PATACON
Policy Analysis Tool Applied to Colombian Needs

Departamento de Modelos Macroeconómicos

Banco de la República

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Outline

1. Introduction
2. Model Features
3. Model Structure
4. Calibration: General Methodology
5. Some Results of the Calibration
6. Estimation
7. Forecast
8. Impulse Response Analysis


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The Model

- PATACON is a DSGE model for policy analysis and forecasting of the Colombian economy.

- The model follows Christiano, Eichenbaum and Evans (2005) and adds characteristics to replicate a small open economy. See González, Mahadeva, Prada and Rodríguez (2011).
General Features I: Nominal and Real Rigidities

- Monopolistic competition in labor and goods market with sticky prices á la Calvo (Erceg, Henderson and Levin (2000) and Kollman (1997)).
- Partial price and wage indexation (past inflation).
- Variable capital utilization with endogenous depreciation as a function of capital utilization. (Greenwood et al, (1988), King and Rebelo (1999), CEE (2001)).
- External habit (Abel (1990), Fuhrer (2000)).
- Adjustment costs are in terms of the change in the flow of investment.
- Incomplete exchange rate pass-through (sticky prices á la Calvo and distribution of imported goods).
- Transforming firms
General Features II: External Shocks and Sources of Fluctuations

**External factors:**
- External demand (prices and quantities).
- Remittances.
- External interest rate.
- Raw materials prices.
- External inflation.
- Prices of imported goods of consumption and investment.

**Internal factors:**
- Monetary policy shocks.
- Exogenous changes in consumption.
- Exogenous changes in labor supply (intensive margin).
General Features III: Exogenous Shocks and Sources of Fluctuations

Internal factors:

- Exogenous movements in Tobin’s Q (investment efficiency).
- Transitory shocks to the productivity in the final good production.
- Permanent shocks to the long run growth of productivity.
- Shocks to the “mark-up” to the final good price and wages.
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Model Structure 4

- Production
- Transforming Firms
- Domestic Consumption
- Domestic Investment
- Exports
- Distribution (T)
- Labor and Capital
- Debt and Remittances
- Households
- Consumption
- Imported Consumption
- Investment
- Imported Investment
- Rest of the World
- Raw Materials

- Transforming Firms to Domestic Consumption
- Domestic Consumption to Domestic Investment
- Domestic Investment to Exports
- Exports to Distribution (T)
- Distribution (T) to Raw Materials

- Production to Transforming Firms
- Transforming Firms to Domestic Investment
- Domestic Investment to Domestic Consumption
- Domestic Consumption to Labor and Capital
- Labor and Capital to Debt and Remittances
- Debt and Remittances to Households

- Households to Consumption
- Consumption to Imported Consumption
- Imported Consumption to Domestic Investment
- Domestic Investment to Investment
- Investment to Imported Investment
- Imported Investment to Rest of the World
- Rest of the World to Distribution (T)
- Distribution (T) to Exports
- Exports to Transforming Firms
Technological progress, population and unemployment

- Total population $N_t$ follows a process
  \[
  \ln \left( \frac{N_t}{N_{t-1}} \right) = \rho_n \ln \left( \frac{N_{t-1}}{N_{t-2}} \right) + (1 - \rho_n) \ln (1 + \bar{n}) + \epsilon_t^n
  \]

- Labor force is defined as
  \[
  L_t = (1 - TD_t) \ TBP_t \ N_t
  \]

- Technological progress (in this model equivalent to trend productivity per hour worked) follows:
  \[
  \ln \left( \frac{A_t}{A_{t-1}} \right) = \rho_a \ln \left( \frac{A_{t-1}}{A_{t-2}} \right) + (1 - \rho_a) \ln (1 + g_a) + \epsilon_t^a
  \]
Model units

- Models such as these are easier to solve if variables can be expressed as stationary, mean zero, deviations for the steady state.
- Therefore we express all variables in model units, effectively adjusting them for the two sources of growth, population and Harrod neutral technological progress.
- Let $J_t$ in uppercase be the total quantity of a real economic variable, such as the volume of consumption.
  - Per-capita terms
    \[
    \tilde{j}_t \equiv \frac{J_t}{N_t}
    \]
  - Model units
    \[
    j_t \equiv \frac{J_t}{Z_t N_t \bar{t}}
    \]
    where $\bar{t}$ is total hours available per person and
    \[
    Z_t = Z_{t-1}^{\gamma g} A_t^{1-\gamma g}
    \]
    and \[
    \frac{Z_t}{Z_{t-1}} = (1 + g_t^Z)
    \]
Continuum of households $j$ of measure one, indexed by $j \in (0, 1)$.

Utility function

$$u(\cdot) = \left[ \left( \frac{Z_t^u}{1-\sigma} \left[ c_t^F(j) - habc_{t-1}^F \right]^{1-\sigma} \right) - \left( \frac{Z_t^h}{1+\eta} \left( (1 - TD_t) TBP_t h_t(j) \right)^{1+\eta} \right) \right] (Z_t^I)^{1-\sigma}$$

where $h_t(j)$ the proportion of total hours that are worked.
Household

- Budget constraint

\[
c_t^F (j) + \frac{p_t^{xF}}{p_t^{cF}} x_t^F (j) + b_t (j) + \int p_{t+1, t}^a (j) a_{t+1} (j) \, d\omega_{t+1, t} (j) + \frac{s_t p_t^{c*}}{p_t^{cF}} \frac{1 + i_{t-1}^*}{1 + \pi_t^{c*}} \frac{b_t^{*} (j)}{(1 + \bar{n})(1 + g_t)} + \frac{r_t^k u_t (j)}{k_{t-1} (j)} \frac{k_{t-1} (j)}{(1 + \bar{n})(1 + g_t)} + \frac{s_t p_t^{c*}}{p_t^{cF}} \frac{1 + i_{t-1}^*}{1 + \pi_t^{cF}} \frac{b_t^{*} (j)}{(1 + \bar{n})(1 + g_t)}
\]

\[
+ \xi_t + a_t (j) + \frac{s_t p_t^{c*}}{p_t^{cF}} tr_t^* + w_t (j) (1 - TD_t) TBP_t h_t (j) + \frac{s_t p_t^{c*}}{p_t^{cF}} b_t^* (j)
\]

\[
+ \frac{b_{t-1} (j)}{(1 + \bar{n})(1 + g_t)} \left( \frac{1 + i_{t-1}^*}{1 + \pi_t^{cF}} \right)
\]
Household

- **Investment cost**
  \[ \Psi^X \left( x_t^F (j), x_{t-1}^F (j) \right) = \frac{\Psi^X}{2} \frac{(x_t^F (j) - x_{t-1}^F (j))^2}{x_{t-1}^F (j)} \]

- **Capital accumulation equation**
  \[ k_t (j) = x_t^F (j) + \frac{(1 - \delta (u_t (j))) k_{t-1} (j)}{(1 + \bar{n}) (1 + g_t)} \]

- **Variable depreciation**
  \[ \delta (u_t (j)) = \bar{\delta} + \frac{b}{1 + \gamma} (u_t (j))^{1+\gamma} \]
\[ \lambda_t^c = z_t^u \left( c_t^F - habc_{t-1}^F \right)^{-\sigma} \]
\[ r_t^k = \frac{\lambda_t^x}{\lambda_t^c} b u_t^r \]
\[ \lambda_t^c p_{t+1}^{xF} \frac{\lambda_t^x}{p_{t+1}^{cF}} = \lambda_t^x - \lambda_t^c \psi^x \frac{x_t^F - x_{t-1}^F}{x_{t-1}^F} \]
\[ + \beta E_t (1 + \bar{n})(1 + g_{t+1})^{1-\sigma} \lambda_{t+1}^c \left( \psi^x (x_{t+1}^F - x_t^F) + \Psi^x (x_{t+1}^F, x_t^F) \right) \]
\[ \lambda_t^x = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c r_{t+1}^k u_{t+1} + \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^x (1 - \delta(u_{t+1})) \]
\[ \lambda_t^c = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c \left( \frac{1 + i_t}{1 + \pi_{t+1}^{cF}} \right) \]
\[ \lambda_t^c = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c (1 + i^*) \left( \frac{1 + d_{t+1}}{1 + \pi_{t+1}^{cF}} \right) \]
Household

- Consumption Bundle

\[
c_t^F (j) = \left[ (\gamma^c)^{\frac{1}{\omega^c}} \left( c_t^{dF} (j) \right)^{\frac{\omega^c - 1}{\omega^c}} + (1 - \gamma^c)^{\frac{1}{\omega^c}} \left( c_t^{mF} (j) \right)^{\frac{\omega^c - 1}{\omega^c}} \right]^{\frac{\omega^c}{\omega^c - 1}}
\]

\[
c_t^{dF} (j) = \gamma^c \left( \frac{p_t^{cdF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F (j)
\]

\[
c_t^{mF} (j) = (1 - \gamma^c) \left( \frac{p_t^{mF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F (j)
\]
Labor market

- Workers are hired by intermediaries firms, which combine the work effort of individual workers and supply a joint labour input

\[
\min_{\{h_t(j)\}} \int_0^1 \tilde{w}_t(j) \tilde{h}_t(j) \, dj \\
\text{s.t} \quad \tilde{h}_t^F \leq \int_0^1 \left( \tilde{h}_t(j) \frac{\theta^w - 1}{\theta^w} \right) \frac{\theta^w}{\theta^w - 1} \, dj
\]

- F.O.C will imply

\[
\tilde{h}_t(j) = \left( \frac{\tilde{w}_t(j)}{\tilde{w}_t} \right)^{-\theta^w} \tilde{h}_t^F \\
\tilde{w}_t \equiv \left[ \int_0^1 \tilde{w}_t(j)^{1-\theta^w} \, dj \right]^{\frac{1}{1-\theta^w}}
\]
Nominal wages are sticky (Calvo wages)

- Given the demand for their differentiated labour, individuals can set their wages. Each individual is only free to renegotiate a salary when they receive a random signal which arrives every quarter with probability $1 - \epsilon^w$.

- If wages are not renegotiated they are set by a rule.

$$w_t^{Rule}(j) = w_{t-1}(j) \left( \frac{1 + \pi_{t-1}^{cF}}{1 + \pi_t^{cF}} \right)$$

- If on the other hand, the $j^{th}$ individual receives the signal to renegotiate her wage at period $t$, that will be set according to:

$$w_t^{opt}(j) = \frac{\theta^w}{\theta^w - 1} \frac{num_t^w(j)}{den_t^w(j)}$$
Nominal wages are sticky (Calvo wages)

where

\[ num_t^w (j) \equiv E_t \sum_{i=0}^{\infty} (\beta \epsilon^w (1 + \bar{n}))^i \prod_{k=1}^{i} [(1 + g_{t+k})^{1-\sigma}] \]

\[
z_{t+i}^h (1 - TD_{t+i}) TBP_{t+i})^{1+\eta} \left( h_{t+i}^F \left( \frac{w_{t+1}^{opt} (j)}{w_{t+i}} \right) \right)^{-\theta^w} \left( \frac{1 + \pi^cF_t}{1 + \pi^cF_{t+i}} \right)^{-\theta^w} 1^ \eta \]

\[ den_t^w (j) \equiv E_t \sum_{i=0}^{\infty} (\beta \epsilon^w (1 + \bar{n}))^i \prod_{k=1}^{i} [(1 + g_{t+k})^{1-\sigma}] \]

\[
\lambda_{t+i}^c (1 - TD_{t+i}) TBP_{t+i} \left( h_{t+i}^F \left( \frac{w_{t+1}^{opt} (j)}{w_{t+i}} \right) \right)^{-\theta^w} \left( \frac{1 + \pi^cF_t}{1 + \pi^cF_{t+i}} \right)^{1-\theta^w} \]

Then, the real wage evolves according to

\[
w_t = \left[ \epsilon^w \left( w_{t-1} \left( \frac{1 + \pi^cF_{t-1}}{1 + \pi^cF_t} \right) \right)^{1-\theta^w} + (1 - \epsilon^w) \left( w_{t}^{opt} \right)^{1-\theta^w} \right]^{\frac{1}{1-\theta^w}} + z_t^w \]
Intermediate Production Firms

Production → Labor and Capital → Households

Households → Domestic Consumption → Domestic Investment → Investment

Investment → Imported Consumption → Imported Investment

Consumption → Domestic Consumption → Domestic Investment

Domestic Investment → Domestic Consumption

Exports → Distribution (T) → Rest of the World

Rest of the World → Imports → Importers

Importers → Debt and Remittances
Intermediate Production Firms

- There are a continuum of firms indexed by \( z \in (0, 1) \)
- Each produces a differentiated product \((z)\) given the following production function which is weakly separable in value-added factors of production

\[
q^C_t (z) = z^q_t \left[ \alpha^\frac{1}{\rho} (va_t (z)) \rho^{-1} + (1 - \alpha) \frac{1}{\rho} \left( rm^F_t (z) \right) \rho^{-1} \right] \frac{\rho}{\rho - 1}
\]

\[
va_t (z) = \left[ \alpha^\frac{1}{\rho^v} (k^s_t (z)) \rho^v - 1 \rho^v \rho^v \rho^v + (1 - \alpha) \frac{1}{\rho^v} \left( (1 - TD_t) TBP_t a_t h_t (z) \right) \rho^v - 1 \right] \frac{\rho^v}{\rho^v - 1}
\]

where \( a_t = \frac{A_t}{Z_t} \) and \( k^s_t = \frac{u_t k_{t-1}}{(1+\bar{n})(1+g_t^z)} \)
Intermediate Production Firms

The first-order conditions are then given as:

\[ w_t = \lambda_t^q (z) z_t^q \left( \frac{\alpha q_t^C (z)}{z_t^q v a_t (z)} \right)^{\frac{1}{\rho}} \left( \frac{(1 - \alpha_v) v a_t (z)}{(1 - TD_t) TBP_t a_t h_t (z)} \right)^{\frac{1}{\rho_v}} \]

\[ r_t^k = \lambda_t^q (z) z_t^q \left( \frac{\alpha q_t^C (z)}{z_t^q v a_t (z)} \right)^{\frac{1}{\rho}} \left( \frac{\alpha_v v a_t (z)}{k_t^s (z)} \right)^{\frac{1}{\rho_v}} \]

\[ \frac{p_t^{rmF}}{p_t^{CF}} = \lambda_t^q (z) z_t^q \left( \frac{(1 - \alpha) q_t^C (z)}{z_t^q r m_t^F (z)} \right)^{\frac{1}{\rho}} \]

where \( \lambda_t^q (z) \) is the real marginal cost in model units and measured in consumption process of these firms.
The real marginal cost in model units and measured in consumption process of these firms $\lambda^q_t(z)$.

$$\lambda^q_t(z) = \frac{1}{(z^q_t)} \left[ \alpha \left( \left[ \alpha_v (r^k_t)^{1-\rho_v} + (1 - \alpha_v) (w_t)^{1-\rho_v} \right]^{\frac{1}{1-\rho_v}} \right)^{1-\rho} + (1 - \alpha) \left( \frac{p^{rmF}_t}{p^{cF}_t} \right)^{1-\rho} \right]$$

The price of raw materials in external currency is exogenous

$$p^{rmC}_t = s_t p^{rm\star}_t$$
Prices are sticky (Calvo pricing)

- Each period firms face a constant probability \((1 - \varepsilon^q)\) of receiving a signal which tells them when they can adjust their price.
- The other \(\varepsilon^q\) firms that are not allowed to reset their prices follow a backward-looking indexation rule:

\[
p_{t}^{\text{rule}}(z) = p_{t-1}^{qF}(z) \left(1 + \pi_{t-1}^{qF}\right)^{\nu^q} \left(1 + \bar{\pi}\right)^{1-\nu^q}
\]

where \(\bar{\pi}\) is average long-run inflation taken to be the central bank’s target, \(\nu^q \geq 0\) is the weight assigned to past inflation as opposed to this target.
Prices are sticky (Calvo pricing)

If on the other hand, the $z^{th}$ receives a signal which tells it that can adjust their price, it will choose

$$p_t^{q_{opt}}(z) = \frac{\theta q}{\theta q - 1}$$

$$\frac{p_t^{qF}}{p_t^{qF}}$$

Therefore the inflation dynamics will evolve according with

$$\left(1 + \pi_t^{qF}\right) = \left[\left(1 - \varepsilon q\right)\left(\frac{p_t^{q_{opt}}}{p_t^{qF}}\right)^{1-\theta q}\left(1 + \pi_t^{qF}\right)^{1-\theta q} + \varepsilon q \left[\left(1 + \pi_{t-1}^{qF}\right)^{\nu q}\left(1 + \pi_t\right)^{i(1-\nu q)}\right]^{1-\theta q}\right]^{1-\theta q} + z_{i}^{\pi}$$
Final production good
Final production good

- There is an aggregation technology

\[
q_t^F = \left[ \int_0^1 \left( q_t^C (z) \right)^{\frac{\theta q - 1}{\theta q}} dz \right]^{\frac{\theta q}{\theta q - 1}}.
\]

- The demand for the intermediate good \((z)\) is given by

\[
q_t^C (z) = \left( \frac{p_t^q (z)}{p_t^{qF}} \right)^{-\theta q} q_t^F
\]

- The output price is given as the aggregate

\[
p_t^{qF} = \left[ \int_0^1 \left( p_t^q (z) \right)^{1-\theta q} dz \right]^{\frac{1}{1-\theta q}}
\]
Transformation of final good
Transformation of final good

At a next stage, the final product $q_t^F$ is transformed into four different types of output: domestic consumption, $c_{t}^{dC}$, intermediate domestic capital goods, $x_{t}^{dC}$, exports, $e_{t}^{C}$, and as distribution services, $dis_{t}^{C}$.

$$q_t^F = \left[ \nu_{nt}^{\omega_{q}^{-1}} (nt_t)^{\omega_{q}} + \nu_{e}^{\omega_{q}^{-1}} (e_{t}^{C})^{\omega_{q}} \right]^{\frac{1}{\omega_{q}}}$$

$$nt_t = \left[ \nu_{c}^{\omega_{nt}^{-1}} (c_{t}^{dC})^{\omega_{nt}} + \nu_{x}^{\omega_{nt}^{-1}} (x_{t}^{dC})^{\omega_{nt}} + \nu_{dis}^{\omega_{nt}^{-1}} (dis_{t}^{C})^{\omega_{nt}} \right]^{\frac{1}{\omega_{nt}}}$$
Transformation of final good

Supplies of each if them are given by

\[
\frac{p_{t}^{cdC}}{p_{t}^{qF}} = \left(\frac{\nu_{nt}nt_{t}}{q_{t}^{F}}\right)^{\omega_{q-1}} \left(\frac{\nu_{cC_{t}}^{dC}}{nt_{t}}\right)^{\omega_{nt-1}}
\]

\[
\frac{p_{t}^{xdC}}{p_{t}^{qF}} = \left(\frac{\nu_{nt}nt_{t}}{q_{t}^{F}}\right)^{\omega_{q-1}} \left(\frac{\nu_{xX_{t}}^{dC}}{nt_{t}}\right)^{\omega_{nt-1}}
\]

\[
\frac{p_{t}^{disC}}{p_{t}^{qF}} = \left(\frac{\nu_{nt}nt_{t}}{q_{t}^{F}}\right)^{\omega_{q-1}} \left(\frac{\nu_{disdis_{t}}^{C}}{nt_{t}}\right)^{\omega_{nt-1}}
\]

\[
\frac{p_{t}^{eC}}{p_{t}^{qF}} = \nu_{e}^{\omega_{q-1}} \left(\frac{e_{t}^{C}}{q_{t}^{F}}\right)^{\omega_{q-1}}
\]
Distribution

- Note that

\[ dis_t^F = dis_t^{cd} + dis_t^{xd} + dis_t^e + dis_t^m \]

- Distribution of the domestic product

\[
j_t(z) = \left[ (\gamma^j) \frac{1}{\omega_j} \left( j_t^C(z) \right)^{\frac{\omega_j - 1}{\omega_j}} + (1 - \gamma^j) \frac{1}{\omega_j} \left( dis_t^i(z) \right)^{\frac{\omega_j - 1}{\omega_j}} \right] \frac{\omega_j}{\omega_j - 1}
\]

where \( j_t \): domestic consumption \( c_t^d \), domestic investment \( x_t^d \), exports \( e_t \) and imports \( m_t^* \).
Distribution

- Demand for $dis^i_t(z)$

$$\frac{p^j_t}{p^cF_t} = \lambda^j_t(z) \left( \frac{\gamma^j j_t(z)}{j^C_t} \right)^{1/\omega^j}$$

$$\frac{p^{disF}_t}{p^{cF}_t} = \lambda^j_t(z) \left( \frac{(1 - \gamma^j) j_t(z)}{dis^i_t} \right)^{1/\omega^j}$$

- Additionally, each of this sectors is subject to a similar price rigidity as the one described above and the price of import goods is also exogenous

$$p^{mc}_t = s_t p^m_t$$
Investment goods produced

\[ x_t^F = z_t^x \left[ (\gamma^x) \frac{1}{\omega^x} \left( x_t^{dF} \right)^{\frac{\omega^x - 1}{\omega^x}} + (1 - \gamma^x) \frac{1}{\omega^x} \left( x_t^{mF} \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}} \]

\[ x_t^{dF} = (\gamma^x) \left( \frac{p_t^{xdF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \]

\[ x_t^{mF} = (1 - \gamma^x) \left( \frac{p_t^{mF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x} \]
Exports

**External demand**

\[ p_t^{eF} = s_t p_t^{eE} \]

\[ e_t^F = \left( \frac{p_t^{e*}}{p_t^{c*}} \right)^{-\mu} c_t^* \]
Interest rates

- **External interest rate**
  
  \[ i^*_t = \bar{i}^* z^*_t \exp \left( \sum_u \left( \frac{s_t p_t^{c*} b_t^*}{p^{cF} y_t} - \bar{b}^* \right) \right) \]

- **Policy rule**
  
  \[ i_t = \rho_s i_{t-1} + (1 - \rho_s) \left( \bar{i} + \varphi_\pi \left( \pi_t^{cF} - \bar{\pi} \right) + \varphi_y \left( \frac{y_t}{y_{t,flex}} - 1 \right) \right) + z^i_t \]
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General Methodology

- We calibrate the steady state by minimizing the following objective function:

\[ f_{obj}(\theta) = \sum_{i=1}^{n} \omega_i \left( x_{i}^{ss}(\theta) - x_{i}^{d-lr} \right)^2 \]

where \( x^{ss} \) the steady state values and \( x_{i}^{d-lr} \) the equivalent ratios in the data.

- In this exercise we target the 21 nominal ratios.
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## Results of the Calibration

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<thead>
<tr>
<th>Ratios and relative prices</th>
<th>Model</th>
<th>Data Colombia</th>
<th>Deviation%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment / GDP</td>
<td>0.22</td>
<td>0.23</td>
<td>0.003</td>
</tr>
<tr>
<td>Imported investment / Total investment</td>
<td>0.37</td>
<td>0.36</td>
<td>0.030</td>
</tr>
<tr>
<td>Domestic inv. without dist. / Gross product</td>
<td>0.13</td>
<td>0.12</td>
<td>0.004</td>
</tr>
<tr>
<td>Consumption / GDP</td>
<td>0.82</td>
<td>0.80</td>
<td>0.023</td>
</tr>
<tr>
<td>Dom. cons. without dist. / Gross product</td>
<td>0.68</td>
<td>0.60</td>
<td>0.035</td>
</tr>
<tr>
<td>Imported consumption / Total consumption</td>
<td>0.12</td>
<td>0.12</td>
<td>0.024</td>
</tr>
<tr>
<td>Labor supply</td>
<td>0.30</td>
<td>0.30</td>
<td>0.000</td>
</tr>
<tr>
<td>Raw materials / Gross product</td>
<td>0.09</td>
<td>0.10</td>
<td>0.029</td>
</tr>
<tr>
<td>FOB Imports / Import. with distribution</td>
<td>0.76</td>
<td>0.73</td>
<td>0.038</td>
</tr>
<tr>
<td>FOB imports + raw material / GDP</td>
<td>0.24</td>
<td>0.23</td>
<td>0.025</td>
</tr>
<tr>
<td>Exports without dist. / Gross product</td>
<td>0.16</td>
<td>0.17</td>
<td>-0.047</td>
</tr>
</tbody>
</table>
Results of the Calibration

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</thead>
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<tr>
<td>Remittances / GDP</td>
<td>0.03</td>
<td>0.04</td>
<td>-0.015</td>
</tr>
<tr>
<td>Dom. consumption dist. / Dom. consumption</td>
<td>0.05</td>
<td>0.06</td>
<td>-0.011</td>
</tr>
<tr>
<td>Dom. investment dist. / Dom. investment</td>
<td>0.04</td>
<td>0.04</td>
<td>-0.005</td>
</tr>
<tr>
<td>Exports dist. / Exports</td>
<td>0.12</td>
<td>0.13</td>
<td>-0.014</td>
</tr>
<tr>
<td>Dom. cons. without dist. / Dom. consumption</td>
<td>0.92</td>
<td>0.94</td>
<td>-0.025</td>
</tr>
<tr>
<td>Dom. inv. without dist. / Dom. investment</td>
<td>0.93</td>
<td>0.96</td>
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<tr>
<td>Exports without dist. / Exports</td>
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<tr>
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<tr>
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<tr>
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<td>0.20</td>
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</table>
Estimation Strategy

We use bayesian methods to estimate

\[
\ln P(\theta|y, \tilde{y}) \propto \ln P(y|\theta) + \ln P(\theta|\tilde{y})
\]

likelihood \quad prior long-run parameters

+ \ln P(\theta)

prior short-run parameters

\[
\ln P(\theta|\tilde{y}) \sim N(\mu, \Sigma)
\]

where \( \mu \) was set to the parameter values found in the calibration stage. The variance \( \Sigma \) was computed by generating draws from the objective function if the calibration stage.

The priors of the short run parameters are characterized by a large variance.
Draws of the calibration objective function
Draws of the calibration objective function

- $gama_x$
- $gama_{sd}$
- $eta_m$
- $hab$
- $sigma_m$
### Estimation

<table>
<thead>
<tr>
<th>Param.</th>
<th>Dist.</th>
<th>Mean</th>
<th>Std</th>
<th>LB</th>
<th>UB</th>
<th>Mode</th>
<th>Mean</th>
<th>Std</th>
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<td>3.0000</td>
<td>$\Sigma$</td>
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<td>$\gamma_m$</td>
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<td>0.99</td>
<td>0.9753</td>
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$$
\Sigma = \begin{pmatrix}
0.08642416 & -0.01378653 & -0.09287633 \\
-0.01378653 & 0.004669188 & -0.01586939 \\
-0.09287633 & -0.015869387 & 0.85507558
\end{pmatrix}
$$
<table>
<thead>
<tr>
<th>Param.</th>
<th>Dist.</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Std</td>
<td>LB</td>
</tr>
<tr>
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</tr>
<tr>
<td>ρ_qv</td>
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<td>0.70083</td>
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<tr>
<td>ε_w</td>
<td>B</td>
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<td>0.14142</td>
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<tr>
<td>ε_m</td>
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<td>0.14142</td>
</tr>
<tr>
<td>ρ_c*</td>
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<td>ρ_πm*</td>
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<td>ρ_πrm*</td>
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<tr>
<td>ρ_tr*</td>
<td>B</td>
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<td>0.14142</td>
</tr>
<tr>
<td>ρ_πc*</td>
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<td>0.14142</td>
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<tr>
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<td>ρ_zx</td>
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<td>0.5000</td>
<td>0.14142</td>
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<td>0.14142</td>
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<td>ψ_x</td>
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<td>0.14142</td>
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</table>
## Estimation

<table>
<thead>
<tr>
<th>Param.</th>
<th>Dist.</th>
<th>Prior</th>
<th>Posterior</th>
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<tbody>
<tr>
<td>( \text{Var}(\pi_{\text{food}}) )</td>
<td>IG</td>
<td>Mean</td>
<td>Std</td>
</tr>
<tr>
<td></td>
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<tr>
<td>( \text{Var}(c^*) )</td>
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<td>( \text{Var}(g) )</td>
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<td>( \text{Var}(\pi^*) )</td>
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<td>( \text{Var}(q^m) )</td>
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<tr>
<td>( \text{Var}(q^{mr}) )</td>
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<td>( \text{Var}(tr) )</td>
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<td>( \text{Var}(z^{i_e}) )</td>
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<tr>
<td>( \text{Var}(z^{\pi q}) )</td>
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<td>100</td>
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<tr>
<td>( \text{Var}(z^{\pi w}) )</td>
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<td>100</td>
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<td>( \text{Var}(z^u) )</td>
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<td>0.0005</td>
<td>100</td>
</tr>
<tr>
<td>( \text{Var}(z^x) )</td>
<td>IG</td>
<td>0.0005</td>
<td>100</td>
</tr>
</tbody>
</table>
Priors - Posteriors

- **$gama_x$**
- **$gama_xd$**
- **$rhoq$**
- **$rhoqv$**
- **$epsq$**
- **$epsw$**
- **$epsm$**
- **$rho_cstar$**
- **$rho_qm$**

DMM (Banco de la República)  
PATACON  
August 3, 2011  
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Priors - Posteriors

![Histograms of various variables](image-url)
Priors - Posteriors

- $\rho_g$
- $\psi_x$
- $\text{var}_{\text{inf\ food}}$
- $\text{var}_{c*}$
- $\text{var}_G$
- $\text{var}_{\pi*}$
- $\text{var}_{qm}$
- $\text{var}_{qmr}$
- $\text{var}_{tr}$

DMM (Banco de la República)
Using a model as a main monetary policy forecasting tool is a very different exercise from simulating a model to answer particular questions.

To forecast for policy, we need to match our model to as much of the useful information, even if that information comes in an awkward variety of shapes and forms.

When forecasting with a DSGE model we have to take into account the following data problems:

- Data uncertainty
- Steady state uncertainty
- Anticipated shocks that can be uncertain
Data Availability

<table>
<thead>
<tr>
<th>Variable</th>
<th>$t$</th>
<th>$t + 2$</th>
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<tbody>
<tr>
<td>Inflación</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Empleo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Salarios</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tasa de interés mundial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Producto mundial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remesas</td>
<td></td>
<td></td>
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</tbody>
</table>

Pronósticos de otros modelos

$t$ $t + 2$
Forecasting Method

- We use a state space representation of the model where the measurement equation relates the available data to the model variables and the transition equation contains the structure of the model.
  - The measurement equation allows us to include the uncertainty of the data through a time varying variance of the measurement error.
  - Also, it allows us to handle an unbalanced data set through the use of a selection matrix.

- The forecast of the model is a mix of the Kalman filter and the filter smoother.
State-space representation

The solution to the log-linearized first order conditions can be written as

\[
\begin{align*}
    c_t &= G p_t \\
    \rho_{t+1} &= H p_t + \epsilon_{t+1}
\end{align*}
\]

Consequently, our transition equation is

\[
    x_t = \begin{pmatrix} c_t \\ \rho_t \end{pmatrix} = \begin{pmatrix} 0 & G H \\ 0 & H \end{pmatrix} \begin{pmatrix} c_{t-1} \\ \rho_{t-1} \end{pmatrix} + \begin{pmatrix} G \\ I \end{pmatrix} \epsilon_t
\]

and our measurement equation is

\[
    W_t y_t = W_t I_s x_t + W_t \Gamma + W_t v_t
\]

where \( W_t \) is a selection matrix and \( I_s \) is matrix where every row contains only one entry different from zero (=1) and every column has at most one entry different from zero. \( \Gamma \) is vector with the steady-state values.
Data uncertainty and off model information
Information about the future

- Endogenous variables

This information is relevant for signal extraction. Which is the state of the economy?

Examples: surveys about expectations, forecasts from other models.

1. Include in the \( y_T \) vector the forecast and in \( x_T \) the expectations of the variable.

Modified measurement equation for time \( T \) (last period of the data set) that includes one period ahead information of the endogenous variable \( x_j \)

\[
\begin{pmatrix}
  y_1, T \\
  \vdots \\
  y_k, T \\
  f_t(y_{j, T+1})
\end{pmatrix}
= \begin{pmatrix}
  x_1, T \\
  \vdots \\
  x_k, T \\
  E_t(x_{j, T+1})
\end{pmatrix}
+ \begin{pmatrix}
  \zeta_1, T \\
  \vdots \\
  \zeta_k, T \\
  \epsilon_f
\end{pmatrix}
\]

2. Calibrate the variance of \( \epsilon_f \) to capture the uncertainty about the forecast.
Information about the future II

- Exogenous variables

This information is used to generate a conditional forecast that assumes a path for the exogenous variables. Example: World GDP forecasts by the IMF

- In this case we include the value of the exogenous variable at time \( T + h \) as an observable variable, consequently this information enters as a surprise in period \( T + h \). At time \( T \) agents are not aware of the value of this variable at time \( T + h \). The measurement equation for the “observable” exogenous variable \( x_l \) at time \( T + h \) becomes

\[
(y_{l,T+h}) = (x_{l,T+h}) + (\nu_{T+h})
\]

where the variance of \( \nu \) determines the weight that \( y_{l,T+h} \) has on the forecast.
Then we have a time varying measurement equation of the form:

\[
\begin{pmatrix}
  y_{1,t} \\
  \vdots \\
  y_{k,t}
\end{pmatrix} = \begin{pmatrix}
  x_{1,T} \\
  \vdots \\
  x_{k,T}
\end{pmatrix} + \begin{pmatrix}
  \zeta_{1,T} \\
  \vdots \\
  \zeta_{k,T}
\end{pmatrix} \quad \text{for } t < T
\]

\[
\begin{pmatrix}
  y_{1,T} \\
  \vdots \\
  y_{k,T}
\end{pmatrix} = \begin{pmatrix}
  x_{1,T} \\
  \vdots \\
  x_{k,T}
\end{pmatrix} + \begin{pmatrix}
  \zeta_{1,T} \\
  \vdots \\
  \zeta_{k,T}
\end{pmatrix} \quad \text{for } t = T
\]

\[
\begin{pmatrix}
  f_t(y_{j,T+1}) \\
  y_{l,t}
\end{pmatrix} = \begin{pmatrix}
  x_{l,t}
\end{pmatrix} + (\nu_t) \quad \text{for } T < t \leq T + H
\]
Forecasting method

Our forecasts for the variables contained in the $y_t$ vector is

$$y_{t+h}^f = l_s x_{t+h}^f + \Gamma$$

with

$$x_{T+h}^f = \begin{cases} x_{T+h}^{T+H} & \text{if } t + h \leq T + H \\ \Phi^{h-H} x_{T+h}^{T+H} & \text{if } t + h > T + H \end{cases}$$

Smoother

Standard Kalman filter forecast

where $x_t^s = E [x_t | Y_s]$ and $Y_s = (y_1, \ldots, y_t, \ldots, y_s)$.

- $E [x_t | Y_s]$ is the Kalman smoother for $s > t$.
- $T + H$ is the last period for which there’s at least data available for one variable in vector $y_t$. 
Productivity Shock
Cost Push Shock
Investment Shock
External Demand Shock

Graphs showing the impact of external demand shocks on various economic indicators:
- $y_{inf\_all\_t}$
- $y_{t}$
- $y_{d\_t}$
- $DA_{ngdp\_pc\_t}$
- $DA_{c\_t}$
- $DA_{x\_t}$
- $DA_{e\_t}$
- $DA_{md\_t}$
- $DA_{rm\_t}$

The graphs illustrate the behavior of these indicators over time, with the y-axis representing the magnitude of the shocks and the x-axis representing time.
Remittances Shock

\begin{align*}
yinf\text{\_all}\_t & \quad 0.05 \\
y\_t & \quad 0.05 \\
yd\_t & \quad 0.5 \\
DA\text{\_ngdp\_pc}\_t & \quad 0.3 \\
DA\text{\_c}\_t & \quad 0.6 \\
DA\text{\_x}\_t & \quad 3.0 \\
DA\text{\_e}\_t & \quad 0.5 \\
DA\text{\_md}\_t & \quad 3.0 \\
DA\text{\_rm}\_t & \quad 1.0 \\
\end{align*}
Risk Premium Shock