



BANCO DE LA REPÚBLICA
CENTRO DE ESTUDIOS ECONÓMICOS REGIONALES (CEER) - CARTAGENA

Population and Zipf's Law in Colombia and the Colombian Caribbean Coast, 1912-1993

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- 1. Introduction**
- 2. Review of Literature**
- 3. Data**
- 4. Methodology**
- 5. Results**
- 6. Final Remarks**

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I. Introduction

- ❖ The importance of the population distribution on urban dynamics.
- ❖ Zipf's Law states that populations are characterized by a Pareto distribution with coefficient 1.
- ❖ Zipf's Law is an empirical regularity.
- ❖ Objectives:
 - ❖ A first analysis on the population distribution dynamics in the Colombian municipalities.
 - ❖ To show a parallel characterization between behavior of the Colombian and the Caribbean Coast municipalities.
 - ❖ Determine the Zipf's Law accomplishment

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I. Introduction

- ❖ A first exercise is to determine if Zipf's Law applies for Colombia and the Caribbean municipalities.
- ❖ The second exercise is an evaluation on the distribution's persistence degree characterizing the city size.
- ❖ Even though some multi-country studies have taken into account Colombian population, this is the first work considering complete census data from the beginning of the twentieth century.

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II. Review of Literature

- ❖ First formal approximation: Zipf (1949) based on Auerbach (1913).
- ❖ Several studies on this subject have been written, not only for individual countries but also in multi-country comparisons.
- ❖ Relationship between Zipf's Law and Gibrat's Law.
- ❖ United States
 - ❖ Ioannides and Dobkins (1999)
 - ❖ Duranton (2002)
 - ❖ Ioannides and Overman (2000), Gabaix (1999), Black and Henderson (2003).

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II. Review of Literature

- ❖ India
 - ❖ Sharma (2002)
- ❖ China
 - ❖ Anderson and Ge (2005)
- ❖ France and Japan
 - ❖ Eaton and Eckstein (1997)
- ❖ Multicountry
 - ❖ Soo (2005)
 - ❖ Rose (2005)
- ❖ Colombia
 - ❖ Soo (2005)

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III. Data

- ❖ Census data corresponding to the following years:
 - ❖ 1912-1918-1938-1951-1964-1973-1985 and 1993.
- ❖ Definitions of "City"
- ❖ Municipal data for Colombia
 - ❖ In Colombia there is not a significant number of "metropolitan areas".
 - ❖ The objective is a general analysis of the Colombian population dynamics, and not only for metropolitan areas or the largest cities.
- ❖ There are considered every municipalities in Colombia, except those inside the 20% with the shortest population.



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IV. Methodology

- ❖ The classical Zipf's Law estimation

$$R(n) = An^{-\alpha}$$

Where:

- ❖ $R(n)$: Rank for the population size "n" after organized from the largest (1) to the shortest (m).

Linear expression:

$$\ln(R(n_{it})) = \ln A - \alpha \ln(n_{it}) + \varepsilon_{it}$$

- ❖ Stochastic kernels

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V. Results

❖ Brief population's description

Colombia

Year	1912	1918	1938	1951	1964	1973	1985	1993
Population Growth* (%)	~	2,6%	2,6%	2,6%	4,1%	3,5%	2,7%	1,5%
Mean of City Size (inhabitants)	7.594,5	8.333,2	12.689,5	16.611,1	23.871,1	27.127,9	35.641,9	38.594,1
Standard Deviation of City Size (inhabitants)	7.117,0	8.393,2	18.591,8	36.597,2	78.961,1	118.750,1	171.379,6	195.841,0
Minimum of City Size (inhabitants)	2.964	2.973	4.402	4.956	5.639	5.334	6.226	5.464
Maximum of City Size (inhabitants)	121.257	143.994	355.502	715.250	1.697.311	2.861.913	4.236.490	4.945.448
# of Municipalities	615	647	647	661	703	816	820	849

Caribbean Coast

Year	1912	1918	1938	1951	1964	1973	1985	1993
Population Growth* (%)	~	2,4%	3,7%	2,7%	5,3%	4,7%	3,0%	1,2%
Mean of City Size (inhabitants)	7.335,7	8.300,1	15.178,8	19.929,2	30.621,4	34.416,4	46.632,9	51.000,2
Standard Deviation of City Size (inhabitants)	6.665,2	8.790,9	19.533,5	32.506,5	55.713,1	71.274,6	99.524,4	110.760,7
Minimum of City Size (inhabitants)	2.862	2.680	4.933	5.703	8.177	8.543	11.054	11.829
Maximum of City Size (inhabitants)	48.907	64.543	152.348	283.238	498.301	703.488	927.233	993.759
# of Municipalities	93	94	89	92	101	128	128	128

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1912				1993			
Department	City	Inhabitants	Position	Department	City	Inhabitants	Position
The most populated cities				The most populated cities			
Bogotá D.C.	Bogotá	121.257	1	Bogotá D.C.	Bogotá	4.945.448	1
Antioquia	Medellín	71.004	2	Valle del Cauca	Cali	1.666.468	2
Atlántico	Barranquilla	48.907	3	Antioquia	Medellín	1.630.009	3
Bolívar	Cartagena	36.632	4	Atlántico	Barranquilla	993.759	4
Caldas	Manizales	34.720	5	Bolívar	Cartagena	656.632	5
The least populated cities				The least populated cities			
Arauca	Arauca	2.973	611	Cundinamarca	Tena	5.532	845
Santander	La Paz	2.972	612	Quindío	Córdoba	5.525	846
Nte. Santander	Mutiscua	2.966	613	Cundinamarca	Tibacuy	5.524	847
Santander	Umapala	2.965	614	Cundinamarca	Gachancipa	5.506	848
Cundinamarca	Jerusalén	2.964	615	Boyacá	Somondoco	5.564	849

1912				1993			
Department	City	Inhabitants	Position	Department	City	Inhabitants	Position
The most populated cities				The most populated cities			
Atlántico	Barranquilla	48.907	1	Atlántico	Barranquilla	993.759	1
Bolívar	Cartagena	36.632	2	Bolívar	Cartagena	656.632	2
Bolívar	Montería	21.521	3	Magdalena	Santa Marta	283.711	3
Bolívar	Lorica	19.005	4	Córdoba	Montería	275.952	4
Bolívar	El Carmen de Bolívar	16.332	5	Cesar	Valledupar	248.525	5
The least populated cities				The least populated cities			
La Guajira	Castilletes	2.928	89	Bolívar	Soplaviento	12.327	124
Atlántico	Suan	2.927	90	Sucre	Guaranda	12.054	125
Magdalena	Barrancas	2.922	91	Magdalena	Remolino	11.966	126
Bolívar	Simití	2.888	92	Atlántico	Santa Lucía	11.944	127
Bolívar	El Guamo	2.862	93	Córdoba	Canalete	11.829	128

Colombia

Caribbean Coast

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V. Results

- ❖ **Population's distribution dynamics**
 - ❖ The next step is to verify the relationship between the rank and the corresponding population in each city.
 - ❖ Zipf's Law states the inverse and homogeneous relationship between the log of the rank and the log of population.
 - ❖ Remember that even if there is not consensus about the type of distribution the population should follow in order to catch up an optimal urban development, Zipf's Law proposed a criterion to which the majority of the developed countries' population tends.

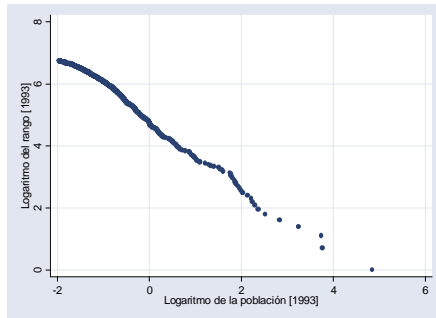
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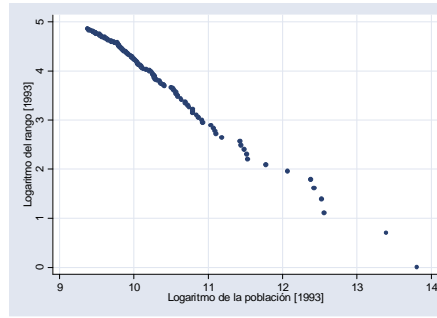
V. Results

Log of rank versus log of city size
(Colombia and the Colombian Caribbean Coast region, 1993)

Colombia



Caribbean Coast



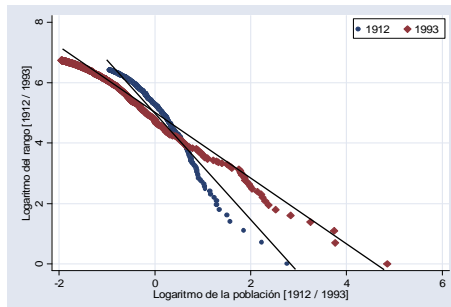
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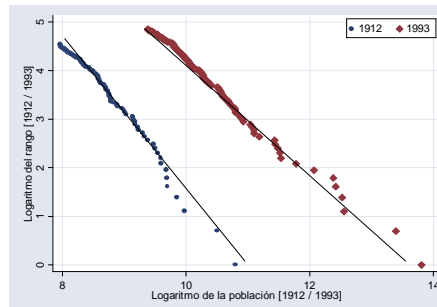
V. Results

Log of rank versus log of city size
(Colombia and the Colombian Caribbean Coast region, 1912 and 1993)

Colombia



Caribbean Coast



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V. Results

Non-Parametric Estimations

Year	Colombia		Caribbean Coast	
	Slope	Confidence Interval	Slope	Confidence Intervals
1912	-1.818 (0.028)	[-1.874, -1.763]	-1.437 (0.011)	[-1.460, -1.414]
1918	-1.788 (0.030)	[-1.847, -1.729]	-1.425 (0.011)	[-1.448, -1.402]
1938	-1.673 (0.017)	[-1.708, -1.639]	-1.346 (0.011)	[-1.369, -1.324]
1951	-1.389 (0.012)	[-1.414, -1.365]	-1.314 (0.011)	[-1.336, -1.292]
1964	-1.203 (0.009)	[-1.222, -1.184]	-1.255 (0.010)	[-1.275, -1.234]
1973	-1.172 (0.007)	[-1.187, -1.156]	-1.220 (0.010)	[-1.242, -1.199]
1985	-1.05 (0.006)	[-1.065, -1.041]	-1.191 (0.010)	[-1.211, -1.170]
1993	-0.991 (0.006)	[-1.004, -0.977]	-1.187 (0.010)	[-1.208, -1.167]
No. of observations: 5.758		No. of observations: 853		
Pseudo R ² : 0.80		Pseudo R ² : 0.87		

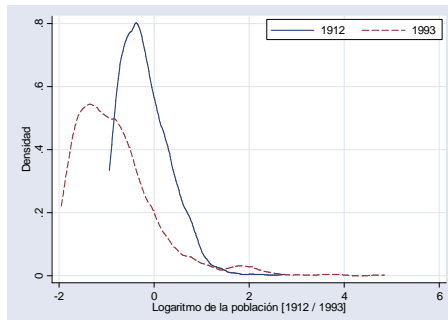
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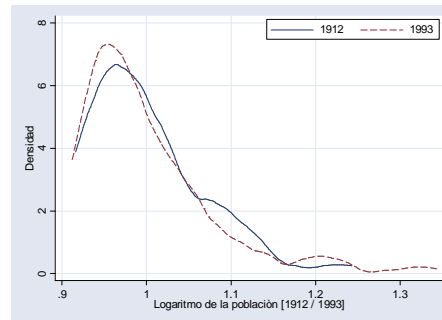
V. Results

Density Function Log of the relative rank size of Population (Colombia and the Colombian Caribbean Coast region, 1912 and 1993)

Colombia



Caribbean Coast



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V. Results

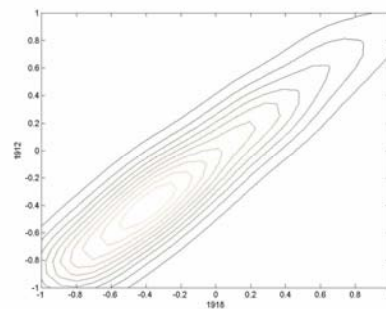
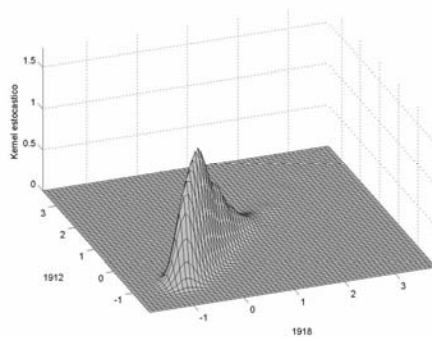
- ❖ **Persistence analysis of the population distribution**
 - ❖ The main objective is to identify the degree of persistence of the city size distribution.

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V. Results

Colombia 1912 - 1918

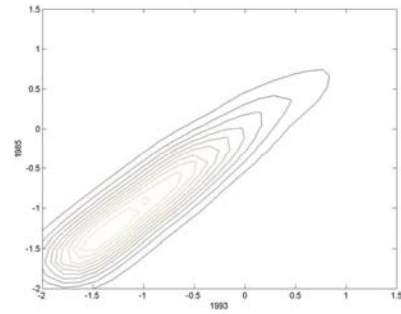
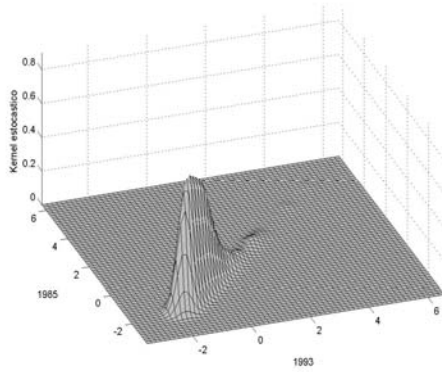


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V. Results

Colombia 1985 - 1993

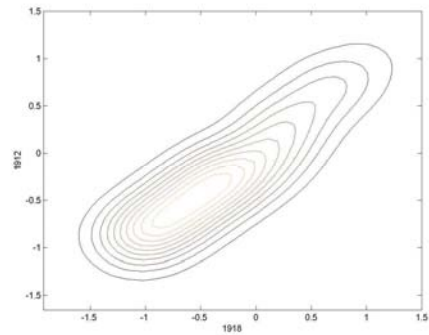
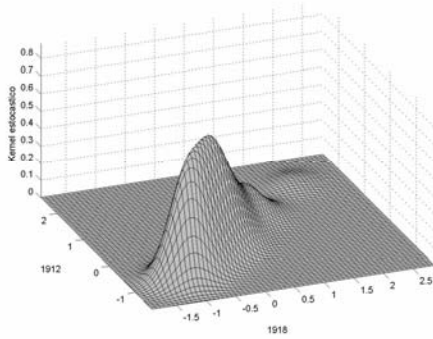


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V. Results

Costa Caribe 1912 - 1918

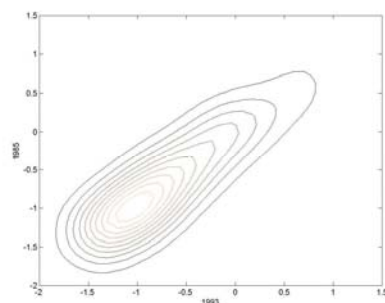
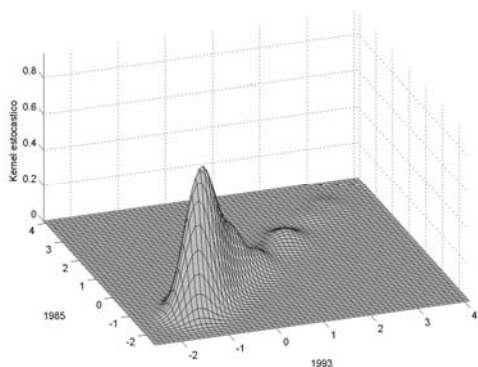


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V. Results

Costa Caribe 1985 - 1993



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V. Results

❖ Persistence analysis of the population distribution

❖ Figures are showing:

- ❖ In both cases, Colombia as a whole and the Caribbean Coast, population distribution is highly persistent.
- ❖ The Caribbean Coast shows a major mobility.

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VI. Final Remarks

- ❖ It was the first time the municipal size distribution was analyzed in Colombia.
- ❖ Colombian municipalities showed different size distributions compared with the Caribbean.
- ❖ Colombian municipalities passed through not to accomplish Zipf's Law in 1912 to do from the 80's. The Caribbean has not achieved Zipf's Law yet.
- ❖ Both Colombian municipalities and the Caribbean showed a high persistence in the population size distributions.

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THANKS!

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