Analysis of Latin American Spot Curves: What Can we Get from the Colombian Spot Curve?

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Abstract

Some authors have worked on the relationship between macroeconomic variables and the term structure of interest rates. In particular, theory says that the shape of the curve reveals valuable insights regarding the behavior of the macro variables such as inflation and growth expectations. Nonetheless, other studies have proven to be effective in the development of forecasts for inflation and growth just based on the information of other economic variables. This paper shows how the forecast for macroeconomic variables can be improved if financial information implicit in the structure of interest rates is included. In particular, we forecast the Colombian inflation with an economic VAR complemented with the spot curve, which is estimated using Nelson and Siegel's model for interest rates. Additionally, the paper studies the factor decomposition for the spot curve estimated with Nelson and Siegel's model for Brazil, Chile, Colombia, Mexico, and Peru, and the correlations for the curve slope among these countries. This analysis reveals two things: i) correlations between empirical and theoretical values for the Nelson and Siegel's model hold over the pre-crisis period and after the crisis those correlations are broken, and ii) when the slopes' correlation among the countries is high, part of it is driven by external factors.

JEL Classification: C3, C5, D4, D9, G1.
Key Words: Spot Curve, Inflation, Economic Growth, Target Interest Rate

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1 Introduction

The behavior of interest rates can be a good proxy of agents expectation about future states of the economy. As such, the identification of their patterns can be associated to different perceptions on the path that macroeconomic variables should follow. For instance, a continuously growing path of interest rate could be interpreted as a signal of future inflationary pressures. Furthermore, the increase in interests rates can be the result of a lower demand for debt showing a preference for current consumption or higher productive capacity investments. These agents' decisions are closely related to an expansive economy and hence a signal anticipating growth. On the other hand, lower long term interest rates can be associated to low economic growth expectations. This view will speed up the need for a more stable future income increasing current savings consequently. As a result, the term structure of interest rates and the assessment of its dynamics provide valuable information about what market believes the economy is heading to. The literature has explored the use of economic data to extract information about the guidelines of macroeconomic variables. Although this information has provided a high degree of explanatory power on the change of other macroeconomic variables, the use of the spot curve could improve the analysis. The purpose of this paper is to explore this hypothesis and to prove that the inclusion of interest rates information, for the particular case of the Colombian market, provides a more accurate forecast of inflation. Although, forecasts of other macroeconomic variables is of interest, the current work focuses on inflation only.

Throughout the paper, spot curves are estimated using Nelson and Siegel's model, which is a simple model able to mirror the structure of interest rates across several maturities. Under this model the behavior of interest rates can be decomposed into three latent factors: level, slope and curvature. Therefore, interest rates move from one period to another following a parallel shift, a steepening or flattening movement and a change in curvature. The first part of this work focuses on the spot curve of Latin American countries that have their domestic public debt most developed, specifically on Brazil, Chile, Colombia, Mexico and Peru. The paper studies the correlations between the latent factors of the spot curve and their empirical counterparts for each country. The analysis finds that: i) the latent factors are well described by the empirical counterparts for Colombia, Chile, Mexico and Peru, but not for Brazil, and ii) the high correlations between empirical and theoretical factors hold over the pre-crisis period but after the crisis those correlations are broken. An additional analysis of correlations for the slopes is carried out among the countries. The analysis is based on the methodology proposed by Bunda et al. [2010] to separate the influence of external and domestic factors. We find that for periods of high correlation, part of it is due to the effect that external factors have on Latin American countries. The second part of the work focuses on Colombia, and on how the spot curve improves the inflation forecast.

2 Modeling the Spot Curve

We use Nelson and Siegel [1987] framework to fit the spot curve on a daily basis for Brazil, Chile, Colombia, Mexico, and Peru. Nelson and Siegel (N&S) proposed a simple parsimonious model able to represent the range of shapes (monotonic, humped, and S shaped) generally associated with spot curves. Their function can be stated as:

$$S_t(m) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-m/\tau_t}}{m/\tau_t}\right) + \beta_{2t}\left(\frac{1-e^{-m/\tau_t}}{m/\tau_t} - e^{-m/\tau_t}\right), \tag{1}$$

where $S_t(m)$ is the spot rate at maturity m (in year terms) and $\beta_{0t}, \beta_{1t}, \beta_{2t}$ and τ are the parameters to be estimated. Diebold and Li [2006] show that the three time-varying Nelson and Siegel's coefficients may be interpreted as factors corresponding to level, slope and curvature. This section explains the framework used, describes the data, and analyzes the results.

2.1 How to Estimate the Spot Curve?

The objective of estimating spot curves is to fit the market data under a model. For a given date and some chosen parameters $(\beta_{0t}, \beta_{1t}, \beta_{2t}, \tau_t)$ the spot curve is generated using equation (1). The price of the bonds traded that date is found using this spot curve. Then, using an optimization routine, the difference between the estimated prices and the actual (market) prices is minimized. The function to minimize is:

$$\min_{\beta_{0t},\beta_{1t},\beta_{2t},\tau_t} \sum_{j=1}^{N} ((p_j - \hat{p}_j)w_j)^2, \tag{2}$$

where N is the number of bonds that were traded in the market, p_j is the observed price of bond j, $\hat{p}_j$ the price of bond j calculated with the spot curve as in (1), and w_j is the bond j's inverse duration fraction given by $w_j = \frac{1/D_j}{\sum_{i=1}^{N} 1/D_i}$. The fit of the spot curve to the observed data is better when equation (2) is minimized including w_j. If equation (2) is minimized setting $w_j = 1\ \forall j$, then the procedure will give higher priority to the match between observed and estimated prices for the bonds with higher duration than to the ones with shorter maturity, given that the price of the formers is more sensible to changes in the interest rate.

To be able to compare the parameters $\beta_{0t}, \beta_{1t}, \beta_{2t}$ overtime, τ needs to be fixed. We fix τ in a similar manner as Nelson and Siegel. Given a historical data set, over a grid of τ[1], for each date and each τ, we estimate the following linear regression by Ordinary Least Squares(OLS):

$$\hat{S} = \hat{\beta}_0 + \hat{\beta}_1 F_1 + \hat{\beta}_{2t} F_2 + \varepsilon, \tag{3}$$

where $\hat{S}$ is the "observed" vector of spot rates[2], and F_1 and F_2 are the loading factors of B_1 and B_2. The τ chosen is the one that provides the lowest sum of square sum of errors (SSE).

2.2 Data and Results

The sample period for all the countries covers until June 30, 2010. Table 1 shows the data set used for each country, the initial date for the sample period[3], and the fixed τ that we use through all the estimations.

Table 1: Data

Country	Sample	τ
Brazil	April 2007	4.6
Chile	Aug 2005	3.5
Colombia	Jan 2003	3.7
Mexico	Aug 2003	4.3
Peru	May 2006	2.1

Source: Bloomberg for all the countries except Colombia.
For Colombia SEN and MEC.

As mentioned before, the parameters $\beta_0, \beta_1, \beta_2$ can be interpreted as the level, the slope and the curvature of the spot curve. Figure 1 through Figure 15 show the estimated parameters for each country versus their empirical counterparts. For some countries (Colombia and Mexico), the data confirms the assertion that the three parameters correspond to level, slope, and curvature, even though not as close as in the US (Diebold and Li [2006]). For Chile, the

[1] Grid with a step of 0.1 over the interval 0.1 to 10.

[2] Yields are the ones observed in the market. Spot rates are estimated by N&S with a flexible τ from the observed yields

[3] The initial date differs by country depending on the availability of the data in Bloomberg.

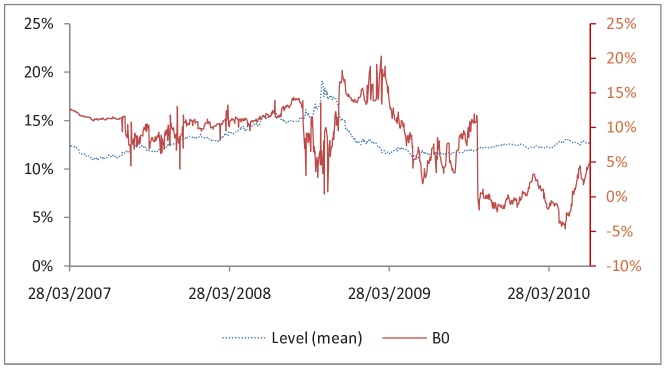

Figure 1: Brazil: B_0 vs. level
Level: mean of the spot curve (0.5 year through 7 year)

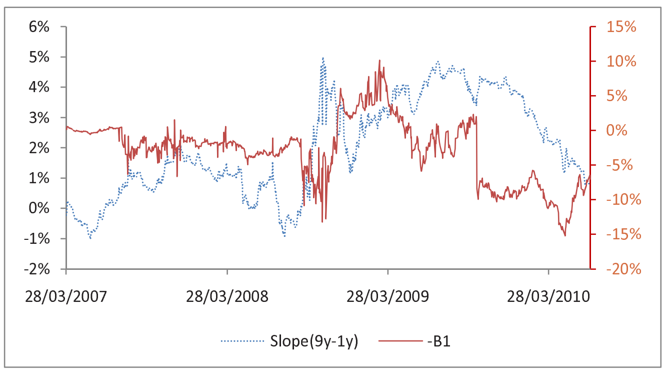

Figure 2: Brazil: $-B_1$ vs. slope
Slope: 7 year spot rate - 1 year spot rate

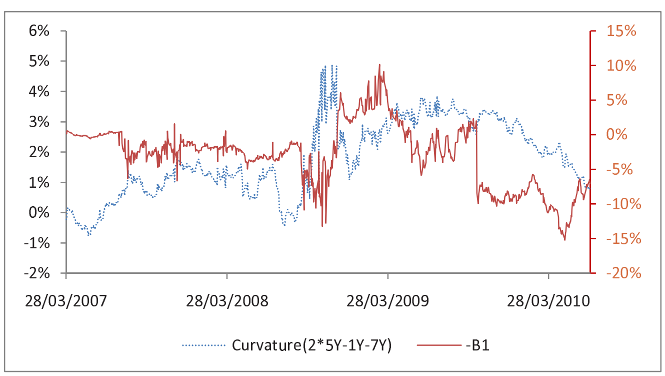

Figure 3: Brazil: B_2 vs. curvature
Curvature: 2*(5 year spot rate)-(1 year spot rate + 7 year spot rate)

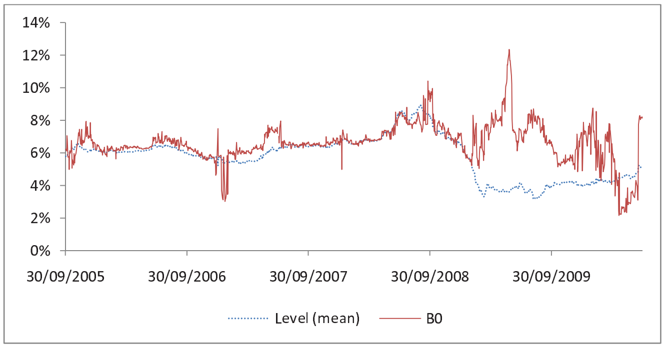

Figure 4: Chile: B_0 vs. level
Level: mean of the spot curve (0.5 year through 9 year)

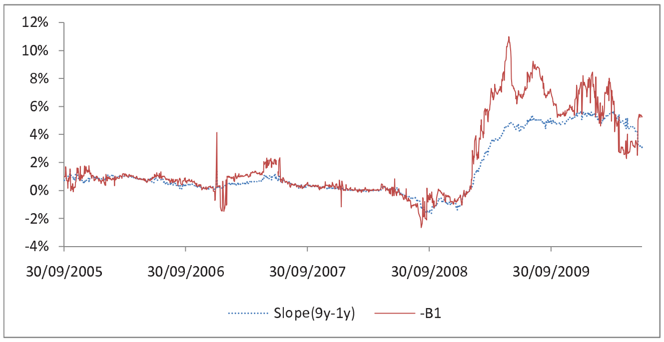

Figure 5: Chile: $-B_1$ vs. slope
Slope: 9 year spot rate - 1 year spot rate

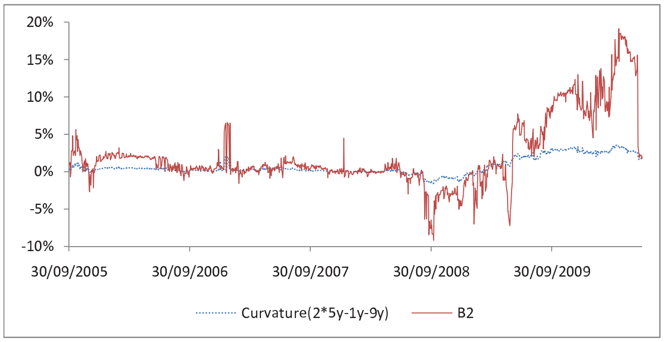

Figure 6: Chile: B_2 vs. curvature
Curvature: 2*(5 year spot rate)-(1 year spot rate + 9 year spot rate)

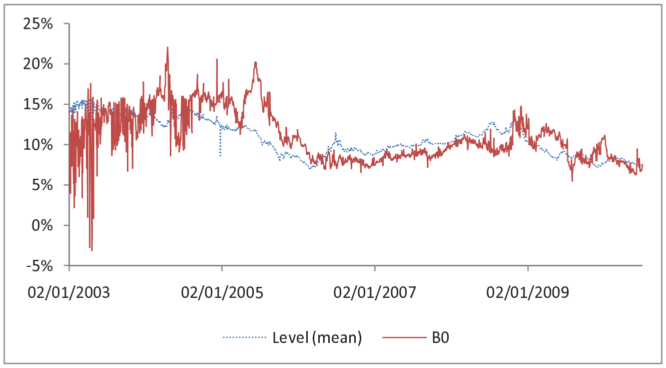

Figure 7: Colombia: B_0 vs. level
Level: mean of the spot curve (0.25 year through 15 year)

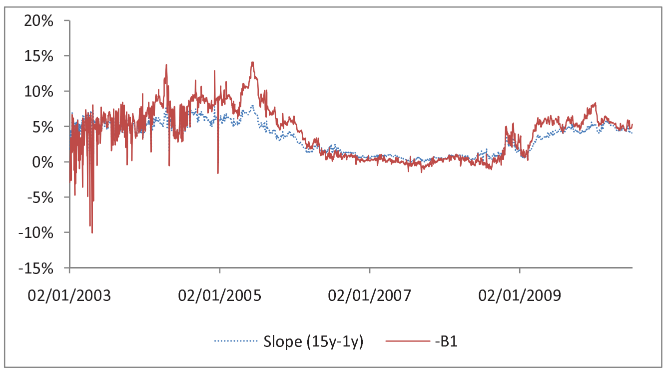

Figure 8: Colombia: $-B_1$ vs. slope
Slope: 15 year spot rate - 1 year spot rate

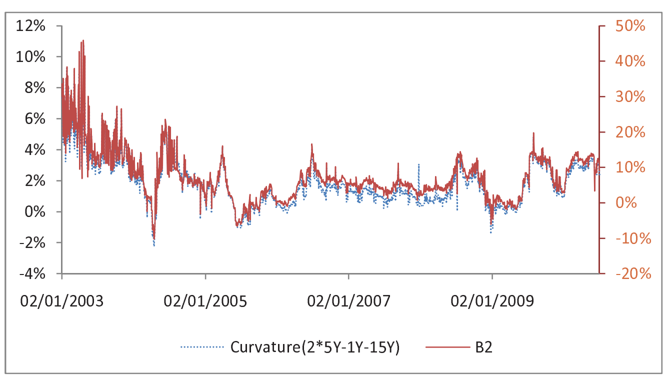

Figure 9: Colombia: B_2 vs. curvature
Curvature: 2*(5 year spot rate)-(1 year spot rate + 15 year spot rate)

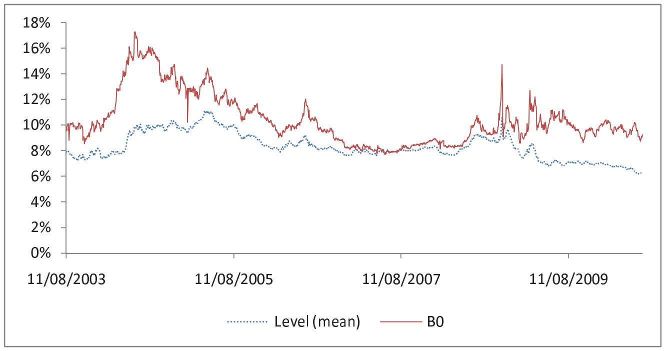

Figure 10: Mexico: B_0 vs. level
Level: mean of the spot curve (0.5 year through 15 year)

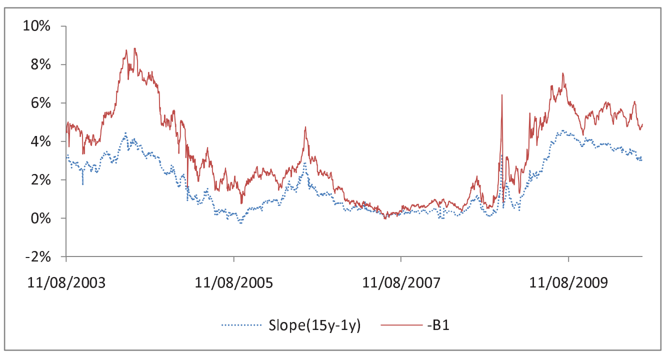

Figure 11: Mexico: $-B_1$ vs. slope
Slope: 15 year spot rate - 1 year spot rate

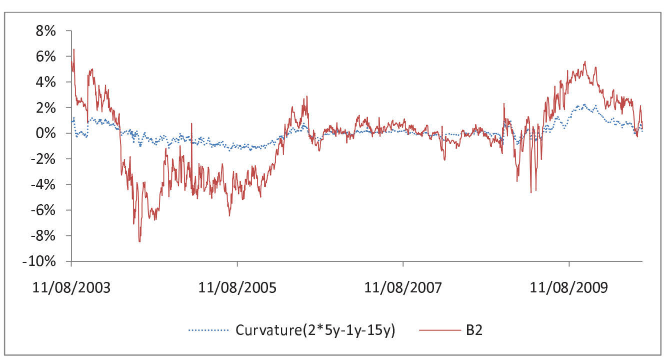

Figure 12: Mexico: B_2 vs. curvature
Curvature: 2*(5 year spot rate)-(1 year spot rate + 15 year spot rate)

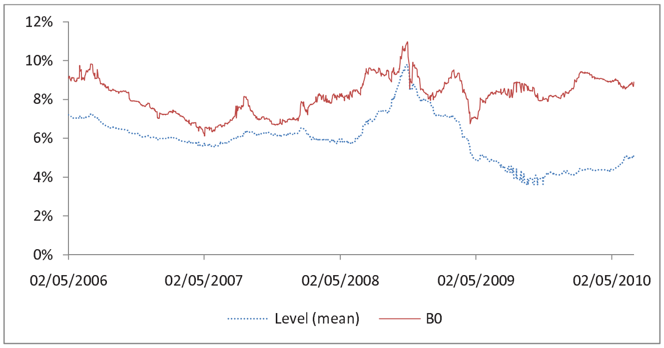

Figure 13: Peru: B_0 vs. level
Level: mean of the spot curve (0.5 year through 15 year)

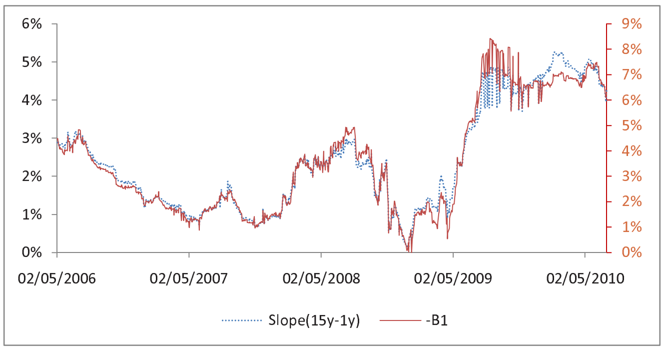

Figure 14: Peru: $-B_1$ vs. slope
Slope: 15 year spot rate - 1 year spot rate

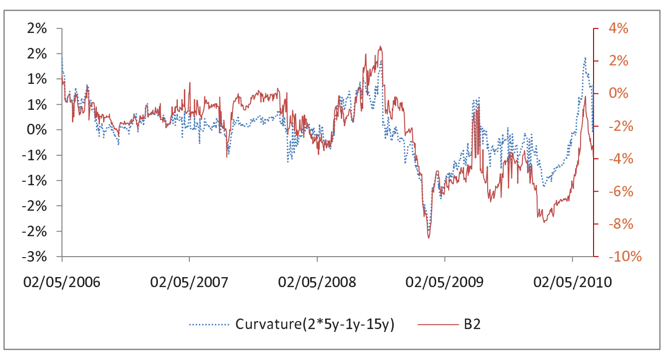

Figure 15: Peru: B_2 vs. curvature
Curvature: 2*(5 year spot rate)-(1 year spot rate + 15 year spot rate)

Table 2: Correlations

Country	β_0-level	β_1-slope	β_2-curvature
Brazil	0.19	0.10	0.73
Chile	0.54	-0.56	0.90
Colombia	0.57	-0.93	0.99
Mexico	0.66	-0.94	0.88
Peru	0.23	-0.99	0.82

estimated parameters have a good fit with their empirical counterparts until October 2008, but then correlations are broken. In Peru, $-\beta_1$ and β_2 follow very carefully the slope and the curvature, but β_0 does not match the level. Brazil, is the only country for which there is not any relationship between the estimated parameters and their empirical counterparts.[4] Table 2, shows that in general β_1 and β_2 have a high correlation with their empirical counterparts, while that is not the case for β_0 and the level. Since not all the time and not for all countries, the estimated parameters from the Nelson and Siegel's model match very closely the level, slope and curvature, from now on in this paper the empirical level, slope and curvature are used.

3 Is There a Common Factor in Latin America?

For reasons that are going to be explained in Section 4, throughout the paper we focus on the level and slope of the spot curve and not on the curvature. Table 3 and 4 show the correlations (Spearman rank order) among the countries for the level and the slope, respectively.[5] For the two factors the correlation is high and positive. The lowest correlation for the level is 0.52 (between Brazil and Mexico), and for the slope is 0.40 (between Brazil and Peru). This simple correlation measure is affected by both global and country-specific factors. Applying the methodology proposed by Bunda et al. to the levels and to the slopes (separately), we try to disentangle the influence of external and domestic factors.

[4] This might be due for the peculiarities of the Brazilian public internal market: i) it has a very short maturity, ii) the market for interest-rate derivatives contracts is more liquid that the bonds market, and iii) most of the transactions are done over the counter (OTC). The longest fixed rate bond until May 2010, was the bond with a maturity of 7 years (In May 2010 the government issued a bond with maturity in 2021). According to Pereira et al. [2010], the average maturity of fixed rate bonds on 2008 was 19 months, and 99.2% of the secondary market trading volume was OTC.

[5] The common sample among the countries is April 2007-June 2010. Figures 16 and 17 present the levels and the slopes of the Latin American countries' spot curves.

Table 3: Level Correlations

	Colombia	Brazil	Chile	Mexico	Peru
Colombia	1.00				
Brazil	0.60	1.00			
Chile	0.87	0.74	1.00		
Mexico	0.88	0.52	0.76	1.00	
Peru	0.79	0.56	0.82	0.82	1.00

Table 4: Slope Correlations

	Colombia	Brazil	Chile	Mexico	Peru
Colombia	1.00				
Brazil	0.71	1.00			
Chile	0.65	0.52	1.00		
Mexico	0.83	0.82	0.69	1.00	
Peru	0.74	0.40	0.61	0.66	1.00

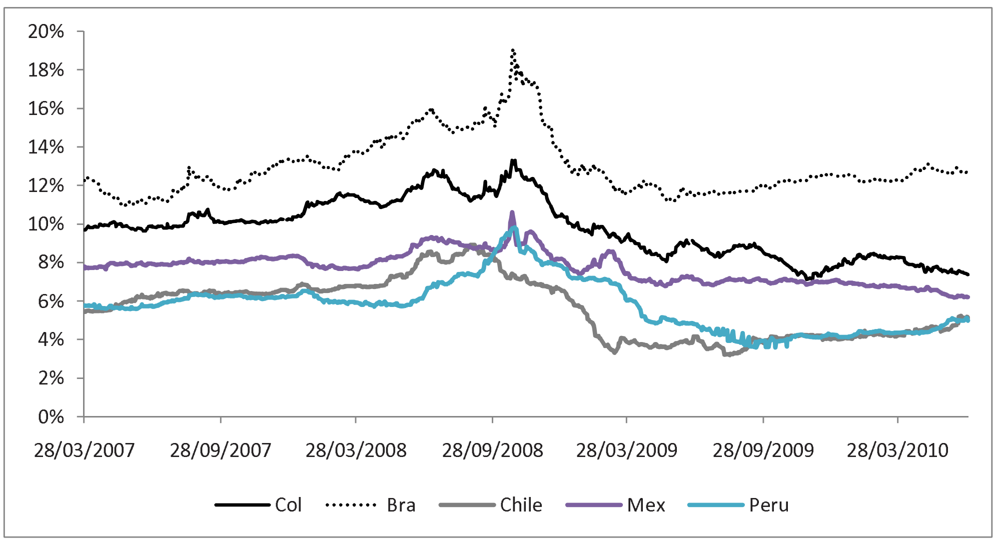

Figure 16: Levels of the Latin America spot curves.

We use the following two factor model for the level (slope) of each country i:

$$Level_{i,t} = \hat{\beta_{i,0}} + \hat{\beta_{i,1}} * VIX_t + \hat{\beta_{i,1}} * USlevel_t + \varepsilon_{i,t}, \quad (4)$$

where i stands for Colombia, Brazil, Chile, Mexico and Peru, VIX is the Chicago Board Options Exchange Volatility Index–usually known as a measure of risk aversion–, and $USlevel$ is the mean level of the US spot curve.[6] As done by Bunda et al., for the common sample period among the countries (April 2007-June 2010), we estimate rolling correlation coefficients over 60-day windows. An average of the simple Spearman's rank order correlation between all the pairs of countries was calculated, ρ. The adjusted correlation, $\hat{\rho}$, is obtained in a similar fashion. The difference is that the Spearman's rank order correlation is between the residuals of each country (from Equation 4) and the ones from the other countries. This correlation coefficient of residuals becomes a measure of the comovement in the levels (slopes) after removing the influence of the common external factors.

Figures 18 and 19 depict the results for the correlation and the adjusted correlation among the level and the slope of the countries in Latin America (Brazil, Colombia, Chile, Mexico, and Peru). The things to notice here are: i) correlations and adjusted correlations are not constant over time, they range between 0.91 and -0.16, ii) the highest correlations for the level and for the slope took place after Lehman Brothers collapse, iii) during the periods of high positive correlation, part of this correlation is due to common external factors (there is a widening gap between the two correlations; $\rho > \hat{\rho}$), and iv) recently the correlation among the levels has been low, while the one for the slopes has been increasing. In conclusion, Latin America government domestic debt does not exhibit a common factor, however the common external factor plays an important role during the periods of high positive correlation.

Appendix A shows the results of Bunda et al.'s methodology applied to credit default swaps (CDS), Emerging Market Bond Indices (EMBI's), and exchange rates of the countries analyzed in this paper.

[6] Bunda et al. use the US Treasury rates, the Standard and Poor Index (S&P), and the US high yield corporate bond index (US-HY), as the external factors. We decided to use the VIX instead of the S&P and the US-HY, because it is highly correlated with the two variables, and it is a more direct measure of risk.

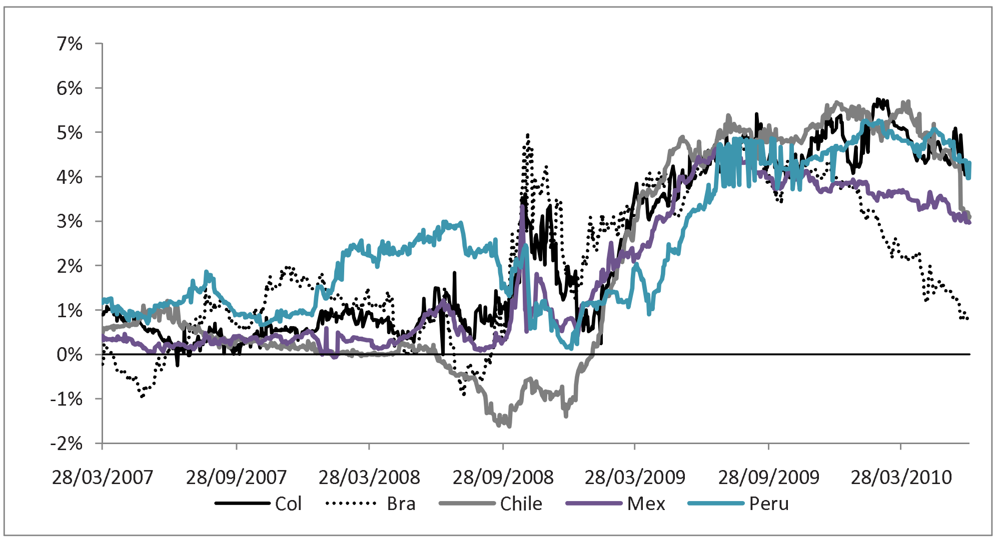

Figure 17: Slopes of the Latin America spot curves.

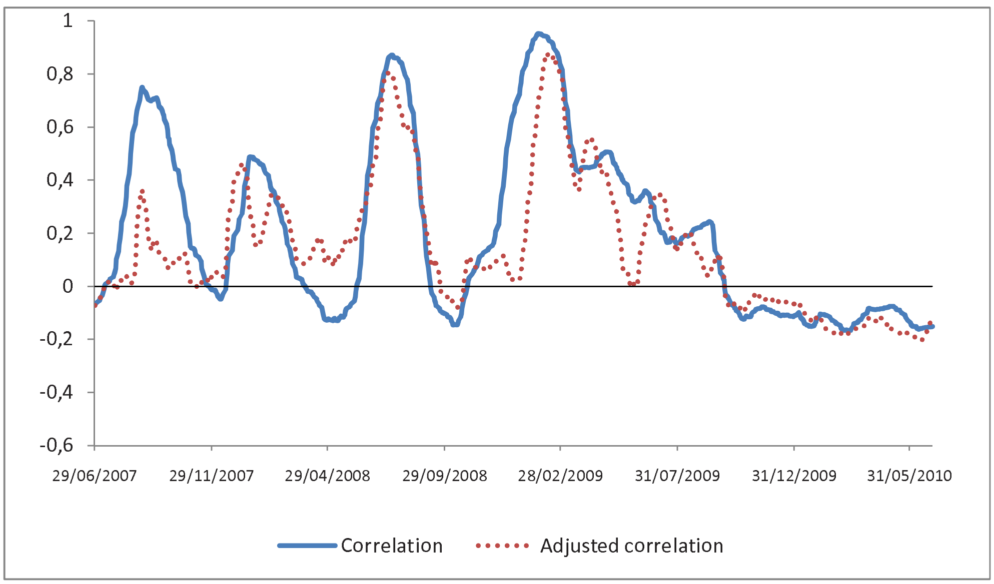

Figure 18: Correlation and Adjusted Correlation for the levels of the Latin America spot curves.

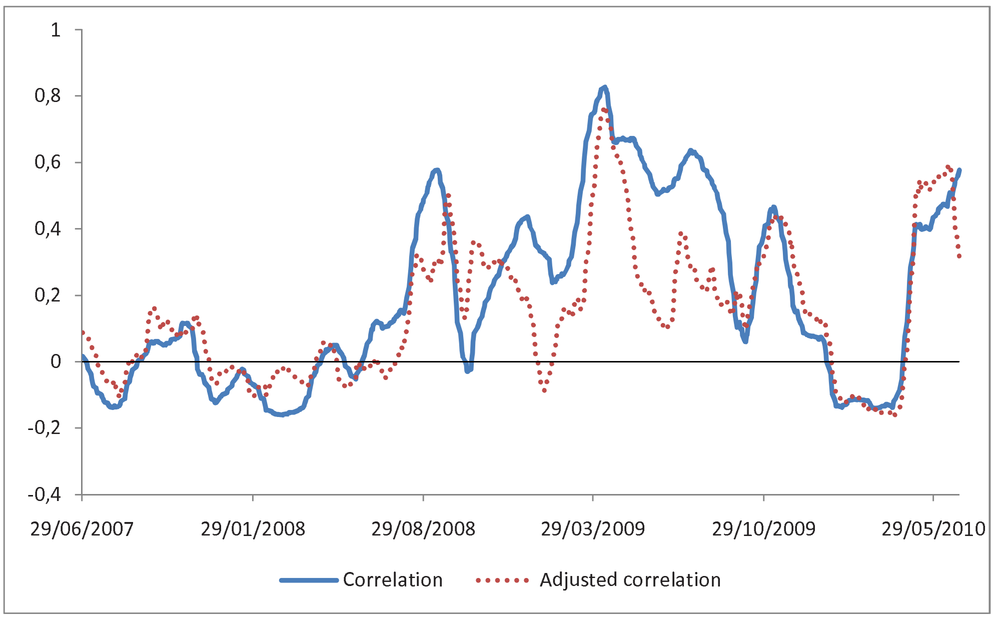

Figure 19: Correlation and Adjusted Correlation for the slopes of the Latin America spot curves.

4 Analysis of the Shape of the Curve and its Relation with Macroeconomic Variables

The behavior of the term structure of interest rates has a close relation with consumption decision as well as the preferences of different agents in the market. Alike preferences for current consumption over late consumption, interest rates reveal also agents' perspectives over the long term performance of the economy and the trend of monetary policy target interest rate. In this sense, studying inter-temporal changes in the shape of the structure of interest rates can provide important signals around what investors believe the economy is coming along with. This section provides an empirical analysis of different macroeconomic variables and three main factors (term structure's shifts) implicit in interest rates' evolution. The current analysis has been split into two components. The first part evaluates the intertemporal relation among macroeconomic variables and the curve factors, and the second part utilizes a VAR (Vector Autoregression) mechanism to study the cross relations and aggregate impact of changes in one of the system's variables. The macro indicators chosen for this analysis correspond to i) annual changes in the CPI, as inflation, ii) annual changes in the Industrial Production Index (IPI), as a proxy of economic growth, and iii) the central bank's target interest rate as the indicator of local monetary policy. The results for the Colombian case are shown as follows.

The first important result, confirms as in the US case, that the third factor (curvature) lacks of any feasible predictive and explanatory power on the behavior of macroeconomic variables. As it is shown in Figure 20, the impulse-response analysis evidences that random shocks in curvature do not have a significant impact neither on future inflation nor on economic growth. On the other hand, a random shock in both inflation and growth does not have an important impact in the curvature of the term structure either. Consequently, any further analysis developed in this paper will not take curvature into account on the sake of more focused and meaningful analysis. Hereafter curve factors will refer to level and slope only.

4.1 IPI vs. Interest Rates Curve Factors

The intertemporal analysis, first measured through cross correlations among the series of IPI, level and slope shows that, in fact there are positive correlations between level and IPI, and slope and IPI. In particular, it is possible to observe how positive shocks in level and slope are correlated to positive changes in the IPI, 30 and 24 months ahead respectively.

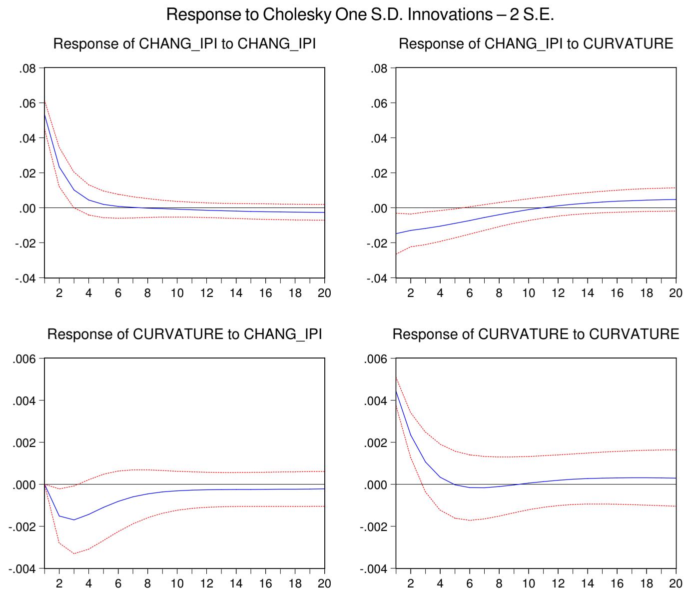

Figure 20: Curvature VS Inflation and IPI

These results can potentially indicate that increments in the level of interest rates as well as an increasing slope of the term structure are correlated to positive results in economic activity.

In general, a drop in short term interest rates can be a mechanism of expansive monetary policy moving towards a more dynamic credit market. The former process will result on a push strategy that incentives all agents in the economy to use cheaper capital for productive use. As a result, a feasible explanation for the relationship between slope and economic growth comes from the fact that a downside shock to short term interest rates will not only affect the slope of the curve, but also produce the adequate ground field for future productivity and growth. The results for the Colombian case confirms this hypothesis and also provides evidence of an effective and credible monetary policy. As it is shown in Figure 22, there is a significant correlation between changes in slope and a positive change in economic activity 2 years later. The same phenomenon can be observed in Figure 21. These results indicate that lower interest rates, in effect incentive the use of productive capital resulting in economic growth.

As we identified a potential relationship among curve factors and an indicator of economic growth, the next step is to validate the direction of the relationship. As it is described in Diebold and Li [2006], there could exist a bidirectional relation between these variables.

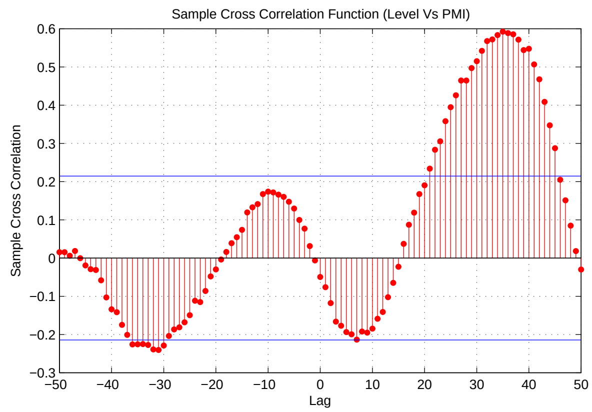

Figure 21: Cross Correlation Level vs. IPI

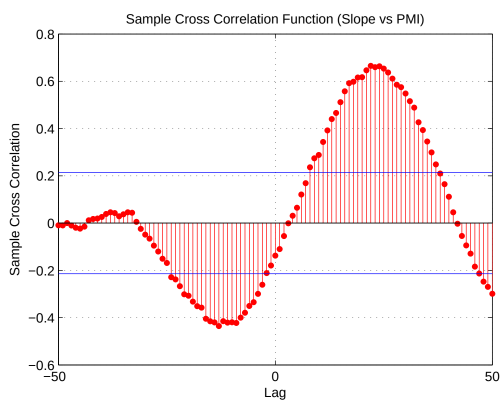

Figure 22: Cross Correlation Slope vs. IPI

The use of a VAR can be meaningful for the study of these relations. Although previous results (the correlograms) indicate the existence of evidence regarding a positive intertemporal relationship between the shape of the term structure and growth, Figure 23 shows that neither shocks on the level nor on slope will have a significant impact on the behavior of the IPI.

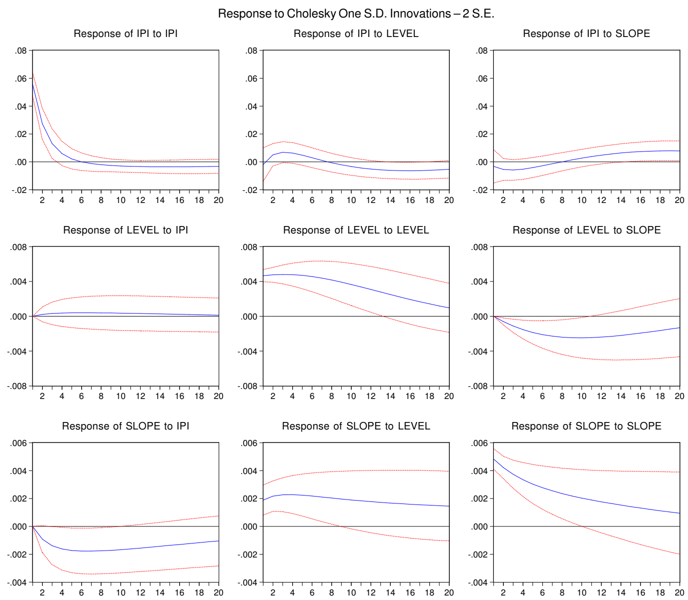

Figure 23: Impulse Response Level, Slope and IPI

The mixed results presented before, can not give any conclusive results for the relationship among growth and changes in interest rates. Although there is statistical significant evidence of positive intertemporal correlation between financial inputs - the shape of interest rates structure -, and a proxy of economic growth, it is not possible to confirm a positive significant impact or consequential behavior as a result of a change in one or the other. A possible explanation for this can be associated to the behavior of the IPI for which volatility can distorts its meaning as a proxy of the evolution of economic activity. In order to cope with this issue, other variables were used (unemployment, M3, monthly adjusted GDP), however it was not possible to identify any answer able to solve the described problem. This dichotomy can open the door for further analysis that is out of the scope of the current work.

4.2 CPI vs. Curve Factors

As in the case of IPI vs curve factors, it is possible to identify a series of relationships among shifts in the interest rates structure and inflation. As expected, in the Colombian case an increase in the level of the curve is positively correlated with an increasing inflation in the long term. The former pattern is consistent with economic theory, showing that as a result of high inflation expectations, the market is able to anticipate increments in interest rates along the entire curve in order to mitigate the harmful effects of excessive inflationary pressures. On the other hand, the cross correlation analysis also depicts the result of an effective monetary policy. In the mid term, it is possible to identify a negative correlation between the level of interest rates and inflation, which implies that increments along the entire term structure could translate into lower inflation. To sum up, Figures 24 and 25 show how and upward trend in the level of the curve is a signal anticipating increments in price levels in the long terms but also the more immediate results could be a drop in short term inflation.

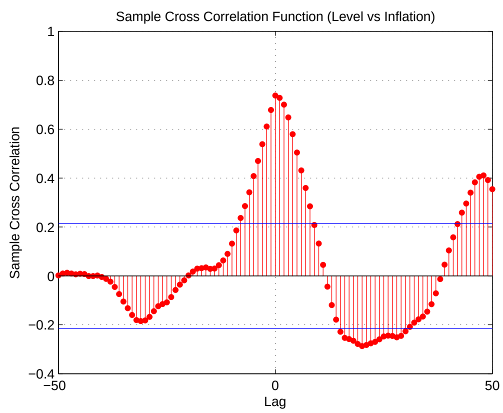

Figure 24: Cross Correlation Level vs Inflation

Clearly, changes in the slope are due to either changes in short or long term interest rates. In contrast to slope vs IPI, the relation between slope and inflation can be particularly associated with changes in long term interest rates. As long as agents expect higher inflation in the future, the risk premium they demand is also increasing. Rational agents will demand higher rates in the long end of the term structure given their need to be compensated for the risk of higher inflation in the future. The decision of substituting current consumption for late consumption will be linked to what those agents expect the prices will be. Therefore, expectations of rising prices will create demand for interest rates high enough to make the substitution appealing. Figure 25 shows an example of this behavior for the colombian case. The cross correlation between the slope and inflation shows that there is a positive relation between current increases in slope, probably associated to long term interest rates, and higher inflation around three and a half years later.

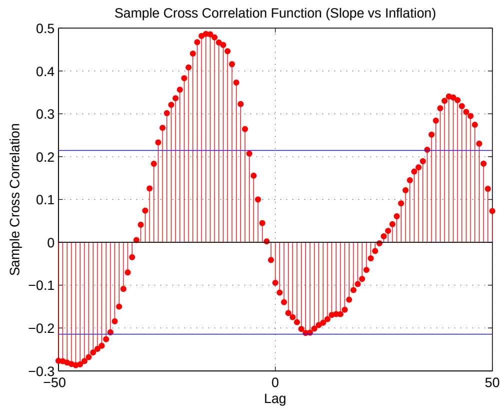

Figure 25: Cross Correlation Slope vs Inflation

Now, given that there is evidence of the intertemporal co-movements between inflation and the information of the term structure, a VAR is utilized to identify the two-ways relationships among the available information (Figure 26). The results of the impulse response analysis shows that any positive shock on the slope of the curve will have a significant impact in short term inflation. The former condition can be the result of materialization of agents' expectations in regard to evolution of market prices. Under this hypothesis, local agents are able to accurately anticipate future changes in inflation and hence, the study of changes in the slope of the curve of interest rates could provide insightful information to feed a forecast model. On the other hand though, unanticipated changes in inflation do not have a significant impact in the slope of the structure of interest rates. Therefore, it would be possible to assert that the bidirectional relationship between slope an inflation is not necessarily clear given that all possible changes could have been already incorporated into the current slope of the curve.

On the other hand, it is possible to observe how the level also has a significant positive positive effect on inflation in the short term. As in the case of slope and inflation, the prevailing relation between level and inflation can be interpreted as an early market reaction to expectation of higher prices in the near future. Hence, the agents are able to adjust the inflation premium accordingly to market expectations on forthcoming inflation.

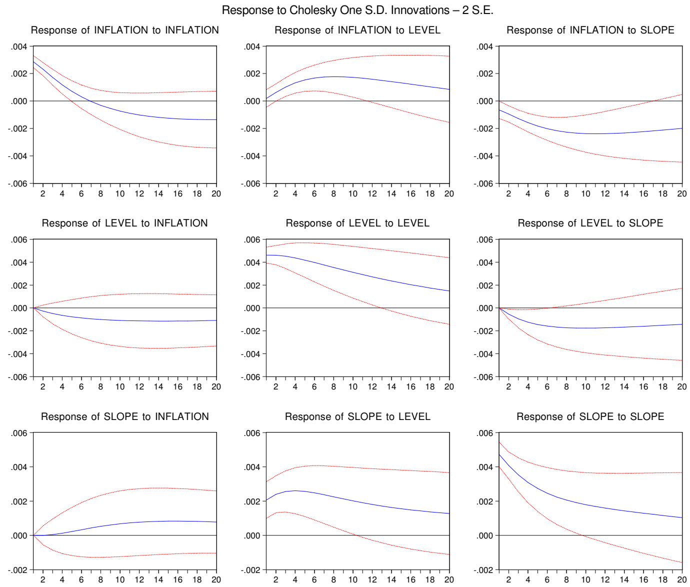

Figure 26: Cross Correlation Slope vs Inflation

4.3 Monetary Policy's Target Interest Rate vs. Curve Factors

Finally, in the same way we interpret the relations among CPI, IPI and curve factors, it is of high interest to identify the reactions on the entire term structure of interest rates as a result of unanticipated changes in the monetary policy target rate or viceversa. In particular, it will be valuable to check if agents in the economy are able to foresee what the benchmark interest rate will be and in this way to price accordingly all the traded securities. The VAR analysis provides an answer to this point. As shown in Figure 27 there is a significant impact of slope over the monetary policy interest rate.

The previous result indicates that the slope of the curve can anticipate later movements in monetary policy interest rates, but it is not clear that changes in this rate will adjust consumption preferences and investment pattern along the curve given that all the changes could have been already adjusted those preferences and hence the impact in the slope.

5 Estimation and Forecast Model. A VAR Approach.

The previous sections showed that there is valuable information about macroeconomic variables in the term structure of interest rates. Then the consistent step to move forward

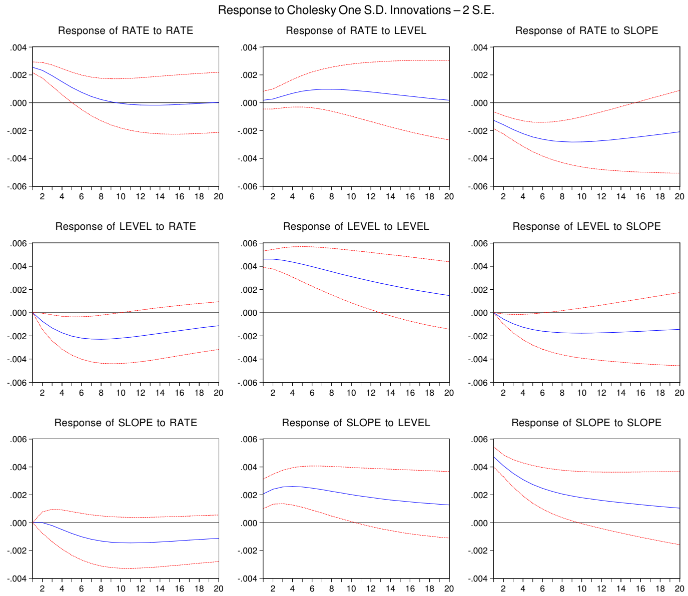

Figure 27: Impulse Response Level, Slope and Target Interest Rate

and use this information is to improve the rational guesses regarding the future behavior of macroeconomic variables by incorporating this information. In the spirit of Diebold and Li [2006],[7] the current work incorporates both economic and financial information to generate estimations particularly focused on inflation. Among the goals of this paper is to show that it is possible to improve the value of a pure economic model by including financial information such as market prices of government securities implicit in the estimation of the term structure. In order to proof this, the current section shows first the general structure of the VAR model used from this point on, using just economic inputs as well as the modified version which includes financial data. A comparison of the results of the best economic model[8]and the model that, besides the economic information, also includes the financial insights is shown at the end of the section.

[7]Diebold and Li's model studies the relation existent among macroeconomic and financial data and their impulse response functions

[8]The best economic model was chosen after running several configurations heading to forecast inflation. The resulting best model is the one involving one month lagged M3, inflation, and unemployment.

5.1 Estimation Model. The Most Accurate Financial-Economic Model

In order to see how macroeconomic information can be used to feed a model able to explain the behavior of the whole system, a general VAR model is used. The structure of this model is described as follows.

For a single variable it is possible to consider the next autoregressive model

$$Y_t = \alpha + \sum_{i=1}^{p} \alpha_i * Y_{t-i} + \varepsilon_t. \quad (5)$$

In case that more than one variable is considered into the system, the following expression provides a more general structure,

$$V_t = \alpha + \sum_{i=1}^{p} \alpha_i * Y_{t-i} + \sum_{i=1}^{q} \beta_i * Z_{t-i} + \sum_{i=1}^{r} \gamma_i * W_{t-i} + \varepsilon_t, \quad (6)$$

where V_t corresponds to a matrix containing the time series for each of the macroeconomic variables studied (Y_t, Z_t and W_t). The following expression provides a useful matrix notation:

$$V_t = \Lambda + V_{t-i} * \Theta + \epsilon_t. \quad (7)$$

Equation (7) allows to jointly estimate the dynamic of each variable based on the interaction of all of them within the system.

As it was stated earlier, the purpose of the model is to develop a forecast for inflation. As such, several model specifications were proved to be significant until coming up with the one that more accurately followed the behavior of the variable of interest. After testing several economic variables in conjunction with inflation, it was found that, for the Colombian case, the best model includes unemployment –as an indicator of economic activity–, M3, and inflation. Equation (8) shows the specification with the best performance.

$$V_t = \alpha + \alpha_1 * Inflation_{t-1} + \beta_1 * M3_{t-1} + \gamma_1 * Unemployment_{t-1} + \varepsilon_t, \quad (8)$$

Where, once again, V_t corresponds to the set of variables of interest in t, whereas the explanatory variables are the same variables lagged 1 period. In Section 5.3 the results of the model are shown on the sake of the validity and efficiency of the model. Table 5 shows the parameters of the model.

Although the coefficients above can not be interpreted as in a regular regression, given

Table 5: VAR parameters, (Standard Errors)

	Unemployment	M3	Inflation
Unemployment(-1)	0.580445	-0.079825	-0.009491
	(0.08240)	(0.16108)	(0.02577)
M3(-1)	-0.053673	0.674424	-0.002087
	(0.04103)	(0.08021)	(0.01283)
Inflation(-1)	0.133071	0.514483	0.988643
	(0.09516)	(0.18603)	(0.02976)
Constant	0.052157	0.030666	0.001545
	(0.01103)	(0.02156)	(0.00345)

that interaction among variables can vary both the direction and the magnitude of the response of each variable, the performance of model is robust on explaining the behavior of the system as a whole. In addition to the fit of the model, it will be shown that the previous specification is able to produce accurate forecasts.

5.2 Forecast Model. Inter-Temporal Financial-Economic Information

Given the nature of the data available in the market, in general it is possible to find that almost real time data is available for financial series, whereas economic indicators, as those used in the current work, are commonly known with some time lag. In this sense a model that works with economic and financial information could create a better estimate. The market and its participants are able to provide, real time prices to all securities, government debt in this case, that will create an instantaneous picture regarding the view of agents' investment preferences. In consumption decision theory these preferences are given by what agents believe the behavior of several underlying variables will be. Among these factors it is possible to identify elements such as inflation expectations, liquidity premia, term premia, etc. Therefore, small changes in the market microstructure will be assimilated and incorporated into agents trading strategies. As such, the use of financial data will be crucial to the development of more accurate and up to date estimates of the studied variables.

In this section the previous phenomenon is incorporated to improve the power and quality of the model. It will be shown first, how it is possible to use two different sets of data (one lagged and the other in real time) in order to capture the inter-temporal effects of pure economic vs financial data, and second how to use those interactions to generate and inflation forecast. Throughout this section the difference between the model with an without financial data is pointed out to highlight the value generated by incorporating this up-to-date information (financial data).

The previous section illustrated the roots of the economic model. In this section, expression (9) modifies the temporality of the previous configuration and introduces the use of two groups of data with different time structures. As it is clear from the equation, financial information evaluated in t is jointly utilized with economic information in $t-1$ to generate estimates of the entire system. $\mathbf{y}_t$ represents the most current information whereas $\mathbf{v}_t$ contains the lagged information.

$$\mathbf{y}_t = A_y - B_{yv}\mathbf{v}_t + \sum_{i=1}^{p} \Phi_{i,y}\mathbf{y}_{t-i} + \sum_{i=1}^{p} \Phi_{i,yv}\mathbf{v}_{t-i} + \Sigma_y \varepsilon_{y,t}, \tag{9}$$

, and

$$\mathbf{v}_t = A_v - B_{vy}\mathbf{y}_t + \sum_{i=1}^{p} \Phi_{i,vy}\mathbf{y}_{t-i} + \sum_{i=1}^{p} \Phi_{i,v}\mathbf{v}_{t-i} + \Sigma_v \varepsilon_{v,t}, \tag{10}$$

where $\varepsilon_{y,t} \sim \text{IID}\mathcal{N}(0, \mathbb{I}_{N_y})$ and $\varepsilon_{v,t} \sim \text{IID}\mathcal{N}(0, \mathbb{I}_{N_v})$. The previous dynamics imply that in t it is possible to observe $\mathbf{y}_t$ and $\mathbf{v}_{t-1}$, however $\mathbf{v}_t$ is still not known.

Now it is possible to group the variables $\mathbf{y}_t$ and $\mathbf{v}_t$ into a new one, $\mathbf{s}_t$, which allows the system to be redefined as follows:

$$B\mathbf{s}_t = A + \sum_{i=1}^{p} \Phi_i \mathbf{s}_{t-i} + \Sigma\varepsilon_t, \tag{11}$$

where

$$\begin{aligned} A &= \begin{pmatrix} A_y \\ A_v \end{pmatrix}, \qquad B = \begin{bmatrix} \mathbb{I}_{N_y} & B_{yv} \\ B_{vy} & \mathbb{I}_{N_v} \end{bmatrix}, \\ \Phi_i &= \begin{bmatrix} \Phi_{i,y} & \Phi_{i,yv} \\ \Phi_{i,vy} & \Phi_v \end{bmatrix}, \quad \text{and} \quad \Sigma = \begin{bmatrix} \Sigma_y & 0 \\ 0 & \Sigma_v \end{bmatrix}. \end{aligned}$$

The system reduced form is expressed as follows

$$\mathbf{s}_t = C + \sum_{i=1}^{p} D_i \mathbf{s}_{t-i} + \Sigma_B \varepsilon_t, \tag{12}$$

where $C = B^{-1}A$, $D_i = B^{-1}\Phi_i$, y $\Sigma_B = B^{-1}\Sigma$. The former presents a general configuration that allows to include any lagged structure of order p.

Now, it is of the main interest of this work to create a forecast model for one of the variables within the system. In particular, the analysis of inflation has been developed in this paper. As such, as shown in Palomino [2009], the basic model can be adjusted and iteratively regrouped to use D_i and Σ_B to create the forecast model.

Again, based on Palomino [2009], the recursive forecast equation is given by

$$\mathbb{E}\left[\mathbf{m}_{t+h} \middle| \mathbf{y}_t, \mathbf{m}_{t-1}\right] = F_{h,m} - G_{h,my}G_{0,y}^{-1}F_{0,y} + G_{h,my}G_{0,y}^{-1}\mathbf{y}_t + \left(G_{h,m} - G_{h,my}G_{0,y}^{-1}G_{0,ym}\right)\mathbf{m}_{t-1},$$

with variance equal to

$$\begin{aligned} \text{var}\left[\mathbf{m}_{t+h} \middle| \mathbf{y}_t, \mathbf{m}_{t-1}\right] &= \left(H_{h,my} - G_{h,my}G_{0,y}^{-1}H_{0,y}\right)\left(H_{h,my} - G_{h,my}G_{0,y}^{-1}H_{0,y}\right)^\top \\ &+ \sum_{i=0}^{h-1} H_{i,my}H_{i,my}^\top + \sum_{i=0}^{h-1} H_{i,m}H_{i,m}^\top. \end{aligned}$$

This variance is helpful to create the confidence intervals for the forecasted variable to several time horizons.

The model presented in this section is able to provide an inflation forecast by using both the last information available and its interaction within the system. As it is derived from the model presented above, the forecast is build based on an autoregressive structure of the variable of interest, however the value of its estimations is referenced to the additional information provided by the interaction of the economic and financial variables. The following section shows the previous assertion.

5.3 Performance Analysis of the Model

The model presented in the previous section was used to produce an inflation forecast for the Colombian case. In this section it will be shown the input data as well as the results of applying the model. A comparison between a model including and without including financial data is also developed in order to establish the impact of using additional information on top of the economic model.

As shown in Section 5.1, the best economic model found to describe the behavior of the system is given by equation (8). In this section the model is enhanced by the inclusion of financial variables, specifically information coming from the term structure of interest rates and risk indicators. Given the evidence presented in previous sections, the financial information set only includes the level and slope of the term structure of interest rates. As it was shown, curvature does not provide any significant relationship or explanatory power on any of the macroeconomic variables of interest. In addition to the financial information implicit in Colombian bond prices, a risk indicator is also included to take into account the price of risk across Colombian securities. In order to do so, monthly series[9] from January 2003 to June 2010 of level and slope, estimated as in Section 2.1, and the monthly level[10] of the EMBI for the Colombian debt were used in the analysis. Economic information corresponds to monthly unemployment rate, M3 and inflation from January 2003 to May 2010. As stated in the previous section, economic information is lagged 1 period, given the more availability of more current financial information. After applying the VAR forecast model[11] showed in the previous section, the results of the model are compared to the actual realizations of the inflation for several time horizons (1 to 11 months). The current analysis utilizes both the mean absolute error (MAE) and the root mean square error (RMSE) to measure the accuracy of the prediction. Table 6 shows the forecast results for both the economic and the financial-economic model.

Table 6: MAE and RMSE for Economic vs. Financial-Economic Model (b.p.)

	Economic Model		Financial-Economic Model	
Time Horizon	MAE	RMSE	MAE	RMSE
1 Month	42	49	36	44
3 Months	99	118	81	94
6 Months	160	189	116	150
12 Months	259	298	201	248

Figures 28 to 31 depict the out of sample forecast for the pure economic and the financial-

[9]Monthly series based on daily averages.
[10]Monthly series based on daily averages.
[11]Given that the interest of the model is the forecast of inflation, the order of the variables in the VAR is not relevant.

economic model. As it is evident from the data and the figures, the performance of the financial-economic model is better than the pure economic one. In general, for all time horizons the back tests show that the inclusion of financial information improves the accuracy of the estimates. In this sense it is possible to conclude that although a pure economic model provides valuable information, the use of more current information implicit in financial data generates an important value proposal. Moreover, for the Colombian case the financial-economic model was proven to be more accurate on predicting the behavior of inflation for all the time periods evaluated.

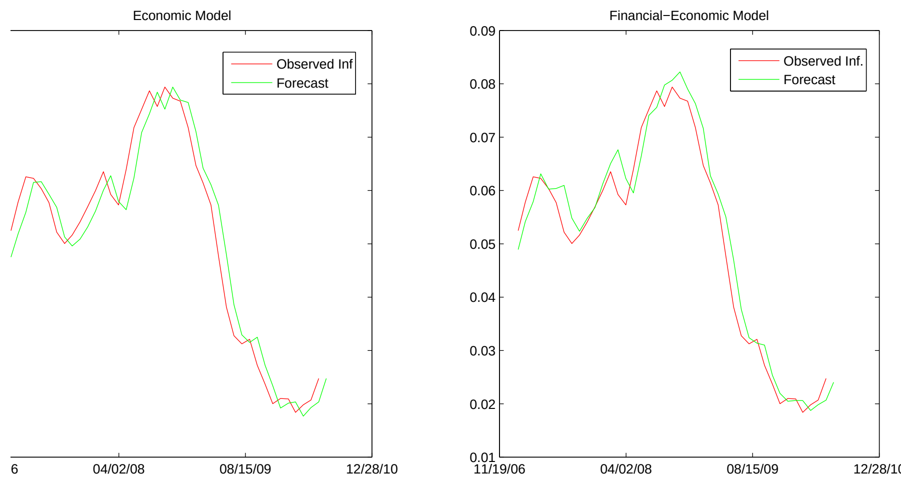

Figure 28: Realized Inflation vs. 1 Month Forecast

According to all above evidence it is clear that the use of inter-temporal information generated by the mismatch between financial and economic data availability, can be materialized into more powerful forecasts. Up-to-date information about market expectations and preferences helps fundamental data to better understand the future path of macroeconomic variables. Specifically for the Colombian case, the consumption preferences and agents' trading strategies can accurately provide a summary of what they believe inflation is moving towards.

6 Conclusions

The current work compiles a methodology for estimating the spot curve and also how to use the information underlying that estimation to understand the market. First of all, it is

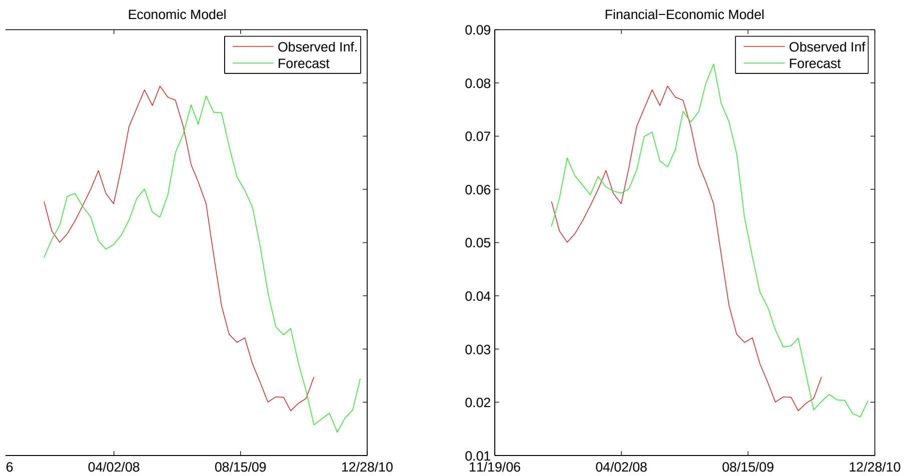

Figure 29: Realized Inflation vs. 6 Months Forecast

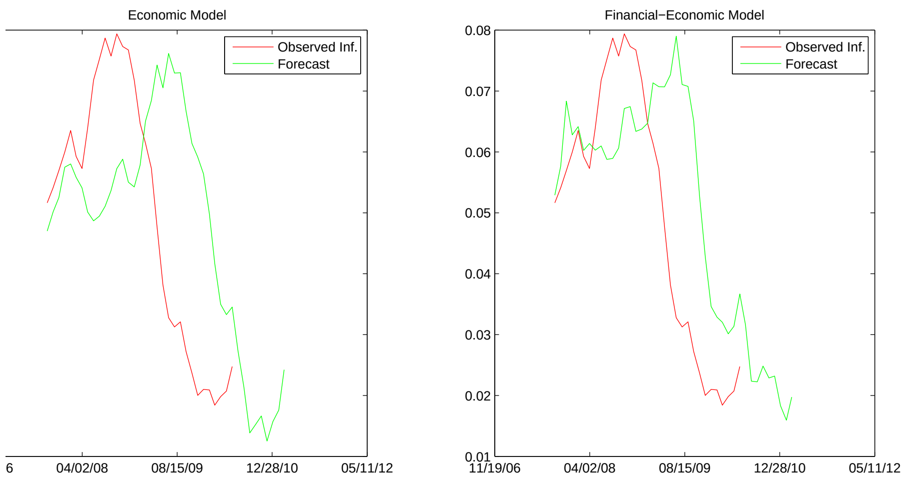

Figure 30: Realized Inflation vs. 9 Months Forecast

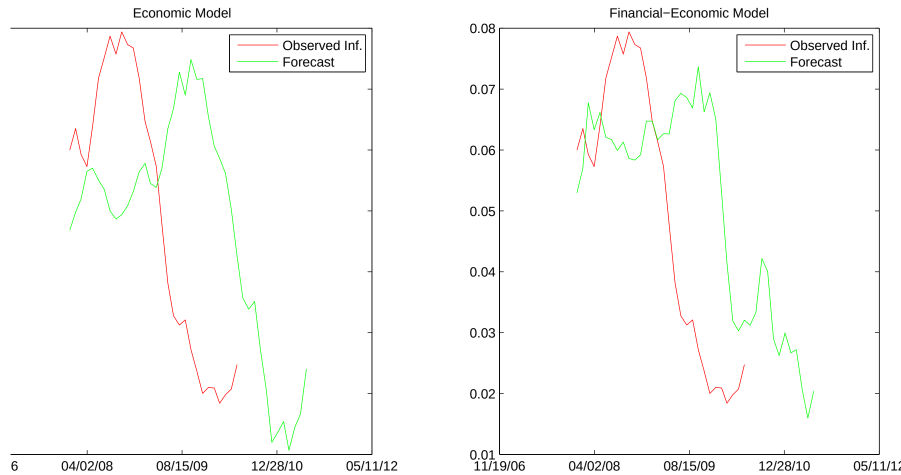

Figure 31: Realized Inflation vs. 12 Months Forecast

possible to observe how the use of Nelson and Siegel [1987] can be successfully applied to the Colombian market to interpret the behavior of interest rates. The work showcases the value of the model to decompose changes of the term structure of interest rates into more intuitive and meaningful information such as the level, slope and curvature factors. Additionally, the estimation methodology presents a robust mechanism for the consolidation of market information, in this case Colombian government securities prices and expectations. Secondly, it was proven the high correlation among the theoretical and empirical factors (level, slope and curvature) reinforcing hence the empirical value of Nelson and Siegel's model.

Agents expectations and preferences, printed in changes of the term structure of interest rates, are associated with macroeconomic variables. Level and slope are proved to have a significant correlation and impact in inflation and monetary policy interest rate, respectively. As such, the study of their behavior can improve the estimates and help to clean up the expectations regarding these economic variables.

The use of current financial data provides valuable information that can make the forecast of macroeconomic variables more accurate. It was proven that, for the Colombian case, the use of financial data along with lagged economic information produces better estimates than those resulting from a pure economic data input model.

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Appendix A

The methodology of Bunda et al. is applied to CDS's, EMBI's, and exchange rates of Colombia, Brazil, Chile, Mexico and Peru. To obtain the adjusted correlation, we use equation 4 with CDS's, EMBI's, and exchange rates weekly returns as the dependent variable.[12] The results are presented in Figures 32 to 34, where from 2006 we see that part of the correlation is due to common external factors (there is a widening gap between the two correlations, $\rho > r\hat{h}o$).

After analyzing the results from the regressions, in the case of Latin America we can conclude that the external factors do help to explain CDS's, EMBI's and exchange rates, and that after controlling for these common external factors we see a significantly market comovement, especially for the CDS's.[13] In other words, for the CDS'S there is no discrimination among countries according to their country risk perception.

[12]Following Bunda et al., returns are computed as $R_{t/t-5} = ln(I_t/I_{t-5})$. The independent variables are also set as weekly returns.

[13]The mean R^2 for the whole sample for the regressions for CDS's, EMBI's, and exchange rates is 0.33, 0.25 and 0.11, respectively.

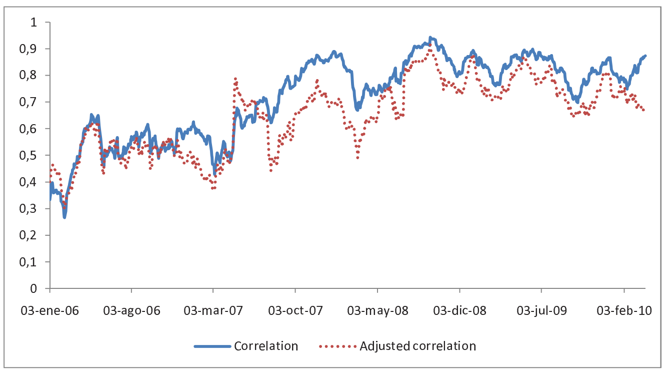

Figure 32: Correlation and Adjusted Correlation for Latin America's CDS.

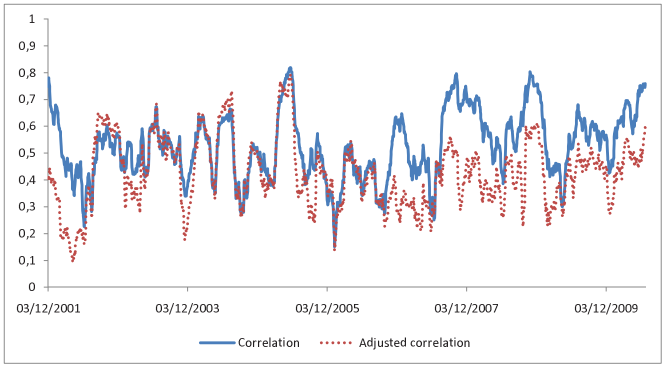

Figure 33: Correlation and Adjusted Correlation for Latin America's EMBI's.

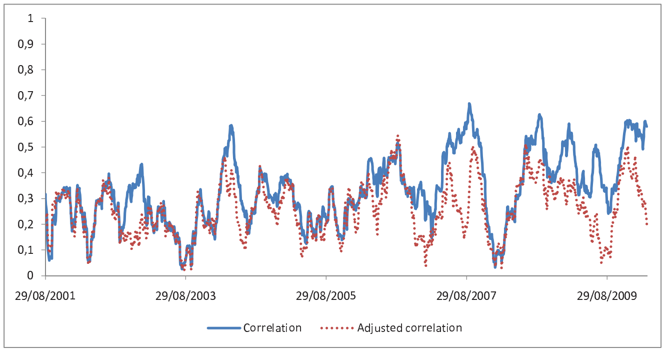

Figure 34: Correlation and Adjusted Correlation for Latin America's exchange rates