Price-Level Targeting: an omelette that requires breaking some Inflation-Targeting eggs?

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Abstract
This paper can be divided into two main parts. The first one, using a simple example by Minford (2004) and Hatcher (2011), gives the reader a basic introduction to understand the comparison between two monetary-policy regimes: Inflation Targeting (IT) and Price-Level Targeting (PLT). The second part, using a model with a New Keynesian Phillips curve and a loss function (both of which incorporate partial indexation to lagged inflation), finds that for standard values of underlying parameters (i) the social loss associated to macroeconomic volatility may decrease about 29% by switching from IT to PLT and (ii) only when the initial level of indexation to lagged inflation is higher than 65% then it is better not to switch to PLT.

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1. Introduction

Since the adoption of Inflation Targeting (IT) by New Zealand in 1990, a growing number of countries have implemented this regime to conduct monetary policy (27 countries according to Hammond, 2012). However, in recent years the financial crisis and the new challenges facing monetary policy have led to a re-examination of IT (Walsh, 2011).

In 2006, the Bank of Canada, one of the IT countries, focused part of its research efforts on the exploration of an alternative regime known as Price Level Targeting (PLT), which intends to stabilise the economy’s price level (rather than inflation) around a predetermined path. In 2011, the Bank of Canada finally decided to stick with IT. The main reason was that it has served Canadians well (Ragan, 2011), and therefore the decision was in the spirit of the common idea that ‘one should not fix something that does not appear to be broken.’ However, the fact that IT works well does not imply that an alternative regime cannot work even better.

PLT is a good candidate to replace IT due to its potential benefits: decreasing long-term price level uncertainty, increasing short-term macroeconomic stability and reducing the probability of falling into liquidity traps. However, these benefits stem from the ability of PLT to better anchor inflation expectations, and therefore this regime requires a high degree of credibility. Furthermore, there is not much practical experience to allow empirical verification of such benefits and it is also considered that a central bank may face more difficulties in communicating price targets rather than inflation targets.

In theory, Vestin (2006) and Roisland (2006) have shown that PLT can be used to implement the IT solution under commitment, and therefore it is already known that PLT can outperform IT under discretion. We intend to find out how big the difference can be.

The present paper analyses the performance of PLT vs. IT, in terms of social wellbeing, using a similar model to that used by Gaspar et al. (2010a), with a New Keynesian Phillips curve and a loss function, both of which include partial indexation to lagged inflation as derived by Woodford (2003). Using standard values of parameters we find that the social loss associated to macroeconomic volatility may decrease about 29% by following PLT rather than IT.
In a second analysis, we compare more practical forms\(^2\) of both PLT and IT in order to determine the critical level of indexation to lagged inflation at which it is better not to switch to PLT. Since the ‘power’ of this regime is based on the effect of future price targets on expectations, it has been remarked by previous literature that inflation inertia reduces its benefits. Using standard parameter values again, we find that the level of indexation has to be larger than 65% to make IT better than PLT in terms of social welfare.

Whether or not inflation targeters should switch to PLT is an ongoing debate. Our results go in the direction of showing that a highly credible PLT regime may represent a great improvement for society compared to a highly credible IT regime. Undoubtedly, since there is not much practical experience there is still a great deal of uncertainty about the potential benefits of PLT; however, we believe we are close to a situation in which the theoretical evidence in favour of this regime will switch the proverb from “IT is not broken so it does not need to be fixed” to the idea that it may be worth risking something that is good so as to get something better: “you can’t make an omelette without breaking a few eggs.”

Before presenting the model and our main findings (Section 4), we make use of a simple analytical example by Minford (2004) and Hatcher (2011) to illustrate some basic features of IT and PLT (Section 2) and to give the reader a basis to understand some of the arguments for and against PLT (which are described in Section 3). Conclusions are detailed in Section 5.

### 2. Inflation Targeting vs. Price-Level Targeting

This section draws heavily on the analytical example used by Minford (2004) and Hatcher (2011) to compare PLT with IT in a very simple way which, only for simplicity and illustrative purposes, ignores expectations and the transmission mechanism of monetary policy. In later sections we incorporate the analysis of such features.

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\(^2\) By ‘more practical forms’ we refer to the fact that to represent the central bank’s objective function we use more common functional forms which do not incorporate the level of indexation, and therefore the implementation of these forms of PLT or IT does not require the monetary authority to know such level.
2.1. Inflation Targeting (IT)

The IT regime is a monetary policy strategy with which a central bank aims to stabilise the economy by announcing an inflation target and publicly committing to stabilisation of inflation (around the target) as the primary goal of monetary policy. Similarly, there are significant efforts to inform the public about the plans and goals of the monetary authorities and mechanisms that reinforce the central bank's responsibility to accomplish its objectives (Bernanke et al., 1999).

For an IT economy with low and stable inflation, the latter is expected to fluctuate around the long-term target:

\[ \pi_t = \pi^* + \varepsilon_t \]

where \( \pi^* \) is the long-term inflation target and \( \varepsilon_t \) an iid shock with zero mean and variance \( \sigma^2 \). Since \( \pi_t \equiv p_t - p_{t-1} \), the level of prices follows a random walk with drift:

\[ p_t = p_{t-1} + \pi^* + \varepsilon_t \]
\[ = p_0 + \pi^* t + \sum_{j=1}^{t} \varepsilon_t \]

In the absence of shocks the expected path for prices can be represented as \( p_t = p_0 + \pi^* t \). If a shock \( \varepsilon_1 > 0 \) occurs in period 1 this path (under IT) changes as shown in Figure 1.

The monetary authority sets the target from period 0 (\( \pi^* \)). The goal is frustrated in period 1: due to the shock of the same period, the price level is \( p_1 \) and inflation is above its target\(^3\). In this case, from period two on, the central bank assumes that "bygones are bygones" and resets the path so as to keep the same target (i.e. the same slope), that is, the monetary authority ignores the deviations in prior periods in order to achieve \( \pi^* \) in each future period. For instance, in the absence of shocks, in period 2 the central bank will achieve its goal with a price level \( p_2 \), which is above \( p_1 \) in \( \pi^* \) units.

\[^3\] The slope between \( p_0 \) and \( p_1 \) is greater than the slope between \( p_0 \) and \( p_1^* \).
Summarising, under IT each deviation of prices from the target will have a permanent effect on the expected path of prices. The (implicit) price level target changes but the inflation target (the path slope) does not change.

**Inflation expectations under IT**

From equation (1), taking the conditional expected value we obtain

\[ E_{t-1} \pi_t = \pi^* \]  

Inflation expectations are equal to the inflation target. Uncorrelated shocks are not important under IT for the future level of inflation. In terms of expectations the shock that matters is that of period \( t \) (whose expected value at \( t - 1 \) is zero).

**2.2. Price-Level Targeting (PLT)**

The monetary policy strategy known as PLT refers to the regime in which the central bank aims to stabilise the aggregate price level around a target path. Under this system,
Macroeconomic shocks do not affect the long-term price path because the primary goal of monetary policymakers is to counteract their effect in order to keep the price level as close as possible to the target. If, for instance, in the current period prices are above the target level, in the near future it is required to achieve below-average inflation in order to stay in the desired price path.

A simple example may help us to understand PLT. If deviations of prices from the target path are fully compensated in the next period and shocks are temporary and uncorrelated, the path of prices in period $t$ can be described by the following equation

\[
p_t = p_0 + \pi^* t + \varepsilon_t
\]

where $p_0$ is the initial price level. We assume that the price-path target implies a long-term inflation target equal to that assumed under IT ($\pi^*$). If a shock $\varepsilon_1 > 0$ occurs in period 1 the expected price path (under PLT) changes as shown in Figure 2.

**Figure 2. Effect of a positive shock on PLT**

![Figure 2](source: Hatcher, 2011)

Similar to the example for IT, the target path starts at $p_0$ and continues its trajectory in period 1 with a price level $p_1^*$. Since in this period the shock $\varepsilon_1$ occurs, the price level rises up to $p_1$ ($p_1 > p_1^*$). Unlike IT, in period 2 the central bank reduces inflation below its average so that prices return to the target level $p_2^*$. This can be seen by the fact that the
increase from $p_1$ to $p_2^*$ implies a flatter slope than the target path. In contrast to IT, the central bank changes the implicit inflation target but does not change the price level target.

**Inflation expectations under PLT**

To describe the behaviour of inflation expectations under the PLT regime we first-difference equation (4):

$$\pi_t = \pi^* + \varepsilon_t - \varepsilon_{t-1}$$  \hspace{1cm} (5)

The level of inflation in period $t$ depends on the implicit inflation target, the shock of the same period and the shock of the previous period.\(^4\) Taking conditional expectations of the foregoing equation we obtain

$$E_{t-1} \pi_t = \pi^* - \varepsilon_{t-1}$$  \hspace{1cm} (6)

It can be seen that past deviations produced by shocks are offset under PLT and rational agents take this fact into account when forming their inflation expectations. It is also important to remark that (6) is obtained under the assumption that the regime is credible, and therefore it requires that private agents trust the central bank to keep prices as close as possible to the target path.

### 3. Some arguments for and against PLT

This section briefly describes some arguments about potential benefits and costs of implementing PLT. For a detailed literature review see Ambler (2009) or Hatcher (2011).

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\(^4\) If deviations were compensated in the following $m$ periods (instead of being completely offset in the next period as we are assuming), inflation in $t$ would depend on the $m$ previous shocks. Thus, for example, if a shock is compensated in the following three periods (33.3% in each period) equation (5) would be $\pi_t = \pi^* + \varepsilon_t - 0.33 \varepsilon_{t-1} - 0.33 \varepsilon_{t-2} - 0.33 \varepsilon_{t-3}$. 
Lower (long-term) inflation volatility and less uncertainty about the price level

It is argued that uncertainty about the future price level and long-run inflation volatility are reduced (compared to those under IT) by implementing PLT. Using the example described in the previous section, this argument can be explained as follows.

Using equation (4), we can express the price level for $n$ periods ahead as

\[ p_{t+n} = p_o + \pi^*(t + n) + \varepsilon_{t+n} \]  

Under PLT, the price level $p_{t+n}$ depends only on the shock in $t + n$ because, as mentioned above, past shocks are neutralised so that prices remain on the target path. Subtracting (4) from (7) and taking conditional variance we obtain

\[ \text{Var}_t(\pi_{t\rightarrow t+n}) = \sigma^2 \]

where $\pi_{t\rightarrow t+n}$ is the $n$-period inflation ($p_{t+n} - p_t$, the inflation rate between $t$ and $t+n$).

The degree of uncertainty under PLT is always the same, no matter how long the forecast horizon is. Although inflationary shocks cannot be predicted, private agents know that any deviation from the price-path target will be offset in the near future (assuming the central bank is truthfully committed to this regime).

In contrast, under IT, past shocks are not eliminated and the possible deviation of the price level with respect to the initial expected path increases over time. This can be seen in the following way. Using equation (2) we can express the price level, under IT, for $n$ periods ahead as

\[ p_{t+n} = p_t + \pi^*n + \sum_{j=1}^{n} \varepsilon_{t+j} \]

Using this equation, the conditional variance of inflation, under IT, is

\[ \text{Var}_t(\pi_{t\rightarrow t+n}) = n\sigma^2 \]

Under IT, the degree of uncertainty over the $n$-period inflation is higher as $n$ increases.

The predictability of prices in the long term is one of the strongest arguments in favour of PLT. Under a highly credible PLT, agents can forecast more accurately the future price
level, and therefore this regime implies welfare gains for the agents who sign mid and long-term nominal contracts.

Of course, there exist alternative instruments to counteract price fluctuations, such as indexed bonds and contingent contracts. Although such instruments may reduce the need for predictability of prices, in practice they are not available for all possible transactions under uncertainty.

*Greater (short-term) macroeconomic stability*

From equation (1) it is easy to see that \( \text{Var}(\pi_t) = \sigma^2 \) and from equation (5) \( \text{Var}(\pi_t) = 2\sigma^2 \), that is, under PLT the unconditional variance of a specific period is twice the corresponding variance under IT. This result illustrates why it used to be considered that setting price targets instead of inflation targets would decrease uncertainty about the future price level at the expense of increasing the short-term variability of both inflation and output.

However, since the example of the previous section disregarded the expectations channel, the aforementioned argument about the unconditional variance is only valid under very particular conditions, for instance, when the proportion of backward-looking expectations is high. As we saw in section 2.2, PLT intends that past shocks do not affect the future price path; thanks to this feature, the adjustment in expectations turns into a stabilising mechanism that helps the central bank to deal with shocks.

Svensson (1999) was the first to show that PLT results in lower short-run inflation variability than IT (while output variability remains equal), under the condition that output gap be moderately persistent and using a neoclassical Phillips curve where inflation in \( t \) depends on inflation expectations for the same period with information available in \( t - 1 \) \((\hat{E}_{t-1}\pi_t)\).

However, nowadays it is more common to analyse the benefits of PLT with a New Keynesian Phillips Curve, which incorporates forward-looking expectations \((\hat{E}_t\pi_{t+1})\) (e.g. Gaspar et al., 2010a). To understand how the stabilising mechanism works in this case
assume there is a shock that will push inflation above average. Private agents expect that in the near future the central bank will reverse such effect on prices by increasing the policy rate and pushing inflation below average. Since agents are confident that the monetary authority will act this way, they reduce their expectations about future inflation. This partially neutralises the effect of the shock on current inflation and helps the central bank to stabilise the economy because a smaller increase in the policy rate is needed.

Let us compare again IT and PLT by another simple example. Now we incorporate expectations. The economy is described by the following equations (aggregate supply and aggregate demand, respectively):

\begin{align}
\pi_t &= E_t \pi_{t+1} + y_t + \varepsilon_t \\
y_t &= -r_t
\end{align}

where \( y \) is the output gap and \( r \) is the deviation of the monetary policy rate with respect to its steady state value (zero). The inflation target is normalised to zero and the period loss function is

\begin{align}
L_t &= \pi_t^2 + y_t^2
\end{align}

i.e. society gives exactly the same importance to inflation and output (we assume the same for the central bank).

Suppose there is a one-off inflationary shock \( \varepsilon_t = 1 \). Since shocks are uncorrelated, under IT, \( E_t \pi_{t+1} = 0 \) and the central bank sets \( r_t = 1/2 \) such that finally \( \pi_t = 1/2 \) and \( y_t = -1/2 \). In \( t + 1 \), no policy action is required and hence \( \pi_{t+1} = 0 \) (as expected by agents) and \( y_{t+1} = 0 \). The total loss (no factor discount assumed) is \( L_t + L_{t+1} = 1/2 \).

Under PLT there is a different story. If inflation is above target this period, agents expect it will be below target the next period. Doing some algebra, it is not difficult to see that in this case \( E_t \pi_{t+1} = -1/3 \) and the central bank sets \( r_t = 1/3 \) such that finally \( \pi_t = 1/3 \) and \( y_t = -1/3 \). In period \( t + 1 \), the central bank sets \( r_t = -1/3 \) (in order to achieve below-average inflation) and, as a result, \( \pi_{t+1} = -1/3 \) (as expected by agents) and \( y_{t+1} = 1/3 \). The total loss is \( L_t + L_{t+1} = 4/9 \), lower than that under IT.
This example illustrates in a very simple way two important results for which the expectations channel is crucial: first, PLT can reduce short-run macroeconomic volatility and second, PLT requires a smaller deviation of the policy rate in response to aggregate shocks, compared to IT.

In Section 4 we use a model with a New Keynesian Phillips curve and a loss function, both of which include partial indexation to lagged inflation and show that, for standard values of parameters, the social loss associated to macroeconomic volatility may decrease about 29% by following PLT rather than IT.

*Lower probability of facing a zero lower bound on interest rates*

As shown above, given an aggregate shock of a specific size, the central bank’s policy response has to be greater under IT than under PLT.

Suppose that, under IT, there is a deflationary shock that requires a large reduction of the policy rate. Such a reduction could require the central bank to set the policy rate at a negative value which, of course, is not possible, and therefore there would be a zero lower bound problem.

Under PLT, the expectations channel helps to reduce the final effect on the economy, and thus there is a lower probability of facing a zero lower bound situation (see, for instance, Eggertsson and Woodford, 2003). In this regime, a deflationary shock also implies that inflation will have to be above average in the future and thus inflation expectations increase. This stabilising mechanism partially neutralises the effect of the shock and alleviates the reduction required on the policy rate.

*Need for credibility and the backward-looking expectations problem*

The expectations channel is an important factor which helps PLT to deliver economic stability. However, this channel only works if there is a high degree of credibility on the regime. If agents are confident that the central bank will soon eliminate the effect of past shocks on prices and will return them to the target path, inflation expectations make the real
interest rate adjusts in a way that it helps to preserve macroeconomic stability (i.e. to reduce the variability of both inflation and output).

If there is not a high level of credibility, the expectations channel effectiveness is reduced and the benefits of PLT are significantly diminished. This happens because agents perceive the central bank is not really committed to this monetary regime and hence will not be willing to carry out the required policy actions to restore prices to their target level. For instance, the central bank may be tempted to deviate from the required policy because compensating past shocks would imply sharp reductions in the aggregate demand.

Besides the regime’s credibility, it is also necessary that the level of indexation to lagged inflation or the proportion of backward-looking expectations be small, otherwise the effect of the expectations channel becomes insignificant and some potential benefits of PLT vanish.

If, for instance, $E_t \pi_{t+1}$ in equation (11) were replaced by $\pi_{t-1}$ it could be seen that PLT would increase short-run macroeconomic volatility. This is also the result obtained for the example with no expectations channel, as explained above.

In practice, both types of expectations (forward and backward-looking) coexist so the question arises about the critical level of indexation to lagged inflation at which it is better not to switch to PLT. We intend to provide an answer to this question in Section 4.

Although inflation inertia may reduce the welfare benefits of PLT, it must also be considered the fact that implementing this regime, and therefore substantially reducing uncertainty about the future level of prices, may change the way some agents form expectations and the way they contract, such that the influence of forward-looking expectations in the economy increases. As a result, a more accurate way of accounting for welfare benefits from PLT should endogenise the degree of indexation. In this regard, Amano et al. (2007) show that the optimal indexation level is lower under PLT.
Lack of international experience and challenges for communication

The costs of moving from one regime to another may be high because the private sector must adapt to the new monetary policy regime. Since there is little and rather outdated experience with PLT, both the central bank and private agents lack a benchmark from which they can learn. According to Böhm et al. (2011) only two countries have set price targets in the past, Czechoslovakia between 1919 and 1923, and Sweden between 1931 and 1937. The speed at which economic agents learn about PLT and credibility can be established will determine the costs of transition from IT to PLT.

Another problem faced by the central bank is to decide the way in which PLT will be communicated to the public. Under IT, people have got used to think and interpret monetary policy issues in terms of inflation, and hence it can be really difficult to make them think in terms of the aggregate price level. However, the central bank can overcome this problem by announcing the implied path for inflation rather than the price-target path. Of course, this does not solve all the communication problems. The implied inflation path for PLT will vary over time; this may be more difficult to understand and embrace when compared to the constant inflation target (which is nowadays common for several inflation targeting countries).

As stated by Kahn (2009), several communication issues can be addressed by a practical and modified version of PLT known as ‘average inflation targeting’ under which the inflation target is defined as a medium-term average (as the size of the moving average window increases this regime converges to PLT). Nessén and Vestin (2005) show that average inflation targeting may provide a good approximation to the optimal policy under commitment and produces similar benefits to those obtained under PLT.

4. Theoretical Framework and Results

Our setup is based on the same model used by Gaspar et al. (2010a) to analyse arguments in favour of PLT. It is composed of two aggregate equations (both derived and explained in detail by Woodford, 2003): a New Keynesian Phillips curve with partial indexation to
lagged inflation and its corresponding loss function. The Phillips curve takes the form

$$\pi_t - \gamma \pi_{t-1} = \beta (E_t \pi_{t+1} - \gamma \pi_t) + \kappa y_t + \varepsilon_t$$

where $\pi_t = p_t - p_{t-1}$ is inflation, $p_t$ is the log of the aggregate price, $y_t$ is the log of the output gap, $\varepsilon_t \sim iid (0, \sigma^2)$ is a supply shock which is assumed to be observed in $t$. $\beta \in (0,1)$ is the discount factor. There is Calvo pricing and $1 - \omega$ is the probability that the firm can adjust its price. The parameter $\kappa$ is equal to $\left[ (1 - \omega)(1 - \omega\beta)/\omega \right] \left[ (\theta^{-1} + \varphi)/(1 + \varphi \theta) \right]$ where $\theta, \varphi$ and $\theta$ are underlying structural parameters representing the elasticity of substitution between goods, the intertemporal elasticity of substitution of households and the elasticity of the real marginal cost with respect to the output level, respectively. The parameter $\gamma \in (0,1)$ captures the degree of indexation so that at the micro level the non-optimising firm $j$ sets its price following $p_{jt} = p_{jt-1} + \gamma\pi_{t-1}$.

The period loss function for this model is derived as a quadratic approximation of the negative of the representative agent’s period utility:

$$L_t = (\pi_t - \gamma \pi_{t-1})^2 + \lambda y_t^2$$

where $\lambda = \kappa/\theta$. The central bank sets the output gap $y$ directly.

4.1. IT under discretion

By defining a variable $z_t = \pi_t - \gamma \pi_{t-1}$, equations (14) and (15) can be rewritten as

$$z_t = \beta E_t z_{t+1} + \kappa y_t + \varepsilon_t$$

$$L^T_t = z_t^2 + \lambda y_t^2$$

To solve this model for IT, under discretion, it is easy to see that we can focus on the one-period problem. Since the central bank does not commit to future policy decisions and the shock $\varepsilon_t$ is white noise, we can consistently assume that rational expectations imply that $E_t z_{t+1} = 0$. Then, by substituting equation (16) into (17) and minimising the loss function with respect to $y$ we obtain
\( y_t^{IT} = -\frac{\kappa}{\kappa^2 + \lambda} \varepsilon_t \)

this is a standard result which shows that, under discretion, the optimal policy is to tighten (loosen) monetary policy when facing a positive (negative) supply shock. Using equations (16)-(18) we can express the period loss function as \( L_t = (\lambda/(\kappa^2 + \lambda)) \varepsilon_t^2 \) and hence the discounted loss is

\( V_t^{IT} = \sum_{i=0}^{\infty} \beta^i E_t L_t^{IT} \rho_t = \frac{\lambda}{\kappa^2 + \lambda} \left( \varepsilon_t^2 + \frac{\beta}{1 - \beta} \sigma^2 \right) \)

where \( V_t^{IT} \) is the value function for the IT case and \( E_t[.] \) corresponds to the expectations operator.

### 4.2. IT under commitment

Details about how to find a solution for IT under commitment (ITC) are provided by previous literature (e.g. Woodford, 2003; Vestin, 2006). In particular, for equations (16) and (17) the solution for \( z_t^{ITC} \) and \( y_t^{ITC} \) are

\( z_t^{ITC} = \frac{\lambda}{\kappa} (y_{t-1}^{ITC} - y_t^{ITC}) \)

\( y_t^{ITC} = \eta \left( y_{t-1}^{ITC} - \frac{\kappa}{\lambda} \varepsilon_t \right) \)

where \( \eta = (\tau - \sqrt{\tau^2 - 4\beta})/2\beta \) and \( \tau = 1 + \beta + \kappa^2/\lambda \). Using the above equations and doing some algebra\(^5\) we can express \( V_t^{ITC} = \sum_{i=0}^{\infty} \beta^i E_t \left( (z_t^{ITC})^2 + \lambda (y_t^{ITC})^2 \right) \) as

\( (z_t^{ITC})^2 + \lambda (y_t^{ITC})^2 + \frac{(1-2\eta)\beta \lambda + \kappa^2 + \lambda}{(1-\beta)(1-\beta \eta^2)\lambda} \beta \sigma^2 + \frac{(1-2\eta)\lambda + (\kappa^2 + \lambda)\eta^2}{(1-\beta)(1-\beta \eta^2)\lambda} \beta (y_t^{ITC})^2 \)

As mentioned in the introduction, Vestin (2006) and Roisland (2006) show that the ITC solution can be fully replicated under discretion by assigning a price level target and a specific weight on output in the central bank loss function. Consequently, the solution of ITC becomes a benchmark to measure the relative performance of PLT.

\(^5\) A similar procedure is followed in Section 4.3, which provides more algebraic details.
As we will see in the next subsection, similarly to the ITC solution (but unlike the IT solution) the PLT regime introduces a history-dependent policy response.

### 4.3. PLT

Let us now focus on the solution for PLT (under discretion). In this case, although the social loss is still the same, the central bank minimises a period loss function of the form

\begin{equation}
L_{t}^{PLT} = (p_{t} - \gamma p_{t-1})^2 + \mu y_{t}^2 \tag{23}
\end{equation}

where \( \mu \) is the weight given by the monetary policymaker to output stabilisation. In this case the central bank is interested in stabilising the path of prices rather than that of inflation. Notice that by defining a variable \( w_{t} = p_{t} - \gamma p_{t-1} \) and taking into account that \( z_{t} = w_{t} - w_{t-1} \), equations (14) and (23) can be rewritten as

\begin{equation}
w_{t} - w_{t-1} = \beta (E_{t} w_{t+1} - w_{t}) + \kappa y_{t} + \varepsilon_{t} \tag{24}
\end{equation}

\begin{equation}
L_{t}^{PLT} = w_{t}^2 + \mu y_{t}^2 \tag{25}
\end{equation}

and we can state the problem of the central bank as

\begin{equation}
Min_{y_{t}} L_{t}^{PLT} + \beta E_{t} V_{t+1}^{PLT} \tag{26}
\end{equation}

The model for PLT is not as simple as that for IT and hence we follow a more complex procedure so as to obtain a solution. First, we postulate functional forms for the law of motion of \( w_{t} \) and the value function \( V_{t+1}^{PLT} \):

\begin{equation}
w_{t} = \phi(w_{t-1} + \varepsilon_{t}) \tag{27}
\end{equation}

\begin{equation}
V_{t+1}^{PLT} = \delta_{0} + \delta_{w} w_{t}^2 \tag{28}
\end{equation}

these functional forms are later verified (and their coefficients \( \phi, \delta_{0}, \delta_{w} \) determined) when finding a final solution to the model.

Solving (24) for \( w_{t} \) yields
And equation (27) implies that

\[(30)\]
\[E_t w_{t+1} = \phi^2 (w_{t-1} + \varepsilon_t)\]

Using (29) and (30) and solving (26) we can obtain the following expression:

\[(31)\]
\[y_{t}^{PLT} = \frac{(1+\beta \delta w)(1+\mu \phi^2)\kappa}{(1+\beta \delta w)\kappa^2+(1+\mu \phi^2)\mu} (w_{t-1} + \varepsilon_t)\]

And substituting this result and (30) into (29):

\[(32)\]
\[w_t = \frac{(1+\beta)(1+\mu \phi^2)\mu}{(1+\beta \delta w)\kappa^2+(1+\mu \phi^2)\mu} (w_{t-1} + \varepsilon_t)\]

We can use equations (28)-(32) and the fact that \(E_t[w_{t-1}\varepsilon_t] = 0\) and \(E_t[\varepsilon_t^2] = \sigma^2\) to find an expression for the value function \(V_{t}^{PLT}\):

\[(33)\]
\[V_{t}^{PLT} = \beta \delta_0 + \frac{(1+\beta \delta w)(1+\mu \phi^2)\mu}{(1+\beta \delta w)\kappa^2+(1+\mu \phi^2)\mu} (w_{t-1}^2 + \sigma^2)\]

Comparing equations (32) and (33) with (27) and (28), we verify the functional forms postulated above and determine the value of the coefficients by solving the following system:

\[(34)\]
\[\phi = \frac{(1+\beta)(1+\mu \phi^2)\mu}{(1+\beta \delta w)\kappa^2+(1+\mu \phi^2)\mu}\]

\[(35)\]
\[\delta_w = \frac{(1+\beta \delta w)(1+\mu \phi^2)\mu}{(1+\beta \delta w)\kappa^2+(1+\mu \phi^2)\mu}\]

\[(36)\]
\[\delta_0 = \beta \delta_0 + \delta_w \sigma^2\]

Remember that we assume, for PLT, that social preferences remain the same, and hence we are not interested in comparing \(V_{t}^{ILT}\) vs. \(V_{t}^{PLT}\) but \(V_{t}^{ILT}\) vs. the discounted loss for society when the central bank minimises a PLT loss function, that is,

\[(37)\]
\[V_{t}^{ILT|PLT} = \sum_{i=0}^{\infty} \beta^i E_t \left( z_{t+i}^{PLT} \right)^2 + \lambda (y_{t+i}^{PLT})^2\]
Using equations (27), (31) and (35) we can express

\begin{equation}
\nu'_{t+i}^{PLT} = \phi y_{t+i-1}^{PLT} - \frac{\kappa \delta w}{(1 + \beta \phi^2) \mu} \varepsilon_{t+i}
\end{equation}

and therefore the policy response is history-dependent, unlike the response for IT under discretion (equation (18)). Using equation (38), the fact that \( z_{t+i} = w_{t+i} - w_{t+i-1} \) and that \( E_{t}w_{t+i}^2 = \phi^2 w_{t}^2 + \phi^2 \sigma (1 - \phi^2)(1 - \phi^2)^{-1} \), after some cumbersome algebra we can express equation (37) in the following way:

\begin{equation}
(z_t^{PLT})^2 + \lambda (y_t^{PLT})^2 + \frac{(1 + \beta (1 - 2 \phi)) (1 + \beta \phi^2)^2 \phi^2 \mu^2 + \lambda \kappa^2 \delta^2_{w}}{(1 - \beta)(1 - \beta \phi^2)(1 + \beta \phi^2)^4 \mu^2} \beta \sigma^2
\end{equation}

\begin{equation}
+ \frac{((1 - \phi)^2 (1 + \beta \phi^2)^2 \mu^2 + \lambda \kappa^2 \delta^2_{w}) \kappa^2 \delta^2_{w}}{(1 - \beta \phi^2)(1 + \beta \phi^2)^4 \phi^2 \mu^4} \beta (y_t^{PLT})^2
\end{equation}

In the next subsection we compare equation (39) with (19) for different parameter values and use (22) as the point of reference for the maximum performance that PLT can attain.

4.4. Parameter values and results

Notice we need to set values to \( \beta, \omega, \theta, \sigma, \vartheta \) and \( \varphi \) in order to compare equations (19) and (39). The values for our benchmark case (\( \beta = 0.99, \omega = 0.66, \theta = 10, \sigma = 0.004, \vartheta^{-1} = 0.16 \) and \( \varphi = 0.47 \)) are taken from Gaspar et al. (2010a, 2010b).\(^6\)

Figure 3 shows the relation between the ratio \( V_{t}^{IT}/V_{t}^{IT|PLT} \) (y-axis) and \( \mu \) (x-axis). For the benchmark values \( \kappa \approx 2.0 \times 10^{-2}, \lambda \approx 2.0 \times 10^{-3} \) (represented by the vertical dotted line in the figure) and \( V_{t}^{IT}/V_{t}^{ITC} \approx 1.29 \) (represented by the horizontal dotted line in the figure), and therefore the social loss is about 29% greater under discretion compared with the commitment solution. Since the latter can be implemented by assigning the appropriate PLT function to the central bank, this result basically says that, for the benchmark case, switching from the optimal IT regime to the optimal PLT regime (both under discretion) may increase social welfare in 29%.

\(^6\)Since Gaspar et al. (2010a, 2010b) do not provide specific values for \( \vartheta^{-1} \) and \( \varphi \), we take them from Woodford (2003, Table 6.1).
Whether or not PLT outperforms IT (and how much) depends on the weight $\mu$ given by the central bank to output stabilisation in the PLT loss function. The maximum is reached at $\mu \approx 1.85 \times 10^{-3}$ (where $V_t^{IT|PLT} \approx V_t^{IT|C}$); then for the benchmark case we can conclude that PLT gets closer to the ITC solution when the weight given to output stabilisation (relative to prices stabilisation) is about $1.85 \times 10^{-3}$, and therefore slightly lower than $\lambda$, the weight given to the same objective (but relative to inflation stabilisation) in the social loss.

Figure 3 also shows that the ratio $V_t^{IT} / V_t^{IT|PLT}$ is lower than one only for either small (lower than $4.5 \times 10^{-4}$, which represents about 22% of the value of $\lambda$) or very large (greater than $5.8 \times 10^{-2}$ –not shown in the figure-, that is, almost thirty times $\lambda$) values of $\mu$, and hence there is a wide range of values which allows the central bank to attain a higher level of social wellbeing under PLT, relative to IT. Only significant deviations of $\mu$ from $\lambda$ can make it possible that IT can do better than PLT.
So far, our results support PLT against IT for the benchmark parameter values. We then explore the robustness of the aforementioned results to changes in four parameter, namely $\theta$, $\vartheta$ and $\varphi$. Figure 4 shows the ratio $V^{IT}_t / V^{IT|PLT}_t$ for different parameter values (we change only one parameter at a time while the others remain constant and equal to their benchmark values). Since $\lambda$ varies with each change, we consider values for $\mu$ between 20% and 400% of the corresponding $\lambda$.

We find that neither the range width (of values of $\mu$ as a proportion of $\lambda$) for which PLT outperforms IT nor the maximum value of the ratio $V^{IT}_t / V^{IT|PLT}_t$ change significantly for different values of $\theta$, $\vartheta$ or $\varphi$. However, they change significantly with $\omega$. Notice that the higher the price rigidity the wider the range at which PLT outperforms IT, but the lower the

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7 It is not usual to allow for variation in $\beta$ so we keep it constant for the analysis in this section. Changes in $\sigma$ do not have a significant impact on the ratio $V^{IT}_t / V^{IT|PLT}_t$. 
maximum ratio $V_t^{IT}/V_t^{IT|PLT}$.

### 4.5. More practical versions of IT and PLT

Notice that equations (15) and (23) incorporate quasi differences of inflation and prices, respectively. In practice, however, it is more common to think of IT as the monetary regime in which the central bank stabilises inflation (rather than its quasi difference) around its target. For the same reason it seems more common, according to practice, to represent the central bank loss function for the IT regime as

\[(40) \quad L_t^{IT'} = \pi_t^2 + \mu y_t^2\]

where we have assumed (as in previous subsections) that the inflation target is zero. Similarly, if a central bank that wants to implement PLT, it is interested in stabilising prices (rather than their quasi difference) around a targeted path\(^8\), then the most appropriate loss function to represent such regime is

\[(41) \quad L_t^{PLT'} = \mu y_t^2\]

These loss functions could also be considered more practical due to the fact that the central bank may have some uncertainty about the level of indexation ($\gamma$) in the economy.

In this subsection we want to compare these versions of IT and PLT which are the ones that have been mainly studied by previous literature, assuming that the social loss is still the same (equation (15)).

As mentioned above, the ITC solution can be implemented by assigning the loss function (23) with the appropriate weight on output (e.g. $\mu = 1.85 \times 10^{-3}$ for the benchmark case presented in the previous subsection). A corollary of that result is the fact that when the level of indexation is too high ($\gamma \rightarrow 1$) the optimal policy under commitment can be implemented by minimising a function of the form (40). In contrast, if the level of indexation is very low ($\gamma \rightarrow 0$) the ITC solution can be implemented by minimising a

\[^8\text{Zero in our case. Remember that } p \text{ is expressed in logs.}\]
function of the form (41). This is also the reason why it is argued that a high level of indexation reduces the benefits of PLT. While indexation had no relevance in the analysis of previous subsections, for the analysis of more practical versions of IT and PLT it becomes crucial.

The question that we intend to answer is how high the level of indexation has to be so as to make IT' better than PLT'. To that purpose we compare the discounted loss for society in both cases, that is,

\[
V_{t}^{IT|IT'} = \sum_{i=0}^{\infty} \beta^{i}E_{t} \left( (\pi_{t+i}^{IT'} - y_{t+i}^{IT'})^{2} + \lambda (y_{t+i}^{IT'})^{2} \right)
\]

VS.

\[
V_{t}^{IT|PLT'} = \sum_{i=0}^{\infty} \beta^{i}E_{t} \left( (\pi_{t+i}^{PLT'} - y_{t+i}^{PLT'})^{2} + \lambda (y_{t+i}^{PLT'})^{2} \right)
\]

For these cases it is more difficult to derive an analytical expression so we obtain numerical approximations taking into account that when the central bank minimises a function of the form (40), the expressions for inflation and output take the following form:

\[
\pi_{t}^{IT'} = \psi_{1}^{IT'} \pi_{t-1}^{IT'} + \psi_{2}^{IT'} \varepsilon_{t}
\]

\[
y_{t}^{IT'} = \psi_{3}^{IT'} \pi_{t-1}^{IT'} + \psi_{4}^{IT'} \varepsilon_{t}
\]

and when the central bank minimises a function of the form (41), the expressions for the aggregate price level and output take the following form:

\[
p_{t}^{PLT'} = \psi_{1}^{PLT'} p_{t-1}^{PLT'} + \psi_{2}^{PLT'} p_{t-2}^{PLT'} + \psi_{3}^{PLT'} \varepsilon_{t}
\]

\[
y_{t}^{PLT'} = \psi_{4}^{PLT'} p_{t-1}^{PLT'} + \psi_{5}^{PLT'} p_{t-2}^{PLT'} + \psi_{6}^{PLT'} \varepsilon_{t}
\]

Where coefficients \(\psi\) in equations (44)-(47) are constants that are determined (numerically) by postulating functional forms, solving the corresponding central bank problems (IT' and PLT') and solving systems of equations in a similar way we did above for PLT (Section 4.3).
We consider again the benchmark case ($\beta = 0.99, \omega = 0.66, \theta = 10, \sigma = 0.004, \theta^{-1} = 0.16$ and $\varphi = 0.47$) and allow for variation in $\gamma$. We set values for $\mu$ between 20% and 300% of $\lambda \approx 2.0 \times 10^{-3}$. Figure 5 shows the ratios $V_t^{IT\mid IT'} / V_t^{ITC}$ (dotted line) and $V_t^{IT\mid PLT'} / V_t^{ITC}$ (solid line) for different values of $\gamma$ from 0.4 to 0.8. Since ITC is the point of reference for the optimal performance, the closer is the line to one the better the performance of the corresponding regime. The fact that these ratios are never equal to 1 illustrates that equations (40) and (41) are optimal only for the extreme values of the indexation level ($\gamma = 1$ or $\gamma = 0$, respectively). However, for the appropriate values of $\mu$, the performance of these regimes can be relatively good when compared to IT under commitment. It can also be seen that IT’ starts to outperform PLT’ with levels of indexation higher than 65% (for values of $\mu$ close to $\lambda$). We denote this critical level by $\gamma^*$. 

Then, as in the previous subsection, we allow for variation of parameters $\omega, \theta, \varphi$ and $\varphi$. Table 1 shows the values of $\gamma^*$ for the benchmark and four extreme cases. These results illustrate the fact that significant changes stem from variation in $\omega$. We can also see that,
even for the cases with high price rigidities, the level of indexation at which IT’ starts to outperform PLT’ is greater than 50%, which is the value of indexation estimated by Sahuc (2006) for the Euro area and the US over the period 1970-2002. Since in the last decades there has been a more stable and lower level of inflation, one should expect that current degrees of indexation for these economies are even lower.

Table 1. Critical values of indexation

<table>
<thead>
<tr>
<th>Parameter values</th>
<th>$\gamma^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = 0.80, \theta = 7, \theta^{-1} = 0.12, \varphi = 0.59$</td>
<td>0.52</td>
</tr>
<tr>
<td>$\omega = 0.80, \theta = 10, \theta^{-1} = 0.16, \varphi = 0.47$</td>
<td>0.54</td>
</tr>
<tr>
<td>$\omega = 0.66, \theta = 10, \theta^{-1} = 0.16, \varphi = 0.47$</td>
<td>0.65</td>
</tr>
<tr>
<td>$\omega = 0.50, \theta = 10, \theta^{-1} = 0.16, \varphi = 0.47$</td>
<td>0.66</td>
</tr>
<tr>
<td>$\omega = 0.50, \theta = 13, \theta^{-1} = 0.21, \varphi = 0.35$</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Furthermore, the foregoing critical values should be regarded as lower bounds if we take into consideration that the model in this paper does not endogenise the indexation level. Even in cases where indexation to lagged inflation is initially higher, it could be beneficial to implement PLT because, as mentioned above, this regime may increase the proportion of forward-looking expectations in the economy (and therefore reduce the influence of backward-looking expectations) by reducing uncertainty about the future level of prices.

5. Conclusion

In recent years the new challenges facing monetary policy have led to a re-examination of Inflation Targeting (IT). Price-Level Targeting (PLT), a monetary policy strategy in which the central bank aims to stabilise the aggregate price level (rather than inflation) around a target path, has been considered a good candidate to replace IT due to its potential benefits: decreasing long-term price level uncertainty, increasing short-term macroeconomic stability and reducing the probability of facing a zero lower bound problem. However, since these benefits become effective through the expectations channel, PLT requires a high degree of credibility. Moreover, there is little and rather outdated practical experience with PLT and there might be significant costs of moving from one regime to another (e.g. due to some
communication issues).

Previous literature has shown that PLT can be used to implement the IT solution under commitment, and therefore it is known that a credible PLT regime can outperform IT under discretion. The present paper intends to measure the welfare gain from switching to PLT.

Using a model with a New Keynesian Phillips curve and a loss function (both of which incorporate partial indexation to lagged inflation) we find, for standard parameter values, that the social loss associated to macroeconomic volatility may decrease about 29% by implementing a credible PLT regime.

In a second analysis, we compare more practical forms of both PLT and IT in order to determine the critical level of indexation to lagged inflation at which it is better not to switch to PLT. Using standard parameter values again, we find that such level is 65%, which is high as the current degree of indexation in US and Europe (from a previous literature result) might be lower than 50%.

Additionally, this critical value (65%) should be regarded as a lower bound since our model does not endogenise the indexation level. In cases where indexation is initially higher, it would still be worth switching to PLT because this regime may increase the influence of forward-looking expectations in the economy by reducing uncertainty about the future level of prices.

References


