Dynamic Moral Hazard, Risk-Shifting, and Optimal Capital Structure

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Motivation: Research Questions

Q1) Does the presence of managerial moral hazard influence the risk-shifting problem?

Q2) How do the optimal policies of the firm in terms of leverage, managerial compensation, and investment decisions change when the two problems are present?
Preview of the Results

- \(\uparrow\) Moral Hazard \(\implies\) \(\uparrow\) Risk-Shifting
  1. Leverage Channel (Standard)
  2. Internal Hedging Channel (New)

- Internal Hedging \(\implies\) Non-monotonicity between leverage and risk-shifting

Corporate Finance Implications:
- \(\uparrow\) Moral Hazard \(\implies\) \(\downarrow\) Leverage

Macroeconomic Implications:
- Amplification mechanism via higher risk-shifting and higher default after bad shocks

Firm survival implications:
- Age effects: Young firms engage in more risk-shifting and have lower survival probabilities
Review of the Literature

Outline

- Motivation

- Model
  - Preferences, Technology, and Timing
  - The Risk-Shifting Problem
  - The Moral Hazard Problem

- Model Solution
  - Model without Moral Hazard
  - Model with Moral Hazard

- Empirical Implications
  - Risk-Shifting
  - Capital Structure

- Conclusions and Extensions
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Preferences and Technology

- Time is continuous and infinite
- Bondholders, shareholders and manager (agent) are risk neutral
- Manager is impatient with discount rate $\gamma > r$
- Cumulative Output $Y_t$ satisfies:
  \[ dY_t = \mu_t a_t dt + \sigma dB_t \]
- The mean-cashflow process $\mu_t$ satisfies:
  \[ d\mu_t = (\Phi - \mu_0) dJ_t \]

where:
- $\Phi \sim N[\mu_0, \sigma_\mu]$
- $J$ is a Poisson process with intensity $\alpha_t$
Timing

- At $t = 0$ initial shareholders issue debt with coupon $C$
- Debt is fairly priced:

$$D_0 = E \left[ \int_0^\tau e^{-rt} Cdt + e^{-r\tau} (1 - \phi) \frac{\mu \tau}{r} \right]$$  \hspace{1cm} (1)

where
- $\tau$ is the endogenous default time chosen by the shareholders
- $\phi$ reflects bankruptcy costs

- Shareholders and manager commit to an optimal incentive compatible contract $\Gamma = \Gamma(\alpha, P, \tau)$ that specifies:
  - Poisson arrival rate $\alpha$ (amount of risk-shifting)
  - Cumulative compensation to the manager $P$
  - Termination time $\tau$
The Risk-Shifting Problem

- Shareholders payoff is given by:

\[ J(\Gamma) = E \left[ \int_{0}^{\tau} e^{-rt} (dY_t - c(\alpha_t)dt - (1 - \psi)C_t dt - dP_t) \right] \]

- \( c(\alpha_t) \) operating cost of investment with arrival rate \( \alpha_t \)
- Shareholders have limited liability \( \implies \) prefer risky projects
  - Shareholders don’t internalize bankruptcy costs incurred by bondholders
- (Justification for \( \alpha \) as the amount of risk-shifting) Let \( \tau_S \) be the time of the shock
  1. \( Y_t|\tau_S \sim N(\mu_0 t, \sigma t + \sigma_{\mu}(t - \tau_S)) \)
  2. \( \text{Var}(Y_t) \) is an increasing function of \( \alpha \).
The Moral Hazard Problem

- The manager exerts unobservable effort $a_t \in \{0, 1\}$ and derives private benefit $\lambda(1 - a_t)\mu_t$
- Shareholders need to provide incentives for the manager to choose work $a_t = 1$
- A contract $\Gamma$ is incentive compatible if choosing work maximizes the manager’s utility:

$$W(\Gamma) = \max_{a \in A} E^a \left[ \int_0^\tau e^{-\gamma t} (\mu_t \lambda(1 - a_t) dt + dP_t) \right]$$  \hspace{1cm} (3)

- Shareholders maximize their utility $J(\Gamma)$ s.t:
  - Incentive compatibility (3)
  - Participation Constraint $W(\Gamma) = W_0 \geq 0$
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Model without Moral Hazard

Perfect alignment of incentives between shareholders and manager
Timing

- At time $t = 0$:
  - The firm issues infinite maturity debt $D(\mu_0)$ with coupon payment $C$
  - The shareholders pay $W_0$ to the manager and she works ($a_t = 1$) until termination
- Between time $t \in (0, \tau_S)$:
  - The shareholders implement the optimal amount of risk-shifting $\alpha^{SB}$, until the shock is realized
- For $t \geq \tau_S$:
  - If $\mu_{\tau_S} < (1 - \psi)C$ is realized then the firm defaults. Bondholders collect $(1 - \phi)\frac{\mu_{\tau_S}}{r}$
  - If $\mu_{\tau_S} \geq (1 - \psi)C$ is realized then shareholders will never default
Solution

- The shareholders value function $F(\mu_0; C)$ satisfies:

$$rF(\mu_0) = \max_{\alpha} \left\{ \mu_0 - (1 - \psi)C - \frac{\theta \alpha^2}{2} + \alpha \left[ \int_{\mathbb{R}} \hat{F}(\hat{\mu})dN(\hat{\mu}) - F(\mu_0) \right] \right\}$$

$$\hat{F}(\mu_{\tau_S}) = \max \left\{ \frac{\mu_{\tau_S} - (1 - \psi)C}{r}, 0 \right\}$$

- Solve for $\alpha$, $F(\mu_0)$, and $D(\mu_0)$
- Calculate Optimal Capital Structure

$$\max_C D(\mu_0; C) + F(\mu_0; C)$$
Takeaway: Leverage Channel (Standard)

A. Initial firm value: $F(C) + D(C)$

B. Risk-shifting: $\alpha(L)$
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Model with Moral Hazard

**BONDHOLDER**
- Receives Coupon C

**SHAREHOLDER**
- Chooses default time \( \tau \)
- Designs optimal contract for the manager

**MANAGER**
- Chooses effort (unobservable) and risk-shifting
- Impatient: Discounts the future at rate \( \gamma > r \)
- Needs incentives to exert effort

**Risk-Shifting**

**Moral Hazard**
Timing

- At time $t = 0$:
  - Firm issues optimal amount of debt
  - Shareholders and manager commit to an incentive compatible contract which implements $a_t = 1$
- For $0 < t < \tau_S$
  - Shareholders implement optimal amount of risk-shifting $\alpha(W_t)$
  - When $W_\tau = 0 \implies$ the manager is either replaced or the firm defaults
- For $\tau_S \leq t$
  - When $W_\tau = 0$:
    - If $\mu_{\tau_S}$ is low $\implies$ the firm defaults
    - If $\mu_{\tau_S}$ is high $\implies$ the manager is replaced
  - Whenever the firm defaults bondholders collect $(1 - \phi)\frac{\mu_{\tau_S}}{r}$ and shareholders get 0
Solution

- Calculate the shareholders value functions $\hat{F}(W, \mu_{\tau_S})$ after the shock. $\hat{F}(W, \mu_{\tau_S})$ satisfies the HJB equation:

$$r\hat{F}(W) = \mu_{\tau_S} - (1 - \psi)C + \hat{F}'(W)\gamma W + \frac{1}{2}\hat{F}''(W)\sigma^2\lambda^2$$

with boundary conditions

$$\hat{F}(0) = \max\{0, \hat{F}(W_0) - R\} \quad \hat{F}'(\bar{W}) = -1 \quad \hat{F}''(\bar{W}) = 0$$

- The manager’s continuation value $W_t$ evolves according to:

$$dW_t = \gamma W_t + \sigma \lambda dB_t - dP_t$$

where $P_t$ are the cumulative payments made to the manager, which reflect $W_t$ at the boundary $\bar{W}$. 
The shareholders value function $F(W, \mu_0)$ before the shock satisfies the HJB equation:

$$rF(W) = \max_{\beta \geq \lambda, \alpha, \Delta W_{\mu_i}} \mu_0 - (1-\psi)C + F'(W)(\gamma W + \rho_t) + \frac{F''(W)\sigma^2 \beta^2}{2}$$

$$+ \alpha \left( \int_{\mathbb{R}} (\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W))dN(\hat{\mu}) \right) - \frac{1}{2} \theta \alpha^2$$

(4)

with boundary conditions:

$$F(0) = \max\{0, F(W_0) - R\} \quad F'(\bar{W}) = -1 \quad F''(\bar{W}) = 0$$
Solution

- The manager’s continuation value $W_t$ evolves according to:

$$dW_t = \gamma W_t - dP_t + \sigma \lambda dB_t + \int 1\{\mu_t = \hat{\mu}\} \Delta W_{\hat{\mu}} d\hat{\mu} + \rho_t dt$$

- Calculate Optimal Capital Structure:

$$\max_C D(\mu_0, W_0; C) + F(\mu_0, W_0; C)$$
Model with Moral Hazard (i.e. With Manager)

A. Shareholder value function: \( F(W) \)

Agent continuation value: \( W \)

B. Risk-shifting \( \alpha(W) \) and \( \alpha^{SB} \)

Risk-shifting \( \alpha(L) \)
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- Model
  - Preferences, Technology, and Timing
  - The Risk-Shifting Problem
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Risk Shifting is amplified by the presence of managerial Moral Hazard:

1. Leverage Channel:
   - Moral Hazard reduces firm value $\implies$ ↑ Leverage
   - Because of Limited Liability ↑ Leverage $\implies$ ↑ Risk-Shifting

2. Internal Hedging Channel:
   - Value of the firm depends on $W$
   - Important for shareholders to benefit from projects with high $\mu$
   - Optimal contract allows shareholders to hedge inefficient liquidation:
     - ↑ $W$ when you draw a project with high $\mu$
     - ↓ $W$ when you draw a project with low $\mu$
   - i.e Relax moral hazard constraint precisely when it is most valuable
Risk-shifting is greater under Moral Hazard: \( \alpha^{SB} \leq \alpha(\bar{W}) \)
Non-monotonic relation between risk-shifting and leverage:

- Isolate role of leverage channel (Preclude adjusting $W$ in response to the shock)
- Potential to reconcile empirical evidence:
  2. Rauh (2009) finds that firms with poorly funded pension plans purchase less risky assets

**Policy Implications:**

- Regulate leverage and managerial compensation
- Under long-term contracts small changes in leverage can increase risk-shifting significantly
- 'If you know someone’s incentives, you have a pretty good idea of what they are going to do?’ Steven Levitt
A. Shareholder value function: $F(W)$

B. Risk-shifting $\alpha(W)$, $\alpha^{NIH}(W)$, $\alpha^{SB}$

C. Risk-shifting $\alpha(L)$, $\alpha^{NIH}(L)$
Macroeconomic Implications:

- Small shocks amplified via higher risk-shifting and deadweight loss of default
- In benchmark case without moral hazard shocks are fully absorbed by shareholders. No amplification

Firm Survival: Age Effects

- Young firms start out financially constrained (low $W$)
- They engage in more risk-shifting and have lower survival probability
- Older firms are less financially constrained (as $W$ grows), and become more stable
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Capital Structure:

- Represents a tradeoff between:
  1. Tax-advantage of debt
  2. Costly bankruptcy

- ↑ Moral Hazard ⇒ ↑ Risk-Shifting ⇒ ↑ Bankruptcy Costs ⇒ ↓ Bond Prices ⇒ ↓ Initial Leverage

- Lower initial leverage for firms with moral hazard
Firm Value $V(C)$

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<th>$C$ (Coupon)</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
<th>0.55</th>
<th>0.6</th>
<th>0.65</th>
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<td>205</td>
<td>210</td>
<td>215</td>
<td>220</td>
<td>225</td>
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Risk-Shifting $\alpha(C)$

<table>
<thead>
<tr>
<th>$L$ (Leverage)</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
<th>0.55</th>
<th>0.6</th>
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<tbody>
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<td>0.5</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
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Moral hazard amplifies the risk-shifting problem:

1. Leverage Channel
2. Internal Hedging Channel

Empirical Implications:
- Non-monotonicity risk-shifting in leverage
- Lower leverage
- Potential amplification mechanism of productivity shocks

Policy Implications:
- Regulations: Capital Requirements + Managerial Compensation