PATACON

Policy Analysis Tool Applied to Colombian Needs

Departamento de Modelos Macroeconómicos

Banco de la República

August 3, 2011

Outline

References

- González, A., L. Mahadeva, J. D. Prada, and D. Rodríguez (2011): “Policy Analysis Tool Applied to Colombian Needs: PATACON Model Description ”, Borradores de economía, 656. Banco de la República.
- Bonaldi, P., González, A. Rodríguez, D. L. E. Rojas, and J. D. Prada (2009): “*A numerical method for calibrating a DSGE model*“, Borradores de economía, 548. Banco de la República
- González, A., Mahadeva, L. D. Rodríguez, and L.E Rojas (2009): “*Monetary Policy Forecasting in a DSGE Model with Data that is Uncertain and Unbalanced*”, Borradores de economía, 559. Banco de la República
- Bonaldi, P., González, A. and D. Rodríguez (2010): Importancia de las rigideces nominales y reales en Colombia: un enfoque de equilibrio general dinámico y estocástico ”, Borradores de economía, 559. Banco de la República

Outline

The Model

- PATACON is a DSGE model for policy analysis and forecasting of the Colombian economy.
- The model follows Christiano, Eichenbaum and Evans (2005) and adds characteristics to replicate a small open economy. See González, Mahadeva, Prada and Rodríguez (2011).

General Features I: Nominal and Real Rigidities

- Monopolistic competition in labor and goods market with sticky prices á la Calvo (Erceg, Henderson and Levin (2000) and Kollman (1997)).
- Partial price and wage indexation (past inflation).
- Variable capital utilization with endogenous depreciation as a function of capital utilization. (Greenwood et al, (1988), King and Rebelo (1999), CEE (2001)).
- External habit (Abel (1990), Fuhrer (2000)).
- Adjustment costs are in terms of the change in the flow of investment.
- Incomplete exchange rate pass-through (sticky prices á la Calvo and distribution of imported goods).
- Transforming firms

General Features II: External Shocks and Sources of Fluctuations

External factors:

- External demand (prices and quantities).
- Remittances.
- External interest rate.
- Raw materials prices.
- External inflation.
- Prices of imported goods of consumption and investment.

Internal factors:

- Monetary policy shocks.
- Exogenous changes in consumption.
- Exogenous changes in labor supply (intensive margin).

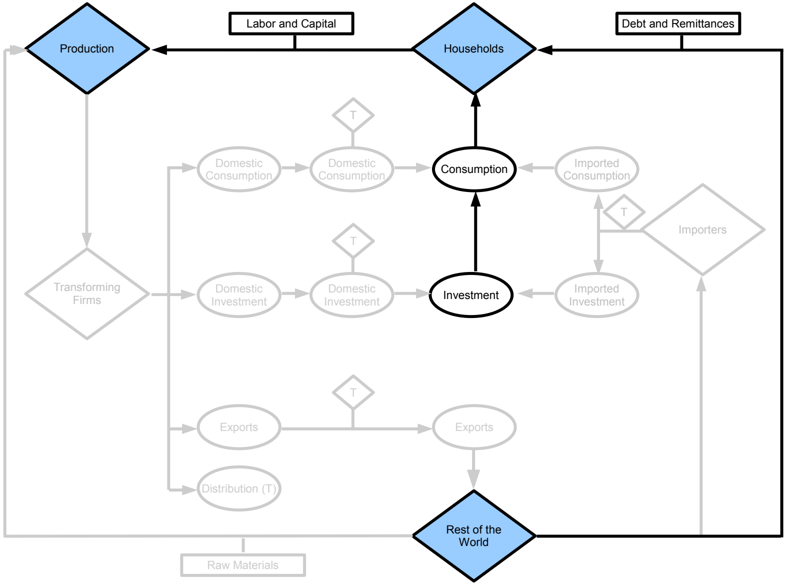
General Features III: Exogenous Shocks and Sources of Fluctuations

Internal factors:

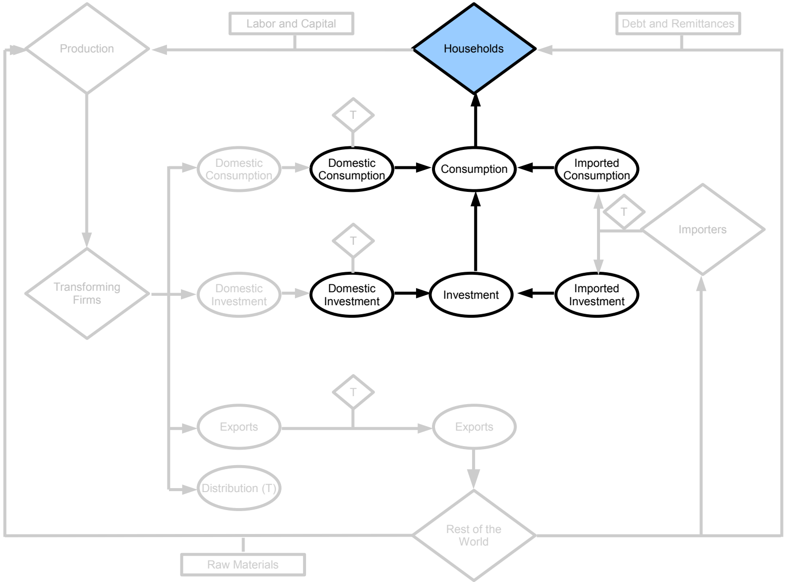
- Exogenous movements in Tobin's Q (investment efficiency).
- Transitory shocks to the productivity in the final good production.
- Permanent shocks to the long run growth of productivity.
- Shocks to the "mark-up" to the final good price and wages.

Outline

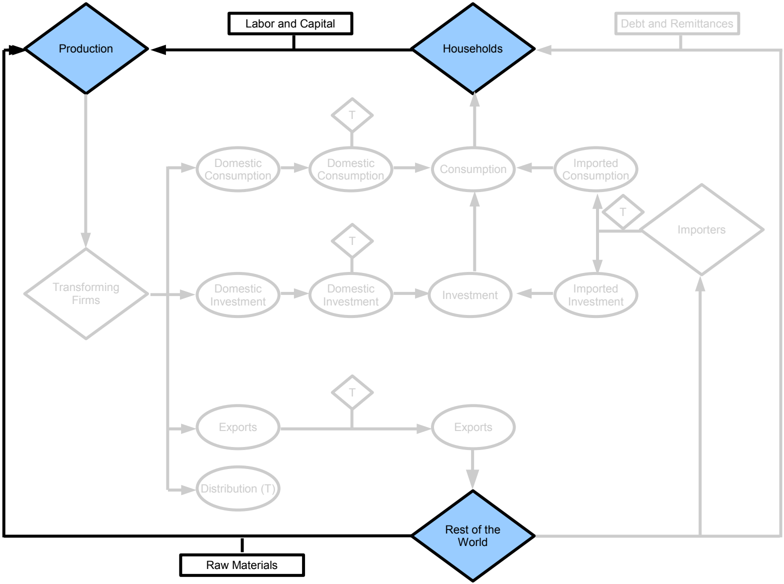
Model Structure 1



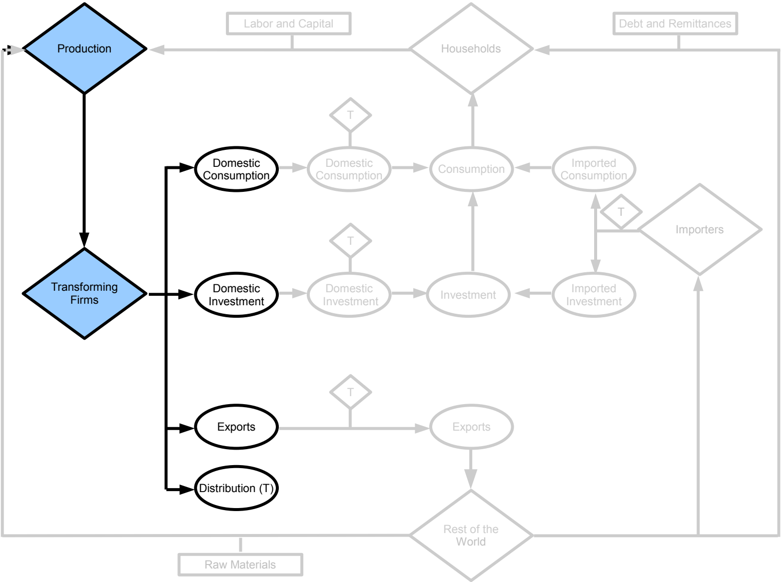
Model Structure 2



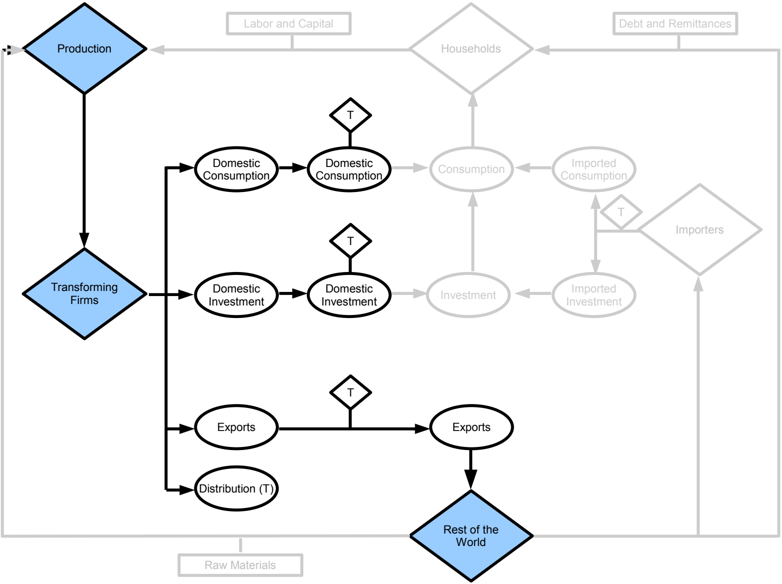
Model Structure 3



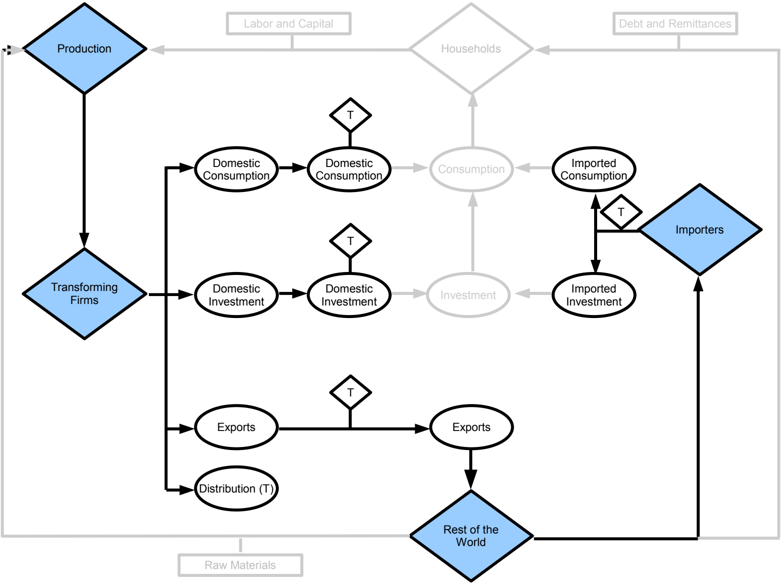
Model Structure 4



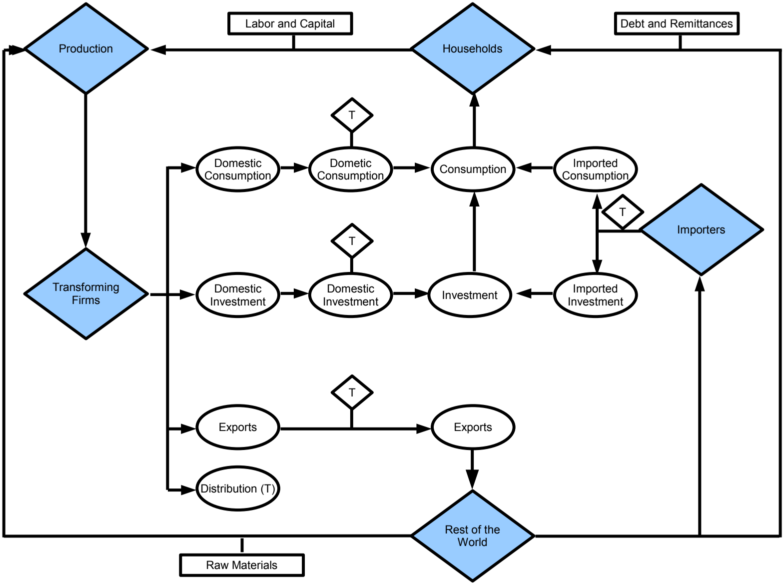
Model Structure 5



Model Structure 6



Model Structure 7



Technological progress, population and unemployment

- Total population N_t follows a process

$$\ln\left(\frac{N_t}{N_{t-1}}\right) = \rho_n \ln\left(\frac{N_{t-1}}{N_{t-2}}\right) + (1-\rho_n)\ln(1+\overline{n}) + \epsilon_t^n$$

- Labor force is defined as

$$L_t = (1 - TD_t)\, TBP_t N_t$$

- Technological progress (in this model equivalent to trend productivity per hour worked) follows:

$$\ln\left(\frac{A_t}{A_{t-1}}\right) = \rho_a \ln\left(\frac{A_{t-1}}{A_{t-2}}\right) + (1-\rho_a)\ln(1+g_a) + \epsilon_t^a$$

Model units

- Models such as these are easier to solve if variables can be expressed as stationary, mean zero, deviations for the steady state
- Therefore we express all variables in model units, effectively adjusting them for the two sources of growth, population and Harrod neutral technological progress
- Let J_t in uppercase be the total quantity of a real economic variable, such as the volume of consumption
  - Per-capita terms

$$\tilde{j}_t \equiv \frac{J_t}{N_t}$$

  - Model units

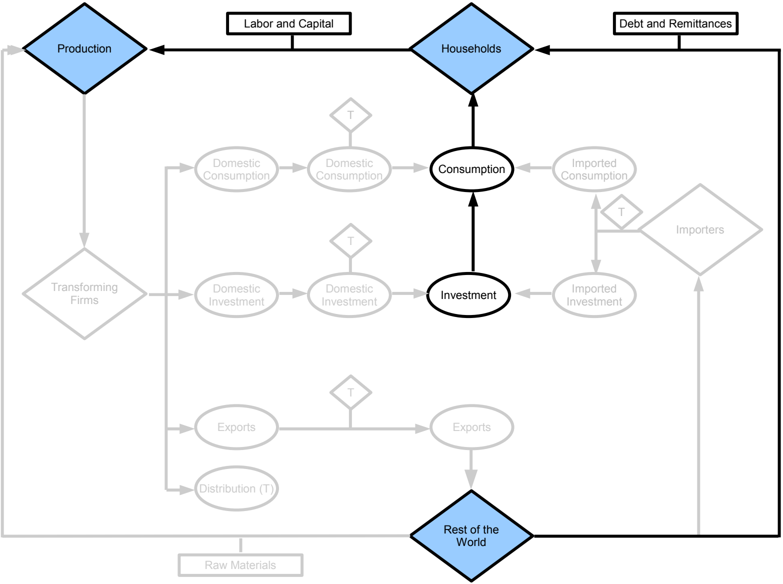
$$j_t \equiv \frac{J_t}{Z_t N_t \bar{l}}$$

where $\bar{l}$ is total hours available per person and

$$Z_t = Z_{t-1}^{\gamma_g} A_t^{1-\gamma_g}$$

and $\frac{Z_t}{Z_{t-1}} = (1 + g_t^z)$

Household



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Domestic Consumption
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Domestic Investment
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Exports
Exports
Distribution (T)
Rest of the World
Raw Materials

Household

- Continuum of households j of measure one, indexed by $j \in (0,1)$.
- Utility function

$$u(\cdot) = \left[\begin{array}{c} \left(\frac{z_t^u}{1-\sigma} \left[c_t^F(j) - hab\bar{c}_{t-1}^F \right]^{1-\sigma} \right) \\ -\left(\frac{z_t^h}{1+\eta} \left((1 - TD_t) \, TBP_t h_t(j) \right)^{1+\eta} \right) \end{array} \right] \left(Z_t \bar{l} \right)^{1-\sigma}$$

- where $h_t(j)$ the proportion of total hours that are worked.

Household

- Budget constraint

$$
\begin{aligned}
c_t^F(j) + \frac{p_t^{xF}}{p_t^{cF}} x_t^F(j) + b_t(j) &+ \frac{s_t p_t^{c\star}}{p_t^{cF}} \frac{1+i_{t-1}^\star}{1+\pi_t^{c\star}} \frac{b_{t-1}^\star(j)}{(1+\overline{n})(1+g_t)} \\
+\int p_{t+1,t}^a(j)\, a_{t+1}(j)\, d\omega_{t+1,t}(j) &+ \Psi^X\left(x_t^F(j), x_{t-1}^F(j)\right) = \\
r_t^k u_t(j) \frac{k_{t-1}(j)}{(1+\overline{n})(1+g_t)} &+ w_t(j)(1-TD_t)\, TBP_t h_t(j) \\
+\xi_t + a_t(j) + \frac{s_t p_t^{c\star}}{p_t^{cF}} tr_t^\star &+ \frac{s_t p_t^{c\star}}{p_t^{cF}} b_t^\star(j) \\
+\frac{b_{t-1}(j)}{(1+\overline{n})(1+g_t)} \left(\frac{1+i_{t-1}}{1+\pi_t^{cF}}\right)
\end{aligned}
$$

Household

- Investment cost

$$\Psi^X\left(x_t^F(j), x_{t-1}^F(j)\right) = \frac{\psi^X}{2}\frac{\left(x_t^F(j) - x_{t-1}^F(j)\right)^2}{x_{t-1}^F(j)}$$

- Capital accumulation equation

$$k_t(j) = x_t^F(j) + \frac{(1-\delta(u_t(j)))\,k_{t-1}(j)}{(1+\overline{n})(1+g_t)}$$

- Variable depreciation

$$\delta(u_t(j)) = \overline{\delta} + \frac{b}{1+\Upsilon}(u_t(j))^{1+\Upsilon}$$

F.O.C s

$$\lambda_t^c = z_t^u \left(c_t^F - hab\bar{c}_{t-1}^F\right)^{-\sigma}$$

$$r_t^k = \frac{\lambda_t^x}{\lambda_t^c}\, b\, u_t^{\Upsilon}$$

$$\lambda_t^c \frac{p_t^{xF}}{p_t^{cF}} = \lambda_t^x - \lambda_t^c\, \psi^X \frac{x_t^F - x_{t-1}^F}{x_{t-1}^F}$$

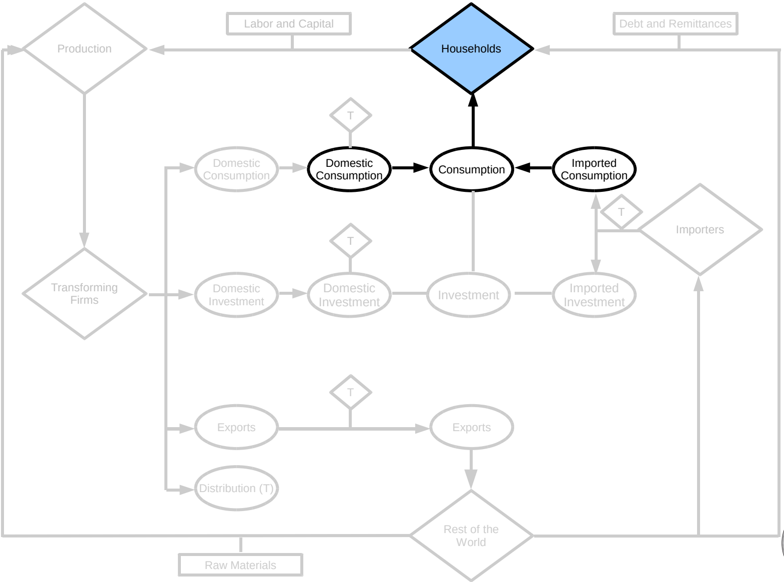
$$+\beta E_t\,(1+\bar{n})\,(1+g_{t+1})^{1-\sigma}\,\lambda_{t+1}^c \left(\frac{\psi^X\left(x_{t+1}^F - x_t^F\right) + \Psi^X\left(x_{t+1}^F, x_t^F\right)}{x_t^F}\right)$$

$$\lambda_t^x = \beta E_t\,(1+g_{t+1})^{-\sigma}\,\lambda_{t+1}^c r_{t+1}^k u_{t+1} + \beta E_t\,(1+g_{t+1})^{-\sigma}\,\lambda_{t+1}^x\,(1-\delta\,(u_{t+1}))$$

$$\lambda_t^c = \beta E_t\,(1+g_{t+1})^{-\sigma}\,\lambda_{t+1}^c \left(\frac{1+i_t}{1+\pi_{t+1}^{cF}}\right)$$

$$\lambda_t^c = \beta E_t\,(1+g_{t+1})^{-\sigma}\,\lambda_{t+1}^c\,(1+i_t^{\star})\left(\frac{1+d_{t+1}}{1+\pi_{t+1}^{cF}}\right)$$

Household



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Household

- Consumption Bundle

$$c_t^F(j) = \left[(\gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{dF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} + (1 - \gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{mF}(j) \right)^{\frac{\omega^c - 1}{\omega^c}} \right]^{\frac{\omega^c}{\omega^c - 1}}$$

$$c_t^{dF}(j) = \gamma^c \left(\frac{p_t^{cdF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j)$$

$$c_t^{mF}(j) = (1 - \gamma^c) \left(\frac{p_t^{mF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j)$$

Labor market

- Workers are hired by intermediaries firms, which combine the work effort of individual workers and supply a joint labour input

$$\min_{\{h_t(j)\}} \quad \int_0^1 \tilde{w}_t(j)\,\tilde{h}_t(j)\,dj$$

$$\text{s.t} \quad \tilde{h}_t^F \leq \int_0^1 \left[\tilde{h}_t(j)^{\frac{\theta^w - 1}{\theta^w}} \, dj \right]^{\frac{\theta^w}{\theta^w - 1}}$$

- F.O.C will imply

$$\tilde{h}_t(j) = \left(\frac{\tilde{w}_t(j)}{\tilde{w}_t} \right)^{-\theta^w} \tilde{h}_t^F$$

$$\tilde{w}_t \equiv \left[\int_0^1 \tilde{w}_t(j)^{1-\theta^w} \, dj \right]^{\frac{1}{1-\theta^w}}$$

Nominal wages are sticky (Calvo wages)

- Given the demand for their differentiated labour, individuals can set their wages. Each individual is only free to renegotiate a salary when they receive a random signal which arrives every quarter with probability $1 - \varepsilon^{w}$.
- If wages are not renegotiated they are set by a rule.

$$w_t^{Rule}(j) = w_{t-1}(j)\left(\frac{1+\pi_{t-1}^{cF}}{1+\pi_t^{cF}}\right)$$

- If on the other hand, the j^{th} individual receives the signal to renegotiate her wage at period t, that will be set according to:

$$w_t^{opt}(j) = \frac{\theta^w}{\theta^w - 1}\frac{num_t^w(j)}{den_t^w(j)}$$

Nominal wages are sticky (Calvo wages)

- where

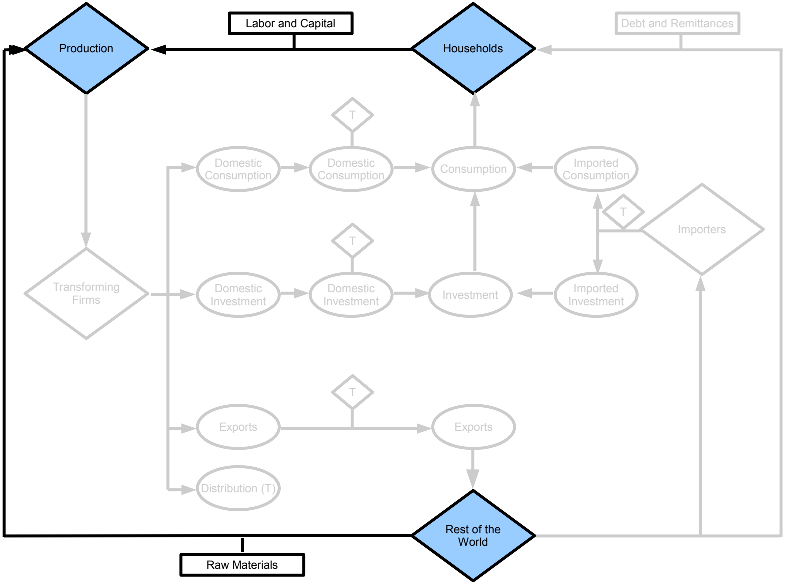
$$
\begin{aligned}
num_t^w(j) \equiv\ & E_t \sum_{i=0}^{\infty} \left(\beta\varepsilon^w (1+\bar{n})\right)^i \prod_{k=1}^{i} \left[(1+g_{t+k})^{1-\sigma}\right] \\
& z_{t+i}^h \left((1-TD_{t+i})\,TBP_{t+i}\right)^{1+\eta} \left(h_{t+i}^F \left(\frac{w_t^{opt}(j)}{w_{t+i}}\right)^{-\theta^w} \left(\frac{1+\pi_t^{cF}}{1+\pi_{t+i}^{cF}}\right)^{-\theta^w}\right)^{1+\eta}
\end{aligned}
$$

$$
\begin{aligned}
den_t^w(j) \equiv\ & E_t \sum_{i=0}^{\infty} \left(\beta\varepsilon^w (1+\bar{n})\right)^i \prod_{k=1}^{i} \left[(1+g_{t+k})^{1-\sigma}\right] \\
& \lambda_{t+i}^c (1-TD_{t+i})\,TBP_{t+i} \left(h_{t+i}^F \left(\frac{w_t^{opt}(j)}{w_{t+i}}\right)^{-\theta^w} \left(\frac{1+\pi_t^{cF}}{1+\pi_{t+i}^{cF}}\right)^{1-\theta^w}\right)
\end{aligned}
$$

- Then, the real wage evolves according to

$$
w_t = \left[\varepsilon^w \left(w_{t-1}\left(\frac{1+\pi_{t-1}^{cF}}{1+\pi_t^{cF}}\right)\right)^{1-\theta^w} + (1-\varepsilon^w)\left(w_t^{opt}\right)^{1-\theta^w}\right]^{\frac{1}{1-\theta^w}} + z_t^w
$$

Intermediate Production Firms



Intermediate Production Firms

- There are a continuum of firms indexed by $z \in (0,1)$
- Each produces a differentiated product (z) given the following production function which is weakly separable in value-added factors of production

$$q_t^C(z) = z_t^q \left[\alpha^{\frac{1}{\rho}} \left(va_t(z)\right)^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} \left(rm_t^F(z)\right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

$$va_t(z) = \left[\alpha_V^{\frac{1}{\rho_V}} \left(k_t^s(z)\right)^{\frac{\rho_V-1}{\rho_V}} + (1-\alpha_V)^{\frac{1}{\rho_V}} \left((1-TD_t)\, TBP_t a_t h_t(z)\right)^{\frac{\rho_V-1}{\rho_V}} \right]^{\frac{\rho_V}{\rho_V-1}}$$

- where $a_t = \frac{A_t}{Z_t}$ and $k_t^s = \frac{u_t k_{t-1}}{(1+\overline{n})\left(1+g_t^z\right)}$

Intermediate Production Firms

- The first-order conditions are then given as:

$$w_t = \lambda_t^q(z)\, z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q va_t(z)}\right)^{\frac{1}{\rho}} \left(\frac{(1-\alpha_v)\, va_t(z)}{(1-TD_t)\, TBP_t a_t h_t(z)}\right)^{\frac{1}{\rho v}}$$

$$r_t^k = \lambda_t^q(z)\, z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q va_t(z)}\right)^{\frac{1}{\rho}} \left(\frac{\alpha_v va_t(z)}{k_t^s(z)}\right)^{\frac{1}{\rho v}}$$

$$\frac{p_t^{rmF}}{p_t^{cF}} = \lambda_t^q(z)\, z_t^q \left(\frac{(1-\alpha)\, q_t^C(z)}{z_t^q rm_t^F(z)}\right)^{\frac{1}{\rho}}$$

- where $\lambda_t^q(z)$ is the real marginal cost in model units and measured in consumption process of these firms.

Intermediate Production Firms

- The real marginal cost in model units and measured in consumption process of these firms $\lambda_t^q(z)$.

$$\lambda_t^q(z) = \frac{1}{(z_t^q)} \left[\alpha \left(\left[\alpha_v \left(r_t^k\right)^{1-\rho_v} + (1-\alpha_v)(w_t)^{1-\rho_v} \right]^{\frac{1}{1-\rho_v}} \right)^{1-\rho} + (1-\alpha)\left(\frac{p_t^{rmF}}{p_t^{cF}}\right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

- The price of raw materials in external currency is exogenous

$$p_t^{rmC} = s_t p_t^{rm\star}$$

Prices are sticky (Calvo pricing)

- Each period firms face a constant probability $(1-\varepsilon^q)$ of receiving a signal which tells them when they can adjust their price.
- The other ε^q firms that are not allowed to reset their prices follow a backward-looking indexation rule:

$$p_t^{rule}(z) = p_{t-1}^{qF}(z)\left(1+\pi_{t-1}^{qF}\right)^{\iota^q}(1+\overline{\pi})^{1-\iota^q}$$

where $\overline{\pi}$ is average long-run inflation taken to be the central bank's target, $\iota^q \geq 0$ is the weight assigned to past inflation as opposed to this target.

Prices are sticky (Calvo pricing)

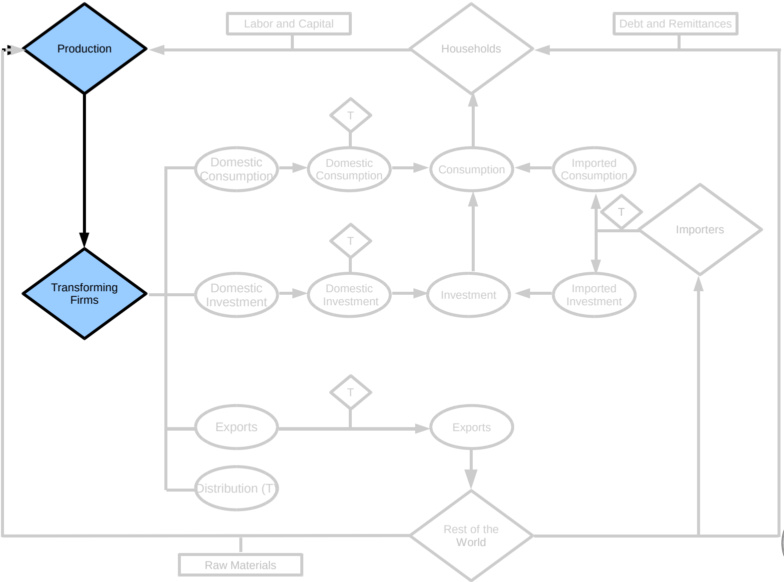
- If on the other hand, the z^{th} receives a signal which tells it that can adjust their price, it will choose

$$\frac{p_t^{qopt}(z)}{p_t^{qF}} = \frac{\theta^q}{\theta^q - 1} \frac{E_t \sum_{i=0}^{\infty} (\varepsilon^q)^i \Delta_{t+i,t} \frac{A_{t+i}}{A_t} \left[\frac{\lambda_{t+i}^q(z) \left(\frac{p_{t+i}^{qF}}{p_t^{qF}} \right)^{\theta^q} q_{t+i}^F}{\left(\prod_{l=1}^{i} \left\{ \left(1+\pi_{t-1+l}^{qF}\right)^{\iota^q} \right\} (1+\overline{\pi})^{i(1-\iota^q)} \right)^{\theta^q}} \right]}{E_t \sum_{i=0}^{\infty} (\varepsilon^q)^i \Delta_{t+i,t} \frac{A_{t+i}}{A_t} \left[\frac{\left(\frac{p_{t+i}^{qF}}{p_t^{qF}} \right)^{\theta^q - 1} \frac{p_{t+i}^{qF}}{p_{t+i}^{cF}} q_{t+i}^F}{\left(\prod_{l=1}^{i} \left\{ \left(1+\pi_{t-1+l}^{qF}\right)^{\iota^q} \right\} (1+\overline{\pi})^{i(1-\iota^q)} \right)^{\theta^q - 1}} \right]}$$

- Therefore the inflation dynamics will evolve according with

$$\left(1+\pi_t^{qF}\right) = \left[\begin{array}{c} (1-\varepsilon^q) \left(\frac{p_t^{qopt}}{p_t^{qF}} \right)^{1-\theta^q} \left(1+\pi_t^{qF}\right)^{1-\theta^q} \\ +\varepsilon^q \left[\left(1+\pi_{t-1}^{qF}\right)^{\iota^q} (1+\overline{\pi})^{i(1-\iota^q)} \right]^{1-\theta^q} \end{array} \right]^{\frac{1}{1-\theta^q}} + z_t^{\pi}$$

Final production good



Final production good

- There is an aggregation technology

$$q_t^F = \left[\int_0^1 \left(q_t^C(z) \right)^{\frac{\theta^q - 1}{\theta^q}} dz \right]^{\frac{\theta^q}{\theta^q - 1}} .$$

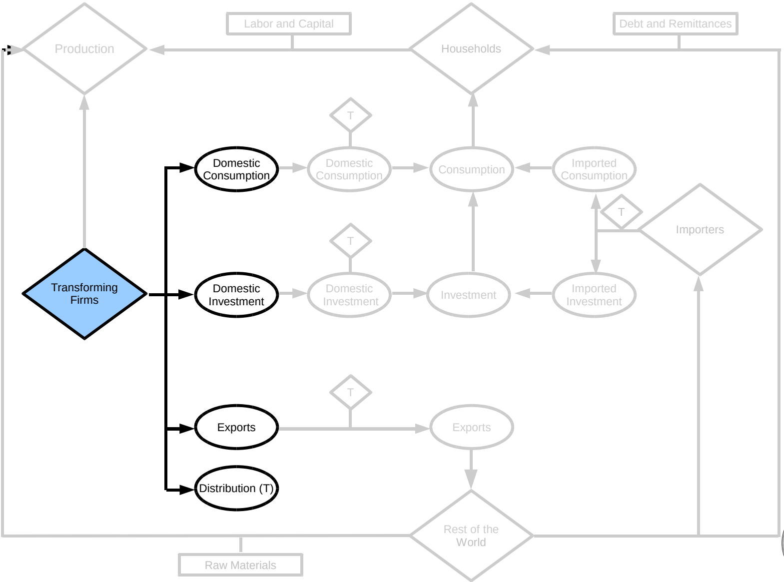
- The demand for the intermediate good (z) is given by

$$q_t^C(z) = \left(\frac{p_t^q(z)}{p_t^{qF}} \right)^{-\theta^q} q_t^F$$

- The output price is given as the aggregate

$$p_t^{qF} = \left[\int_0^1 \left(p_t^q(z) \right)^{1-\theta^q} dz \right]^{\frac{1}{1-\theta^q}}$$

Transformation of final good



Transformation of final good

- At a next stage, the final product q_t^F is transformed into four different types of output: domestic consumption, c_t^{dC}, intermediate domestic capital goods, x_t^{dC}, exports, e_t^C, and as distribution services , dis_t^C.

$$q_t^F = \left[\nu_{nt}^{\omega_q - 1} (nt_t)^{\omega_q} + \nu_e^{\omega_q - 1} \left(e_t^C\right)^{\omega_q}\right]^{\frac{1}{\omega_q}}$$

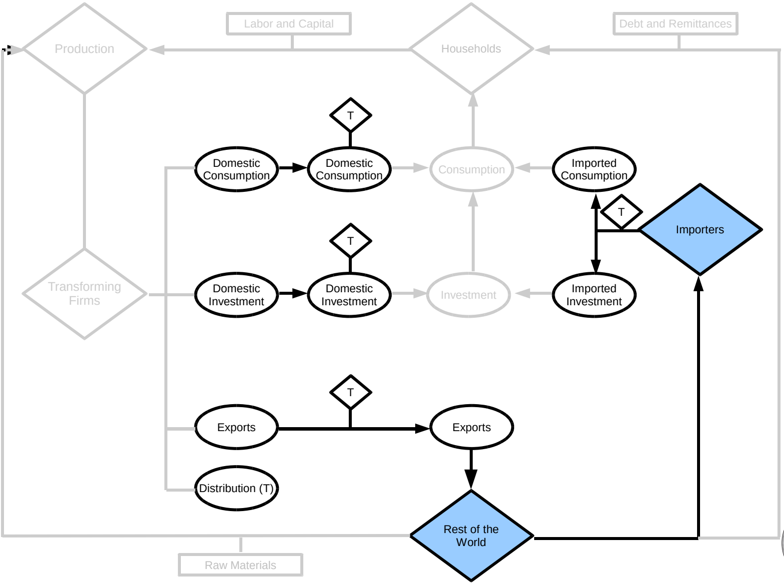
$$nt_t = \left[\nu_c^{\omega_{nt} - 1} \left(c_t^{dC}\right)^{\omega_{nt}} + \nu_x^{\omega_{nt} - 1} \left(x_t^{dC}\right)^{\omega_{nt}} + \nu_{dis}^{\omega_{nt} - 1} \left(dis_t^C\right)^{\omega_{nt}}\right]^{\frac{1}{\omega_{nt}}}$$

Transformation of final good

- Supplies of each if them are given by

$$
\begin{aligned}
\frac{p_t^{cdC}}{p_t^{qF}} &= \left(\frac{\nu_{nt} nt_t}{q_t^F}\right)^{\omega_q - 1} \left(\frac{\nu_c c_t^{dC}}{nt_t}\right)^{\omega_{nt} - 1} \\
\frac{p_t^{xdC}}{p_t^{qF}} &= \left(\frac{\nu_{nt} nt_t}{q_t^F}\right)^{\omega_q - 1} \left(\frac{\nu_x x_t^{dC}}{nt_t}\right)^{\omega_{nt} - 1} \\
\frac{p_t^{disC}}{p_t^{qF}} &= \left(\frac{\nu_{nt} nt_t}{q_t^F}\right)^{\omega_q - 1} \left(\frac{\nu_{dis} dis_t^{C}}{nt_t}\right)^{\omega_{nt} - 1} \\
\frac{p_t^{eC}}{p_t^{qF}} &= \nu_e^{\omega_q - 1} \left(\frac{e_t^{C}}{q_t^F}\right)^{\omega_q - 1}
\end{aligned}
$$

Distribution



Distribution

- Note that
$$dis_t^F = dis_t^{cd} + dis_t^{xd} + dis_t^e + dis_t^m$$
- Distribution of the domestic product

$$j_t(z) = \left[\left(\gamma^j\right)^{\frac{1}{\omega^j}} \left(j_t^C(z)\right)^{\frac{\omega^j - 1}{\omega^j}} + \left(1 - \gamma^j\right)^{\frac{1}{\omega^j}} \left(dis_t^j(z)\right)^{\frac{\omega^j - 1}{\omega^j}} \right]^{\frac{\omega^j}{\omega^j - 1}}$$

where j_t: domestic consumption c_t^d, domestic investment x_t^d, exports e_t and imports $m_t^\star$.

Distribution

- Demand for $dis_t^j(z)$

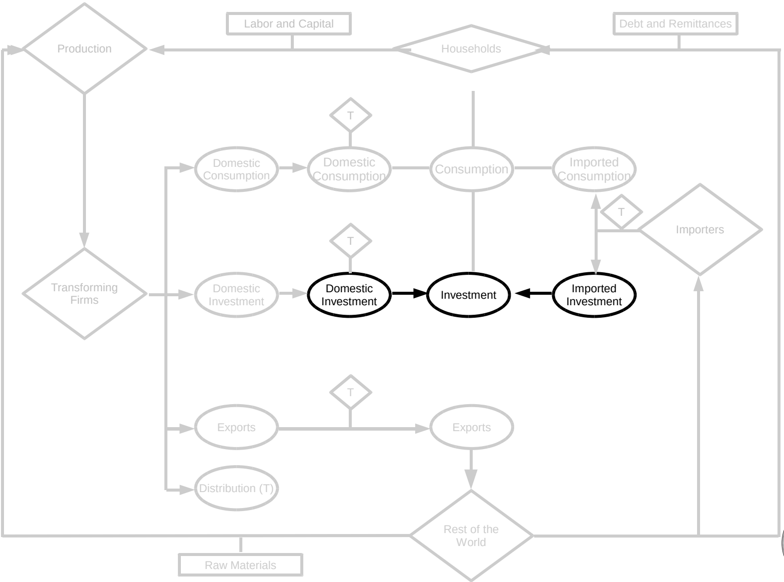
$$\frac{p_t^{jC}}{p_t^{cF}} = \lambda_t^j(z)\left(\frac{\gamma^j j_t(z)}{j_t^C}\right)^{\frac{1}{\omega^j}}$$

$$\frac{p_t^{disF}}{p_t^{cF}} = \lambda_t^j(z)\left(\frac{\left(1-\gamma^j\right) j_t(z)}{dis_t^j}\right)^{\frac{1}{\omega^j}}$$

- Additionally, each of this sectors is subject to a similar price rigidity as the one described above and the price of import goods is also exogenous

$$p_t^{mC} = s_t p_t^{m\star}$$

Investment



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Exports
Distribution (T)
Rest of the World
Raw Materials

Investment

- **Investment goods produced**

$$x_t^F = z_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{dF} \right)^{\frac{\omega^x - 1}{\omega^x}} + (1 - \gamma^x)^{\frac{1}{\omega^x}} \left(x_t^{mF} \right)^{\frac{\omega^x - 1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x - 1}}$$

$$x_t^{dF} = (\gamma^x) \left(\frac{p_t^{xdF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x}$$

$$x_t^{mF} = (1 - \gamma^x) \left(\frac{p_t^{mF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x}$$

Exports

- **External demand**

$$p_t^{eF} = s_t p_t^{eE}$$

$$e_t^F = \left(\frac{p_t^{e\star}}{p_t^{c\star}}\right)^{-\mu} c_t^{\star}$$

Interest rates

- **External interest rate**

$$i_t^\star = \bar{i}^\star z_t^{i\star} \exp\left(\Omega_u\left(\frac{s_t p_t^{c\star}}{p_t^{cF}}\frac{b_t^\star}{y_t} - \overline{b^\star}\right)\right)$$

- **Policy rule**

$$i_t = \rho_s i_{t-1} + (1-\rho_s)\left(\bar{i} + \varphi_\pi\left(\pi_t^{cF} - \overline{\pi}\right) + \varphi_y\left(\frac{y_t}{y_t^{flex}} - 1\right)\right) + z_t^i$$

Outline

General Methodology

- We calibrate the steady state by minimizing the following objective function:

$$f_{obj}(\theta) = \sum_{i=1}^{n} \omega_i \left(x_i^{ss}(\theta) - x_i^{d\_lr} \right)^2$$

where x^{ss} the steady state values and $x_i^{d\_lr}$ the equivalent ratios in the data.

- In this exercise we target the 21 nominal ratios.

Outline

Results of the Calibration

Ratios and relative prices	Model	Data Colombia	Deviation%
Investment / GDP	0.22	0.23	0.003
Imported investment / Total investment	0.37	0.36	0.030
Domestic inv. without dist. / Gross product	0.13	0.12	0.004
Consumption / GDP	0.82	0.80	0.023
Dom. cons. without dist. / Gross product	0.68	0.60	0.035
Imported consumption / Total consumption	0.12	0.12	0.024
Labor supply	0.30	0.30	0.000
Raw materials / Gross product	0.09	0.10	0.029
FOB Imports / Import. with distribution	0.76	0.73	0.038
FOB imports + raw material / GDP	0.24	0.23	0.025
Exports without dist. / Gross product	0.16	0.17	-0.047

Results of the Calibration

Ratios and relative prices	Model	Data Colombia	Deviation%
Remittances / GDP	0.03	0.04	-0.015
Dom. consumption dist. / Dom. consumption	0.05	0.06	-0.011
Dom. investment dist. / Dom. investment	0.04	0.04	-0.005
Exports dist. / Exports	0.12	0.13	-0.014
Dom. cons. without dist. / Dom. consumption	0.92	0.94	-0.025
Dom. inv. without dist. / Dom. investment	0.93	0.96	-0.026
Exports without dist. / Exports	0.85	0.88	-0.028
Domestic consumption / Gross product	0.68	0.64	0.061
Domestic investment / Gross product	0.13	0.12	0.031
Exports / Gross product	0.19	0.20	-0.019

Outline

Estimation Strategy

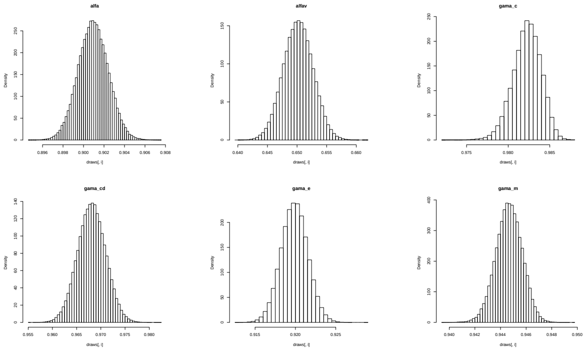
- We use bayesian methods to estimate

$$\ln P(\theta|y,\tilde{y}) \propto \underbrace{\ln P(y|\theta)}_{\text{likelihood}} + \underbrace{\ln P(\theta|\tilde{y})}_{\text{prior long-run parameters}} + \underbrace{\ln P(\theta)}_{\text{prior short-run parameters}}$$

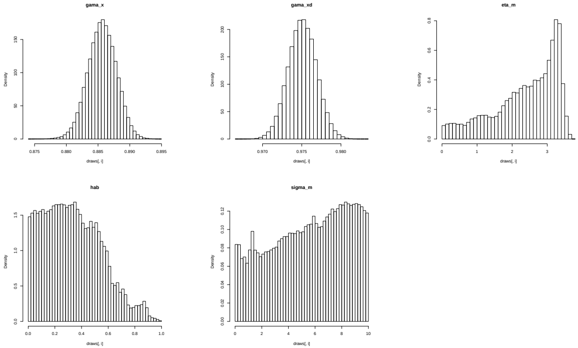
$$\ln P(\theta|\tilde{y}) \sim N(\mu, \Sigma)$$

  where μ was set to the parameter values found in the calibration stage. The variance Σ was computed by generating draws from the objective function if the calibration stage.
- The priors of the short run parameters are characterized by a large variance.

Draws of the calibration objective function



Draws of the calibration objective function



Estimation

Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
η	MN	3.0000	Σ	0.01	5.00	2.475	2.5054	0.31853
hab	MN	0.2100	Σ	0.00	0.99	0.315	0.3160	0.07109
σ	MN	4.0000	Σ	0.01	10.00	4.9	5.0393	0.85355
α	N	0.9008	0.00147	0.10	0.99	0.9011	0.9011	0.00146
α_v	N	0.6501	0.00251	0.10	0.99	0.65025	0.6501	0.00250
γ_c	N	0.9823	0.00169	0.00	1.00	0.9838	0.9838	0.00174
γ_{cd}	N	0.9679	0.00290	0.01	0.99	0.9683	0.9679	0.00289
γ_e	N	0.9197	0.00164	0.01	0.99	0.9198	0.9197	0.00165
γ_m	N	0.9448	0.00103	0.01	0.99	0.9447	0.9446	0.00103
γ_x	N	0.8859	0.00225	0.01	0.99	0.8858	0.8856	0.00225
γ_{xd}	N	0.9751	0.00180	0.01	0.99	0.9753	0.9751	0.00181

$$\Sigma = \begin{pmatrix} 0.08642416 & -0.013786532 & -0.09287633 \\ -0.01378653 & 0.004669188 & -0.01586939 \\ -0.09287633 & -0.015869387 & 0.85507558 \end{pmatrix}$$

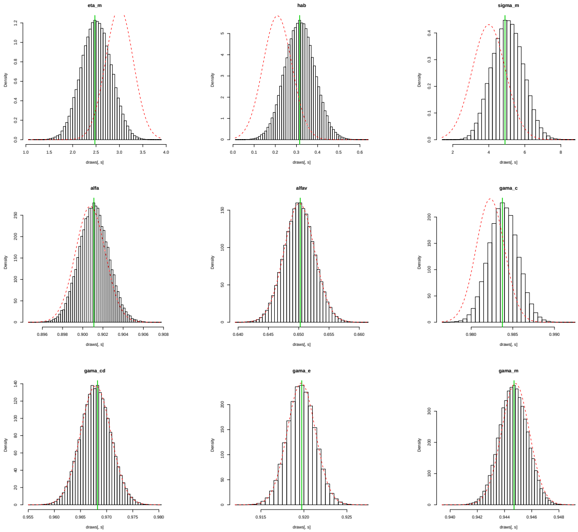
Estimation

Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
ρ_q	U	1.5500	0.70083	0.10	3.00	0.9255	0.9231	0.00560
ρ_{qv}	U	1.5500	0.70083	0.10	3.00	0.8900	0.9023	0.06242
ε_q	B	0.5000	0.14142	0.00	1.00	0.3225	0.3216	0.02142
ε_w	B	0.5000	0.14142	0.00	1.00	0.4050	0.4136	0.07221
ε_m	B	0.5000	0.14142	0.00	1.00	0.1125	0.1156	0.02654
$\rho_{c^\star}$	B	0.5000	0.14142	0.00	1.00	0.7100	0.6590	0.12524
$\rho_{\pi^{m\star}}$	B	0.5000	0.14142	0.00	1.00	0.8650	0.8501	0.04590
$\rho_{\pi^{rm\star}}$	B	0.5000	0.14142	0.00	1.00	0.5700	0.5536	0.07968
$\rho_{zi^\star}$	B	0.5000	0.14142	0.00	1.00	0.3450	0.3463	0.09084
$\rho_{tr^\star}$	B	0.5000	0.14142	0.00	1.00	0.8550	0.8508	0.04209
$\rho_{\pi^{c\star}}$	B	0.5000	0.14142	0.00	1.00	0.1650	0.1732	0.05215
$\rho_{z^{\pi q}}$	B	0.5000	0.14142	0.00	1.00	0.7900	0.6585	0.19998
$\rho_{z^{\pi w}}$	B	0.5000	0.14142	0.00	1.00	0.2300	0.2515	0.08583
$\rho_{\pi_{food}}$	B	0.5000	0.14142	0.00	1.00	0.2950	0.2916	0.08151
ρ_{z^x}	B	0.5000	0.14142	0.00	1.00	0.4050	0.4013	0.06727
ρ_{z^u}	B	0.5000	0.14142	0.00	1.00	0.8650	0.8371	0.07080
ρ_g	B	0.5000	0.14142	0.00	1.00	0.3250	0.3118	0.07713
ψ_x	B	0.5000	0.14142	0.00	1.00	0.4550	0.4483	0.06787

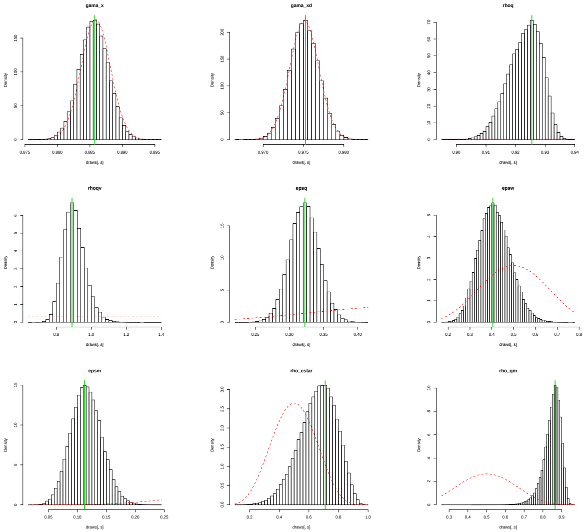
Estimation

Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
Var (π_{food})	IG	0.0005	100	1E-07	0.05	0.0004	0.0004	0.00008
Var $(c^\star)$	IG	0.0005	100	1E-07	0.05	0.0000	0.0001	0.00001
Var (g)	IG	0.0005	100	1E-07	0.00	0.0000	0.0001	0.00001
Var $(\pi^\star)$	IG	0.0005	100	1E-07	0.05	0.0001	0.0001	0.00003
Var (q^m)	IG	0.0005	100	1E-07	0.05	0.0005	0.0007	0.00016
Var (q^{mr})	IG	0.0005	100	1E-07	0.05	0.0040	0.0042	0.00097
Var (tr)	IG	0.0005	100	1E-07	0.05	0.0235	0.0252	0.00512
Var (z^i)	IG	0.0005	100	1E-07	0.05	0.0005	0.0005	0.00012
Var (z^{ie})	IG	0.0005	100	1E-07	0.05	0.0000	0.0000	0.00000
Var $(z^{\pi q})$	IG	0.0005	100	1E-07	0.05	0.0001	0.0001	0.00002
Var $(z^{\pi w})$	IG	0.0005	100	1E-07	0.05	0.0004	0.0007	0.00032
Var (z^u)	IG	0.0005	100	1E-07	0.05	0.0023	0.0045	0.00222
Var (z^x)	IG	0.0005	100	1E-07	0.05	0.0006	0.0006	0.00019

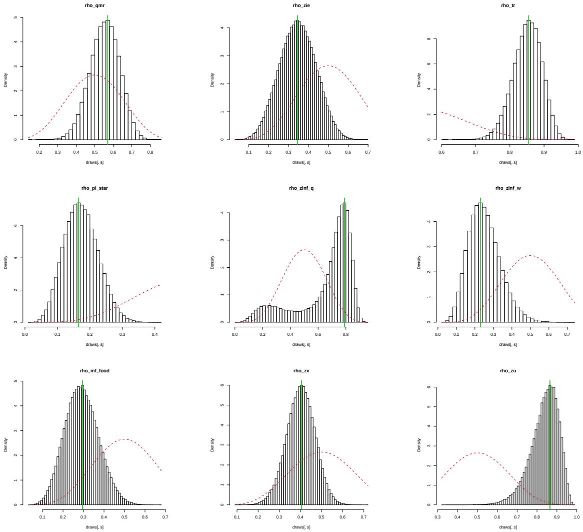
Priors - Posteriors



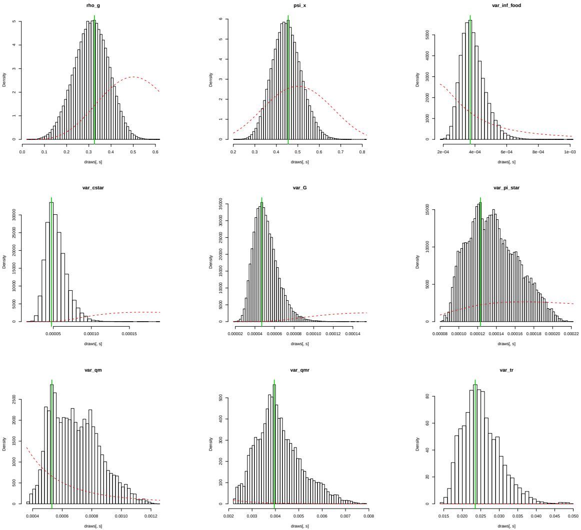
Priors - Posteriors



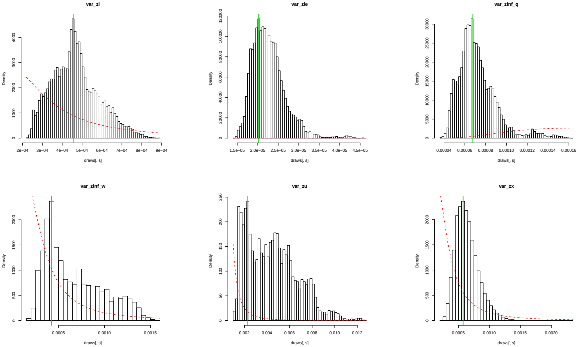
Priors - Posteriors



Priors - Posteriors



Priors - Posteriors

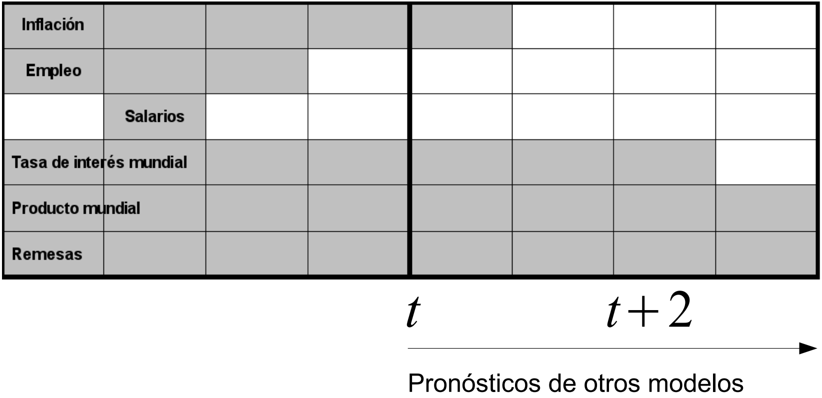


Outline

Motivation

- Using a model as a main monetary policy forecasting tool is a very different exercise from simulating a model to answer particular questions.
- To forecast for policy, we need to match our model to as much of the useful information, even if that information comes in an awkward variety of shapes and forms.
- When forecasting with a DSGE model we have to take into account the following data problems:
  - Data uncertainty
  - Steady state uncertainty
  - Anticipated shocks that can be uncertain

Data Availability



Forecasting Method

- We use a state space representation of the model where the measurement equation relates the available data to the model variables and the transition equation contains the structure of the model.
  - The measurement equation allows us to include the uncertainty of the data through a time varying variance of the measurement error.
  - Also, it allows us to handle an unbalanced data set through the use of a selection matrix.
- The forecast of the model is a mix of the Kalman filter and the filter smoother.

State-space representation

The solution to the log-linearized first order conditions can be written as

$$c_t = Gp_t \qquad\qquad c_t = GHp_{t-1} + G\epsilon_t$$
$$p_{t+1} = Hp_t + \epsilon_{t+1} \qquad\qquad p_t = Hp_{t-1} + \epsilon_t.$$

- Consequently, our transition equation is

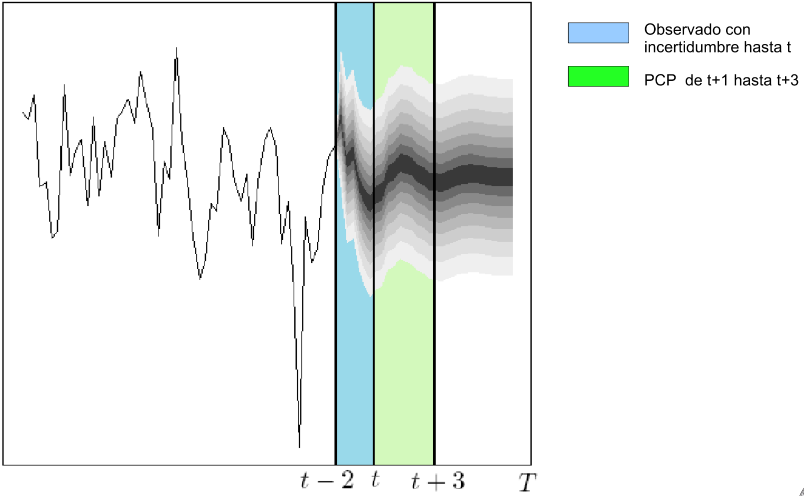
$$x_t = \begin{pmatrix} c_t \\ p_t \end{pmatrix} = \begin{pmatrix} \mathbf{0} & GH \\ \mathbf{0} & H \end{pmatrix} \begin{pmatrix} c_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} G \\ I \end{pmatrix} \epsilon_t$$

  and our measurement equation is

$$W_t y_t = W_t I_s x_t + W_t \Gamma + W_t v_t$$

  where W_t is a selection matrix and I_s is matrix where every row contains only one entry different from zero (=1) and every column has at most one entry different from zero. Γ is vector with the steady-state values.

Data uncertainty and off model information



Information about the future I

- Endogenous variables

This information is relevant for signal extraction. Which is the state of the economy?
Examples: surveys about expectations, forecasts from other models.

1. Include in the y_T vector the forecast and in x_T the expectations of the variable.
   Modified measurement equation for time T (last period of the data set) that includes one period ahead information of the endogenous variable x_j

$$\begin{pmatrix} y_{1,T} \\ \vdots \\ y_{k,T} \\ f_t(y_{j,T+1}) \end{pmatrix} = \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \\ E_t(x_{j,T+1}) \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \\ \epsilon_f \end{pmatrix}$$

2. Calibrate the variance of ϵ_f to capture the uncertainty about the forecast.

Information about the future II

- Exogenous variables

This information is used to generate a conditional forecast that assumes a path for the exogenous variables.
Example: World GDP forecasts by the IMF

- In this case we include the value of the exogenous variable at time $T + h$ as an observable variable, consequently this information enters as a surprise in period $T + h$. At time T agents are not aware of the value of this variable at time $T + h$. The measurement equation for the "observable" exogenous variable x_l at time $T + h$ becomes

$$\left(y_{l,T+h}\right) = \left(x_{l,T+h}\right) + \left(\upsilon_{T+h}\right)$$

where the variance of υ determines the weight that $y_{l,T+h}$ has on the forecast.

Summary of I and II

Then we have a time varying measurement equation of the form:

$$\begin{pmatrix} y_{1,t} \\ \vdots \\ y_{k,t} \end{pmatrix} = \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \end{pmatrix} \quad \text{for } t < T$$

$$\begin{pmatrix} y_{1,T} \\ \vdots \\ y_{k,T} \\ f_t(y_{j,T+1}) \end{pmatrix} = \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \\ E_t(x_{j,T+1}) \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \\ \epsilon_f \end{pmatrix} \quad \text{for } t = T$$

$$(y_{l,t}) = (x_{l,t}) + (\upsilon_t) \quad \text{for } T < t \leq T + H$$

Forecasting method

Our forecasts for the variables contained in the y_t vector is

$$y_{t+h}^f = I_s x_{t+h}^f + \Gamma$$

with

$$x_{T+h}^f = \begin{cases} x_{T+h}^{T+H} & \text{if } t+h \leq T+H \quad \text{Smoother} \\ \Phi^{h-H} x_{T+H}^{T+H} & \text{if } t+h > T+H \quad \text{Standard Kalman filter forecast} \end{cases}$$

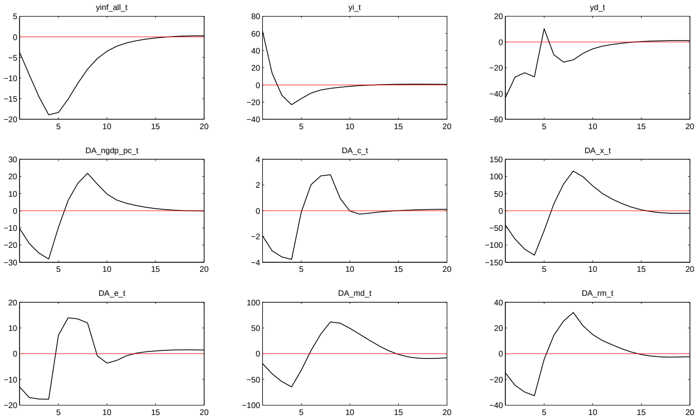
where $x_t^s = \mathrm{E}\left[x_t | Y_s\right]$ and $Y_s = (y_1 \ldots, y_t, \ldots, y_s)$.

- $\mathrm{E}\left[x_t | Y_s\right]$ is the Kalman smoother for $s > t$.
- $T + H$ is the last period for which there's at least data available for one variable in vector y_t.

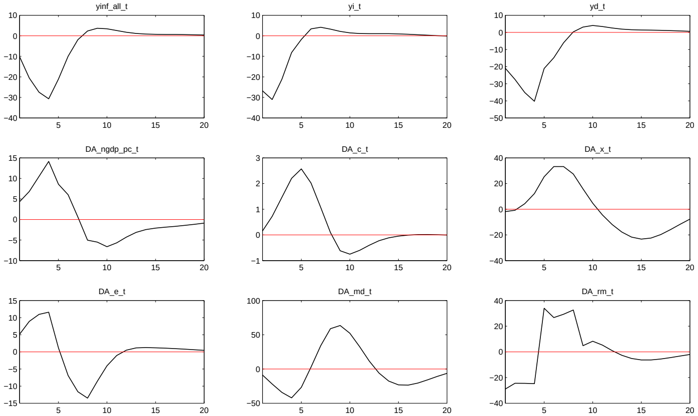
Outline

1. Introduction
2. Model Features
3. Model Structure
4. Calibration: General Methodology
5. Some Results of the Calibration
6. Estimation
7. Forecast
8. Impulse Response Analysis

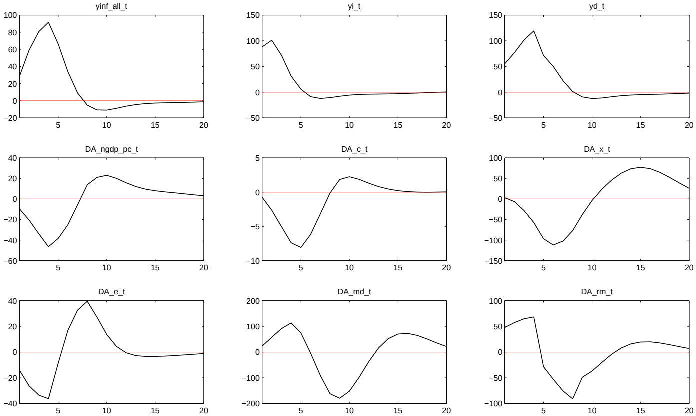
Monetary Shock



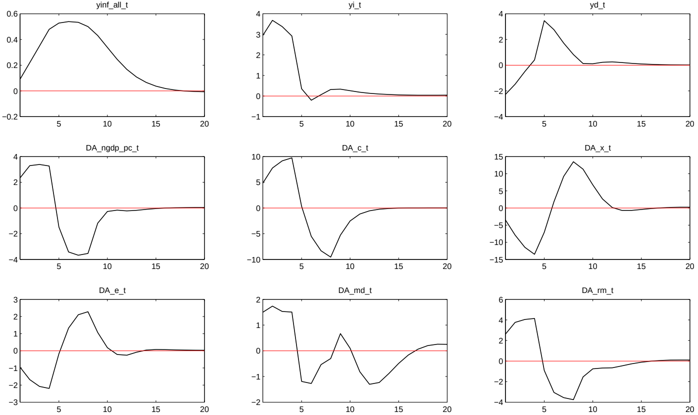
Productivity Shock



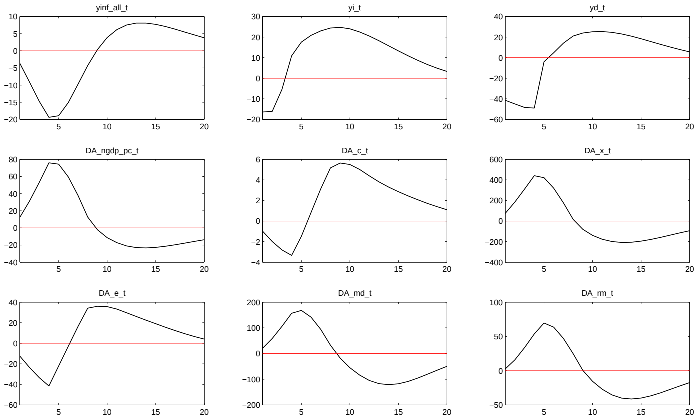
Cost Push Shock



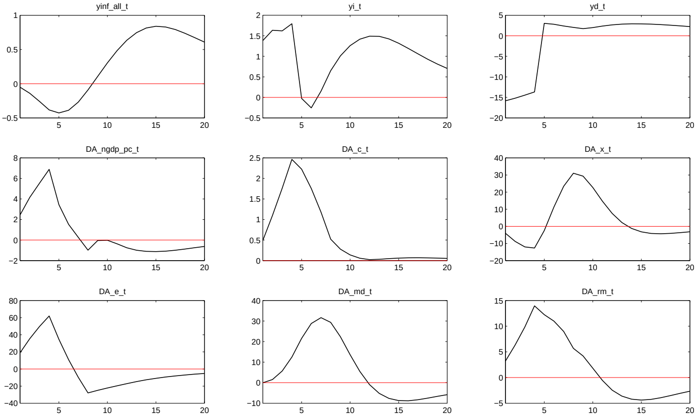
Preferences Shock



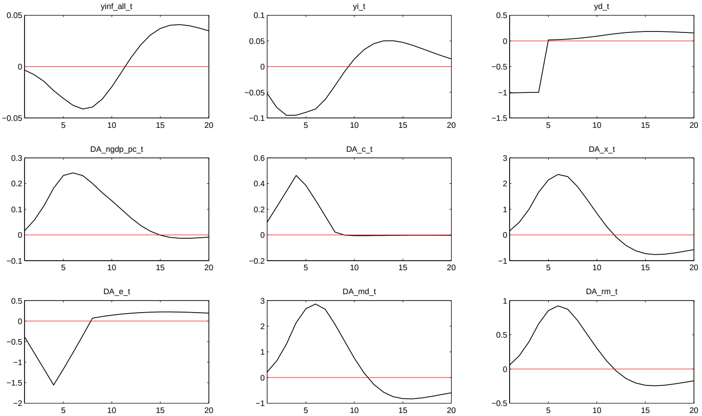
Investment Shock



External Demand Shock



Remittances Shock



Risk Premium Shock