Working Papers Economics - Density Estimation and Specification Testing for Nonparametric Regression Models

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The classical nonparametric kernel estimator converges slowly to the density function. To address this limitation, this study proposes more efficient estimators that, in addition to exhibiting several properties absent from the classical kernel estimator, converge more rapidly to the true density as the number of observations increases. The paper also develops a more powerful density-based specification test for the nonparametric regression function.

Publication Date: Monday, 3 of August 2026 **Approach**
Motivated by the slow convergence of the classical nonparametric kernel density estimator commonly used in the literature to estimate density functions, this paper investigates more efficient estimators of the density function and its derivative for the marginal density in the context of nonparametric regression models. We show that, in the presence of an unknown nonparametric regression function, the proposed estimators of the density and its derivative achieve a parametric rate of convergence and possess several desirable properties that the classical estimator lacks. Moreover, in the absence of a nonparametric regression function, under normality, the proposed method performs as well as if the model were known and the density were estimated using maximum likelihood estimation. Building on the new density estimator, the paper also proposes a more powerful density-based specification test for the nonparametric regression function.

Contribution
This study contributes to the literature on more efficient estimation of the density function and its derivative for the marginal density in nonparametric regression models. Specifically, it introduces a density estimator that is more efficient than the classical kernel estimator traditionally employed for this purpose.

Several advantages of the proposed estimators are highlighted. First, we establish that the proposed density estimator can attain the parametric rate of convergence despite its nonparametric nature. Second, in addition to converging faster than the traditional density estimator commonly used in the literature, the new estimator exhibits several attractive properties, including being uniformly bounded, having uniformly bounded derivatives, and possessing smoothness properties that are absent from the classical kernel estimator. Finally, beyond providing information on the distribution of the data, density functions have also been widely used in model specification testing. Accordingly, based on the proposed density estimator, the paper develops a more powerful density-based specification test for the nonparametric regression function.

Results

We demonstrate that the proposed estimators of the density and its derivative are more efficient than existing alternatives, meaning that they converge more rapidly to the true density as the sample size increases. In addition, the proposed density estimator in the context of nonparametric regression models exhibits several properties not shared by the classical kernel estimator, including being uniformly bounded, having uniformly bounded derivatives, and possessing desirable smoothness properties.

Furthermore, extensive numerical analyses show that the proposed estimators of the density and its derivative, as well as the proposed model specification test, significantly outperform existing approaches.

Fuente: https://www.banrep.gov.co/en/publications-research/working-papers-economics/density-estimation-specification-testing