Preliminary and Incomplete

Public Goods, Penalties and the Informal Sector

- Competitive Models of the Informal Sector: the Role of Detection and Prevention -

by

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Abstract

Recent models of the Informal Sector emphasize the free choice of sectors by firms or workers and define informal firms as those that do not pay taxes. One dominant group of models examines identical firms producing a homogeneous final good, where legal firms pay taxes and receive a public good, while non-legal firms do not pay taxes and receive a smaller amount of the public good. These models may be consistent with a stable equilibrium where there are both Formal (legal) and Informal (non-legal) firms, as observed in developing countries.

This paper first presents a generic and somewhat general version model with public goods and taxation ("g-t model"), finding that most equilibria are unstable, so that all firms choose to be either legal or non-legal, which is inconsistent with the observed facts for developing countries. However, when the government finances detection of non-legal firms and fines those firms, stable 'internal' solutions may dominate. This requires a probability of detection that falls as the number of legal firms increases.

A general model of detection, penalties and their public finance is presented and analyzed, finding that the probability of detection often, though not always, falls with the number of legal firms. The role of exogenous financing of detection is emphasized, where greater exogenous financing reduces informality and if high enough constitutes complete prevention, inducing all firms to be formal. This is both a *positive* and *normative* finding, potentially explaining the absence of informal firms in some countries, and the presence in others, while also constituting a policy instrument for eradicating informality.

As in the g-t model, the Loayza model of informality requires a probability of detection that falls as the number of legal firms rises for a stable internal solution. Thus, this general model of detection partially validates the Loayza model. It is also observed that in the Loayza model, governments may eliminate informality without penalties, by lowering the tax rate sufficiently. This also has *positive* and *normative* implications.

The paper concludes by sketching how the detection model can be incorporated into a model of the labor market with payroll taxes and penalties, where in equilibrium both legal and non-legal firms may coexist.

I. INTRODUCTION

Many recent theories of the Informal Sector have turned from non-competitive segmentation theories, such as Todaro (1969), to competitive models emphasizing the payment of taxes and adherence to norms dictated by the State, where workers and firms freely choose to be legal or non-legal. Here the "Informality Sector" is understood to refer to non-legal firms that do not pay taxes or adhere to State norms. This focus on competitive models reflects growing evidence against segmentation theories [e.g. Magnac (1991), MacIsaac and Rama (1997), Heckman and Hotz (1986)].

Recent competitive theories of the Informal Sector emphasize the role of public goods, often in the absence of any penalties for non-legal behavior. One important competitive model is that of Marcouiller and Young (1995). That model assumes that legal firms produce one good and non-legal firms produce a different good. They seek to show how the State may be empowered to impose high taxes and extract high levels of rents, without resorting to persecuting and penalizing non-legal firms. While rising tax rates induce firms to move to the non-legal sector, this is limited because this leads to high relative supplies of the non-legal good. And this rising relative supply of the non-legal good depresses the prices of that good, vis-à-vis the legal good. This constitutes a self-regulating mechanism that determines the size of the non-legal sector and limits its growth in the face of a "predatory" State.

Another group of models assumes a homogenous final good. Grossman and Yoshiaki (2003), for example, present a model of firms producing a homogeneous final good, requiring a public good for production. Non-legal firms produce a substitute public good. This model, assumes that firm's sectoral choices are governed by differences in exogenous endowments of capital and the structure of taxation. Another often cited model is Loayza (1996). That model is similar to the Grossman-Yoshiaki model, but differs in that it assumes that firms are identical and that non-legal firms have access to some part of the public good, because the public good is not entirely "excludable".

The Loayza model seeks to explain the simultaneous presence of both legal and non-legal firms in a stable equilibrium. To achieve this result, the model assumes that non-legal firms face penalties and that the State dedicates resources towards the detection of non-legal firms, avoiding tax payments. The model crucially assumes a structure of the probability of detection that is neither convincingly motivated nor modeled. Moreover, the model also assumes that the public good is strictly rival.

This paper focuses upon competitive models of the Informal Sector, where firms are identical and produce a homogeneous final good. Section II develops a simple generic model of public goods (g) and taxation (t), the g-t model. It models the public finance of public good production and offers a general formulation that allows for variations in three key parameters: the returns to scale in the production of the public good; the degree of rivalry of the public good; and the degree of excludability of the public good. Analysis of different parameter structures reveals some stable internal equilibria, but a preponderance of unstable equilibria. As such, the g-t model cannot adequately explain the dominant facts for developing countries, where both legal and non-legal firms coexist (a stable internal solution, or SIS). These results point towards the need for endogenous detection and penalties as a key to understanding SIS in some countries. Section II ends by showing how the existence of endogenous penalties, where the probability of detection varies with the size of the non-legal sector, can explain the coexistence of legal and non-legal firms (the g-t-p model).

Section III develops a model of detection and penalties, and their public finance. Detection is assumed to always be partly self-financed, from fines on non-legal firms. In addition, detection may be financed by general tax revenues. Under most parameter assumptions, this model leads to a probability of detection that increases with the share of non-legal firms. That result is required by Loayza and by the g-t-p model for stable internal solutions.

Section IV synthesizes the Loayza (1996) model, L96. While L96 assumes the existence of detection and penalties, we point out that lowering the tax rate below a threshold level will eliminate non-legal firms, or 'eradicate the Informal Sector'. If the tax rate is downwardly rigid, penalties may be needed for a stable internal solution, however. This is the case analyzed by L96. As explained, L96 assumes, but does not adequately explain or model, that the probability of detection increases with the share of non-legal firms. Thus, the model in Section III provides key support to the L96 model.

Section V applies the lessons of the detection model in Section III and the Loayza model of Section IV to the labor market and payroll taxes. It is shown that these two models provide a basis for modeling how payroll taxes accompanied by efforts to identify and fine firms shirking payroll taxes, can lead to the coexistence of both legal and non-legal firms.

A variety of normative and positive conclusions emerge from these explorations. In particular, we find that increases in general tax funds dedicated to taxation can raise the initial probability of detection when there are no non-legal firms and thereby either lower or reduce to zero the share of non-legal firms in equilibrium. Elimination of 'informality', should that be a desired social goal, could be achieved initially by designating high levels of general tax funds to detection, while more gradually working to raise the efficiency of detection (increasing β), through technological innovation and modernization. Increasing the level of the fine charged to the non-legal firm that is detected, M, will increase the slope of the probability of detection, and shift that curve leftwards. This will lower the level of non-legal firms in equilibrium. However, this will not by itself lead to the complete elimination of non-legal firms. Increases in β or T are required to eliminate entirely non-legal firms, or to entirely eradicate 'informality'.

II. THE GENERIC $g - t$ MODEL OF PUBLIC GOODS, TAXATION AND INFORMALITY (NON-LEGALITY)

Here we present a simple, generic model of the finance of public good production that allows for variations in three key parameters: the returns to scale in the production of the public good, α (as alpha rises, returns to scale increase); the degree of rivalry of the public good, *m* (as *m* falls from 1, rivalry falls); and the degree of excludability of the public good, γ (as gamma rises from 0, excludability falls).

Total profits V_L and V_N in legal and non-legal sectors, respectively, are given by:

$$V_L = (pQ - wE - rK) + (g - t), \tag{2.1}$$

and

$$V_N = (pQ - wE - rK) + \gamma g. \tag{2.2}$$

Where '*g*' is the public good per firm in the legal sector. γg is the per firm public good in the non-legal sector. If gamma is zero, non-legal firms receive no public goods.

$$g = \frac{G}{[L + \gamma N]^m}. \tag{2.3}$$

Here,

L = number of legal firms;

N = number of non-legal firms;

γ = coefficient that indicates the degree of exclusion of the non-legal firms[1], $0 \le \gamma \le 1$;

m = coefficient that indicates the degree of rivalry of the public good[2], $0 \le m \le 1$.

Production of the aggregate public good, *G*, is a function of available funding, which equals the lump sum per firm tax, *t*, times the number of legal firms. Thus,

$$G = (Lt)^\alpha, \tag{2.4}$$

with $\alpha > 0$ the coefficient that indicates the returns to scale and where *t* is a lump-sum tax per legal firm, $t \ge 1$. The per firm public good for legal firms is then:

[1] A value of $\gamma = 0$ indicates that the public good is completely excludable; $\gamma = 1$ implies that the public good is completely non-excludable.

[2] When $m = 0$ the public good is non-rival; when $m = 1$ the public good is rival.

$$g = \frac{(Lt)^{\alpha}}{[L+\gamma N]^{m}}. \qquad (2.5)$$

Assuming that the total number of firms is constant and normalizing such that $L, N \in [0,1]$ and $L + N = 1$, we can rewrite the (2.5) as:

$$g = \frac{(Lt)^{\alpha}}{[L(1-\gamma)+\gamma]^{m}}. \qquad (2.6)$$

The condition of equilibrium (profits equalization) between legal and non-legal sectors is given by:

$$g - t = \gamma g \text{. Equivalently, } g = \frac{t}{1-\gamma}. \qquad (2.7)$$

We will denote the right-hand side expression, $\frac{t}{1-\gamma}$, by $\tilde{t}$.

Note that if $g > \tilde{t}$ then profits in the legal sector are higher than profits in the non-legal sector and vice versa. In equilibrium, the fraction of legal (and non-legal) firms depends on the function *g(L).* If the slope of *g(L)* is negative, then there may exist an internal solution, *L**, that is stable (see Figure 1, column 1). At any initial distribution of firms below (above) *L**, profits in the legal sector are higher (lower) than profits in the non-legal sector, because $g > \tilde{t}$ ($g < \tilde{t}$). Therefore more firms will choose to be legal (non-legal). This continues until reaching the point *L**, where firms are indifferent between being legal or being non-legal.

On the other hand, if the function *g(L)* has a positive slope, while there could be an internal equilibrium, it will be unstable (see Figure 1, column 2). Then, if L falls below *L**, $\tilde{t}$ will exceed the per firm public good, *g*, so that *L* will fall to zero. And if *L* were to rise above *L**, *g* will exceed $\tilde{t}$, so that *L* increases to one. Thus, when *g'(L)*>0, while there may exist an internal solution, it will be unstable. It will collapse to either of two cases, where *L*=0 or *L*=1.

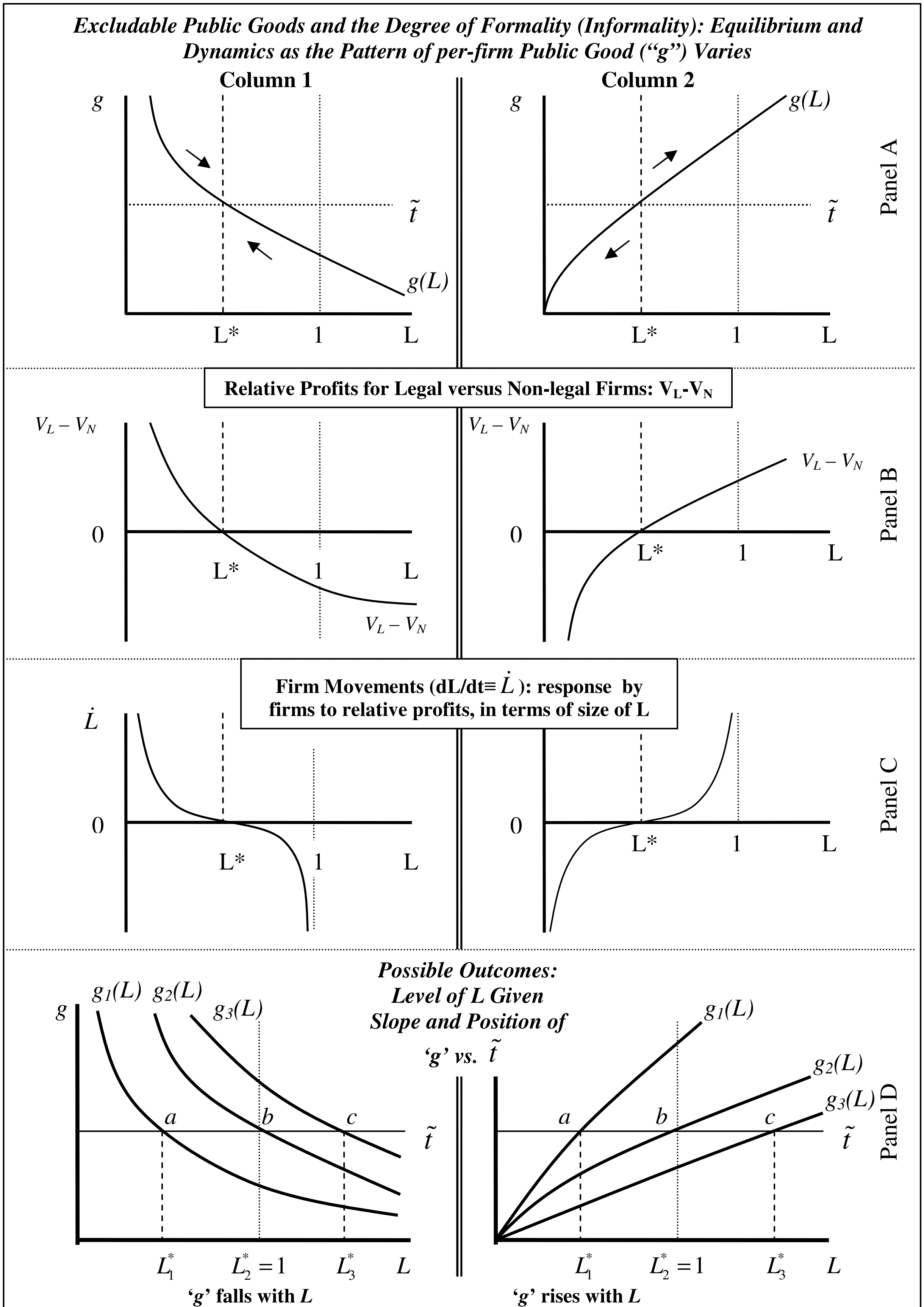

Figure 1. Graphic analysis of the fraction of firms that choose to be legal in equilibrium.

The slope of *g(L)* is given by:

$$\frac{\partial g}{\partial L}=\frac{\alpha L^{\alpha-1}t^{\alpha}[L(1-\gamma)+\gamma]^{m}-L^{\alpha}t^{\alpha}m[L(1-\gamma)+\gamma]^{m-1}(1-\gamma)}{[L(1-\gamma)+\gamma]^{2m}}. \quad (2.8)$$

Because the denominator is typically positive[3], the sign of this derivative depends on the sign of the numerator:

$$\frac{\partial g}{\partial L}\gtreqless 0 \text{ as } \alpha L^{\alpha-1}t^{\alpha}[L(1-\gamma)+\gamma]^{m}\gtreqless L^{\alpha}t^{\alpha}m[L(1-\gamma)+\gamma]^{m-1}(1-\gamma). \quad (2.9)$$

This expression simplifies to:

$$\alpha\gamma\gtreqless L(1-\gamma)(m-\alpha).^{4} \quad (2.10)$$

If $\alpha>m$ or $\alpha=m$ and $\gamma>0$, *g'(L)* is positive. For other cases, the sign of the derivative is indeterminate, without assuming specific parameter values. The synthesis for the sign of the derivative *g'(L)* is reported in Table 1.

Table 1. Sign of the derivative *g'(L)*

α versus *m*	**Values of γ**	
	$\gamma=0$ (1)	$\gamma\in(0,1)$ (2)
$\alpha>m$ (includes $m=0$)	(+)	(+)
$\alpha=m$	0	(+)
$\alpha<m$	(-)	$g'(L)\gtreqless 0$ as $\alpha\gamma\gtreqless L(1-\gamma)(m-\alpha)$

Analysis of the Equilibrium Fraction of Legal Firms when there is no Closed Form Solutions for *L**

When $\gamma\in(0,1)$ and $m\in(0,1]$ there are no closed form solutions for *L**. Next, we analyze the empirically likely case where the per firm public good is partially excludable ($\gamma\in(0,1)$), allowing m to vary.

The per capita public good, *g*, is always equal to zero, when *L* equals zero ($g(L=0)=0$, from (2.5)) and the derivative of *g(L)* at *L*=0 is positive.[5]

[3] It is zero if $\gamma=L=0$.

[4] If we do not normalize L and N, the restriction for the derivative being positive or negative is $\alpha\gamma\gtreqless\frac{L}{\bar{L}}(1-\gamma)(m-\alpha)$, where $\bar{L}=L+N$. This restriction is equal to that when *L* and *N* are normalized.

[5] From (2.10),

Depending on whether α is greater or less than *m*, $g(L)$ is monotonically increasing ($\alpha > m$); or $g(L)$ is non-monotonic ($\alpha < m$) with a maximum at *L'*, and the function *g(L)* tends to be concave.

Case A: $\alpha > m$

From (2.10) and Table 1 we know that when $\gamma \in (0,1)$ and $\alpha > m$, the derivative *g'(L)* is always positive (see Figures 2a to 2c).

Case B: $\alpha < m$

Here, as mentioned above, *g(L)* is non-monotonic, rising to a point *L'* and falling afterwards. *L'* is given by:

$$L' = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{\alpha}{m-\alpha}\right). \qquad (2.11)$$

From (2.11), we see that *L'* is not necessarily restricted to the interval (0,1). *L'* may lie in the interval (0,1) if γ is low, α is low, or both γ and α are low. Simulations show that the possible patterns for *g(L)* in this case can be summarized in Figures 2d and 2e.[6]

$$\alpha\gamma \gtreqless L(1-\gamma)(m-\alpha) .$$

Or, with *L*=0,

$$\alpha\gamma > 0 .$$

Given $\alpha > 0$ in general and $\gamma > 0$ here, then *g'*(*L*=0)>0.

[6] Note that in simulations, the case where *g(L)* is non-monotonic and $\tilde{t}$ crosses below its maximum (twice) is not observed.

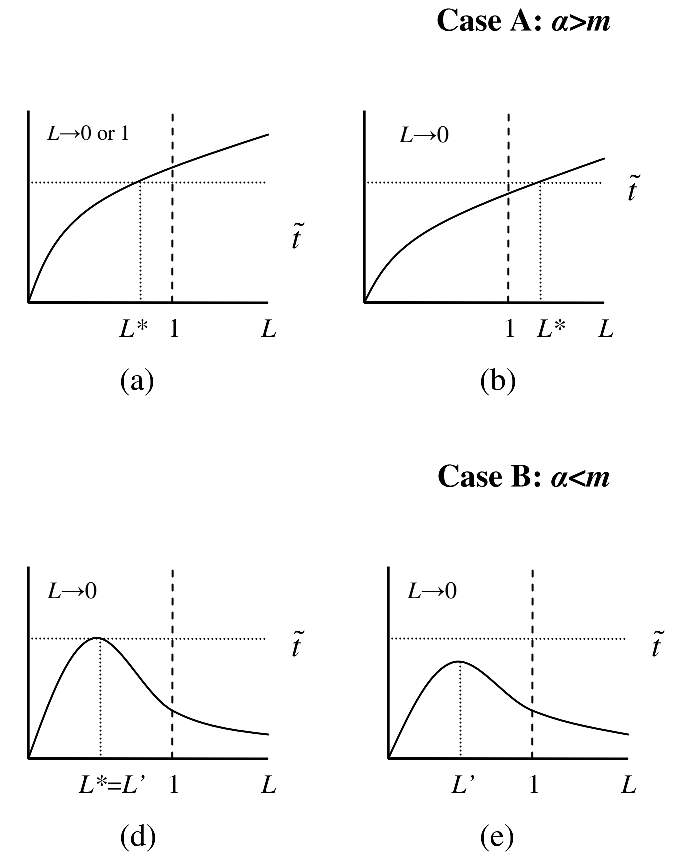

Figure 2. Possible patterns for the public good per firm (g) when the public good is partially excludable ($\gamma \in (0,1)$) - Varying α and m.

We examined a range of specific parameter combinations. Table 2, below, reports where there are internal solutions for permutations of: γ= 0.2, 0.8; m= 0, 0.5, 1; t=1, 2, 3, 5; α=0.5, 1, 1.5, 2.5, 3, 4.

Table 2. Partially Excludable Public Good ($\gamma \in (0,1)$) **- Cases Where Internal Solutions Exist -**				
Notation: Spaces " ": there is no internal solution "*": $L^* \in (0,1)$ and $L \to 0$ or 1 (unstable) "**": $L^* \in (0,1)$ and $L \to 0$ (unstable)				
Highly Excludable Public Good: $\gamma = 0.2$				
	Tax (t)			
	1	**2**	**3**	**5**
$m = 1$				
$\alpha = 0.5$	**			
$\alpha = 1$				
$\alpha = 1.5$		*	*	*
$m \in [0,1)$				
$\alpha = 0.5$				
$\alpha = 1$				
$\alpha = 1.5$		*	*	*
Weakly Excludable Public Good: $\gamma = 0.8$				
$\forall m$				
$\alpha = 2.5$				*
$\alpha = 3$			*	*
$\alpha = 4$		*	*	*

Summary and Interpretation

The foregoing g-t model is one where legal firms pay taxes, receive some of the public good and where non-legal firms do not pay taxes and receive less of the public good (none or a smaller share than legal firms). The model is somewhat general in that it contemplates differing returns to scale in the production of the public good, differing levels of rivalry of the public good and differing degrees of excludability of the public good. In theory, no penalties are required for a stable internal solution, with both legal and non-legal firms, to exist.

However, analysis of this model yields few cases where a stable internal solution exists. Complex interactions between returns to scale, rivalry, and excludability exist. In the empirically important case of a partially excludable public good, in general there is no stable internal solution. These results are at odds with the reality of most developing countries, where legal and non-legal firms coexist.

However, if we add detection and penalties for non-legal firms to this model, the modified model can lead to stable internal solutions. Moreover, while many authors (notably Grossman and Yoshiaki, 2003) have focused on models with public goods without penalties, in practice most governments do dedicate resources to detecting and penalizing non-legal firms.

The g-t Model with Detection and Penalties: g-t-p Model

Here we modify the basic g-t model by introducing detection and penalties. If the State dedicates resources to detection, so that there is a positive probability of detecting non-legal firms, ρ, and fines detected non-legal firms in the amount *M*, then the equalization of profits between legal and non-legal sectors implies:

$$g - t = \gamma \cdot g - \rho M \ , \tag{2.12}$$

or

$$\tilde{g} \equiv \left(g + \frac{\rho M}{(1-\gamma)} \right) = \left(\frac{t}{(1-\gamma)} \right) \equiv \tilde{t} \ . \tag{2.13}$$

This is the same condition for equilibrium as before, except for that instead of '*g*' on the left hand side we have $\tilde{g}$, which is equal to '*g*' plus the term $\frac{\rho M}{(1-\gamma)}$. It follows directly that if the derivative of the probability of detection with respect to the fraction of legal firms, or $\frac{\partial \rho}{\partial L}$, is negative and large relative to the derivative of '*g*' with respect to *L*, then the derivative of $\tilde{g}$ with respect to *L* can be negative. Then if there is an internal solution it will be stable.

$$\text{For } \frac{\partial \tilde{g}}{\partial L} < 0 \text{, requires } \frac{\partial \rho}{\partial L} < -\frac{\partial g}{\partial L}\left(\frac{1-\delta}{M} \right) < 0 \, . \tag{2.14}$$

Thus, for $\left| \frac{\partial \rho}{\partial L} \right|$ sufficiently large, the introduction of detection and penalties will lead to a stable internal solution.

In the following section we develop a simple, general model of detection and penalties. For a number of important parameter values, the penalty falls with the fraction of non-legal firms, as required in the g-t-p model, above.

III. A GENERAL MODEL OF THE PROBABILITY OF DETECTION AND ITS PUBLIC FINANCE

This section develops a model of endogenous detection with penalties. Above we saw that a probability of detection that falls with the share of legal firms (rises with the share of non-legal firms) may lead to stable internal solutions with both legal and non-legal firms, in the g-t-p model. And in the subsequent section, we will examine the Loayza (1996) model which assumes, but does not prove, this to be the case.

We begin by assuming that there are *H* firms, *N* non-legal and *L* legal, or *L*+*N*=*H*. The State dedicates resources to generating a random sample of all firms, *n*. This random sample is a function of resources, *Y*. Because the sampling is random and once a firm is sampled its nature is revealed (to be an *L* or *N* firm), the probability of detection of a non-legal firm is given by the ratio of the sample to the overall population:

$$\rho = \frac{n(Y)}{H}. \tag{3.1}$$

The $n(Y)$ is a function of the resources, *Y*, available to produce that sample. We assume a general (AK type) production function for the generation of the sample:

$$n(Y) = \beta Y^{\lambda}, \tag{3.2}$$

The parameter β reflects the efficiency in the production of the sample. As β goes to infinity, information becomes perfect and the State knows the actions of all firms without cost or resort to sampling. Returns to scale in the production of the sample are reflected by λ.

Government resources for detecting firms that do not pay taxes may derive from general tax funds, *T*, and from funds collected from non-complying firms which are caught and penalized. Thus, the probability of detection is endogenous and will depend upon the number of non-legal firms, *N*.

The expected amount of penalties collected is equal to the probability of detection, ρ, times the fine, *M*, times the number of non-legal firms, *N*. We also include a corruption term via ε, where 0<ε<1. Then,

$$Y = T + \varepsilon \rho N M .[7] \tag{3.3}$$

Thus:

[7] Or $Y = (T - FC) + \varepsilon \rho N M$, where FC are fixed costs.

$$\rho = \frac{\beta\left[T + \varepsilon\rho NM\right]^{\lambda}}{H}\ .^{8} \qquad (3.4)$$

Anticipating results below, note briefly that as T or β, or both, rise, the intercept of the probability of detection rises. Thus, for high values of T or β the expected penalty could be high enough to prevent firms from ever contemplating non-legality. In a *normative* sense, this offers the State policy instruments for preventing *informality,* and in a positive sense may help explain variations in the share of firms that are non-legal, or informal.

Case A: No Autonomous Financing of Detection (T=0)

When there is no autonomous financing, the closed form solution for ρ is as follows:

$$\rho^* = \left[\frac{\beta}{H}(\varepsilon NM)^{\lambda}\right]^{\frac{1}{1-\lambda}}. \qquad (3.5)$$

The derivative of ρ^* with respect to N is given by:

$$\frac{\partial\rho^*}{\partial N} = \frac{\lambda}{1-\lambda}\left(\frac{\beta}{H}(\varepsilon M)^{\lambda}\right)^{\left(\frac{1}{1-\lambda}\right)} N^{\left(\frac{\lambda}{1-\lambda}-1\right)}. \qquad (3.6)$$

Thus $\frac{\partial\rho^*}{\partial N} \gtrless 0$ as $\lambda \lessgtr 1$[9]. (3.7)

Case B: Autonomous Financing of Detection (T>0)

When detection is financed by funds from general taxes, so that $T > 0$, there is no explicit solution for the probability of detection, ρ^*. Therefore, we analyze the slope of $\rho^*(N)$ by calculating the implicit derivative of ρ^* with respect to *N*. Define the implicit function, *F*, as follows:

$$F = \rho - \frac{\beta}{H}\left[T + \varepsilon\rho NM\right]^{\lambda}. \qquad (3.8)$$

Then,

$$\frac{d\rho^*}{dN} = -\frac{F_N}{F_\rho}, \qquad (3.9)$$

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[8] Technically, $\rho = \min\left[\frac{\beta[T+\varepsilon\rho NM]^{\lambda}}{H}\ ,\ 1\right]$.

[9] $\frac{\partial\rho^*}{\partial N}$ is not defined for λ=1.

where F_N and F_ρ are the partial derivatives of F with respect to N and ρ, respectively. These partial derivatives are:

$$F_N = -\frac{\lambda\beta\varepsilon\rho M}{H}\left[T+\varepsilon\rho NM\right]^{\lambda-1}, \tag{3.10}$$

and:

$$F_\rho = 1-\frac{\lambda\beta\varepsilon NM}{H}\left[T+\varepsilon\rho NM\right]^{\lambda-1}. \tag{3.11}$$

Because the numerator of the implicit derivative of ρ with respect to N, $(-F_N)$, is positive, the sign of $\frac{\partial\rho^*}{\partial N}$ is equal to the sign of the denominator F_ρ:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } 1 - \frac{\lambda\beta\varepsilon NM}{H}\left[T+\varepsilon\rho NM\right]^{\lambda-1} \gtrless 0. \tag{3.12}$$

Multiplying (3.12) through by $\frac{H}{\beta\left[T+\varepsilon\rho NM\right]^{\lambda}}$, we obtain:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{H}{\beta\left[T+\varepsilon\rho NM\right]^{\lambda}} - \frac{\lambda\varepsilon NM}{T+\varepsilon\rho NM} \gtrless 0. \tag{3.13}$$

Noting that the term $\frac{H}{\beta\left[T+\varepsilon\rho NM\right]^{\lambda}}$ is equal to ρ^{-1}, we may rewrite (3.13) as follows:

$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{(T/\varepsilon NM)+\rho}{\rho} \gtrless \lambda, \tag{3.14}$$

or, equivalently:

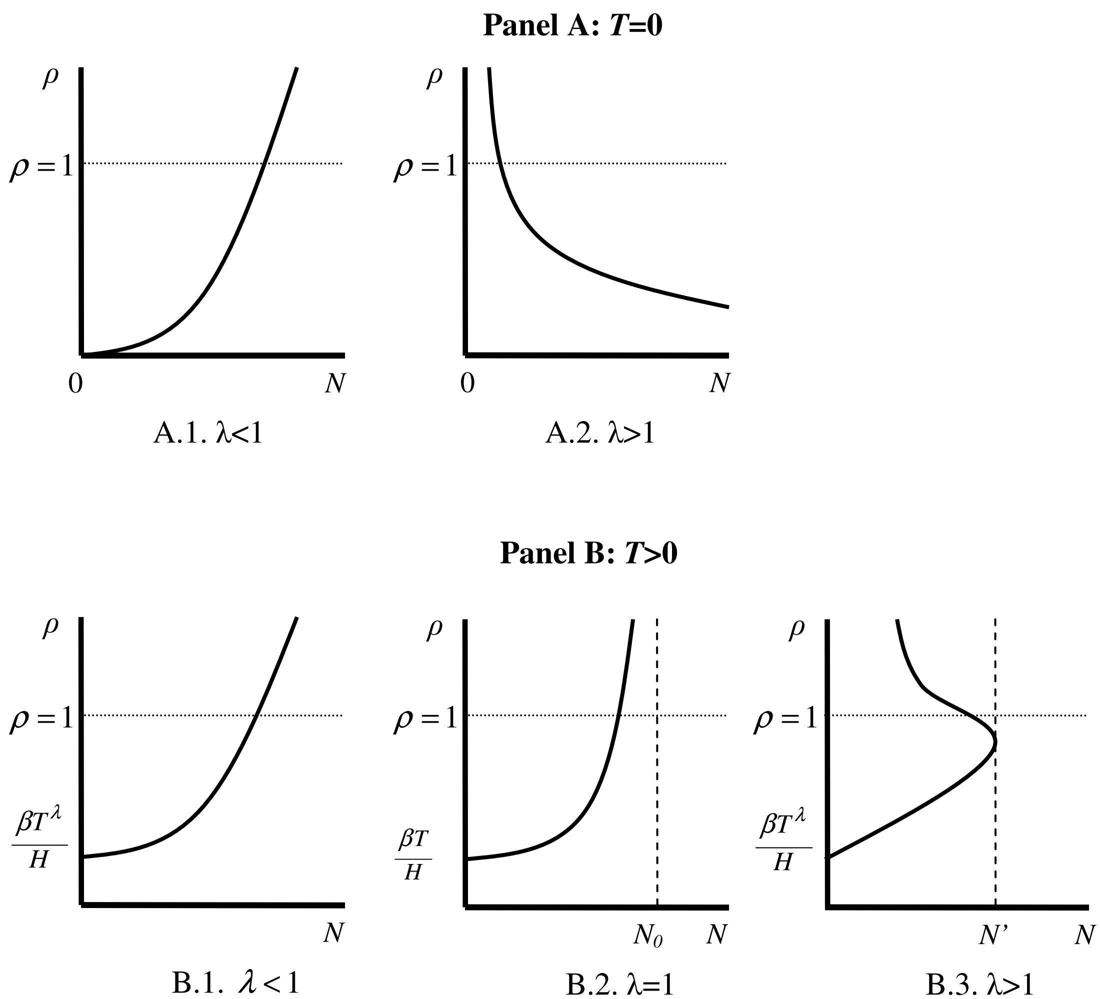
$$\frac{\partial\rho^*}{\partial N} \gtrless 0 \text{ as } \frac{T}{\varepsilon\rho NM} \gtrless \lambda-1. \tag{3.15}$$

Noting that in the empirically likely case, where detection receives some exogenous financing, or $T > 0$, the term $\frac{(T/\varepsilon NM)+\rho}{\rho}$ exceeds one. Thus, when $\lambda \le 1$, the derivative of the probability of detection with respect to N will be positive.

These results are summarized below in Table 3. The dynamics of the model is illustrated in figure 3.

Table 3. Derivative of the Probability of Detection with Respect to the Size of the Non-legal Sector

T: quasi exogenous financing of sample production	λ: returns to scale in production of sample	Sign of $\frac{d\rho}{dN}$ with $N>0$
$T=0$	>1	-
	$=1$	?
	<1	+
$T>0$	>1	$\frac{\partial\rho^*}{\partial N} \gtrless 0$ as $\frac{T}{\varepsilon\rho NM} \gtrless \lambda-1$
	$=1$	+
	<1	+



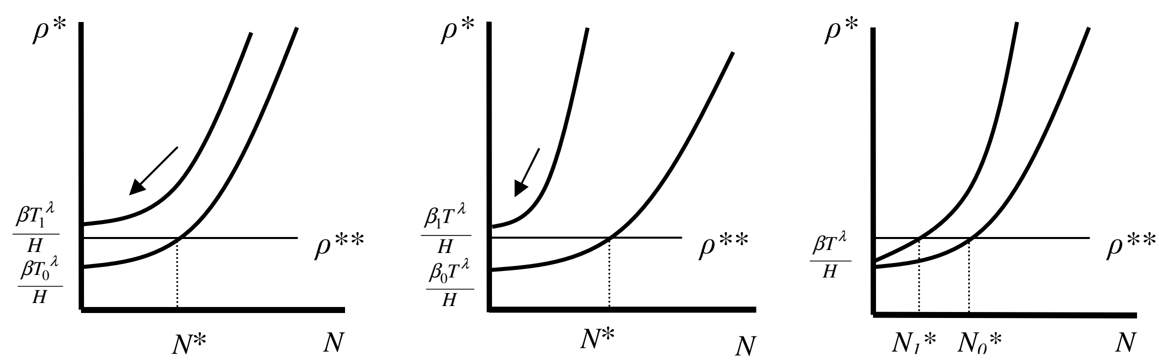
Notation:

N: number of non-legal firms;
ρ: probability of detection;
T: autonomous financing;
λ: returns to scale in the production of the sample;
β: efficiency in the production of the sample; and
H: total number of firms.

Figure 3. The probability of detection.

We can integrate this detection probability model into different models, such as g-t Model and Loayza's (1996). Assume that in the absence of detection and fines, legal firms have higher profits than non-legal firms. Then the addition of detection and penalties for non-legal firms will lower profits for non-legal firms, while not affecting profits of legal firms. Thus, it will often be the case that there exists a threshold level of the probability of detection at which profits in both sectors are equalized. If the probability of detection varies with the share of legal (or non-legal) firms, as in the model above, the intersection of the probability of detection curve, $\rho^*(N)$, and the threshold level of the probability, ρ^{**}, determines the equilibrium level of non-legal (legal) firms, *N* (L*)*. And if *N** is less than the total number of firms, *H*, this constitutes an internal solution. Finally, if, as we have shown is often the case, the probability of detection rises with *N* (falls with *L*), this internal solution will be stable.

This logic is illustrated in figure 4, below. V_L and V_N are the net profits of a firm choosing to be in the legal (*L*) or non-legal (*N*) sector. Assume there is a ρ ** such that $V_L = V_N$. Figure 4 shows a possible equilibrium and the dynamics when *T*, *β* or *M* vary.



a) Increases in *T*: $T_1 > T_0$ b) Increases in *β*: $\beta_1 > \beta_0$ c) Increases in M

Figure 4. Probability of detection and possible equilibriums varying parameters: *T*, *β* and *M*.
When *T* or *β* increases, the intercept of the function *ρ*(N)* shifts upward. Additionally, when *β* increases, there is a large increase in the slope of the curve. When *M* increases, the slope increases, but the intercept remains unchanged.

Normative and Positive Implications of the Detection Model – Early Deterrence

It is important to note that if the intercept of the probability function is sufficiently high, there will never be any non-legal firms. This has both *positive* and *normative* implications. Countries without "informal" firms may be those which attain high probabilities of detection even when there are no non-legal firms. The intercept of the probability function rises with β or *T*. Thus, the autonomous detection financing, *T*, constitutes a central policy instrument for the State, to achieve normative goals. By increasing this autonomous financing high enough, the State can eliminate "informality", defined here as non-legal firms. Increases in the degree of perfection of information, β, also shift the intercept of the probability function upwards. However, the State's ability to affect the degree of information perfection would arise from technological innovation, modernization, or structural reforms affecting detection, which are more difficult to effect than changes in exogenous financing of detection, *T*. Elimination of 'informality', should that be a desired social goal, could be achieved initially by designating high levels of general tax funds to detection, while more gradually working to raise the efficiency of detection (increasing β), through technological innovation and modernization.

Note that increasing the level of the fine charged to the non-legal firm that is detected, *M*, will increase the slope of the probability of detection, and shift that curve leftwards. This will lower the level of non-legal firms in equilibrium. However, this will not by itself lead to the complete elimination of non-legal firms. Increases in β or *T* are required to eliminate entirely non-legal firms, or to entirely eradicate 'informality'.

Summary

The foregoing model may provide insights into how different structures of public goods, taxes and penalties lead to differing shares of non-legal firms across countries and within countries over time. In general, high levels of exogenous funding for detection will lower or even eliminate the percentage of firms which are non-legal. This may be used to explain differences over countries and over time, or as policy instruments to change outcomes.

In the next section we synthesize Loayza (1996), which presents a model of public goods, taxation and detection. That model assumes a probability of detection but neither explains nor models it. The above model of detection, therefore, is a crucial complement to the Loayza (1996) model, lending it greater clarity and legitimacy. At the same time, because we have found that the probability of detection is not always an increasing function of the share of non-legal firms, we find that the conclusions of Loayza (1996) are not completely robust to different assumptions regarding the detection process.

IV. LOAYZA: MODELING PENALTIES WITH PROBABILITY MODEL

One much noted article which models firm sector choice with public goods, taxation and penalties for firms that do not pay taxes is Loayza (1996), or L96. L96 assumes a homogeneous final good and identical firms. L96 assumes an AK production function where the public good is necessary for production, regardless of the firm's sector choice, and that the public good is partially non-excludable, $\gamma \in (0,1)$. Unlike Grossman and Yoshiaki (2003), non-legal firms do not produce a substitute public good, so that given the assumption that the public good is a necessary input to production, it follows that the partial non-excludability of the public good is necessary if non-legal firms are to exist.

The profits of legal and non-legal firms given by V_L and V_N are the net profits for firms if they choose to be legal or non-legal:

$$V_L = (1-\tau)Ag^b K - rK \text{ (legal)} \tag{4.1}$$

and:

$$V_N = (1-\pi)A(g\gamma)^b K - rK \text{ (non-legal).} \tag{4.2}$$

Here g is the per firm public good and A is an exogenous productivity parameter. Gross output in the legal sector is $Ag^b K$ and $A(g\gamma)^b K$ in the non-legal sector, where the non-legal firm receives $(g\gamma)$ units of the public good, and where Loayza assumes that gamma is positive but less than one. τ is the tax rate and π is the expected penalty rate. This expected penalty consists of the probability of detection, ρ, times the penalty rate, μ (the total per firm penalty for firms caught and fined, M, is equal to the penalty rate times the non-legal firms production: $\mu \cdot (g\gamma)^\lambda K$).

Note that, in addition to Loayza's formulation, it is possible here to bring to bear the formulation for the public good and the per firm public good presented in the g-t model, above:

$$G = \left[c\tau g^b K - T\right]^\alpha \text{. Then } g = \frac{[c\tau g^b K - T]^\alpha}{[L(1-\gamma)+\gamma]^m}. \tag{4.3}$$

In contrast to L96, this formulation models more explicitly the public finances of the production of the public good, and introduces a net deduction of total tax funds generated to be applied to the detection of non-tax paying firms, *T*.

The equilibrium where firms are indifferent between being legal or non-legal is where net profits equalize: $V_L = V_N$. Thus:

$$(1-\tau)g^b K - rK \;=\; (1-\pi)(g\gamma)^b K - rK. \tag{4.4}$$

Noting that $\pi = \rho\mu$, we have:

$$1-\tau=(1-\rho\mu)\gamma^{b}\,. \tag{4.5}$$

This equality defines a threshold level of the probability of detection, ρ^{**}, which will equalize profits in both sectors:

$$\rho^{**}=\frac{1}{\mu}\left(1-\frac{(1-\tau)}{\gamma^{b}}\right). \tag{4.6}$$

Note that for the threshold probability to be positive, ρ^{**}>0, this requires $\gamma^{b}>(1-\tau)$. If the public good is "fully non-excludable" ($\gamma=1$), the equilibrium probability becomes $\rho^{**}=\frac{\tau}{\mu}$. If the public good is completely excludable ($\gamma=0$), the non-legal sector disappears, because they have no access to the necessary public good.

Loayza focuses upon a potential stable internal equilibrium, where both legal and non-legal firms coexist, consistent with the observed reality in developing countries. However, it is useful to note that this model points to a normative result. If the government lowers the tax rate enough, it can induce all firms to choose to be legal, without the introduction of penalties. If there are no penalties, equation (4.5) simplifies to $(1-\tau)=\gamma^{b}$, which defines a threshold level of taxation, τ^{*}:

$$\tau^{*}=1-\gamma^{b}\,. \tag{4.7}$$

If the government sets the tax rate below τ^{*} ($\tau=\tau^{*}-\varepsilon$), then all firms will choose to be legal.

Thus, for Loayza's model to be relevant, in a *positive* sense, there must be reasons why the government would not lower taxes enough to induce full compliance with the law. And in a *normative* sense, it is important to keep this potential policy avenue in mind: non-legal firms, or "Informality", could be potentially eliminated by lowering the tax rate sufficiently. If the tax rate is rigid and/or the public good is almost non-excludable (γ is near to one), penalties would be necessary. There is an internal solution of the model with penalties, if the following condition is satisfied:

$$\rho^{**}>0 \text{ as } \gamma^{b}+\tau>1. \tag{4.8}$$

The equilibrium and dynamics that L96 emphasizes are summarized in the following figure:

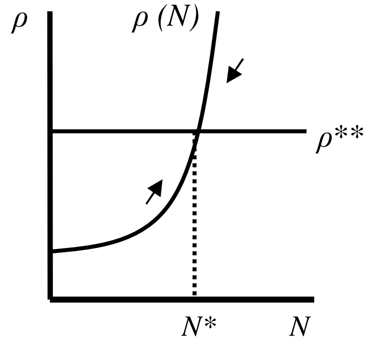


Figure 5. Probability of detection and equilibrium in the Loayza's model**.**

Equilibrium occurs where the probability $\rho(N)$ crosses with the threshold value of the probability, consistent with the equalization of profits across sectors, ρ^{**}. This equilibrium is stable, because for *N*<*N**, profits in the non-legal sector exceed those of firms in the legal sector, leading to an increase in *N* and convergence to *N**. For *N*>*N** the opposite occurs, with *N* falling to *N**.

The Probability of Detection in Loayza

L96 *assumes* that the probability of detection rises with the percent of all firms that are non-legal firms (or falls as the percent of legal firms rises). This assumption is key to that model. If the probability of detection falls with the number of non-legal firms, there may be an internal solution where the equilibrium fraction of legal firms is between zero and one, but this equilibrium will not be stable. The fraction of legal firms would converge to zero or one, and the model would not lead to the observed reality of developing countries.

However, L96 neither provides a clear intuitive justification for this assumption, not makes an attempt at modeling it.[10] In the prior section we have developed a simple, general model of detection of non-legal firms, and the underlying public finances.

Implications of the General Model of Detection for the Loayza Model, L96

As shown in the probability model (previous section), the probability of detection often, though not always, has a positive slope. Thus, combining the general model of detection with the L96 model leads to a more complete model, where as in L96, stable internal solutions may dominate. Below we extend the L96 model in combination with the probability model for the case of constant returns to scale in the production of the sample in the presence of both exogenous (T>0) and endogenous financing.

[10] Some mention is made of the expected tax rate rising as the size of the non-legal sector increases, as a reaction by legal firms. No probability model is provided and not discussion of the financing of detection is developed.

Applying the Generic Probability Model to Loayza (1996)

In this section, we apply the generic probability model developed in Section III to the model of Loayza (1996).

Given that M can be expressed as

$$M = \mu \cdot y_I \tag{4.11}$$

where y_I is output per firm in the informal economy, we can rewrite (4.6) as:

$$\rho^{**} = \frac{y_I}{M}\left(1 - \frac{(1-\tau)}{\gamma^b}\right). \tag{4.12}$$

To solve for N^*, we equalize ρ from (3.4) and ρ^{**} from (4.12), and obtain:

$$N^* = \frac{1}{\varepsilon \cdot y_I \cdot F}\left[\left(F \frac{y_I}{M}\frac{H}{\beta}\right)^{\frac{1}{\lambda}} - T\right], \tag{4.13}$$

where $F \equiv \left(1 - \frac{(1-\tau)}{\gamma^b}\right)$.

The sign of the partial derivatives of N^* are:

$$\frac{\partial N}{\partial M} < 0\,;\ \frac{\partial N}{\partial \beta} < 0\,;\ \frac{\partial N}{\partial T} < 0\,;\text{ and }\ \frac{\partial N}{\partial \varepsilon} < 0\,.$$

$\frac{\partial N}{\partial \tau}$ is positive if $\lambda<1$, and it is negative if $\lambda>1$ and simultaneously $T=0$. The sign depends on parameter values in any other case.

V. MODELING PAYROLL TAXES AND 'INFORMALITY'

Payroll taxes are taxes paid principally by firms to finance health, pension and other benefits. When workers value firms' payroll tax contributions completely, base wages for non-legal firms should be higher in that amount than base wages in legal firms, and the net profits of both legal and non-legal firms will be equal, without the need for penalties.

However, when workers values firms' contributions by less than 100 percent, the firm's payroll tax contribution will constitute a net cost to the firm. While with public goods, taxes paid by the firm lead to a compensating benefit in the form of the public good, this is not the case with payroll taxes.[11]

To illustrate the general logic of payroll taxes with detection, we will assume a generally artificial model where workers do not contribute to payroll taxes and do not value those benefits. Assume that the marginal payroll tax rate is δ. Then legal firms' profits can be written as sales minus wages*(1+ δ) minus costs of capital:

$$V_L \equiv PQ - wE(1+\delta) - rK \, . \tag{5.1}$$

It is not obvious how identical firms that do not need to pay payroll taxes can exist alongside legal firms. And in the absence of penalties, all firms would prefer to be non-legal. Thus, penalties are needed to explain a stable internal solution with both legal and non-legal firms. However, many penalty structures will lead to the complete domination of either legal or non-legal firms.

The foregoing models of detection and penalties, however, provide a structure to modelling payroll taxes with stable internal solutions, consistent with the observed facts for developing countries. What is needed is to assume that non-legal firms face expected penalties that are proportional to wages, in parallel with payroll taxes. And that the expected penalties rise with the share of non-legal firms.

Thus, the net profits of non-legal firms, V_N, are:

$$V_N \equiv PQ - wE(1+\pi) - rK \, , \tag{5.2}$$

were π is the expected fine if caught, and consists of the probability of detection, ρ, times the fine, M, or $\pi = \rho M$. The probability of detection is explained by the model of detection developed in Section III.

In equilibrium, firms are indifferent between being legal or non-legal, or:

[11] It is, however, conceivable that payment of payroll taxes might lead to benefits to the firm, in the form of higher worker productivity, less turnover, etc.

$$V_L \equiv PQ - wE(1+\delta) - rK \qquad = \qquad PQ - wE(1+\pi) - rK \equiv V_N \, . \tag{5.3}$$

This implies that in equilibrium:

$$(1+\delta) = (1+\pi)\,, \tag{5.4}$$

or,

$$\delta = \rho M \; . \tag{5.5}$$

This defines the threshold level of the probability at which there exists an internal solution, ρ^{**}:

$$\rho^{**} = \delta / M \; . \tag{5.6}$$

Because in the generic detection model, the probability of detection often rises with the share of non-legal firms, this solution will be stable.

This model provides a basis for analyzing the impact of payroll taxes on wages, employment and the size of the non-legal or informal sector.[12]

[12] For lack of space, we do not include the full development of this model and its variants here. See my forthcoming volume, *"Payroll Taxes, Labor Reform and Unemployment – Models and Simulations for Colombia"*, for a full treatment of these models.

VI. CONCLUSION

Recent models of the Informal Sector emphasize the free choice of sectors by firms or workers and define informality firms as those that do not pay taxes. One dominant group of models examines identical firms producing a homogeneous final good, where legal firms pay taxes and receive a public good, while non-legal firms do not pay taxes and receive a smaller amount of the public good. These models may be consistent with a stable equilibrium where there are both Formal (legal) and Informal (non-legal) firms, as observed in developing countries.

This paper first presents a generic and somewhat general version model with public goods and taxation model ("g-t model"), finding that most equilibria are unstable, so that all firms choose to be either legal or non-legal, which is inconsistent with the observed facts for developing countries. However, when the government finances detection of non-legal firms and fines those firms, stable 'internal' solutions may dominate. This requires a probability of detection that falls as the number of legal firms increases.

A general model of detection, penalties and their public finance is presented and analyzed, finding that the probability of detection often, though not always, falls with the number of legal firms. The role of exogenous financing of detection is emphasized, where greater exogenous financing reduces informality and if high enough constitutes complete prevention, inducing all firms to be formal. This is both a *positive* and *normative* finding, potentially explaining the absence of informal firms in some countries, and the presence in others, while also constituting a policy instrument for eradicating informality.

As in the g-t model, the Loayza model of informality requires a probability of detection that falls as the number of legal firms rises for a stable internal solution. Thus, this general model of detection partially validates the Loayza model. It is also observed that in the Loayza model, governments may eliminate informality without penalties, by lowering the tax rate sufficiently. This also has *positive* and *normative* implications.

Finally, in Section V we presented a model of payroll taxes with detection, based on the model of detection in Section III and the structure of taxation and detection in Loayza (1996). This provides the basis of a consistent model of payroll taxes and the size of the non-legal or 'informal' sector.

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APPENDICES

A.1. Analysis of the Equilibrium Fraction of Legal Firms when There Exist Closed Form Solutions for *L**: Fully Excludable Public Goods (γ=0) and Strictly Non-Rival Public Goods (*m*=0)

<u>A.1.1 Fully Excludable Public Goods (γ=0)</u>

Replacing $\gamma = 0$ in the expression for *g* (2.5) yields:

$$g = L^{\alpha - m} t^{\alpha} . \qquad \text{(A.1.1.1)}$$

From the equilibrium condition, $g = \tilde{t}$, the solution for *L* is:

$$L^* = t^{\left(\frac{1-\alpha}{\alpha - m}\right)} . \qquad \text{(A.1.1.2)}$$ [13]

Replacing γ=0 in expression (2.10), the conditions for the sign of the derivative *g'(L)* are:

$$g'(L) \gtreqless 0 \text{ as } 0 \gtreqless L(m - \alpha) . \qquad \text{(A.1.1.3)}$$

We see that the derivative of *g* with respect to *L* is positive or negative according to $\alpha \gtrless m$. If $\alpha < m$ and there exists an internal solution, this solution will be stable. On the other hand, if $\alpha > m$ [14] and there exists an internal solution, this solution will be unstable, so that *L* will fall to zero or rise to one.

In the case where $\alpha = m$, we have $g'(L) = 0$. The public good per firm, *g*, is $g = t^{\alpha}$, which does not depend on *L*. The only value of *t* that satisfies the condition of equilibrium is

---

[13] We have the following derivatives of *L** with respect to *m*, *t* and *α*:

$$\frac{\partial L^*}{\partial m} = \frac{t^{\left(\frac{1-\alpha}{\alpha-m}\right)} \cdot (1-\alpha) \cdot \ln t}{(\alpha - m)^2} ; \; \frac{\partial L^*}{\partial m} \gtreqless 0 \text{ as } \alpha \lesseqgtr 1 .$$

$$\frac{\partial L^*}{\partial t} = \left(\frac{1-\alpha}{\alpha - m}\right) \cdot t^{\left(\frac{1-2\alpha+m}{\alpha-m}\right)} ; \text{ if } \alpha > 1 , \; \frac{\partial L^*}{\partial t} < 0 . \text{ If } \alpha < 1 , \; \frac{\partial L^*}{\partial t} \gtreqless 0 \text{ as } \alpha \gtreqless m .$$

$$\frac{\partial L^*}{\partial \alpha} = \frac{t^{\left(\frac{1-\alpha}{\alpha-m}\right)} \cdot \ln t \cdot (m-1)}{(\alpha - m)^2} ; \; \frac{\partial L^*}{\partial \alpha} < 0 \text{ if } m<1 \text{, and } \frac{\partial L^*}{\partial \alpha} = 0 \text{ if } m=1.$$

There are internal solutions such that Legal and Non-Legal firms coexist under γ=0 in the following cases:

*L**>0 implies $t^{\left(\frac{1-\alpha}{\alpha-m}\right)} > 0$. This condition is satisfied because *t* is positive.

*L**<1 implies $t^{\left(\frac{1-\alpha}{\alpha-m}\right)} < 1$. Given that $t \geq 1$ then $\left(\frac{1-\alpha}{\alpha - m}\right)$ should be negative for the internal solution to exist. This will occur in only two cases: $\alpha > 1$ (implying $\alpha > m$) or $\alpha < m$. If $\alpha > 1$, the derivative of *g(L)* is positive and the internal solution is unstable; if $\alpha < m$, the derivative of *g(L)* is negative and the internal solution is stable.

[14] This is always true with *α*>1, so that there exists increasing returns to scale in the production of *G*.

t=1[15]. In this case, L^* can take any value between 0 and 1. If $t > 1$, the tax exceeds the per firm public good, so that all firms will prefer to be non-legal (see Figure 2).

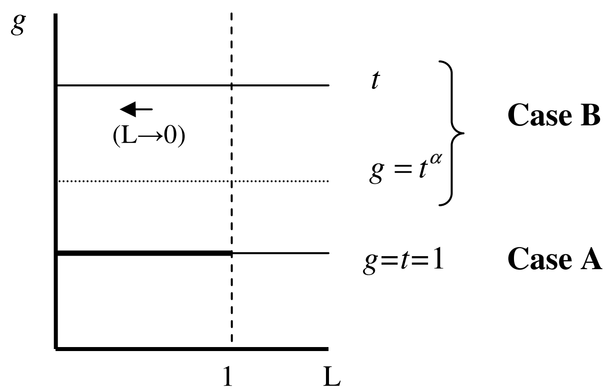

Figure A.1.1. Equilibrium of L with fully excludable public goods (γ=0) and α=m (Case A: t>1; Case B: t=1).

The results for $\gamma = 0$ are presented in Table A.1.1, below.

[15] However, if α=1, the solution is g=t, which is valid for all values of t.

Table A.1.1. Excludable ($\gamma = 0$) Partially Rival Public Goods ($m \in (0,1)$)
Fraction of Legal Firms
(Solutions for *L, varying *m*, α and *t*)**

Returns to scale: α versus 1	***Returns to scale: α versus m***	***Sign of $g'(L)$***	***Tax level (t)***	***Fraction of Legal Firms in Equilibrium: L****	***Properties of Solution (CS: corner solution; IS: Internal Solution)***		***References to figures***
$\alpha > 1$	$\alpha > m$	(+)	$t>1$	$L^* \in (0,1)$ $L \to 0$ or 1	Unstable	IS	Figure 1, column 2, Panel D, point *a*
			$t=1$	$L^*=1$ $L \to 0$		CS	Figure 1, column 2, Panel D, point *b*
$\alpha = 1$	$\alpha > m$	(+)	$\forall t$	$L^*=1$ $L \to 0$	Unstable	CS	Figure 1, column 2, Panel D, point *b*
$\alpha < 1$	$\alpha > m$	(+)	$t > 1$	$L^* > 1$ $L \to 0$	Unstable	CS*	Figure 1, column 2, Panel D, point *c*
			$t=1$	$L^*=1$ $L \to 0$		CS	Figure 1, column 2, Panel D, point *b*
	$\alpha < m$	(-)	$t>1$	$L^* \in (0,1)$ $L \to L^*$	Stable	IS	Figure 1, column 1, Panel D, point *a*
			$t=1$	$L^*=1$ $L \to 1$		CS	Figure 1, column 1, Panel D, point *b*
	$\alpha = m$	0	$t>1$	$\nexists L^*$ $L \to 0$	There is no solution		Figure A.1, Case B
			$t=1$	$L^* \in [0,1]$	Non unique solution		Figure A.1, Case A
* The solution, L*, is outside the interval [0, 1].							

A.1.2. Strictly Non-Rival Public Goods (*m*=0)

In this case,

$$L^* = t^{\left(\frac{1-\alpha}{\alpha}\right)} \cdot (1-\gamma)^{-\frac{1}{\alpha}}.^{16} \tag{A.1.2.1}$$

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16 The derivatives of *L** with respect to γ, *t* and α are:

$$\frac{\partial L^*}{\partial \gamma} = \left(\frac{1}{\alpha}\right) \cdot t^{\left(\frac{1-\alpha}{\alpha}\right)} \cdot (1-\gamma)^{\left(-\frac{1}{\alpha}-1\right)} > 0;$$

$$\frac{\partial L^*}{\partial t} = \left(\frac{1-\alpha}{\alpha}\right) \cdot t^{\left(\frac{1-\alpha}{\alpha}-1\right)} \cdot (1-\gamma)^{-\frac{1}{\alpha}}; \quad \frac{\partial L^*}{\partial t} \overset{\geq}{<} 0 \text{ as } \alpha \overset{\leq}{>} 1;$$

$$\frac{\partial L^*}{\partial \alpha} = t^{\left(\frac{1-\alpha}{\alpha}\right)} \cdot (1-\gamma)^{-\frac{1}{\alpha}} \cdot \left[\frac{\ln(1-\gamma) - \ln t}{\alpha^2}\right] < 0 \cdot$$

Replacing *m*=0 in expression (2.10), the derivative of *g* with respect to *L* is:

$$g'(L) \gtreqless 0 \text{ as } \gamma \gtreqless -L(1-\gamma). \qquad \text{(A.1.2.2)}$$

Given that $(-L(1-\gamma))$ is negative, when *m* is zero $g'(L)$ is always positive. Thus, when *m*=0, though there may exist internal solutions[17], these solutions will be unstable, because $g'(L) > 0$.[18]

[17] The conditions for the existence of internal solutions are:

L^*>0 implies $t^{\left(\frac{1-\alpha}{\alpha}\right)} \cdot (1-\gamma)^{-\frac{1}{\alpha}}$>0. Because $(1-\gamma)$ and *t* are positive this condition is satisfied.

L^*<1 implies $t^{\left(\frac{1-\alpha}{\alpha}\right)} \cdot (1-\gamma)^{-\frac{1}{\alpha}}$<1. Equivalently, $\gamma < 1 - t^{1-\alpha}$. If $\alpha < 1$, this condition is not satisfied.

[18] When L*>1, L→0. When $L^* \in (0,1)$, L→0 or 1.

Table A.1.2. Synthesis of Results for "g-t Model" (1)

Notation:

L^* (2): fraction of legal firms in equilibrium $(g = \tilde{t})$.

$L \rightarrow 0$ or 1: fraction of legal firms goes to 0 if $L < L^*$, or it goes to 1 if $L > L^*$.

$L \rightarrow 0$: fraction of legal firms goes to 0.

Cases	Conditions	Sign of g'(L)	Result	Comments	Reference to Figures
I. Fully Excludable Public Good, $\gamma = 0$	There are internal solutions when $\alpha \neq m$, depending on the level of α and t.				
A. $\alpha < m$	$t > 1$	(-)	$L^* \in (0,1)$	Internal solution(3) (stable)	Figure 1, column 1
B. $\alpha > m$	$(t, \alpha) > 1$	(+)	$L^* \in (0,1)$ and $L \rightarrow 0$ or 1	Internal solution(3) (unstable)	Figure 1, column 2
C. $\alpha = m$: $\alpha = 1$		0	$L^* \in [0,1]$	Non unique solution	Figure A.1
C. $\alpha = m$: $\alpha < 1$	$t = 1$	0	$L^* \in [0,1]$	Non unique solution	Figure A.1
C. $\alpha = m$: $\alpha < 1$	$t > 1$	0	$\nexists L^*$ $L \rightarrow 0$	There is no solution	Figure A.1
II. Non-excludable Public Good, $\gamma = 1$	$L \rightarrow 0$ because $\left[g = (L\,t)^{\alpha} \right] < \left[\tilde{t} = \frac{t}{1-\gamma} = \infty \right]$				
III. Partially Excludable Public Good, $\gamma \in (0,1)$	There are no internal solutions for γ medium or high except when α takes unrealistically high values (α > 1.5). For γ low, there are some unstable internal solutions when α and t are high (> 1), regardless of m. When there exist internal solutions they are unstable, with $L \rightarrow 0$ or 1, as $dg/dL > 0$.				
A. $\gamma = 0.2$	$(t, \alpha > 1)$, $\forall m$	(+)/(-)(4)	$L^* \in (0,1)$ and $L \rightarrow 0$ or 1	Internal solution (unstable)	
B. $\gamma = 0.8$	$(t > 1) \cap \left(\alpha > \frac{\ln \tilde{t}}{\ln t} \right)$, $\forall m$	(+)/(-)(4)	$L^* \in (0,1)$ and $L \rightarrow 0$ or 1	Internal solution(5) (unstable)	

(1) $\alpha \in$ [0.5, 4], $t \in$ [1, 5], $m \in$ [0, 1], and γ takes four specific values: 0, 0.2, 0.8, 1.

(2) L^* denotes the mathematical equilibrium where $g = \tilde{t}$.

(3) When t=1, L^*=1.

(4) There are cases like Figures 2d and 2e, where though the derivative becomes negative at a value L', there are no stable internal solutions.

(5) α must be very high to obtain an internal solution with γ=0.8.