Grading standards under monopolistic provision of education

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Abstract: In this paper I study the grading standards used by an unregulated school that maximizes profits. To this end I set up a model in which firms in the labor market do not observe individual productivities. Instead, they observe whether the individual attended school and whether he achieved the grading standard. I show that the school will always set a grading standard such that all individuals attend school and *all* achieve the degree. In the model I consider this is an inefficient outcome. The efficient grading standard is such that some individuals achieve the degree and some others do not.

Keywords: Education, Grading standards, Monopoly provision

JEL codes: I21

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1 Introduction

In many countries there is an ongoing debate about the need to reform education regulatory frameworks in order to raise students' achievement. The proposals discussed include instruments such as the introduction of state or national level exit exams to award degrees or to regulate schools (v.g. in the US[1], in Britain[2] or in Brazil[3]), minimum curriculum content (v.g. in Latin American countries[4]) or the introduction of competition between schools.

One of the main determinants of the optimal regulatory instruments for the education market is the information that firms have about workers' productivities. The usefulness of different instruments relies, mainly, on the observability of individuals productivities in the labor market. The most important policy instruments that fall in this distinction are those that affect the way degrees are awarded. These forms of regulating schools are most relevant when individual productivities are private information.

Degrees and grades have recently received considerable attention in the economics literature mostly following seminal contributions by Costrell (1994) and Betts (1998). The question they address is how social optimal standards depend on the objective function of the government.

Despite the recent interest in degrees and grades the issue of how self-interested teachers or school managers set grading standards has not received any attention in theoretical literature. In this paper I will consider this issue. The problem is important for, at least, two reasons. First, there are many countries where these forms of regulation are not used. In most countries the degree awarding decision is decentralized at the school level. Obviously, this would not be a problem with perfect information labor markets and fully rational individuals since degrees would be useless. However, the evidence of whether the effects of investment in education in the labor market are best described by a human capital or a signaling model is mixed.[5] Consequently, the policy instruments that are useful under the signaling hypothesis should not be ignored.

Second, answering the question of how self-interested schools set grading standards is a prerequisite for the good design of regulatory instruments when one

1. The Wall Street Journal (2000)

2. The Economist (2003)

3. World Bank (1999, p. 63)

4. World Bank (1999, pp. 23-26)

5. Some empirical studies of this issue are Kroch and Sjoblom (1994), Lang and Kropp (1986), Riley (1979) and Wolpin (1977).

thinks that the relation between the labor market and education system is affected by signaling issues. Not only the way standards are set by schools must be considered. Also, the interaction between standards and school inputs must be considered. Incentives to increase teachers' effort or direct spending are also crucially affected by asymmetries of information in the labor market. If there is a strong effect of degrees in the labor market, increasing expenditure in education or setting incentives to increase teachers' effort will have negligible effects if they are not accompanied by other measures carefully chosen to make these efforts profitable.

In these pages my aim is to make a first step in the issue of how a self-interested school sets grading standards in an unregulated environment. This will allow, in future work, to address other issues such as the optimal regulation strategies when governments have imperfect instruments and on the effect of competition on the rules used by schools to award degrees.

In Costrell (1994) and Betts (1998) schools are immaterial; there is no object for schools besides awarding degrees and the behavior of schools is not modelled explicitly. Both use human capital-signaling models where, in schools, students increase their labor market productivity. Since individual labor market productivity is not observed by firms, schools must give degrees to signal their labor market productivity to firms. As they appear in those papers these models are not suitable for the analysis of the problem in hand. They lack assumptions to make demand and supply of education endogenous. In this paper I extend the Betts (1998) model accordingly.

Formally, I assume that individual productivity depends on ability, individual effort and quality of education. Wages are set according to the signaling hypothesis; they depend on whether the individual attended school and on whether he achieved the degree. Effort is costly for individuals and quality of education is costly for the school. I analyze the problem of how a school chooses a threshold level that signals individuals with higher and lower productivity together with tuition and quality of education.

The main result in this paper is that the optimal standard set by a monopoly school is always such that all individuals attend school and *all* achieve the degree. The extreme inefficiency of this result is not difficult to see. In stable equilibria this results from a very low standard. Since the effort needed to achieve the degree is decreasing in ability, a very low grading standard is also associated to low achievement of high-ability individuals. It is not surprising, then, the conclusion that the efficient grading standard is generally such that not all students achieve the degree.

A second result concerns the difference in the efficient quality and tuition with those set by the monopoly. The social optimal quality is given by a modified Samuelson rule in which the welfare of all students is taken into account. However, since the monopoly can only extract the surplus of the *marginal individual* (the one who is indifferent between attending school and not attending) its quality choice only takes into account his utility.[6] The tuition set by the monopoly is also very different from the efficient one. The efficient tuition solves the sorting problem, while the monopoly's tuition extracts the maximum surplus from students.

The grading literature to which this paper belongs is similar to the broader literature on certification. Probably the closest model in this literature is that by Lizzeri (1999). There are, however, important differences between the two papers. First, Lizzeri (1999) considers a different technology for the intermediary (in this paper, the school) that allows to perfectly observe the quality of a good sold by one agent and can choose to transmit this information to potential buyers. Second, he uses a pure adverse selection model. The assumption that the intermediary can perfectly observe individuals' productivity makes the efficient outcome an unappropriate benchmark for the monopoly's problem. Under this assumption the benevolent school could fully reveal individual productivity to the labor market making degrees useless. Additionally, it seems realistic to assume that grades cannot reflect the complete distribution of productivities but that they can only capture intervals.

The introduction of moral hazard is important for my purposes since with a pure adverse selection model and exogenous labor supply, efficiency does not depend on how individuals are separated in the labor market. With the effort variable determining individual productivity a particular grading standard has not only the effect of separating individuals according to their ability but also inducing some effort which enhances productivity and welfare.

Despite these differences the main result in this paper is similar to the main finding of Lizzeri (1999): the self interested intermediary (in this paper the school) chooses not to transmit any information. However, as a result of the difference in the technology of the intermediary the result in this paper does not depend on the distribution function of types as in Lizzeri (1999). Moreover, in this educational setting the result has a more serious implication than in the pure adverse selection setting of Lizzeri (1999). Namely, the fact that the school chooses not to reveal any information means that students will make a very low effort at school and this reduces individual and aggregate welfare.

6. Sheshinski (1976) and Spence (1973) analyze models where a monopoly sells a good of variable quality. However, those models do not include certification technologies nor moral hazard which are crucial in this paper.

The plan of the paper is the following. Section 2 sets up the model. Section 3 describes individuals' behavior and the labor market. Section 4 analyzes the second-best efficient outcome. Section 5 describes the school's behavior. I leave the analysis of the first-best for the appendix. The final section concludes.

2 The model

2.1 Individuals

There is a continuum of individuals indexed by i with utility

$$U^i = w^i - t - v(e^i), \tag{1}$$

where w^i is the labor market wage, t is the tuition payed to school and e^i is the effort level made at school. $v(\cdot)$ is convex, increasing and satisfies the Inada conditions, $v(0) = 0$ and $\lim_{e \to 0} v'(e) = 0$.

Labor market productivity of an individual is enhanced at school. An individual with ability ϕ^i who exerts effort e^i and attends a school of quality q will have a labor market productivity of $\omega(\phi^i, e^i, q)$. ϕ^i is drawn from the interval $\left[\phi^L, \phi^H\right]$ with a continuous and differentiable probability function $F(\cdot)$. The function $\omega(\phi^i, e^i, q)$ is concave in (e^i, q), strictly increasing in all its arguments and satisfies

$$\omega_{\phi q}(\phi^i, e^i, q) > 0. \tag{2}$$

2.2 The school

There is a monopoly school with two tasks. First, it provides the quality of education q. Second, it transmits information to firms about the productivity of the individuals who attended the school.

The school is not able to observe perfectly the labor market productivity of each student. It can only observe if the productivity of a student is above or below a certain level, $\underline{\omega}$. Let the variable

$$D^i = \begin{cases} d \text{ if } \omega\left(\phi^i, e^i, q\right) \geq \underline{\omega} \\ f \text{ otherwise} \end{cases}$$

represent whether a student achieved the degree (d) or not (f).

The threshold level $\underline{\omega}$ is chosen by the school. The costs of serving a proportion π of students and providing a school quality of q are $\pi\psi(q) + C$. Where C is the fixed cost of running a school and $\psi(\cdot)$ is convex and increasing. If the school receives t from each student its payoff will be given by:[7]

$$\Pi = \pi\Big[t - \psi(q)\Big] - C. \tag{3}$$

2.3 Firms

There is a large number of risk neutral competitive firms in the labor market. Firms do not observe individual labor market productivity nor any of the arguments on which it depends. They only observe whether an individual attended school and whether he obtained the degree.

Since the labor market is competitive and firms are risk neutral, wages will equal expected productivities conditional on available information. Let

$$A^i = \begin{cases} a \text{ if the individual attended school} \\ u \text{ otherwise.} \end{cases}$$

Since firms only observe A^i and D^i the wage received by an individual will be:

$$w^i = w(A^i, D^i) = E\left[\omega(\phi^i, e^i, q) \mid A^i, D^i\right]. \tag{4}$$

Wages will take one of three possible values. Let these values be $w^i = w^d$ if $D^i = d$, $w^i = w^f$ if $D^i = f$ and $w^i = w^u$ if $A^i = u$.

2.4 Timing and strategies

I model the unregulated market game as a three stage game with the following succession of events:

Stage 1	The school offers a triplet $(q, \underline{\omega}, t)$
Stage 2	After observing $(q, \underline{\omega}, t)$ students choose whether to attend school
Stage 3	Students choose their effort level e^i.

7. When modelling school behavior a common problem is the objective function assumed for schools. There is a wide range of possibilities, among others, one can think of schools as profit maximizing units, as altruistic or non-for-profit ones. The chosen formulation builds on the observation that schools are generally labor managed firms and assuming that quality is costly and that schools care for the tuition level is a reasonable approximation to reality.

At the end of Stage 3 individuals earn $w(A^i, D^i)$ as defined in (4).

Consequently, a strategy for the school is given by $(q, \underline{\omega}, t)$, i.e. quality, grading standard and tuition fee. A strategy for the individuals is given by a pair (A^i, e^i) for each $(q, \underline{\omega}, t)$ possibly chosen by the school. Given the strategy of the school individuals decide whether to attend school and their effort level if they attend. As usual in this type of problems, the solution is found by backward induction (I assume sequential equilibrium). In the third and second stages students choose effort level and demand for schooling anticipating the wage they will receive in the labor market. In the first stage the school chooses the tuition level, the grading standard and the school quality anticipating the decisions by students in subsequent stages.

3 Individuals' behavior and the labor market

This section considers the behavior of individuals and its consequences on the equilibrium in the labor market conditional on the decisions made by the school on $(q, \underline{\omega}, t)$.

3.1 The behavior of individuals

In the appendix I show that given the strategy of the school, $(q, \underline{\omega}, t)$, the optimal behavior of individuals satisfies the following properties.

Property 1 *Let ε^i denote the optimal effort level of a student of type i. If the student achieves the degree, ε^i will be equal to $\max\{\underline{e}^i, 0\}$, where $\underline{e}^i$ is given by $\omega(\phi^i, \underline{e}^i, q) = \underline{\omega}$. If the student does not achieve the degree or it does not attend school, ε^i is equal to zero.*

Property 2 *If a student of type ϕ^i chooses to achieve the degree all students with abilities $\phi^h \geq \phi^i$ will also choose to achieve the degree. Conversely, if a student with type ϕ^i chooses not to achieve the degree all students with abilities $\phi^k < \phi^i$ will also choose not to achieve the degree. Consequently, there exists a* marginal student *with ability $\tilde{\phi}$ who is indifferent between achieving and not achieving the degree.*

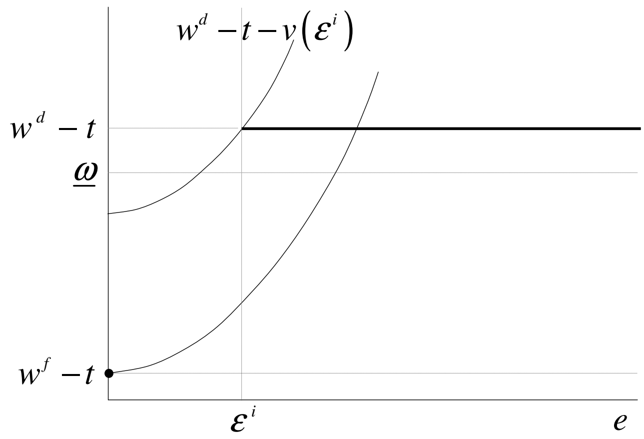

Figure 1: The students choosing to achieve the degree. The choice set of an individual of type i, once he has chosen to attend school, is given by the point $(0, w^f - t)$ together with the bold line starting in $(\varepsilon^i, w^d - t)$. Note that, since ε^i decreases in ϕ^i, the lower the ability level, the further to the right the point $(\varepsilon^i, w^d - t)$ will be.

Property 3 *All individuals not exerting any effort have the same utility if they receive the same wage. Consequently all individuals who would find optimal not to achieve the degree, if they attended school, would either find optimal to attend school or not to attend. Moreover, it may be that some students who would achieve the degree in the case they attended school, may find optimal not to attend. Let $\hat{\phi}$ represent the ability level of the* marginal individual. *$\hat{\phi}$ must satisfy*

$$\hat{\phi} \geq \tilde{\phi} \text{ and } \hat{\phi} \geq \phi^L \quad \text{or} \quad \hat{\phi} = \phi^L \text{ and } \tilde{\phi} \geq \phi^L.$$

Property 1 follows from the observation that a change in effort can only affect utility if it changes productivity around $\underline{\omega}$. Otherwise, since wages cannot be conditioned on effort, all individuals have incentives to free ride and exert the minimum effort to be in their desired category (d, f or u). Property 2 follows from a standard revealed preference argument that results form the monotonicity properties of utility. Property 3 follows from the fact that utility for individuals who attend school and fail and for individuals who do not attend school do not depend on ability since they exert no effort. These three properties are illustrated in figures 1 and 2.

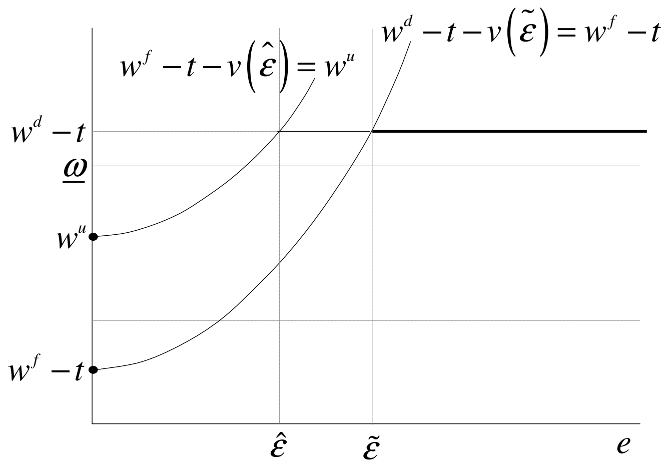

Figure 2: Some students who prefer achieving the degree may choose not to attend school. The bold line is part of the choice set of the marginal student and the light line part of the choice set of the marginal individual. All those individuals who need to exert effort $\varepsilon^i \in [\hat{\varepsilon}, \tilde{\varepsilon}]$ will not attend school even though they would achieve the degree in case they attended.

According to these properties equilibria can be of two types. In the first type, labelled **Regime 1**, not all individuals attend school, all those attending achieve the degree and there is a marginal individual indifferent between these two options. In the second type, labelled **Regime 2**, all individuals attend school and there is a marginal student indifferent between achieving the degree or not. In both regimes effort level is minimal. Individuals who achieve the degree exert the minimal effort needed to attain the productivity level $\underline{\omega}$, individuals who do not achieve the degree (either attending or not attending school) exert no effort. It should be noted here that both regimes admit corner solutions. The possibility of having all individuals achieving the degree and thus, attending school is not precluded from Regime 1. Similarly, in Regime 2 it can be that all individuals achieve the degree or that no individual does so. Note also that although in these corner solutions both regimes resemble, they are still differentiated by the fact that in Regime 1 there are no students who attend school and do not achieves the degree while in Regime 2 this is not precluded.

An additional property regarding the effort level of the marginal individual in Regime 1 and of the marginal student in Regime 2 will be useful to understand the results in this paper.

Property 4 *(Regime 1) Let $\hat{\varepsilon}$ be the effort exerted by the marginal individual to achieve the degree. It must be that*

$$\omega_e(\hat{\phi}, \hat{\varepsilon}, q) < v'(\hat{\varepsilon}).$$

(Regime 2) Similarly, let $\tilde{\varepsilon}$ be the effort exerted by the marginal student to achieve the degree. It must be that

$$\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q) < v'(\tilde{\varepsilon}).$$

Property 4 is proven in the appendix. The property is very intuitive. To achieve the degree the marginal individual (and the marginal student) will need to exert an effort level which exceeds the efficient level since the wage level which corresponds to his productivity when he exerts the efficient effort level is not feasible.[8]

Before continuing let me make a remark on the terminology used in the paper. As in the previous paragraphs, I will continue to use the expression *marginal individual* for that who is indifferent between attending school and not attending in Regime 1 and the expression *marginal student* for that who is indifferent between achieving and not achieving the degree in Regime 2.

3.2 The labor market equilibrium wages and some comparative statics

Given the optimal strategies of individuals described above, and the school choice of $(q, \underline{\omega}, t)$, the equilibrium in the labor market will be described by two wage functions and a function determining the marginal individual (in Regime 1) or the marginal student (in Regime 2). These will differ according to the regime.

Regime 1

Consider first the case in which only some students attend school. This happens when, for any $(q, \underline{\omega})$, t is so big that it is better not to attend school than to attend and fail. The equilibrium will be described by the wages and the ability of the

8. The first-best efficient effort for each individual is the e^i that solves $\omega_e \phi^i, e^i, q) = v'(e^i)$.

marginal individual. Specifically, Properties 1 and 3 imply that wages are given by:

$$\begin{aligned}\hat{w}^d(\hat{\phi}, q, \underline{\omega}) &= E\left[\omega(\phi^i, \varepsilon^i, q) \mid \phi^i \geq \hat{\phi}\right] = \int_{\hat{\phi}}^{\phi^H} \omega(\phi^i, \varepsilon^i, q)\frac{dF(\phi^i)}{1 - F(\hat{\phi})} \\ &= \underline{\omega}\frac{\left[F(\Phi) - F(\hat{\phi})\right]}{1 - F(\hat{\phi})} + \int_{\Phi}^{\phi^H} \omega(\phi^i, 0, q)\frac{dF(\phi^i)}{1 - F(\hat{\phi})}\end{aligned} \tag{5}$$

and

$$\begin{aligned}\hat{w}^u(\hat{\phi}) &= E\left[\omega(\phi^i, 0, 0) \mid \phi^i < \hat{\phi}\right] = \int_{\phi^L}^{\hat{\phi}} \omega(\phi^i, 0, 0)\frac{dF(\phi^i)}{F(\hat{\phi})} \\ &= \int_{\phi^L}^{\hat{\phi}} \omega(\phi^i, 0, 0)\frac{dF(\phi^i)}{F(\hat{\phi})}.\end{aligned} \tag{6}$$

where $\Phi(q, \underline{\omega})$ is the ability level of the individual who needs to exert no effort to achieve the degree, i.e., $\Phi(q, \underline{\omega})$ is the inverse of $\omega(\Phi, 0, q) = \underline{\omega}$.

To write the condition that defines the ability level of the marginal individual define the function

$$\hat{\Delta}(\phi^i, q, \underline{\omega}, t) \equiv w^d(\phi^i, q, \underline{\omega}) - w^u(\phi^i) - v(\varepsilon^i) - t.$$

corresponding to the value of achieving the degree for a student with ability ϕ^i when he is the student with the lowest ability to do so. Note that ε^i is a function of ϕ^i, q and $\underline{\omega}$, explaining why ε^i does nor apear as argument of $\hat{\Delta}$. The ability level of the marginal individual will satisfy

$$\hat{\Delta}(\hat{\phi}, q, \underline{\omega}, t) = 0 \tag{7}$$

and has the following comparative statics:

$$\hat{\phi}_t(q, \underline{\omega}, t) = -\frac{\hat{\Delta}_t(\hat{\phi}, q, \underline{\omega}, t)}{\hat{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega}, t)}, \tag{8}$$

$$\hat{\phi}_{\underline{\omega}}(q, \underline{\omega}, t) = -\frac{\hat{\Delta}_{\underline{\omega}}(\hat{\phi}, q, \underline{\omega}, t)}{\hat{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega}, t)} \tag{9}$$

and

$$\hat{\phi}_q(q, \underline{\omega}, t) = -\frac{\hat{\Delta}_q(\hat{\phi}, q, \underline{\omega}, t)}{\hat{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega}, t)}. \tag{10}$$

The signs of these derivatives are stated in the following property.

Property 5 *The function* $\hat{\phi}(q, \underline{\omega}, t)$ *satisfies*

$$\begin{array}{ll} (a) & sign[\hat{\phi}_t(q,\underline{\omega},t)] = sign[\hat{\Delta}_{\hat{\phi}}(\hat{\phi},q,\underline{\omega},t)], \\ (b) & sign[\hat{\phi}_{\underline{\omega}}(q,\underline{\omega},t)] = sign[\hat{\Delta}_{\hat{\phi}}(\hat{\phi},q,\underline{\omega},t)] \ \textit{and} \\ (c) & sign[\hat{\phi}_q(q,\underline{\omega},t)] = -sign[\hat{\Delta}_{\hat{\phi}}(\hat{\phi},q,\underline{\omega},t)]. \end{array}$$

Although the results in this paper are not restricted by any assumption on the sign of $\hat{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega})$ it is important to note the implications of it being positive. As noted already, the main condition describing the conditional equilibrium in this model is equation (7). That equation is a complicated polynomial on $\hat{\phi}$ which can have several solutions not all of which will be stable. However, it can be shown that if $\hat{\Delta}_{\hat{\phi}}(\phi^i, q, \underline{\omega}) > 0$ the equilibrium will be stable.[9] This condition is by itself intuitive. It says that, the "better" the pool of individuals that achieves the degree, the higher the benefits from achieving a degree. Moreover, it has intuitive implications. In the stable equilibria, increasing the tuition will reduce demand for school (part (a)), increasing $\underline{\omega}$ reduces the number of individuals achieving the degree (part (b)) and increasing q increases school demand (part (c)).

Regime 2

When, for some $(q, \underline{\omega}, t)$ all individuals attend school, wages will be given by

$$\begin{aligned} \tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) &= E\left[\omega(\phi^i, \varepsilon^i, q) \mid \phi^i \geq \tilde{\phi}\right] = \int_{\tilde{\phi}}^{\phi^H} \omega(\phi^i, \varepsilon^i, q) \frac{dF(\phi^i)}{1 - F(\tilde{\phi})} \\ &= \underline{\omega} \frac{\left[F(\Phi) - F(\tilde{\phi})\right]}{1 - F(\tilde{\phi})} + \int_{\Phi}^{\phi^H} \omega(\phi^i, 0, q) \frac{dF(\phi^i)}{1 - F(\tilde{\phi})} \end{aligned} \tag{11}$$

and

$$\tilde{w}^f(\tilde{\phi}, q) = E\left[\omega(\phi^i, \varepsilon^i, q) \mid \phi^i < \tilde{\phi}\right] = \int_{\phi^L}^{\tilde{\phi}} \omega(\phi^i, 0, q) \frac{dF(\phi^i)}{F(\tilde{\phi})}. \tag{12}$$

With $\Phi = \Phi(q, \underline{\omega})$.

9. See Betts (1998) and Costrell (1994) for very similar proofs.

To write the condition that describes the ability of the marginal student, $\tilde{\phi}$, I proceed as before and define the function

$$\tilde{\Delta}(\phi^i, q, \underline{\omega}) \equiv w^d(\phi^i, q, \underline{\omega}) - w^f(\phi^i, q) - v(\varepsilon^i) \tag{13}$$

corresponding to the value of achieving the degree for a student with ability ϕ^i when he is the student with the lowest ability to do so.[10] The marginal individual will be that for whom

$$\tilde{\Delta}(\tilde{\phi}, q, \underline{\omega}) = 0. \tag{14}$$

Consequently, the ability of the marginal student will depend on q and $\underline{\omega}$ only. For any $(q, \underline{\omega})$, $\tilde{\phi}(q, \underline{\omega})$ will represent the equilibrium ability level of the marginal individual.

Differentiating (14) yields

$$\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega}) = -\frac{\tilde{\Delta}_{\underline{\omega}}(\tilde{\phi}, q, \underline{\omega})}{\tilde{\Delta}_{\tilde{\phi}}(\tilde{\phi}, q, \underline{\omega})} \tag{15}$$

and

$$\tilde{\phi}_q(q, \underline{\omega}) = -\frac{\tilde{\Delta}_q(\tilde{\phi}, q, \underline{\omega})}{\tilde{\Delta}_{\tilde{\phi}}(\tilde{\phi}, q, \underline{\omega})}. \tag{16}$$

The signs of the comparative statics are given by the following property, proven in the appendix.

Property 6 *The function $\tilde{\phi}(q, \underline{\omega})$ satisfies:*

$$\begin{array}{ll} (a) & sign[\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega})] = sign[\tilde{\Delta}_{\hat{\phi}}(\tilde{\phi}, q, \underline{\omega})] \textit{ and} \\ (b) & sign[\tilde{\phi}_q(q, \underline{\omega})] = -sign[\tilde{\Delta}_{\hat{\phi}}(\tilde{\phi}, q, \underline{\omega})] \textit{ if } \omega_{\phi q}(\phi^i, e^i, q) > 0. \end{array}$$

The stability issue in this regime is analogous to that in Regime 1. If $\tilde{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega}) > 0$ the equilibrium will be stable. Since in this case there is no effect of tuition on the ability of the marginal student the interpretation is slightly different. Suppose that for some reason there is a fixed cost for students to achieve the degree. In such a case, $\tilde{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega})$ can be interpreted as the inverse of the effect of an increase in the fixed cost on the ability of the marginal student. It is likely that this effect is positive. As in Regime 1 stability has the intuitive implication that increasing $\underline{\omega}$ and reducing q reduces the number of students achieving the degree.

10. Note that ε^i is a function of ϕ^i, q and $\underline{\omega}$. This explains why ε^i does no appear as an argument of $\tilde{\Delta}$.

4 The second-best efficient outcome

As a benchmark for the behavior of the monopolist I turn to the analysis of the efficient outcome of this model. This second-best situation is not the only relevant one. However, it is the more appropriate benchmark for the problem solved in the following section. In the appendix I present an analysis of the first-best problem which, although also interesting, is not as relevant since there is no role for degrees.

I assume that the government has full information about the actions of the school (i.e. it observes t, $\underline{\omega}$ and q) but does not observe the parameters nor the actions of individuals (i.e. it does not observe ϕ^i, e^i nor ω^i).

The government plays in the first stage of the game presented in section 2. It anticipates individuals' behavior and predicts labor market wages. This means that the government's choice is subject to the optimality of student's choices in subsequent periods. To deal with the issue of the two types of equilibria that may hold in this model I will analyze sequentially the efficient outcomes in each of the two types of equilibria. I will then discuss which of the two outcomes will be the efficient one.

Before going on, an additional issue must be resolved. In both types of equilibria the government must satisfy individual participation constraints to induce its desired school demand. These participation constraints differ according to which of the two types of equilibria is the efficient one. Moreover, in both cases the constraints involve the computation of the utility obtained from suboptimal choices. Thus, to write the constraints I need to define the out of equilibrium beliefs of firms in this economy.

I will assume that the hypothetical expected productivity of an individual who attends school but does not achieve the degree, when this is a suboptimal choice, is given by

$$\hat{w}_c^f(\hat{\phi}, q) = E\left[\omega(\phi^i, 0, q) \mid \phi^i < \hat{\phi}\right]. \tag{17}$$

Similarly, the hypothetical expected productivity of an individual who did not attend school, when this is suboptimal, is given by

$$\tilde{w}_c^u(\tilde{\phi}) = E\left[\omega(\phi^i, 0, 0) \mid \phi^i < \tilde{\phi}\right]. \tag{18}$$

In both cases, I am assuming that beliefs about ability of the individuals who did not attend school and those that attended school but did not achieve the degree are drawn from the same interval.

4.1 Solving the welfare maximization problem

In this section I solve the problem in each of the two Regimes that can hold in this model. In the following section I discuss which of the two solutions is the efficient one.

Regime 1

We can now write the welfare maximization problem in each of the two cases considered. I start with the case in which not all individuals attend school. The problem of the government can be written as follows:

$$\begin{aligned}
\max_{q,\underline{\omega},t} \quad & \hat{w}^d(\hat{\phi}, q, \underline{\omega})\left[1 - F(\hat{\phi})\right] + \hat{w}^u(\hat{\phi})F(\hat{\phi}) \\
& \qquad - \int_{\hat{\phi}}^{\phi^H} v(\varepsilon^i)dF(\phi^i) - \left[1 - F(\hat{\phi})\right]\psi(q) - C \\
\text{s.t.} \quad & \hat{w}^u(\hat{\phi}) \geq \hat{w}^f_c(\hat{\phi}, q) - t \\
& \hat{\phi} = \hat{\phi}(q, \underline{\omega}, t) \text{ and } \hat{\phi}(q, \underline{\omega}, t) \text{ given by equation (7)} \\
& \varepsilon^i \text{ given by Property 1.}
\end{aligned} \tag{P1}$$

Several comments about the objective function are in order. As a result of quasilinearity the government is indifferent not only to the distribution of utilities but also to the distribution of net income. The government is only interested in the maximization of total income net of the costs associated to effort and school provision. This has two consequences. First, the tuition payed by those individuals attending school does not affect welfare directly. Second, it may be that the optimal tuition is not enough to finance the school. However, this is not a problem since the government can also use lump-sum taxes to cover the deficit. The same assumption of distribution-neutrality implies that these taxes will not affect welfare.

Introducing the values of the wages and the optimal effort in the objective function the Lagrangian can be written as:

$$\begin{aligned}
\hat{L} = & \int_{\Phi}^{\phi^H} \omega(\phi^i, 0, q)dF(\phi^i) + \underline{\omega}\left[F(\Phi) - F(\hat{\phi})\right] \\
& + \int_{\phi^L}^{\hat{\phi}} \omega(\phi^i, 0, 0)dF(\phi^i) - \int_{\hat{\phi}}^{\Phi} v(\varepsilon^i)dF(\phi^i) - C - \left[1 - F(\hat{\phi})\right]\psi(q) \\
& + \hat{\lambda}\left\{\frac{1}{F(\hat{\phi})}\int_{\phi^L}^{\hat{\phi}} \left[\omega(\phi^i, 0, 0) - \omega(\phi^i, 0, q)\right]dF(\phi^i) + t\right\}
\end{aligned}$$

with $\hat{\phi} = \hat{\phi}(q, \underline{\omega}, t)$, $\Phi = \Phi(q, \underline{\omega})$, where $\hat{\lambda}$ is a Lagrange multiplier and ε^i is given by Property 1. The derivatives of the Lagrangian with respect to the control variables are:

$$\begin{aligned}
\frac{d\hat{L}}{dt} &= \Big[-\underline{\omega} + \omega(\hat{\phi}, 0, 0) + v(\hat{\varepsilon}) + \psi(q)\Big] f(\hat{\phi})\hat{\phi}_t \\
&\quad + \hat{\lambda}\left\{\Big[\Big(\omega(\hat{\phi}, 0, 0) - \omega(\hat{\phi}, 0, q)\Big) - \Big(w^u(\hat{\phi}) - \tilde{w}_c^f(\hat{\phi}, q)\Big)\Big] \times \frac{f(\hat{\phi})}{F(\hat{\phi})}\hat{\phi}_t + 1\right\}, \\
\frac{d\hat{L}}{dq} &= \int_{\Phi}^{\phi^H} \omega_q(\phi^i, 0, q)dF(\phi^i) + \int_{\hat{\phi}}^{\Phi} v'(\varepsilon^i)\frac{\omega_q(\phi^i, \varepsilon^i, q)}{\omega_e(\phi^i, \varepsilon^i, q)}dF(\phi^i) \\
&\quad + \Big[-\underline{\omega} + \omega(\hat{\phi}, 0, 0) + v(\hat{\varepsilon}) + \psi(q)\Big] f(\hat{\phi})\hat{\phi}_q - \Big[1 - F(\hat{\phi})\Big]\psi'(q) \\
&\quad + \hat{\lambda}\left\{\Big[\Big(\omega(\hat{\phi}, 0, 0) - \omega(\hat{\phi}, 0, q)\Big) - \Big(\hat{w}^u(\hat{\phi}) - \hat{w}_c^f(\hat{\phi}, q)\Big)\Big]\right. \\
&\qquad\qquad \left. \times \frac{f(\hat{\phi})}{F(\hat{\phi})}\hat{\phi}_q - \int_{\phi^L}^{\hat{\phi}} \omega_q(\phi^i, 0, q)dF(\phi^i)\right\}
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\hat{L}}{d\underline{\omega}} &= \Big[F(\Phi) - F(\hat{\phi})\Big] + \Big[-\underline{\omega} + \omega(\hat{\phi}, 0, 0) + v(\hat{\varepsilon}) + \psi(q)\Big] f(\hat{\phi})\hat{\phi}_{\underline{\omega}} \\
&\quad - \int_{\hat{\phi}}^{\Phi} v'(\varepsilon^i)\frac{1}{\omega_e(\phi^i, 0, q)}dF(\phi^i) \\
&\quad + \hat{\lambda}\Big[\Big(\omega(\hat{\phi}, 0, 0) - \omega(\hat{\phi}, 0, q)\Big) - \Big(\hat{w}^u(\hat{\phi}) - \hat{w}_c^f(\hat{\phi}, q)\Big)\Big] \times \frac{f(\hat{\phi})}{F(\hat{\phi})}\hat{\phi}_{\underline{\omega}}.
\end{aligned}$$

Where I have omitted the arguments of $\hat{\phi}(q, \underline{\omega}, t)$ and $\Phi(q, \underline{\omega})$ and have again used the implicit function theorem to write the derivatives of ε^i.

The meaning of these equations is best understood under the assumption that $\hat{\lambda}$ will be equal to zero. This happens when the corresponding constraint is slack, i.e. when at the optimal $(q, \underline{\omega}, t)$, $\hat{w}^u(\hat{\phi}) > \hat{w}_c^f(\hat{\phi}, q) - t$. Using (7), which defines the marginal individual, the first-order condition for t can be written as

$$\left\{\Big[\hat{w}^d(\hat{\phi}, q, \underline{\omega}) - \underline{\omega}\Big] + \Big[\omega(\hat{\phi}, 0, 0) - \hat{w}^u(\hat{\phi})\Big] - t + \psi(q)\right\} f(\hat{\phi})\hat{\phi}_t = 0.$$

Thus, provided $f(\hat{\phi})\hat{\phi}_t \neq 0$, the optimal tuition level will be given by

$$t = \psi(q) + \Big[\hat{w}^d(\hat{\phi}, q, \underline{\omega}) - \hat{w}^u(\hat{\phi})\Big] - \Big[\underline{\omega} - \omega(\hat{\phi}, 0, 0)\Big], \tag{19}$$

which is strictly positive and higher than the marginal cost of serving an additional student at school. The government sets the tuition to extract the gain of the marginal individual from attending school, $w^d(\hat{\phi}, q, \underline{\omega}) - \hat{w}^u(\hat{\phi})$, discounting the increase in total productivity when the marginal individual attends school, $\underline{\omega} - \omega(\hat{\phi}, 0, 0)$. This optimal tuition reflects the fact that social and individual gains from education are different due to asymmetries of information. To better understand the meaning of the optimal tuition one can ask for the ability level of the marginal individual under this tuition. Using equation (7) one finds that, with the tuition in (19),

$$\underline{\omega} - \omega(\hat{\phi}, 0, 0) - v(\hat{\varepsilon}) = \psi(q).$$

Which means that the social gains (right hand side) and costs (left hand side) of letting the individual with ability $\hat{\phi}$ attend school will be equalized.

Using equation (19) the first order conditions for q and $\underline{\omega}$ can be written as follows:

$$\int_{\Phi}^{\phi^H} \omega_q(\phi^i, 0, q) dF(\phi^i) + \int_{\hat{\phi}}^{\Phi} v'(\varepsilon^i) \frac{\omega_q(\phi^i, \varepsilon^i, q)}{\omega_e(\phi^i, \varepsilon^i, q)} dF(\phi^i) = \left[1 - F(\hat{\phi})\right] \psi'(q) \quad (20)$$

and

$$\left[F(\Phi) - F(\hat{\phi})\right] = \int_{\hat{\phi}}^{\Phi} v'(\varepsilon^i) \frac{1}{\omega_e(\phi^i, \varepsilon^i, q)} dF(\phi^i). \quad (21)$$

Equation (20) is a Samuelson condition. The left hand side gives the marginal benefits of an increase in q. The first term in the expression is the marginal increase in the productivities of the students who do not exert effort at school. The students who exert a strictly positive effort at school adjust their effort level to continue meeting the standard after an increase in q, thus, for them, there is no change in productivity. However, there is a benefit associated with these students which is the reduction in effort needed to achieve the degree. The second term in equation (20) is exactly the value of this reduction in effort. The right hand side gives the marginal cost of an increase in q. Equation (20) says that at the optimal q, marginal costs and benefits must be equated.

Equation (21) gives the level of the efficient standard. It can also be interpreted in terms of the equality of marginal costs and marginal benefits. As before the marginal benefits are given in the left hand side and the marginal costs in the right hand side. The marginal cost of an increase in the standard is given by the value of the additional effort that must be made to achieve the degree. The marginal benefit is given by the number of students who will have a greater productivity.

Unfortunately, while $\hat{\lambda}$ can be equal to zero it is not necessary that this will be so. If the tuition in (19) is not high enough to make the option of attending school and not achieving the degree unattractive the participation constraint will bind. Two possibilities arise here. First, restricting the government to the instruments we have allowed, namely $(q, \underline{\omega}, t)$, the tuition will be equal to $\hat{w}_c^f - \hat{w}^u$ and the quality of education and the grading standard must be used as sorting instruments. Second, note that allowing the government to punish individuals that attend school and do not achieve the degree would relax the participation constraint. Moreover, the use of this additional instrument would not have any impact on welfare since no individual would attend and would not achieve the degree. This punishment may be related to a fee (for example charging an initial tuition which is higher than that in (19) and refunding the difference with the optimal t to students who achieve the degree) but it can also be a denial of an attendance certificate, this will impede individuals from having the wage for individuals who attended school but did not succeed at school.

Regime 2

I consider now the case in which the government will set the school variables if it wants all individuals to attend school. There are two differences between this problem and the previous one. First, the objective function is different since now it includes the wage of the individuals who attend school without achieving the degree instead of the wage of the individuals who do not attend school. Second, the solution to the problem must satisfy a participation constraint that guarantees that those who will not achieve the degree prefer to attend school. The problem of the government is

$$
\begin{aligned}
\max_{q,\underline{\omega},t} \quad & \tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) \left[1 - F(\tilde{\phi})\right] + \tilde{w}^f(\tilde{\phi}, q) F(\tilde{\phi}) \\
& \qquad - \int_{\tilde{\phi}}^{\phi^H} v(\varepsilon^i) dF(\phi^i) - \psi(q) - C \\
\text{s.t.} \quad & \tilde{w}^f(\tilde{\phi}, q) - t \geq \tilde{w}_c^u(\tilde{\phi}) \\
& \tilde{\phi} = \tilde{\phi}(q, \underline{\omega}) \text{ and } \tilde{\phi}(q, \underline{\omega}) \text{ given by equation (14)} \\
& \varepsilon^i \text{ given by Property 1.}
\end{aligned}
\tag{P2}
$$

The same comments on the objective function as for the case when not all individuals attend school hold in this case. The government is indifferent to the distribution of net income. Consequently the tuition does not affect welfare directly and school may be financed with the proceeds of lump-sum taxes which are implicit in the problem.

To write the Lagrangian of this problem introduce the expressions for wages and the ability of the marginal student in the objective function and in the participation constraint. The Lagrangian is given by:

$$\begin{aligned}\tilde{L} = &\int_{\Phi}^{\phi^H} \omega(\phi^i, 0, q) dF(\phi^i) + \underline{\omega} \left[F(\Phi) - F(\tilde{\phi}) \right] \\ &+ \int_{\phi^L}^{\tilde{\phi}} \omega(\phi^i, 0, q) dF(\phi^i) - \int_{\tilde{\phi}}^{\Phi} v(\varepsilon^i) dF(\phi^i) - \psi(q) - C \\ &+ \tilde{\lambda} \left[\int_{\phi^L}^{\tilde{\phi}} \omega(\phi^i, 0, q) \frac{dF(\phi^i)}{F(\tilde{\phi})} - t - \int_{\phi^L}^{\tilde{\phi}} \omega(\phi^i, 0, 0) \frac{dF(\phi^i)}{F(\tilde{\phi})} \right].\end{aligned} \tag{22}$$

With $\tilde{\phi} = \tilde{\phi}(q, \underline{\omega})$, $\Phi = \Phi(q, \underline{\omega})$ where $\tilde{\lambda}$ is a Lagrange multiplier and ε^i is given by Property 1.

Since $\tilde{\phi}$ does not depend on t it can be easily seen that the derivative of the Lagrangian with respect to t is

$$\frac{d\tilde{L}}{dt} = -\tilde{\lambda}$$

which means that the participation constraint will not bind. The tuition fee set by the government can be anything as long as it is strictly lower than $\tilde{w}_c^f - \tilde{w}^u$. Consequently $\tilde{\lambda}$ will be equal to zero. Alternatively, this means that school provision should be financed with the proceeds of the taxation system. Taking this into account, the derivatives of $\tilde{L}$ with respect to q and $\underline{\omega}$ are (again I have omitted the arguments of $\tilde{\phi}(q, \underline{\omega})$ to lighten notation):

$$\begin{aligned}\frac{d\tilde{L}}{dq} = &\int_{\Phi}^{\phi^H} \omega_q(\phi^i, 0, q) dF(\phi^i) + \int_{\phi^L}^{\tilde{\phi}} \omega_q(\phi^i, 0, q) dF(\phi^i) \\ &+ \int_{\tilde{\phi}}^{\Phi} v'(\varepsilon^i) \frac{\omega_q(\phi^i, \varepsilon^i, q)}{\omega_e(\phi^i, \varepsilon^i, q)} dF(\phi^i) - \psi'(q) \\ &+ \left[-\underline{\omega} + \omega(\tilde{\phi}, 0, q) + v(\tilde{\varepsilon}) \right] f(\tilde{\phi}) \tilde{\phi}_q\end{aligned}$$

and

$$\begin{aligned}\frac{d\tilde{L}}{d\underline{\omega}} = &\left[F(\Phi) - F(\tilde{\phi}) \right] - \int_{\tilde{\phi}}^{\Phi} v'(\varepsilon^i) \frac{1}{\omega_e(\phi^i, \varepsilon^i, q)} dF(\phi^i) \\ &+ \left[-\underline{\omega} + \omega(\tilde{\phi}, 0, q) + v(\tilde{\varepsilon}) \right] f(\tilde{\phi}) \tilde{\phi}_{\underline{\omega}}.\end{aligned}$$

The first-order conditions describing the interior optimal levels of q and $\underline{\omega}$ can be written as

$$\begin{aligned}\int_{\Phi}^{\phi^H} \omega_q(\phi^i,0,q)dF(\phi^i) + \int_{\phi^L}^{\tilde{\phi}} \omega_q(\phi^i,0,q)dF(\phi^i) \\ + \int_{\tilde{\phi}}^{\Phi} v'(\varepsilon^i)\frac{\omega_q(\phi^i,\varepsilon^i,q)}{\omega_e(\phi^i,\varepsilon^i,q)}dF(\phi^i) \\ + \left[-\underline{\omega} + \omega(\tilde{\phi},0,q) + v(\tilde{\varepsilon})\right] f(\tilde{\phi})\tilde{\phi}_q = \psi'(q)\end{aligned} \tag{23}$$

and

$$\begin{aligned}\left[F(\Phi) - F(\tilde{\phi})\right] + \left[-\underline{\omega} + \omega(\tilde{\phi},0,q) + v(\tilde{\varepsilon})\right] f(\tilde{\phi})\tilde{\phi}_{\underline{\omega}} \\ = \int_{\tilde{\phi}(q,\underline{\omega},t)}^{\Phi} v'(\varepsilon^i)\frac{1}{\omega_e(\phi^i,\varepsilon^i,q)}dF(\phi^i).\end{aligned} \tag{24}$$

These first-order conditions resemble those in equations (20) and (21) with one important difference. Since the tuition stops being a useful instrument to sort individuals, school quality and the standard must be used for this purpose. This is shown by the fourth term in (23) and the second one in (24). Note that these correction terms are very similar. The term $-\underline{\omega} + \omega(\tilde{\phi},0,q) + v(\tilde{\varepsilon})$ appears in equations (23) and (24). From the equation defining the ability of the marginal student, 14, it can be seen that this expression is equivalent to

$$\left[\tilde{w}^d - \tilde{w}^f\right] - \left[\underline{\omega} - \omega(\tilde{\phi},0,q)\right]$$

which is positive. It is also equal to the difference between the private and social gains from attending school for the marginal individual. Thus, the direction in which q and $\underline{\omega}$ are adjusted depends on the sign of $\tilde{\Delta}_{\tilde{\phi}}(\tilde{\phi},q,\underline{\omega})$ (recall Property 2). In the stable equilibrium, $\tilde{\phi}_{\underline{\omega}} > 0$ and $\tilde{\phi}_{\underline{q}} < 0$, which imply that $\tilde{\phi}$ will be lower than if there were enough instruments to determine sorting of individuals in an independent way from the incentives to learn.

4.2 The efficient outcome

My aim in this section has been to transmit two messages. First, and most important, the second-best efficient allocation admits a grading standard such that individuals are separated according to their ability with individuals of high ability achieving the degree and those of low ability not achieving the degree or not attending school. The main reason for a grading standard that sorts individuals into two groups is to give incentives to students for exerting effort; grading standards that do not separate individuals induce too little effort. This has already been pointed by Costrell (1994), and Betts (1998). However, they did not considered the problem taking into account the additional instruments of tuition and school quality.

The second message is related to these additional instruments. Tuition is used only when it is useful as a sorting device; this is when it is optimal to leave some individuals outside from school. The quality of education follows a Samuelson rule modified by the fact that benefits for those individuals whose productivity is equal to $\underline{\omega}$ is not to increase their productivity but to reduce their effort. These two messages are summarized in the following proposition.

Proposition 1 *(a) The efficient grading standard is given by* (20) *or by* (24). *These equations admit solutions such that not all individuals achieve the degree. (b) In both cases the optimal quality of education is given by modified Samuelson rules. (c) If a tuition is charged, it will be used only to achieve an efficient sorting of individuals between attending and not attending school.*

Additionally it would be desirable to know which of the two outcomes considered in this section, Regime 1 and Regime 2, is the efficient one. This is a difficult issue and not very interesting for my main purpose in this paper which is to understand the implications of profit maximizing behavior on the optimal standard. I will, thus, not pursue with this objective and go on with the analysis of the unregulated monopolist.

5 The behavior of the monopolist school

In this section I address the analysis of the first stage of the game presented in section (2). The difference with the analysis in the previous section is that school choice will be determined to maximize the payoff of an egoistic agent that maximizes profits. Property 3 allows me to write the school's objective function as

$$\Pi = \left[1 - F(\hat{\phi})\right] \left[t - \psi(q)\right] - C. \tag{25}$$

I will first show the solution to the problem under the assumption that not all individuals attend school. Then I will show the solution under the assumption that all individuals attend school. In each of these cases the school is subject to the same participation constraints as those that constrained the welfare maximization problem. Finally, I compare both solutions and show which is the one that maximizes the payoff of the school.

5.1 Solving the problem of the monopolist

As for the efficient outcome, I first solve the problem in each of the two Regimes and then show which is the one that maximizes the monopolist's payoff.

Regime 1

Consider the case in which not all individuals attend school. The problem of the monopoly school will be given by

$$\begin{aligned} \max_{\hat{\phi},q,\underline{\omega},t} \quad & \hat{\Pi} = \Big[1 - F(\hat{\phi})\Big]\Big[t - \psi(q)\Big] - C \\ \text{s.t.} \quad & \hat{w}^u(\hat{\phi}) > \hat{w}^f_c(\hat{\phi}, q) - t \\ & \phi^H \geq \hat{\phi} \geq \phi^L \\ & \hat{\phi} = \hat{\phi}(q,\underline{\omega},t) \text{ and } \hat{\phi}(q,\underline{\omega},t) \text{ given by equation (7).} \end{aligned} \tag{P3}$$

To solve this problem I solve (implicitly) equation (7) to find the ability level of the marginal individual $\hat{\phi}(q,\underline{\omega},t)$ and introduce it in the objective function. Regarding the participation constraint, I solve the problem neglecting it and check ex-post whether the solution satisfies the restriction. The constraints on $\hat{\phi}$ will be considered explicitly. Under these conditions the Lagrangian of the problem of the monopolist is

$$\begin{aligned} \hat{\Gamma} = & \Big[1 - F(\hat{\phi}(q,\underline{\omega},t)\Big]\Big[t - \psi(q)\Big] - C \\ & + \overline{\eta}(\phi^H - \hat{\phi}(q,\underline{\omega},t)) + \underline{\eta}(\hat{\phi}(q,\underline{\omega},t) - \phi^L) \end{aligned}$$

where $\overline{\eta}$ and $\underline{\eta}$ are Lagrange multipliers.

The derivatives of the Lagrangian with respect to the control variables t, q and $\underline{\omega}$ are

$$\begin{aligned} \frac{d\hat{\Gamma}}{dt} = & - f\left(\hat{\phi}(q,\underline{\omega},t)\right)\hat{\phi}_t(q,\underline{\omega},t)\left[t - \psi(q)\right] \\ & + \left[1 - F\left(\hat{\phi}(q,\underline{\omega},t)\right)\right] + (\underline{\eta} - \overline{\eta})\hat{\phi}_t(q,\underline{\omega},t) \end{aligned} \tag{26}$$

$$\begin{aligned} \frac{d\hat{\Gamma}}{dq} = & - f\left(\hat{\phi}\left(q,\underline{\omega},q\right)\right)\hat{\phi}_q(q,\underline{\omega},t)\left[t - \psi(q)\right] \\ & - \left[1 - F\left(\hat{\phi}(q,\underline{\omega},t)\right)\right]\psi'(q) + (\underline{\eta} - \overline{\eta})\hat{\phi}_q(q,\underline{\omega},t) \end{aligned} \tag{27}$$

and

$$\frac{d\hat{\Gamma}}{d\underline{\omega}} = -\, f\left(\hat{\phi}(q,\underline{\omega},t)\right)\hat{\phi}_{\underline{\omega}}(q,\underline{\omega},t)\left[t-\psi(q)\right]+(\underline{\eta}-\overline{\eta})\hat{\phi}_{\underline{\omega}}(q,\underline{\omega},t). \tag{28}$$

Initially, I will ignore the first order condition for q and concentrate on the optimal standard. Suppose that the ability of the marginal individual will be at an interior level, i.e. $\phi^H > \hat{\phi} > \phi^L$. With this assumption $\overline{\eta}$ and $\underline{\eta}$ will be equal to zero. Consequently, the optimal tuition can be obtained from equation (26) equated to zero and is given by

$$t = \psi(q) + \frac{1 - F\left(\hat{\phi}(q,\underline{\omega},t)\right)}{f\left(\hat{\phi}(q,\underline{\omega},t)\right)\hat{\phi}_t(q,\underline{\omega},t)}. \tag{29}$$

Equation (28) can be simplified using (29) as follows:

$$\frac{d\hat{\Gamma}}{d\underline{\omega}} = -\left[1 - F\left(\hat{\phi}(q,\underline{\omega},t)\right)\right]\frac{\hat{\phi}_{\underline{\omega}}(q,\underline{\omega},t)}{\hat{\phi}_t(q,\underline{\omega},t)}. \tag{30}$$

Equation (30) is equal to zero at $\hat{\phi}(q,\underline{\omega},t) = \phi^H$, but this cannot be a maximum since in such a case there is no individual attending school and, thus, there are no profits for the monopoly school. At any other point, Property 5 implies that

$$\frac{d\hat{\Gamma}}{d\underline{\omega}} < 0.$$

This means that the grading standard is such that all individuals will attend school and all achieve the degree, i.e., $\hat{\phi}(q,\underline{\omega},t) = \phi^L$. In the stable equilibria, this implies a very low grading standard. In the unstable equilibria, it implies the counterintuitive result of a very high grading standard achieved by all students.

The reason for this extreme result can be found in Properties 4 and 5. Property 4 says that the effort of the marginal individual is in a range in which marginal disutility is higher than the marginal effect on productivity. This implies that for the marginal individual, the direct effects of an increase in the grading standard on the utility when achieving the degree are always negative. Consequently, increasing the standard will always reduce the willingness to pay for attending school. Property 5 states that increasing the tuition and the grading standard have effects of equal directions on the ability of the marginal individual. This is a consequence of Property 4. As usual, a change in tuition has two effects on profits, the direct effect and the induced change in demand. A change in the grading standard only affects demand. Thus, given an increase in the tuition its consequent reduction in demand can be neutralized reducing the grading standard. With the grading standard the monopolist can increase tuition without reducing the demand for school. The intuition in the unstable equilibrium is very similar, however, the grading standard will be increased to increase the demand for school.

Finally, I must verify whether the participation constraint is satisfied by this solution. This is a simple consequence of the fact that the monopoly school sets the triplet $(q, \underline{\omega}, t)$ in order to have all individuals optimally choosing to attend school and achieving the degree. If all individuals attend school the stated participation constraint is trivially satisfied.

Regime 2

Consider the situation where all individuals attend school (i.e. $\hat{\phi} = \phi^L$). The problem of the school is given by

$$\begin{aligned} \max_{q,\underline{\omega},t} \quad & \tilde{\Pi} = t - \psi(q) - C \\ \text{s.t.} \quad & \tilde{w}^f(\tilde{\phi}, q) - t \geq \tilde{w}_c^u(\tilde{\phi}) \\ & \phi^H \geq \tilde{\phi} \geq \phi^L \\ & \tilde{\phi} = \tilde{\phi}(q, \underline{\omega}) \text{ and } \tilde{\phi}(q, \underline{\omega}) \text{ given by equation (14).} \end{aligned} \tag{P4}$$

The Lagrangian of this problem is

$$\begin{aligned} \tilde{\Gamma} = & \tilde{w}^f(\tilde{\phi}(q, \underline{\omega}), q) - \tilde{w}_c^u(\tilde{\phi}(q, \underline{\omega})) - \psi(q) - C \\ & + \overline{\mu}(\phi^H - \tilde{\phi}(q, \underline{\omega})) + \underline{\mu}(\tilde{\phi}(q, \underline{\omega}) - \phi^L) \end{aligned}$$

where I have replaced t in the objective function using the participation constraint and $\overline{\mu}$ and $\underline{\mu}$ are Lagrange multipliers.

The derivatives of the Lagrangian of this problem are

$$\frac{d\tilde{\Gamma}}{dq} = \frac{d}{dq}\left[\tilde{w}^f(\tilde{\phi}, q) - \tilde{w}_c^u(\tilde{\phi})\right] - \psi'(q) + (\underline{\mu} - \overline{\mu})\tilde{\phi}_q \tag{31}$$

and

$$\frac{d\tilde{\Gamma}}{d\underline{\omega}} = \frac{d}{d\underline{\omega}}\left[\tilde{w}^f(\tilde{\phi}, q) - \tilde{w}_c^u(\tilde{\phi})\right] + (\underline{\mu} - \overline{\mu})\tilde{\phi}_{\underline{\omega}}. \tag{32}$$

As before ignore (31) and suppose $\phi^H > \hat{\phi} > \phi^L$. In these cases $\underline{\mu} = \overline{\mu} = 0$ and equation (32) can be written as follows:

$$\frac{d\tilde{\Gamma}}{d\underline{\omega}} = \frac{\partial}{\partial\tilde{\phi}}\left[\tilde{w}^f(\tilde{\phi}, q) - \tilde{w}^u_c(\tilde{\phi})\right]\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega}).$$

This expression will not be equal to zero. This follows from two facts. First, if $q > 0$,

$$\frac{\partial}{\partial\tilde{\phi}}\left[\tilde{w}^f(\tilde{\phi}, q) - \tilde{w}^u_c(\tilde{\phi})\right] = \omega(\tilde{\phi}, 0, q) - \omega(\tilde{\phi}, 0, 0) > 0$$

since $\omega(\phi^i, e^i, q)$ is increasing in q. Second, Property 6 implies that $\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega}, t)$ is either positive or negative but it is always different than zero. This means that the grading standard will be set such that $\tilde{\phi} = \phi^H$. This will be achieved with a very high grading standard, if $\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega}, t) > 0$, or with a very low one, otherwise. In both cases no student will achieve the degree. Consequently in the optimum $\overline{\mu} \neq 0$ and $\underline{\mu} = 0$.

In this case the result is explained by a more conventional reason. Since we are in a regime in which all individuals attend school, the monopolist will set the grading standard in such a way that the tuition is maximized. This is achieved when the difference between the wage of going to school and not achieving the degree and that of not going to school is maximized. This happens when the marginal student is that with the highest ability. The reason behind this is very simple, since the productivity of a given individual is always higher when attending school than when not attending (because of the effect of school quality) the wage differential between attending school and not achieving the degree and not attending school is always increasing in the ability of the marginal student.

5.2 The optimal choice of the school

Regarding the grading standard the solutions of problems P3 and P4 are very similar; as a result of the grading standard set by the monopolist individuals will not be differentiated in the labor market. However, in the case in which the solution is a standard that all students can achieve, there will be a mass of students that will exert positive effort while in the case in which no student achieves the optimal standard, students will exert no effort. This means that the wage rate seen in the labor market will be (weakly) higher in the case where all students achieve the degree than in the case no student does.

Finally, I must compare the payoffs of the school in each of these two situations. To find the payoff in Regime 1, when all students achieve the degree, one must have an expression for the tuition when $\hat{\phi} = \phi^L$. Strictly, what one should do is to use the first order conditions of this problem to find a value for $\overline{\eta}$ and use this value together with the first order condition for t to find the optimal tuition. However, once it is known that the school will set the grading standard to have all individuals attending school and achieving the degree, $\hat{\phi}$ can be treated as a fixed value and one can use equation (7) to find the maximum tuition that can be charged. Thus, the value of the objective function of problem P3 is:

$$\hat{\Pi} = \int_{\phi^L}^{\phi^H} \omega(\phi^i, \varepsilon^i, q) dF(\phi^i) - \omega(\phi^L, 0, 0) - v(\varepsilon^L) - \psi(q) - C$$

with q set at its optimal level and ε^i given by Property 1.

In Regime 2 the payoff is given by

$$\tilde{\Pi} = \int_{\phi^L}^{\phi^H} \left[\omega(\phi^i, 0, q) - \omega(\phi^i, 0, 0)\right] dF(\phi^i) - \psi(q) - C$$

with q given set at the optimal level.

To compare profits in these two situations first consider the extreme case in which $\omega(\phi^L, e^i, q) = 0$ for all pairs (e^i, q). This represents the case in which, by no means, the individual with the lowest ability will have a strictly positive productivity. Under this assumption, in the case in which every body achieves the degree, the grading standard is so low that the individual with the lowest ability must make no effort to achieve the degree. In this case profits will be given by

$$\hat{\Pi}_* = \int_{\phi^L}^{\phi^H} \omega(\phi^i, 0, q) dF(\phi^i) - \omega(\phi^L, 0, 0) - \psi(q) - C.$$

Now compare $\hat{\Pi}_*$ and $\tilde{\Pi}$. Since the only difference between them is a constant it must be that the optimal q must be the same in both problems. Therefore,

$$\hat{\Pi}_* - \tilde{\Pi} = \int_{\phi^L}^{\phi^H} \omega(\phi^i, 0, 0) dF(\phi^i) - \omega(\phi^L, 0, 0) > 0. \tag{33}$$

Thus, under the assumption that $\omega(\phi^L, e, q) = 0$ for all pairs (e, q) the optimal choice of the monopolist is a very low standard that can be achieved by all students.

Sticking to the assumption that $\omega(\phi^i, e^i, q)$ is strictly increasing in all its arguments (i.e. if $\omega_q(\phi^L, 0, q) > 0$), this argument continues to hold. To see this, note that the profits $\hat{\Pi}_*$ can always be obtained by the monopolist when all students achieve the degree. It is enough for the monopolist to set a very low level of $\underline{\omega}$ such that no student makes effort. Thus, it must be that

$$\hat{\Pi} \geq \hat{\Pi}_* > \tilde{\Pi}. \tag{34}$$

Actually, the first inequality in (34) could be written as a strict inequality since under Property 4 and the assumptions that $\omega_{e^i}(\phi^i, e^i, q) > 0$ and the Inada condition on $v(e)$, individuals with ability ϕ^L will exert a non-zero effort.

Two facts lie behind this result. First, because of the beliefs of firms, the surplus that can be extracted in the case all students achieve the degree is higher when all students achieve the degree than when no student achieves it. Second, the effect of beliefs could vanish under a different assumption,[11] however, higher effort results in a higher extractable surplus as a result of the the concavity assumptions which imply that the monopolist school benefits form individuals exerting a non-zero effort.

Finally, consider the quality of education chosen by the school. It must be clear that I only need to consider the case in which $\hat{\phi} = \phi^L$. From equations (27) and (29) it follows that:

$$\psi'(q) = -\frac{\hat{\phi}_q(q, \underline{\omega}, t)}{\hat{\phi}_t(q, \underline{\omega}, t)}.$$

Which, using the expressions for $\hat{\phi}_t(q, \underline{\omega}, t)$, $\hat{\phi}_q(q, \underline{\omega}, t)$ and $\hat{\Delta}_q(\hat{\phi}, q, \underline{\omega})$ given by (8), (10) and (47) can be written as

$$\int_{\hat{\phi}}^{\phi^H} \omega_q(\phi^i, \varepsilon^i, q) \frac{dF(\phi^i)}{1 - F(\hat{\phi})} + \frac{v'(\hat{\varepsilon})}{\omega_e(\hat{\phi}, \hat{\varepsilon}, q)} \omega_q(\hat{\phi}, \hat{\varepsilon}, q) = \psi'(q). \tag{35}$$

In the optimum school quality will be given by (35) evaluated at $\hat{\phi} = \phi^L$. The monopolist only takes into account, in setting q, the effect it has in the utility of the marginal individual since that determines the surplus it will be able to extract from students.

I summarize the main result in this section in the following proposition.

Proposition 2 *(a) The monopolist school will set a grading standard such that all individuals attend school and all achieve the degree. (b) The quality of the school is set to equate the marginal gain from going to school of the marginal student with the marginal cost of quality. (c) The monopolist sets a tuition that extracts all the surplus of the marginal student.*

Propositions 1 and 2 show the inefficiency of the choices made by the monopolist. The most important implication of the propositions is that while it may be efficient to have some separation according to productivity in the labor market the

11. For example if the contrafactual wages are assumed to be equal to the productivity of the individual with the lowest ability as in Lizzeri (1999)

monopolist will not allow this to happen. This is because the grading standard can be used by the monopolist school to increase demand with out needing to reduce the tuition that the marginal individual is willing to pay and without affecting the costs faced by the monopolist.

Additionally, the monopolists school does not set the level of q according to a Samuelson rule, it only considers the effect on the utility of the marginal individual. Finally, the social optimal pricing rule and that of the monopolist are also different. The school wants to extract the whole surplus of the marginal individual while the social optimal pricing rule takes into account the divergence between the social and the private values of education to achieve an optimal sorting of individuals between those that attend school and those that do not attend.

6 Concluding comments

In this paper I have addressed the issue of how an unregulated self-interested school would set grading standards. I have done so, assuming the role of education is to increase individual labor market productivity and that the school and the labor market have imperfect information about students productivities and of its determinants. I have shown that the monopoly school always sets the grading standard such that all individuals attend school and all achieve the degree. This means that with a monopoly school all individuals in the labor market will be undifferentiated and they will all have the same wage. The efficient outcome, however, differs from this one in that individuals are differentiated in the labor market.

The inefficiency of the monopolist grading standard comes from the divergence of its objective with social welfare. The monopolist sets the grading standard to maximize the extractable surplus. This is equal to the maximum tuition that the marginal individual is willing to pay and is maximized allowing all individuals to achieve the degree. What explains this is that, for the marginal individual, the direct effect of an increase in the grading standard always reduces his utility of achieving the degree and consequently reduces his marginal willingness to pay for education. The efficient grading standard maximizes average utility, which is maximized with an intermediate grading standard.

The results in this paper are extreme with respect to what is observed in reality. The level of drop outs and the attendance level in secondary and university education is far from that predicted by this model. However extreme, the results do reflect an important fact observed by empirical researchers on the economics of education. It has been noticed by many authors that graduation rates in private schools are higher than in public schools.[12] I do not argue that the objective function used in Section 4 is a realistic choice for the objective of public schools, however it does say that benevolent governments would want schools to set more rigorous grading standards than those set by an unregulated monopoly school.

Moreover, the model analyzed in this paper is a first step towards other issues. In future research I plan to consider the effect of competition between schools in grading standards. This is particularly important to answer whether the extreme results on the decentralized grading standards results from the lack of competition or from the asymmetries of information in the labor market. Other lines of research that I plan to address are that of the regulation of schools, particularly the introduction of competition together with the use of vouchers.

There are other important situations that should be analyzed besides the first- and second-best analyzed in this paper. Particulary it would also be important to understand the optimal policies when the government faces additional information restrictions. It is likely that the government does not observe the threshold used by the school to award degrees nor the effort of teachers. However, the government may have access to other sorts of information like the number of students attending school and the number of individuals who achieve the degree.

12. This literature is surveyed partially by Hanushek (2002, pp. 2107-2114). It is strange to see that, virtually, in all of the empirical studies addressing the issue, graduation rates are seen as a measure of success of schools. However, this model implies that the graduation rate is a bad predictor of school quality particularly when grading standards are set by schools as is done in most of the States in the US or in most countries in Latin America.

Appendix

A Full information Pareto optimal solution

Under conditions of perfect information, if the government and the firms observe perfectly the parameters and choice variables of individuals and the school, degrees are not needed. Individuals will receive a wage equal to their productivity. Although in this paper I am mostly interested in the optimal level of the grading standard I include this appendix to highlight the inefficiencies due to asymmetric information.

The most important problem that must be resolved here is that of the attendance to school problem. The quality level of the school q will be given by a traditional Samuelson condition. However, it is no longer clear that the monotonic rules about attendance to school are optimal

Since the labor market observes directly the productivity of each individual there is no role for the signaling device. The problem of the government is thus to decide the optimal attendance rule, the quality level of the school and the optimal effort of each individual.

To write the problem of the government let me introduce some additional notation. Let $\alpha^i \in \{0,1\}$ be equal to one if it is optimal to let individual of type ϕ^i attend school and equal to zero otherwise. With this definition I can write the efficiency maximization problem as follows:

$$\begin{aligned} \max_{q,\{\alpha^i,e_u^i,e_a^i\}_{i=L}^{i=H}} \quad & \int_{\phi^L}^{\phi^H} \alpha^i \left[\omega(\phi^i, e_a^i, q) - v(e_a^i) - \psi(q)\right] dF(\phi^i) \\ & + \int_{\phi^L}^{\phi^H} [1-\alpha^i] \left[\omega(\phi^i, e_u^i, 0) - v(e_u^i)\right] dF(\phi^i) - C \end{aligned} \tag{P5}$$

where e_a^i and e_u^i are, respectively, the effort levels of an individual of ability ϕ^i when and when he does not attend school.

The first order conditions for an interior optimum are:

$$[q]: \quad \int_{\phi^L}^{\phi^H} \alpha^i \left[\omega_q(\phi^i, e_a^i, q) - \psi'(q)\right] dF(\phi^i) = 0, \tag{36}$$

$$\left[e_a^i\right]: \quad \omega_e(\phi^i, e_a^i, q) - v'(e_a^i) = 0, \tag{37}$$

$$\left[e_u^i\right]: \quad \omega_e(\phi^i, e_u^i, 0) - v'(e_u^i) = 0 \tag{38}$$

and

$$[\alpha]: \qquad \alpha^i = \begin{cases} 1 & \text{if} \begin{array}{l} [\omega(\phi^i, e_a^i, q) - v(e_a^i)] \\ -[\omega(\phi^i, e_u^i, 0) - v(e_u^i)] \geq \psi(q) \end{array} \\ 0 & \text{otherwise.} \end{cases} \tag{39}$$

These conditions have standard interpretations. Equation (36) is a standard Samuelson condition, it says that sum of the marginal benefits of school quality should be set to equalize its marginal costs. Equations (37) and (38) say that the marginal disutility of effort should be set to equalize the marginal gains of effort.

Equation (39) rules whether an individual attends school. This depends on the difference of his utility levels in both situations. In the case this difference is big enough (bigger or equal to the cost of attending a student at school) α^i will be set to one. In any other case it will be set to zero.

These equations, particularly the rules about optimal effort level and school attendance are very different than all the others I have found in the paper. Equations (37) and (38) differ with the decentralized and second best outcomes presented in the paper in three ways. First, in the first-best solution, effort level will only be zero if there is no interior solution. Contrary to this, I have shown that in the second best and in the monopoly solution effort is zero for a wide range of individuals.

Second, in the second-best and in the decentralized outcomes effort level is a decreasing function of ability for those students who achieve the degree. Particularly in those two problems whether the effort level increases or decreases with ability is independent of the form of $\omega(\cdot, \cdot, \cdot)$. However, in the first-best outcome, the effort level will only be decreasing if effort and ability are substitutes while if they are complements it will be increasing.

Finally, the first best solution differs with the others considered in this paper in the way students are allocated between attending school or not. Particularly with out further restrictions in $\omega(\cdot, \cdot, \cdot)$ it is not possible to have a monotonic rule as in the second-best. To have a rule like that one I would need to impose sufficient conditions to assure that the difference in the utility between attending school and not attending is increasing in ability.[13]

The inefficiency of the level of attendance to school found in the decentralized outcome in this model is due to the divergence between the private and the social

13. I am aware that if $\psi(q)$ is very small it is possible to have a situation in which it is optimal to let all individuals attend school. Although this does not seem to be a very interesting case, note that there is a big difference between the reason why this happens here and the reason why it happens in the decentralized outcome.

returns to education. Individuals decide whether to attend school based on the comparison between the wage rate and the tuition level. However, the government decides whether to allow and individual to attend school on the basis of the comparison between the productivity and the marginal cost of attending a student at school. The fact that the wage rate (the private return) does not correspond with individual productivity (the social return) is the responsible of the inefficiency of the second-best outcome.

B Proofs

B.1 Proof of Property 1

I consider only the first part of the property since the proof of the second one is even simpler. Suppose that a student of type i decides to achieve the degree and that for this individual $\underline{e}^i > 0$. This means that there is some effort level e^i_* for which $w^d - t - v(e^i_*) \geq w^f - t$ that satisfies $\omega(\phi^i, e^i_*, q) \geq \underline{\omega}$. Since wages cannot be conditioned on the effort level, if $e^i_* > \underline{e}^i$ the student will be better off exerting effort $\underline{e}^i$ than e^i_*. If for this individual, $\omega(\phi^i, 0, q) \geq \underline{\omega}$ he would not exert an effort level different than zero since doing so would only reduce his utility.

B.2 Proof of Property 2

Suppose an individual with ability ϕ^i (weakly) prefers to achieve the degree. This means that $w^d - t - v(\varepsilon^i) \geq w^f - t$. Consider now an individual with type $\phi^h > \phi^i$. His utility will be $w^d - t - v(\varepsilon^h)$ if he achieves the degree and $w^f - t$ if he does not. Since $\omega(\phi^i, e^i, q)$ is increasing in ϕ^i and in e^i it should be that $\varepsilon^i > \varepsilon^h$ and thus $w^d - t - v(\varepsilon^h) > w^d - t - v(\varepsilon^i)$. Clearly $w^d - t - v(\varepsilon^h) > w^f - t$. The proof for the second part is along the same lines.

B.3 Proof of Property 3

Given $(q, \underline{\omega}, t)$ the demand for schooling will depend on the ordering of the utility levels achieved when attending school and when not attending. If an individual of type i attends school his utility will be

$$U^i = \max\{w^d - t - v(\varepsilon^i), w^f - t\}.$$

If he does not attend school it will be w^u since they will not exert any effort. The individual will attend school only if $U^i \geq w^u$. Since the utility of attending school and not achieving the degree does not depend on ability all students attend school if

$$w^f - t \geq w^u. \tag{40}$$

Otherwise, those who would not achieve the degree do not attend school. Moreover, when $w^u > w^f - t$ there is a group of individuals who would achieve the degree if they had attended school but prefer not to attend school since for them $w^u > w^d - t - v(\varepsilon^i)$.

The proof for the monotonicity of the attendance decision is along the same lines as the proof for Property 2.

B.4 Proof of Property 4

I provide the proof only for Regime 2. The proof for Regime 1 requires minor changes on that presented here.

Notice that a student exerting no effort can not be marginal (unless $\tilde{\phi} = \phi^L$) since his utility when achieving the degree, $\tilde{w}^d(q, \tilde{\phi}, \underline{\omega}) - t$, is strictly grater than that if he does not achieve the degree, $w^f(\tilde{\phi}, q) - t$. Consequently, I only need to consider students who exert a strictly positive effort to achieve the degree.

For the marginal student it must be that

$$\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) \geq \omega(\tilde{\phi}, \tilde{\varepsilon}, q)$$

and

$$\omega(\tilde{\phi}, 0, q) \geq \tilde{w}^f(\tilde{\phi}, q)$$

with at least one of the inequalities being strict (both expressions will hold simultaneously only in the case in which $\phi^H = \phi^L$). The first of these inequalities implies that

$$\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - t - v(\tilde{\varepsilon}) \geq \omega(\tilde{\phi}, \tilde{\varepsilon}, q) - t - v(\tilde{\varepsilon}) \tag{41}$$

and the second that

$$\omega(\tilde{\phi}, 0, q) - t \geq \tilde{w}^f(\tilde{\phi}, q) - t. \tag{42}$$

Now fix q and let $\tilde{e}^*$ satisfy

$$\frac{v'(\tilde{e}^*)}{\omega_e(\tilde{\phi}, \tilde{e}^*, q)} = 1.$$

Suppose that the marginal student needs to exert an ability level $\tilde{\varepsilon} \leq \tilde{e}^*$ to achieve the standard. Since $\omega(\phi^i, e^i, q) - v(e^i)$ is concave and $\tilde{\varepsilon} > 0$, $\tilde{\varepsilon} \leq \tilde{e}^*$ implies that

$$\omega(\tilde{\phi}, \tilde{\varepsilon}, q) - t - v(\tilde{\varepsilon}) > \omega(\tilde{\phi}, 0, q) - t. \tag{43}$$

Equations (41), (42) and (43) imply that

$$\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - t - v(\tilde{\varepsilon}) > \tilde{w}^f(\tilde{\phi}, q) - t.$$

Which contradicts the fact that for the marginal student it must be that

$$\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - t - v(\tilde{\varepsilon}) = \tilde{w}^f(\tilde{\phi}, q, \underline{\omega}) - t.$$

This implies that the marginal student must exert an effort level $\tilde{\varepsilon} > \tilde{e}^*$. Thus,

$$\frac{v'(\tilde{\varepsilon})}{\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q)} > 1.$$

from the concavity of $\omega(\phi^i, e^i, q) - v(e^i)$.

B.5 Proof of Property 5

Using the expressions:

$$\hat{\Delta}_t(\hat{\phi}, q, \underline{\omega}, t) = -1 \tag{44}$$

$$\hat{\Delta}_{\hat{\phi}}(\hat{\phi}, q, \underline{\omega}, t) = \frac{\partial}{\partial \hat{\phi}}\left[\hat{w}^d(\hat{\phi}, q, \underline{\omega}) - \hat{w}^u(\hat{\phi})\right] + v'(\hat{\varepsilon})\frac{\omega_{\hat{\phi}}(\hat{\phi}, \hat{\varepsilon}, q)}{\omega_e(\hat{\phi}, \hat{\varepsilon}, q)}, \tag{45}$$

$$\hat{\Delta}_{\underline{\omega}}(\hat{\phi}, q, \underline{\omega}, t) = \frac{\partial}{\partial \underline{\omega}}\hat{w}^d(\hat{\phi}, q, \underline{\omega}) - v'(\hat{\varepsilon})\frac{1}{\omega_e(\hat{\phi}, \hat{\varepsilon}, q)} \tag{46}$$

and

$$\hat{\Delta}_q(\hat{\phi}, q, \underline{\omega}, t) = \frac{\partial}{\partial q}\hat{w}^d(\hat{\phi}, q, \underline{\omega}) + v'(\hat{\varepsilon})\frac{\omega_q(\hat{\phi}, \hat{\varepsilon}, q)}{\omega_e(\hat{\phi}, \hat{\varepsilon}, q)}. \tag{47}$$

the proof follows along the same lines as those of the proof for Property 6 which follows.

B.6 Proof of Property 6

(a) The expression for $\tilde{\phi}_{\underline{\omega}}(q, \underline{\omega})$ is given by equation (15). According to this equation I need to prove that $\tilde{\Delta}_{\underline{\omega}}(\tilde{\phi}, q, \underline{\omega}) < 0$. First, from (13):

$$\tilde{\Delta}_{\underline{\omega}}(\tilde{\phi}, q, \underline{\omega}) = \frac{\partial}{\partial \underline{\omega}}\left[\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - \tilde{w}^f(\tilde{\phi}, q)\right] - v'(\tilde{\varepsilon})\frac{1}{\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q)}, \tag{48}$$

and equations (11) and (12) one obtains

$$\frac{\partial}{\partial \underline{\omega}}\left[\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - \tilde{w}^f(\tilde{\phi}, q)\right] = \frac{F(\Phi) - F(\tilde{\phi})}{1 - F(\tilde{\phi})}.$$

This term is clearly smaller or equal to one. Second from Property 4

$$v'(\tilde{\varepsilon})\frac{1}{\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q)} > 1.$$

These two facts and equation (48) yields the stated result.

(b) The expression for $\tilde{\phi}_q(q, \underline{\omega})$ is given by equation (16). The expression for $\tilde{\Delta}_q(\tilde{\phi}, q, \underline{\omega})$ is

$$\tilde{\Delta}_q(\tilde{\phi}, q, \underline{\omega}) = \frac{\partial}{\partial q}\left[\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - \tilde{w}^f(\tilde{\phi}, q)\right] + v'(\tilde{\varepsilon})\frac{\omega_q(\tilde{\phi}, \tilde{\varepsilon}, q)}{\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q)}. \tag{49}$$

Consequently I need to prove that $\tilde{\Delta}_q(\tilde{\phi}, q, \underline{\omega}) > 0$. Equations (11) and (12) yield

$$\frac{\partial}{\partial q}\left[\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - \tilde{w}^f(\tilde{\phi}, q)\right]$$
$$= \int_{\Phi}^{\phi^H} \omega_q(\phi^i, 0, q)\frac{dF(\phi^i)}{1 - F(\tilde{\phi})} - \int_{\phi^L}^{\tilde{\phi}} \omega_q(\phi^i, 0, q)\frac{dF(\phi^i)}{1 - F(\tilde{\phi})}$$

which is positive if $\omega_{\phi q}(\phi^i, e^i, q) > 0$.

Equations (16) and (49) and the previous equation yield the stated result.

Finally, just for the record, let us state here the expression for $\tilde{\Delta}_{\tilde{\phi}}(\tilde{\phi}, q, \underline{\omega})$.

$$\tilde{\Delta}_{\tilde{\phi}}(\tilde{\phi}, q, \underline{\omega}) = \frac{\partial}{\partial \tilde{\phi}}\left[\tilde{w}^d(\tilde{\phi}, q, \underline{\omega}) - \tilde{w}^f(\tilde{\phi}, q)\right] + v'(\tilde{\varepsilon})\frac{\omega_\phi(\tilde{\phi}, \tilde{\varepsilon}, q)}{\omega_e(\tilde{\phi}, \tilde{\varepsilon}, q)}. \tag{50}$$

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