C_{t}^{s})}{1 - \sigma_{s}^{-1}} \]

\[ - \frac{\psi}{1 + \nu} \left( \frac{\tilde{A}_{t}}{\lambda_{t}} \right)^{1+\nu} \bar{H}_{t}^{-\nu} \left( \frac{Y_{n,t}^{h}}{Z_{t}} \right)^{1+\omega_{\nu}} \Delta_{n,t}^{h} \]  

(H.8)

Finally the Central Planner Objective is to maximize the following expected welfare function, defined as:

\[ W_{t} = \tilde{U}_{t} + \beta E_{t} W_{t+1} \]  

(F.9)

References


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Figure 1: Model Dynamics - 1% Foreign Interest Rate Shock I
Figure 2: Model Dynamics - 1% Foreign Interest Rate Shock II
Figure 3: Model Dynamics - 1% Foreign Interest Rate Shock III
Figure 4: Model Dynamics - 1% Foreign Interest Rate Shock IV
Figure 5: Model Dynamics - 1% Credit Spread Shock I
Figure 6: Model Dynamics - 1% Credit Spread Shock II
Figure 7: Model Dynamics - 1% Credit Spread Shock III
Figure 8: Model Dynamics - 1% Credit Spread Shock IV
Figure 9: Model Dynamics - 1% Productivity Shock I
Figure 10: Model Dynamics - 1% Productivity Shock II
Figure 11: Model Dynamics - 1% Productivity Shock III
Figure 12: Model Dynamics - 1% Productivity Shock IV