

# Why does natural resource abundance not always lead to better outcomes? Limited financial development versus political impatience.

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## **Abstract**

Is the failure of natural resource abundance to achieve better economic outcomes due to limited financial development or fiscal policy short-termism? I answer this question in a precautionary savings model where both resource revenues and asset returns are uncertain. Calibrating for Colombia, I find that under policy impatience, welfare costs are large, net assets are insufficient and net discretionary expenditures are too sensitive to resource revenues. If financial markets are underdeveloped, we can generate welfare costs of the same magnitude but not also explain why there are insufficient net effective assets, nor the heightened sensitivity to revenues.

**JEL-Classification:** E44, E60, H60, Q32

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# 1 Introduction

When a country discovers new reserves of marketable natural resources, its citizens are gifted with a new source of revenue that though potentially capable of beneficial economic transformation also brings volatility. History has shown that an abundance of tradable natural resources can often turn out to lower average growth through more volatility.<sup>1</sup> van der Ploeg and Poelhekke (2009)'s estimations on a cross-country panel for 1970 to 2003 reveal that the ill effects of having natural resources are related to a greater unpredictability of per capita growth. Collier and Goderis (2008) find strong evidence from a dynamic panel estimates that high-rent non-agricultural commodity booms have only short-lived favourable effects on output and the lower average growth rate of commodity-exporting economies is almost entirely due to a higher incidence of sharp slowdowns.

Revenue flows from the exploitation of important natural resource reserves typically accrue to the state and are therefore a fiscal matter. The assets and liabilities of the state can be used to manage the unpredictability of resource rents. Most of the earnings should be parked with an immediate reduction in debt or as an inflow into a wealth fund and spent on private consumption at later dates. There should be investment in a higher effective public capital stock. And the fiscal outgoings which constitute injections into consumers' utility through transfers or subsidised public services, should be not only on average higher but also be smoothed. These functions might be complicated by the fact that returns to these assets and liabilities are themselves uncertain. But nevertheless, through use of the state balance sheet, the natural resource earnings should translate into private benefit.

As volatile and low growth has been a feature of many resource-rich countries, it seems likely that there has not been enough efficient precautionary saving, or not of the right sort. The transfer of revenue to the private sector has been too low, too volatile and too poorly timed. It is important to understand what is the main driver of this failure so that policies can be aligned to anticipate and correct the condition. In this paper, two prominent likely causes of this failure are tested against each other. First, that the portfolio of investments and loans available to the state are so limited that they cannot adequately diversify the income risk away from its citizens. Second, that due to political economy failures, fiscal decisions are discounted at a higher rate than is socially optimal.

The literature contains many different definitions of limited financial development. Most commonly, limited financial development is defined in terms of outcomes, such as a low average ratio of private credit to GDP. By limited financial development, I mean that the government's portfolio of assets and liabilities are unequal to the task. For one thing, this could mean that the returns on the government's investment options are on average too low and excessively volatile. And, as that portfolio also includes liabilities, borrowing rates could be on average too high and also too volatile. Another aspect of limited development that I explore is that the state's portfolio returns are correlated with its citizen's income risk, the natural resource revenue. Standard finance theory argues that under these circumstances, the undiversifiable component of risk in the government's portfolio is greater and its potency

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<sup>1</sup>The natural resource curse is surveyed by, for example, van der Ploeg (2011) and Collier (2010).

diluted. Hence, a government constrained by a poorly developed financial sector finds it difficult to diversify away its exogenous income risk. One of the implications, though not explored here, could well be a low ratio of private credit to GDP.

Driessen and Laeven (2007) estimate that there are large cross country differences in the portfolios of local investors, such that developing country investors would benefit most from investing abroad, even after taking account of currency effects. Indeed many resource exporting countries are developing countries, and one of the endemic features of a lower level of development is a paucity of efficient domestic assets that collectively offer an adequate combination of risk and return. Consistently, van der Ploeg and Poelhekke (2009) also found in their panel estimates that ill effects of volatility depend on the extent of underdevelopment of domestic financial markets. Broner and Rigobon (2005) estimated the unconditional standard deviation of capital flows into less developed nations is on average eighty percent times larger than to developed counterparts partly because disturbances to financial market conditions for these countries tend to be more persistent.

As for net returns correlated with income risk, many authors have found that the lending terms on developing countries' external liabilities are more lax when GDP is strong,<sup>2</sup> and that the appetite of foreign investors tends to be positively related to the terms of trade or export prices (Reinhart and Reinhart (2008)). In fact, capital inflows are increasingly associated with FDI into the commodity producing sector.<sup>3</sup>

Not only foreign borrowing terms, but also *domestic* rates wax and wane with export prices. A common argument is that this is due to possibly suboptimal policy decisions, for example towards resisting exchange rate appreciations. But under another version of events, limited financial development in the sense of an inherently procyclical banking sector is instead the dominant cause. As it likely that capital inflows and buoyant resource revenues stimulate a greater inflow of deposits into the banking sector, procyclical domestic banks will respond by offering credit at lower rates. As bank credit is typically skewed to the non-tradable sector, the real exchange rate, being the price of non-tradables to tradables, appreciates strongly. This real appreciation also makes policymakers less able (or willing) to tighten domestic currency rates, preferring instead to rely on reserve requirements and other macroprudential tools.<sup>4</sup> Thus procyclicality due to frictions in the domestic financial sector is consistent with another regularity about when a dependence of natural resources implies low growth, that the real exchange is correlated with commodity price (Papyrakis and Gerlagh (2004)).

The leading competing explanation for poor fiscal outcomes following resource discoveries to a lack of financial development is a political economy failure. Nearly all of the empirical papers that identify a significant negative effect of natural resource abundance, directly or through volatility, onto growth also find that this is alleviated by some measure of the

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<sup>2</sup>See Kaminsky, Reinhart, and Végh (2004), for example, for evidence.

<sup>3</sup>UNCTAD (2007) report that 'in 2005, the 10 largest FDI recipient countries in Africa were rich in oil or metal minerals; and in Latin America, most economies with significant natural resources saw increases in FDI in primary industries.'

<sup>4</sup>See Hausmann and Rigobon (2003) for a formal model of this mechanism in commodity exporting countries.

quality of policy institutions.<sup>5</sup> And political failures can explain all the other features of the fiscal problem: for example evidence of poor risk-adjusted returns to investments may be attributed to political economy driven misallocation rather than a limited set of investment opportunities. Tornell and Lane (1999) suggest that if fiscal discipline is weak, interest groups may scramble to secure expenditure commitments. Alesina, Campante, and Tabellini (2008) show that suboptimally-timed procyclical spending may be the outcome if voters do not know how much of the oil rents are being appropriated by a corrupt state. They forcedly argue that financial frictions are not the main reason why fiscal policy in developing countries is so procyclical. A straightforward interpretation of the political economy argument is that the rate of discount applied to fiscal policy decisions is higher than the social rate, and thus that future consequences are suboptimally disregarded.

In this paper I contribute to the literature by comparing limited financial development against political impatience. These two hypotheses are impossible to distinguish without reference to an encompassing model. I build a model where the government receives an uncertain exogenous revenue stream, an important part of which is down to a natural resource. It can invest in an aggregate net asset, but the returns on those assets are themselves uncertain, and may be correlated with revenue. There is also the possibility of unalterable exogenous expenditure commitments. From this, the government has to extract a stream of net expenditure which it transfers to private consumers. The government can be more impatient than is socially optimal. I derive the welfare costs of these different sources of uncertainty and political impatience. I calibrate this to match modern day Colombia, a good case study because the central expectation is that oil revenues will play an ever greater role in determining the country's economic future.

As a by product, I implement an important technical innovation in modelling the natural resource problem. I apply Coeurdacier, Rey, and Winant (2011)'s risky steady state concept to determine the optimal level of net assets. The risky steady state is defined intuitively where all variables are constant, or grow at a constant rate, while uncertainty is expected in the future. This is the unconditional mean of the ergodic distribution of variables and is related to the stochastic steady state or long-run invariant distribution in optimal growth models. There is an explicit recognition that natural resource revenues, expenditure commitments, debt costs and returns to public capital can be uncertain. Fittingly, it permits a welfare analysis of the profound consequences of natural resource abundance, as volatility affects the steady state and not just the dynamics.

The bulk of the literature on fiscal policy for resource rich countries uses either a perfect foresight or a log-linear solution. A famous example is the Hartwick rule (Hartwick (1977)) which prescribes a path where fiscal saving is exactly equal to current resource extraction revenue with the objective of keeping consumption constant at some unspecified level. Certainty equivalence excludes the prudential motivation to limit net borrowing or hold assets. Thus net debt is indeterminate in these models and has to be either left indeterminate or imposed. The first option implies the unpalatable property that where debt settles depends on the initial level and the history of *temporary* shocks (Schmitt-Grohé and Uribe (2003)).

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<sup>5</sup>See Collier and Goderis (2008) or Arezki and Bückner (2012) for example.

The second leaves a crucial part of the problem unexplained.

The importance of precautionary saving for the fiscal problem of resource-producing countries has been tackled by for example van der Ploeg (2010), Bems and de Carvalho Filho (2011) and recently van den Bremer and van der Ploeg (2012). None of these papers allow for welfare comparisons as I do, however.

Another relative novelty of this paper is that I allow public capital to be consider an asset of the state. The returns to public investment are certainly risky. Gupta, Kangur, Papageorgiou, and Wane (2011) find that investment is highly inefficient in creating capital stock on average across all emerging market countries while Bhattacharyya and Collier (2011) show that this seems to be especially true of natural resource exporting reporting countries. But despite its riskiness, the expected return on public capital is still usually high enough to make a vital contribution to the welfare of its citizens. Macroeconomists often discuss the countercyclical role of public investment, as an injection into private domestic incomes at the time of making the expense, independent of its economic function. The controversy generated by this short-term boost masks that an effective public capital stock also yields utility to private citizens in the form of stream of future services, just as a consumer durable.<sup>6</sup> As these dividends only occur after a long and uncertain horizon following investment, it is not realistic to think of public investment as being manipulated systematically to counteract previously unanticipated shocks to resource returns. But portfolio theory tells us that as long the service flow injection into private utility is uncorrelated with other shocks such as resource rents, there could still be social benefit from effective public investment (see Collier and Venables (2011) page 23).

Most of the literature on the fiscal management of natural resources has sidestepped this issue by restricting attention to financial assets that earn safe returns, such as reserves. Debt is occasionally incorporated (often as negative reserves — by assuming that the rate of repayment is certain). But public capital is nearly always disregarded. Indeed, over the recent two decades, resource revenues have been parked in foreign reserves with low risk (at least in dollar terms) and debt liabilities have also lowered. While this is prudent, it is also important to recognize that purely financial instruments imply a very low real rate of social return, quite possibly insufficient to compensate even for the elimination of risk.<sup>7</sup> Hence large level of reserves or low level of debt are not necessarily consistent with an optimal level of state assets; we should also consider the effective public capital stock in making this assessment.

In the next section, Section 2, I go on to describe the model of the fiscal problem and provide some intuition as to what we might expect from a numerical solution. Section 4 explains how I measure welfare. In Section 5, the calibrations for Colombia are presented and the baseline solution of the model is discussed. Sections 6 and 7 explore the effects of financial shallowness and then political impatience. Section 8 compares these competing

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<sup>6</sup>Another argument in favour of including public capital that I do not include here is that many resource exporting countries are capital constrained (van der Ploeg and Venables (2011) and van den Bremer and van der Ploeg (2012)).

<sup>7</sup>See Bernstein, Lerner, and Schoar (2009) for evidence of and explanations for low returns from sovereign wealth funds.

causes. Section 9 concludes.

## 2 The fiscal policy problem

A government typically has many different types of items on its current balance sheet. A first distinction is between those which are reproducible and those which are not. Reproducible assets earn a rate of return, while reproducible liabilities such as debt require interest payment. A full list of the assets of the state would be public sector physical capital, domestic financial investments, foreign reserves and wealth funds. Reproducible liabilities would be sovereign debt and bank loans. In what follows, I aggregate these into a net asset, allowing for imperfect substitutability.

As for non-reproducible expenditure and revenue streams, these are further distinguished into those which can be adjusted and those which cannot. The former are called discretionary items, the latter are non-discretionary. The discretionary transfers between the private sector and the government are in net terms. This is called *net* discretionary expenditure, the amount that the government chooses to spend on private individuals minus what it chooses to take from them and is denoted by  $C_t$ .

The logic behind non-discretionary flows is similar to that underlying the *structural budget*, a popular decomposition which removes cyclical and other elements that are not controlled by the government, or at least those which respond passively to the environment. Accordingly, non-discretionary revenues (NDR) include the royalties from natural resource windfalls. Within non-discretionary expenditures (NDE), one could include pension liabilities, compensation for conflict, the resolution of bad banks or contributions to a disaster fund or more generally, the large notoriously rigid component of total government expenditure. Here finite natural resources are not depletable capital and the simultaneous decision of how much to extract is not modelled. The right to raise tax revenue could also be conceived as a capital stock whose capacity requires investment and yields a return. But here tax incomes are from a non-reproducible source.

Formally, the fiscal authority maximises the following additive, time-separable, infinite horizon welfare function:

$$\mathbb{E}_t U(C_t, \dots, C_\infty) = \mathbb{E}_t L_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{L_s}{L_t} \frac{\left(\frac{C_s}{L_s}\right)^{1-\gamma} - 1}{1-\gamma} \quad (1)$$

with  $\gamma > 0$ . Here  $C_t$  is discretionary net transfer from the government to its citizens and  $L_t$  is the population at time  $t$ .  $\beta$  is the government's rate of discount which is assumed to be greater than the social rate of discount ( $\hat{\beta}$ ). The government is supposed to care about future generations weighted by their population size. There are three other important assumptions inherent in these preferences. First, that privately purchased consumption is

additive in social welfare to net discretionary expenditure.<sup>8</sup> Second, that it is *net* public expenditure that matters; i.e. taxes do not matter independently. This implies that tax rates are assumed to be fixed in this paper. It is possible to generalize the model to allow for more realistic choice of tax rates. But this would come at the cost of greater complexity and with little difference to the main result. Third, that non-discretionary expenditures and revenues of the government are also additive in utility to the discretionary component.

The budget constraint of the state is

$$C_t = r_t \left( \frac{W_{t-1}}{\Lambda_t L_t \tilde{w}} \right)^{-\delta} W_{t-1} - W_t + X_{1,t} - X_{2,t}. \quad (3)$$

For the sake of argument, non-resource GDP grows at an exogenous rate  $\tau = \frac{\Lambda_t L_t}{\Lambda_{t-1} L_{t-1}}$  and  $\Lambda_0 = 1$ .

To finance net discretionary expenditures ( $C_t$ ), the government receives a non-discretionary income in real dollar terms of  $X_{1,t}$ , faces committed spending plans of  $X_{2,t}$  and can save in net an amount equal to  $W_t$  which earns a real gross rate of return in domestic currency. The rate of return on net assets is the product of two influences: an exogenous rate,  $r_t$ , as well as an endogenous component  $\left( \frac{W_{t-1}}{\Lambda_t L_t \tilde{w}} \right)^{-\delta}$ . The latter depends on the size of the net asset position,  $W_t$ , relative to a benchmark,  $\tilde{w}\tau^t$  and the parameter  $\delta$  measures the sensitivity of real return to this imbalance.

In appendix A, I explain how the endogenous term can be interpreted as the outcome of the opposing influences of a declining marginal product of capital and the importance of a net asset cushion for production. I show that there are good reasons to argue that the first effect dominates such that  $\delta$  should be small and positive, though it should be closer to zero in economies with more financial frictions. This term is not required for there to be an endogenous equilibrium level of net assets, but it does mean that, plausibly, governments might respond to lower returns on net assets by investing less in efficient public sector capital. This exacerbates the consequences of procyclical debt rates and interferes with the functioning of a countercyclical rate policy.

The budget constraint (3) can be written as

$$c_t = x_{1,t} - x_{2,t} + \frac{w_{t-1}}{\tau} \left( \frac{w_{t-1}}{\tau \tilde{w}} \right)^{-\delta} r_t - w_t \quad (4)$$

where lower case denotes model units; i.e. that all real volume variables have been divided by time  $t$  technical progress and population size which together grow at a constant rate;

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<sup>8</sup>Thus if private agents' utility were instead of the following form

$$\mathbb{E}_t[U(C_t, \dots, C_\infty) + V(H_t, \dots, H_\infty)] = \mathbb{E}_t L_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{L_s}{L_t} \left[ \frac{\left( \frac{C_s}{L_s} \right)^{1-\gamma} - 1}{1-\gamma} + \frac{\left( \frac{H_s}{L_s} \right)^{1-\gamma} - 1}{1-\gamma} \right] \quad (2)$$

with  $H_t$  indicating privately purchased consumption items, the solution to the government's problem would be the same as in the rest of the paper.

i.e  $y_s \equiv \frac{Y_s}{\Lambda_t} \tau^{t-s}$  for any real volume variable,  $y_s$ .<sup>9</sup> Thus the variables in equation 4 can be considered to be stationary.

At the risky steady state, equation 4 becomes

$$\bar{c} = \bar{x}_1 - \bar{x}_2 + \frac{\bar{w}}{\tau} \left( \frac{\bar{w}}{\tau \bar{w}} \right)^{-\delta} \bar{r} - \bar{w}, \quad (5)$$

where  $\bar{c}$  and  $\bar{w}$  are to be determined.

A crucial feature is that both non-discretionary income and expenditure as well as the real rate of return are stochastic.  $x_{1,s}$ ,  $x_{2,s}$ , and  $r_s$  all follow autocorrelated log-normally distributed processes:

$$\begin{aligned} \ln(x_{1,s}) &= (1 - \kappa_1) \bar{\ln} x_1 + \kappa_1 \ln(x_{1,s-1}) + u_{1,s+1} \\ \ln(x_{2,s}) &= (1 - \kappa_2) \bar{\ln} x_2 + \kappa_2 \ln(x_{2,s-1}) + u_{2,s+1} \\ \ln(r_s) &= (1 - \kappa_r) \bar{\ln} r + \kappa_r \ln(r_{s-1}) + u_{r,s+1} \\ u_{r,s+1} &= \theta u_{1,s+1} + u_{3,s+1} \end{aligned} \quad (6)$$

Defining

$$\mathbf{u}_s \equiv (u_{1,s}, u_{2,s}, u_{r,s}) \stackrel{\text{d}}{\sim} N(0, \mathbf{D}), \quad (7)$$

with  $\mathbf{D} \equiv [d_{ij}]$ . The last line of 6 stipulates that real rates vary counter or procyclically with surprises in non-discretionary natural resource revenues, through varying  $\theta = \frac{\text{Cov}_t[r_{t+1}, u_{1,t+1}]}{\text{Var}_t[u_{1,t+1}]}$ .

The risky steady state values are the unconditional means of the ergodic distributions, which up to a second-order approximation are:

$$\begin{aligned} \bar{x}_1 &= e^{\left( \bar{\ln} x_1 + \frac{d_{11}}{2(1-\kappa_1^2)} \right)} \\ \bar{x}_2 &= e^{\left( \bar{\ln} x_2 + \frac{d_{22}}{2(1-\kappa_2^2)} \right)} \\ \bar{r} &= e^{\left( \bar{\ln} r + \frac{d_{33}}{2(1-\kappa_r^2)} \right)}. \end{aligned} \quad (8)$$

Expressions for the conditional mean and variances of these exogenous processes that are needed to solve the model can be derived following Granger and Newbold (1976).

### 3 Solution

The first-order Euler equation for maximising the objective 1 subject to the constraint 4 is

$$f(c_{t+1}, r_{t+1}, c_t) \equiv \beta(1 - \delta) \mathbb{E}_t \left[ \left( \frac{c_{t+1} \tau}{c_t} \right)^{-\gamma} r_{t+1} \left( \frac{w_t}{\tau \bar{w}} \right)^{-\delta} \right] - 1 = 0, \quad (9)$$

written in model units. The transversality condition is such that

$$\lim_{s \rightarrow \infty} \beta^s W_{t+s} (C_{t+s})^{-\gamma} = 0 \quad (10)$$

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<sup>9</sup>This is akin to normalising by dividing by non-resource GDP.

which after expressing in model units and substituting from the Euler equation becomes

$$\begin{aligned}
r_t(\tau\tilde{w})^{-\delta}c_t^{-\gamma}\lim_{s\rightarrow\infty}\left(\frac{\tau^{(1-\delta)}}{(1-\delta)(\tilde{w})^\delta}\right)^s\frac{1}{\mathbb{E}_t[w_{t+s}^{\delta-1}]\prod_{v=0}^s\frac{r_{t+v}}{w_{t+v}^\delta}} &= 0, \\
\Rightarrow (1-\delta)\bar{r}\left(\frac{\bar{w}}{\tau\tilde{w}}\right)^{-\delta} &> \tau.
\end{aligned} \tag{11}$$

This solvency condition — a lower bound on the real rate adjusted for default risk — is necessary to ensure that the government cannot increase net assets faster than the real returns it pays on them. Compared to the solvency condition in certainty equivalent models, which is simply  $\bar{r} > \tau$ , here solvency also depends on the uncertainties faces by the exporting country. As we shall see, an additional impatience restriction is needed to ensure that there exists a positive flow of net discretionary expenditure on average, which places an upper bound on the real rate.

A second-order approximation of the first-order condition  $f$  (defined in equation 9) yields

$$\begin{aligned}
\mathbb{E}_t[f(c_{t+1}, r_{t+1}, c_t, w_t)] &\approx \hat{\Phi}(\mathbb{E}_t[c_{t+1}, r_{t+1}, c_t, w_t]) \\
\Rightarrow 1 - \underbrace{\left(\frac{\mathbb{E}_t[c_{t+1}]\tau}{c_t}\right)^\gamma \frac{1}{\beta\mathbb{E}_t[r_{t+1}]}}_{\text{Smoothing}} \underbrace{\frac{1}{1-\delta}\left(\frac{w_t}{\tau\tilde{w}}\right)^\delta}_{\text{Capital}} &\underbrace{-\gamma\frac{Cov_t[c_{t+1}, r_{t+1}]}{\mathbb{E}_t[c_{t+1}]\mathbb{E}_t[r_{t+1}]}}_{\text{Risk Premium}} \underbrace{+\frac{\gamma(\gamma+1)}{2}\frac{Var_t[c_{t+1}]}{\mathbb{E}_t[c_{t+1}]^2}}_{\text{Prudence}} \approx 0,
\end{aligned} \tag{12}$$

with the complete derivation in appendix B.<sup>10</sup>

The first term in equation 12  $\left(\left(\frac{\mathbb{E}_t[c_{t+1}]\tau}{c_t}\right)^\gamma \frac{1}{\beta\mathbb{E}_t[r_{t+1}]}\right)$  is all what we would see in a standard (linearised) description of the fiscal policy problem. As Harding and van der Ploeg (2009) explain, it is optimal to deviate from the Hartwick rule and spend some of the windfall now. How much smoothing takes place depends on the contribution that the resource makes to permanent income. If it is expected to be a temporary boom, there will be little current spending.

The next term reflects the importance of capital in lowering the rate of return. On one hand, more capital lowers the marginal productivity of public capital. On the other, more capital potentially represents more net worth, supporting returns.

The last two terms appear because uncertainty matters.<sup>11</sup> The third term is related to risk aversion while the fourth is related to prudence. Consistently, while the second derivative of the utility function — an elasticity of  $\gamma$  — determines a preference for risk aversion, the third derivative — here an elasticity of one plus the coefficient of risk aversion, or  $\gamma + 1$

<sup>10</sup>Written as in 9, errors in approximating the first-order condition are likely to be bounded. As we seek to minimise the maximum error to the solution system, this automatically satisfies the diagnostic for perturbation methods suggested by Jin and Judd (2002).

<sup>11</sup>In consumption based asset pricing models, consumption is considered exogenous and the price of risky assets is determined within an equation similar to 12 with uncertainty terms.

— determines the proclivity for prudence (Kimball (1990)).<sup>12</sup> In order to provide some intuition, I consider equation 12 at the risky steady state:

$$\bar{c}^2 \hat{\Phi} = \left(1 - \frac{\tau^\gamma}{\beta(1-\delta)\bar{r}} \left(\frac{\bar{w}}{\tau\bar{w}}\right)^\delta\right) \bar{c}^2 - \gamma Cov_t [c_{t+1}, r_{t+1}] \frac{\bar{c}}{\bar{r}} + \frac{\gamma(\gamma+1)}{2} Var_t [c_{t+1}] \approx 0. \quad (13)$$

This quadratic formula in  $\bar{c}$  will have real solutions if

$$\beta(1-\delta)\bar{r} \left(\frac{\bar{w}}{\tau\bar{w}}\right)^{-\delta} < \tau^\gamma. \quad (14)$$

14 is an impatience condition, an intrinsic feature of the buffer stock problem (Carroll (2004)). If the rate of return on assets, the growth rate is too low, the government is too risk averse or does not discount the future highly enough, then there will be no risky steady state because the government will only keep saving.<sup>13</sup>

Combining this with the steady-state implication of the solvency condition (11) gives a range:

$$\beta(1-\delta)\bar{r}\tau^{-\gamma} < \left(\frac{\bar{w}}{\tau\bar{w}}\right)^\delta < \frac{(1-\delta)\bar{r}}{\tau}, \quad (15)$$

In what follows, this is always satisfied.

The next step is to posit that the only state variable,  $w_t$ , follows the expectations rule

$$w_{t+1} = \bar{w} + G_{ww}(w_t - \bar{w}) + G_{wr}(r_{t+1} - \bar{r}) + G_{w1}(x_{1,t+1} - \bar{x}_1) + G_{w2}(x_{2,t+1} - \bar{x}_2), \quad (16)$$

where the coefficients  $G_{ww}$ ,  $G_{wr}$ ,  $G_{w1}$  and  $G_{w2}$  as well the risky steady state value of net assets  $\bar{w}$  are to be determined. When combined with the budget constraint, this gives

$$\begin{aligned} c_{t+1} = & (1 - G_{w1})x_{1,t+1} - (1 + G_{w2})x_{2,t+1} \\ & + \left(\frac{w_t}{\tau} \left(\frac{w_t}{\tau\bar{w}}\right)^{-\delta} - G_{wr}\right)r_{t+1} - G_{ww}w_t \\ & + (G_{ww} - 1)\bar{w} + G_{wr}\bar{r} + G_{w1}\bar{x}_1 + G_{w2}\bar{x}_2. \end{aligned} \quad (17)$$

Equation 17 clarifies that the expression 12 is a partial description of our solution. Not only is the term in expected consumption endogenous as in a linearised Euler equation, the variance and covariance also need to be solved out jointly with the risky steady state and the rational expectation coefficients.

We can use economic intuition to guide the values of the coefficients of this rule. First we could expect that  $0 < G_{ww} < 1$ , as then net assets are more likely to converge. Second and third,  $0 < G_{w1} < 1$ ,  $-1 < G_{w2} < 0$ , but with both being closer to one in absolute value,

<sup>12</sup>In more general utility functions, it is possible to have these parameters determined independently (van der Ploeg (2010)).

<sup>13</sup>This is only for the risky steady state. There is another condition in dynamics: see Arrau and Claessens (1992) for an example. A necessary condition for the impatience restriction and the transversality condition 11 to be mutually satisfied is that the economy is expected to grow in the long run and there is prudence such that  $\tau^{\gamma-1} > \beta$  always.

because this implies that most of extra oil revenue will be saved on accrual and that extra expenses are not offset one for one with lower discretionary expenditure. In fact, we shall see that the values of these coefficients do respect these bounds, even for a wide range of parameter values where the solution converges.

If  $\frac{w_t}{\tau} \left(\frac{w_t}{\tau \tilde{w}}\right)^{-\delta} > G_{wr} > 0$ , then we would expect that governments which are net savers to respond to lower returns on net assets by decreasing discretionary expenditure (adding to net asset stocks). With public capital included in  $w$ , most governments will be net savers. This would then potentially be an important transmission channel by which through raising debt interest rates and lowering the return on net assets, a countercyclical rate policy stimulates a build up of net assets in boom times. But with  $\delta$  even slightly positive and high level of net assets, governments may be bound by diminishing returns to capital to do the opposite, save less when rates of return fall. The model therefore contains the inherent constraint on a countercyclical interest rate policy represented by diminishing marginal returns.

In the following sections I quantify the strength of these effects. But before that I establish a metric for quantifying the importance of uncertainty.

## 4 A welfare metric

In this section, I explain how I judge outcomes in terms of both the mean and conditional variance of consumption, through an approximation for utility-based social welfare function. As I shall show, it makes a difference that steady state levels in this framework are responsive to uncertainty parameters as they come to play a dominating role in the welfare comparison. Crucially, the discount rate is  $\hat{\beta}$  and can be lower than that used by the government in formulating its fiscal plans.

A second-order approximation of the utility function in equation 1 is as follows:

$$\begin{aligned}
\mathbb{E}_t U(C_t, \dots, C_\infty) &\approx V(\mathbb{E}_t c_t, \dots, \mathbb{E}_t c_\infty) \\
&\equiv L_t \sum_{s=t}^{\infty} \hat{\beta}^{s-t} \frac{L_s}{L_t} \tau^{(s-t)(1-\gamma)} \frac{\bar{c}^{1-\gamma} - 1}{(1-\gamma)} \\
&+ L_t \sum_{s=t}^{\infty} \hat{\beta}^{s-t} \frac{L_s}{L_t} \tau^{(s-t)(1-\gamma)} \bar{c}^{-\gamma} (\mathbb{E}_t[c_s] - \bar{c}) \\
&- \frac{1}{2\gamma} L_t \sum_{s=t}^{\infty} \hat{\beta}^{s-t} \frac{L_s}{L_t} \tau^{(s-t)(1-\gamma)} \bar{c}^{-(1+\gamma)} \text{Var}_t[c_s] \\
&= \frac{(\bar{c})^{1-\gamma} - 1}{(1-\gamma)(1 - (1+n)\tau^{1-\gamma})} \\
&- \frac{\gamma \bar{c}^{-(1+\gamma)}}{2} \sum_{s=t}^{\infty} \hat{\beta}^{s-t} (1+n)^{s-t} \tau^{(s-t)(1-\gamma)} \text{Var}_t[c_s]. \tag{18}
\end{aligned}$$

I am assuming that the population grows at a constant rate and is normalised at  $L_t = 1$ . Note also that the term in the expected deviation of net discretionary expenditure from its

risky steady state mean ( $\mathbb{E}_t[c_s] - \bar{c}$ ) is dropped as net discretionary expenditure is assumed to begin at the risky steady state and expected to be there on average forever.

Being a convolution of jointly log-normal variables, the process  $c_t$  is  $h$ -step ( $h > 1$ ) conditionally heteroscedastic and it is not easy to determine the conditional variance of future consumption,  $Var_t[c_{t+s}]$ . I propose to approximate welfare further by

$$\begin{aligned} \mathbb{E}_t U(C_t, \dots, C_\infty) &\approx V_2(\mathbb{E}_t c_t, \dots, \mathbb{E}_t c_\infty) \\ &\equiv \frac{(\bar{c})^{1-\gamma} - 1}{(1-\gamma)(1-\hat{\beta}(1+n)\tau^{1-\gamma})} \\ &\quad - \frac{\gamma \bar{c}^{-(1+\gamma)}}{2} \sum_{s=t}^{\infty} \hat{\beta}^{s-t} (1+n)^{s-t} \tau^{(s-t)(1-\gamma)} Var_t[\hat{c}_s] \end{aligned} \quad (19)$$

where  $Var_t[\hat{c}_s]$  is the variance of an approximation to consumption calculated using linearised versions of the budget constraint and the exogenous processes. This approximation is derived in appendix C and exactly equal to the analytical expression in the next period (when  $s = t + 1$ ).

Expression 19 balances the gains of a high mean level of discretionary expenditure against the benefits of lower volatility within the metric of utility-based welfare. As the scaling factor on the volatility term ( $-\frac{\gamma \bar{c}^{-(1+\gamma)}}{2}$  in 19) depends on the mean value of discretionary expenditure and through that, uncertainty parameters, in the risky steady state framework, welfare evaluation is more of a comparison between different steady states rather than a comparison around a fixed state.

I assess total welfare across different scenarios by an *ex-ante compensating variation* defined as the value of an increase in discretionary expenditure (fixed in percentage terms) that is expected to yield the same utility as in the baseline scenario. This is the value of  $\alpha$  that solves the following equation:

$$(1 + \alpha)^{1-\gamma} V_2(\mathbb{E}_t c_t, \dots, \mathbb{E}_t c_\infty) = V_2(\mathbb{E}_t \hat{c}_t, \dots, \mathbb{E}_t \hat{c}_\infty) \quad (20)$$

where  $\hat{c}_s$  defines the baseline expenditure stream. One criticism to this could be that the approximation errors involved under each scenario may differ in magnitude and sign to that under the baseline, and thus the welfare comparison could be severely distorted. Given the size of the welfare differences I find, the approximation errors would individually have to be very large for that to be true.<sup>14</sup>

## 5 Matching the model to the Colombian data

### 5.1 Introduction and methodology

In this section, the model is calibrated to the Colombian data. The aim is to try and find the parameter values that bring the optimal value of net assets as predicted by the model

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<sup>14</sup>A Monte Carlo reproduction of the conditional mean and variance of  $w_{t+s}$  gives similar values.

closer to the Colombian data. As there are separate parameters to capture limited financial development and political impatience, the exercise of matching model to data represents a valid test between these two competing explanations of real net asset positions.

The parameter values are chosen to match a risky steady state for Colombia at an annual frequency. Indeed Colombia presents a good case study for this approach. For while it is not among the countries completely dominated by its energy sector, energy and coal production matters for macroeconomic outcomes. External sales from petroleum and other mineral products have risen to 69% by the end of 2011 from 40% of exports in 2001. The dominance of energy is also reflected in the capital account: 73% of the FDI inflow into Colombia in 2010 was destined for mineral and energy sector. More saliently, commodity-related revenues also play a large role in fiscal policy. Between 2007 and 2010, central government received 1.4%, and regional governments, 1.2%, of GDP per year on average from energy and mineral royalties. As the flow of oil extracted from Colombian soil is set to keep increasing at least until 2020 and, if the oil price remains not too far below its current price of nearly 100 dollars a barrel, these fiscal revenues should become even more important — reaching 3 to 4% of GDP until 2020. From then on, one possible scenario is they will peter out rapidly. Other forecasts are for continued strong revenue streams. Yet another scenario is that a deteriorating security situation renders the costs of extracting and piping oil prohibitive curtailing the windfall. In other words, as high as revenues are expected to be, there are large risks associated with this outlook. The challenge is to make the most of this possibly temporary but certainly uncertain windfall and significantly improve the wellbeing of current and future generations of Colombians.

## 5.2 Calibration of model parameters

As the first calibration, I used Iregui and Melo (2010)'s estimate of  $\gamma$ , the coefficient of risk aversion for Colombia, which is about 2.5 and so indicates positive prudence.<sup>15</sup> My calibration of the long-run growth rate for Colombia will unavoidably be controversial. In the 1990s, Colombian GDP grew at a low average annual rate of 2.9%. But this recovered to 4.1% over the proceeding decade and not all of this extra increase is even mechanically due to energy, whose contribution to GDP increased by only a percentage point between the two decades.<sup>16</sup> Hence I choose 4% — a rate close to a fifty year historical average — as an estimate of the long-run growth rate in the absence of temporary resource windfalls.

The mean real interest rate on net government financial assets was set at 10% (in log terms). In keeping with equation 33 in section A, this reflects a weighted average of the real rate on public capital and the rate of return of net financial assets. The rate of return on net financial assets is broadly consistent with Colombia's recent external debt spread over US treasury bills, earnings of foreign assets, both adjusted for appreciation, the domestic currency bond rate as measured by Galindo and Hofstetter (2008) for 2002-2006 and an

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<sup>15</sup>Their estimate has the merit of conditioning on the limited access of the many poorer Colombians to financial services. It also falls within the range of the previous estimates they survey.

<sup>16</sup>I am using World Bank data on the contribution of resource rents to GDP in nominal terms to make this rough calculation. Growth in the 1990s was affected by a boom and bust cycle.

estimated rate of earnings on government's bank deposits. The rate of return on public capital was based on the mean rate of 12% for World Bank projects estimated by Pohl and Mihaljek (1992) adjusted for a 2.5% depreciation rate used by Gupta, Kangur, Papageorgiou, and Wane (2011).

Galindo and Hofstetter (2008) estimate the unconditional variance of domestic bond rates at 7pp for 2002-2006. And I calculated the unconditional variance of the real external finance rate using World Bank data to be of the same magnitude. These are surprising volatile, quite possibly more volatile than we could expect for the future of Colombian state debt. On the other hand my aim is to estimate the volatility of the rate of return in net assets, allowing for the rate of return on public capital. Thus a conservative assumption would be that the unconditional variance of returns to net assets including public capital is 7pp. The autoregressive parameter on a simple regression of real lending rates was 0.9 (much of Colombian debt is of long maturity), which if we can carry over to assets, implies a conditional standard deviation of real returns of  $3\text{pp} = (7^2 \times (1 - 0.9^2))^{0.5}$ . This implies very unpredictable returns on net assets: the 95% confidence interval of next year's rates of return is 4 to 17% (in gross rate terms).

The means, conditional variances and persistence of non-discretionary fiscal revenues and expenditures were calibrated using fiscal data as follows. First I took annual data series on the nominal peso revenues of the consolidated Colombian public sector from the Banco de la Republica from 1986 to 2010, and divided them by annual nominal GDP. The average of this series was taken to be the mean level,  $\bar{l}x_1$ . I regressed the log of this series on a constant, a lag of itself, the lagged GDP output gap and the detrended world oil price cycle. The standard deviation of this regression became a measure of the conditional volatility,  $\sqrt{d_{11}}$ , while the coefficient on the lagged dependent variable was used for  $\kappa_1$ . This implies a 95% confidence interval for next period revenues of between 24.5 and 30.7% of GDP.

I then regressed the log of the ratio of nominal non-interest expenses of the consolidated Colombian public sector to GDP on a constant, its lagged value, the lagged GDP output gap and the log of revenues to GDP. The average of the dependent variable was taken to be  $\bar{l}x_2$  and the standard deviation of the regression was taken to be  $\sqrt{d_{22}}$ . The coefficient on the lagged dependent variable was used for  $\kappa_2$ . Covariances were assumed to be zero in the baseline case.<sup>17</sup> Table 1 below summarises these values.

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<sup>17</sup>In fact, I found a strong positive correlation between my crude measures of non-discretionary expenditures and revenues (0.6).

**Table 1: Calibrations of Exogenous Variables**

Variables:	Non-Discretionary Revenues	Non-Discretionary Expenditures	Real Rate of Return
	$\ln(N - DR)$	$\ln(N - DE)$	$\ln(RR)$
Mean values:	$\bar{l}x_1 = 0.98$	$\bar{l}x_2 = 0.80$	$\bar{l}r = 0.1$
AR(1) params.:	$\ln(N - DR)$	$\ln(N - DE)$	$\ln(RR)$
	$\kappa_1 = 0.07$	$\kappa_2 = -0.04$	$\kappa_r = 0.90$
Covariances:	$\ln(N - DR)$	$\ln(N - DE)$	$\ln(RR)$
$\ln(N - DR)$	$d_{11} = 0.06^2$	-	-
$\ln(N - DE)$	$d_{12} = 0$	$d_{22} = 0.06^2$	-
$\ln(RR)$	$d_{13} = 0$	$d_{23} = 0$	$d_{33} = 0.0077^2$

$\delta$  was set at 0.01, following the arguments set out in Section A, and  $\tilde{w}$  was chosen to be 1, with the justification of these calibrations deferred to later on. I set the social impatience  $\hat{\beta}$  to discount at 2% and population growth to be 1.5%. At these values, both the transversality condition and the impatience restriction are satisfied.

These should be taken to be rough calibrations. Nevertheless, it is reassuring that the solution values are reasonably robust to different values within a plausible range of most of the parameters. The exceptions were  $\delta$  and  $\tilde{w}$ , whose values we have little prior idea and to which the solution can be sensitive. Shortly I discuss how I justify the calibrations for these parameter values.

### 5.3 Risky steady state values under the baseline calibration

In appendix B I explain how I apply Coeurdacier, Rey, and Winant (2011)'s method to obtain numerical solutions to  $G_{ww}$ ,  $G_{wr}$ ,  $G_{w1}$ ,  $G_{w2}$ ,  $\bar{w}$  and  $\bar{c}$ . The method belongs to the class of increasingly popular perturbation methods (Judd (1996)).

The endogenous risky steady state values in our baseline case are contained in Table 2 below.

**Table 2: Outcome of calibrations (baseline)**

Variables:	Non-Discretionary Revenues	Non-Discretionary Expenditures	Real Rate of Return
	N-DR	N-DE	RR
Risky steady state values:	$\bar{x}_1 = 27.6\%$	$\bar{x}_2 = 25.4\%$	$\bar{r} = 11\%$
Covariances:	N-DR	N-DE	RR
N-DR	$\sigma_{11}^2 = 2.51$	-	-
N-DE	$\sigma_{12} = 0$	$\sigma_{22}^2 = 1.74$	-
RR	$\sigma_{13} = 0$	$\sigma_{23} = 0$	$\sigma_{33}^2 = (3.4pp)^2$
Mean outcomes	Assets	DE	Primary deficit
	$\bar{w} = 28.8\% \text{ GDP}$	$\bar{c} = 3.1\%$	$\bar{c} - \bar{x}_1 + \bar{x}_2 = 0.9\%$

The value of optimum net assets is found to be 29% of GDP. To compare this against data, I take public sector capital to be the only non-financial asset of the Colombian government. Gupta, Kangur, Papageorgiou, and Wane (2011)'s estimates are that the ratio between Colombia's real public sector capital stock and real GDP was 0.71 in 2007 at 2005 prices

and has been falling. Botero and Ramirez Hassan (2010)'s estimates indicate that the real user cost of capital of all investment has also been falling, at about 6% year in Colombia.<sup>18</sup>. Extrapolating these trends gives us an estimate of the nominal share of the public capital stock to GDP of approximately 40% in 2011. The total net financial debt of the consolidated Colombian public sector was 28% in 2011, which would leave net assets at just over 10% GDP. Hence, the estimate in the data is well below our calibrated value of 29%. But the recently announced plans of the Colombian government are indeed to lower net financial debt permanently and to raise the value of the public capital stock. I thus consider 29% to be the optimal level of average net assets, implying a small primary surplus in expectation.

Crucially, it is difficult to get a much *lower* mean level of net assets with any plausible assumptions about mean returns. Thus even if the mean rate of return on net assets were 7% in terms of log returns (or 7.5% in gross terms), optimal net assets would only fall to around 25%. Raising mean non-discretionary revenues by 20% would indeed lower optimal mean net assets (as a smaller asset cushion is needed) to 10%. But this would be extremely unlikely, at a probability of 0.01% according to a log-normal distribution. The conditional one step ahead standard deviation of net assets in Colombia in the risky steady states is estimated to be 1.9 pp of GDP. For those countries where volatile resource revenue contributes more to fiscal revenues, there would appropriately be a higher standard deviation around the mean value of net assets.

The point is then that the value of optimal net assets is quite insensitive below to the choices of mean returns within a plausible range. In the following sections, I experiment with different aspects of financial development and also political impatience to isolate the reason why net assets in the Colombia data are much lower than the optimal value.

This asymmetric sensitivity of net assets also applies to the two parameters which are most difficult to calibrate. A smaller value of  $\tilde{w}$  (at 0.01 as opposed to the baseline value of 1) lowers net assets to 25% (while a value of 150 raises mean net assets to 80%). Similarly if  $\delta$  is at  $-0.04$  compared to  $-0.01$  in the baseline, net assets fall to 20%. If  $\delta$  is  $-0.001$ , average net assets rise to 50%. Values of  $\delta$  outside this bound generate unstable solutions depending on the value of  $\tilde{w}$ , as the solvency and impatience conditions (bounds 15) are not satisfied. In the absence of further work to develop out economic understanding of this sensitivity, this supports the baseline choices for these two tricky parameters.

## 5.4 Coefficients of the policy functions

The implied linear conditional dependences between variables are presented in Table 3.

Our baseline calibration implies that surprises to net assets will be positively related to non-discretionary (natural resource) revenue surprises, with an multiplier of 0.84: a 1pp surprise rise in revenues should lead to a rise in net assets of the order of 0.84pp of GDP on average. This saving is the combined effect of consumption smoothing, capital and precautionary saving effects. Another indication is that the primary deficit is negatively related to revenue surprises by the same magnitude. This implies that a rise in revenues of

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<sup>18</sup>Much (but not all) is due to a one-off fall in the real interest rate that is unlikely to continue.

**Table 3: Predicted linear conditional dependences (baseline)**

Of $\rightarrow$	Net assets	Primary deficit	(Net) Discretionary Expenditure (NDE)	Non-discretionary Revenue	Real rate ( $\div 100$ )
	$w_t$	$c_t - x_{1,t} + x_{2,t}$	$c_t$	$x_{1,t}$	$r_t$
On $\downarrow$					
Net assets	-	-0.98	4.90	0.84	0.25
Primary deficit	-0.79	-	-4.49	-0.84	0.02
NDE	0.18	-0.21	-	0.16	0.02

1pp of GDP should imply a 0.16pp increase in net discretionary expenditure (the value of  $G_{w1}$ ), and thus not all of the extra revenue is saved when they are realised.<sup>19</sup> Note also that in the baseline, surprises in the real return on net assets are mostly saved. A surprise 1pp rise in rates above the mean value leads to only a 0.02pp of GDP rise in net discretionary spending and the primary deficit, with a 0.25pp GDP rise in net assets. Unreported here, the persistence of debt,  $G_{ww}$ , is positive and less than one and  $G_{w2}$  in the baseline is such that a sudden 1pp expenditure commitment is only offset with an 0.21pp reduction in net discretionary expenditure. These values fall within the ranges suggested in Section 2.

Experiments reveal that, as revenue volatility increases, there is an important effect on optimal behaviour, an effect that is completely ignored in certainty equivalent frameworks. A lower proportion should be spent immediately and likewise, a lower proportion of a surprise expenditure commitment should be absorbed by the discretionary budget. With greater revenue volatility, there is a larger stock of net assets held and a bit more of the rate rise is saved. More net assets imply a higher stream of revenue to sustain discretionary expenditure, and a larger deficit. However not even with a very predictable revenue stream, can we reach net asset levels of 10%.

The persistence of revenue streams also matters. In the baseline, the autoregressive parameter on the log of non-discretionary revenue is 0.7. If expected streams of non-discretionary revenue are more short-lived — a sequence of bonanzas that come and go — then volatility falls and a somewhat smaller net asset position is warranted (21% when there is no autoregression). But more is saved from each surprise bonanza, reflecting that each rise is less likely to affect permanent income. Indeed with no autoregression, only 6pp of a 1pp rise in revenue is spent as opposed to 16pp in the baseline. Conversely more persistent revenue streams imply that the country should be less of a saver.

## 6 The effect of limited financial development

In the baseline, the standard deviation of the return on net assets was set at 3pp. In the introduction and in appendix A, I argued that a government that operates in a market with limited financial depth is likely to face more volatile rates. To explore the implications of

<sup>19</sup>Rincón Castro (2010)'s estimates of the optimal fiscal rule for Colombia also imply substantial counter-cyclicality at an overall level — he favours that a 1pp rise in the total output gap should lead to a concurrent fall in the deficit of 0.3pp of GDP.

financial shallowness, I vary the uncertainty in real rates.

Table 4 reports the results. Higher return uncertainty makes it more costly to use net assets as a cushion, and the size of the net stock held shrinks slightly to 28.1%. However it never falls below 28% of GDP. Conversely, with very predictable asset returns, the optimal level of net assets rises dramatically. When volatility is at 1pp, the optimal level of net assets is just over 40% of GDP. The coefficient of rates of return onto discretionary expenditure (the coefficient on  $r_{t+1}$  in equation 17) also rises substantially when return volatility falls. In a low volatility scenario, a rise in returns of 1pp would lead to a 0.08pp rise in discretionary expenditure, four times the effect in the baseline. This is yet another permanent income effect; only returns that are not likely to be reversed are spent. While these results are illustrative, it should be noted that given that public capital is an important component of net assets, a 1pp asset return standard deviation should be ruled out as implausibly low.

**Table 4: Effect of varying degree of financial development**

Of →	Baseline	High Rate of Return Volatility	Low Rate of Return Volatility	Procyclicality of Real Rates	Countercyclicality of Real Rates
	$\sigma_{33} = 3.4\%, \frac{\sigma_{13}}{\sigma_{11}^2} = 0$	$\sigma_{33} = 5.7$	$\sigma_{33} = 1.1$	$\frac{\sigma_{13}}{\sigma_{11}^2} = 0.8$	$\frac{\sigma_{13}}{\sigma_{11}^2} = -0.8$
On ↓					
Net Assets ( $\bar{w}$ )	28.8%	28.1	40.4	22.9	32.7
Net Discretionary Expenditure ( $\bar{c}$ )	3.1%	3.2	3.2	3.0	3.2
Coefficient of Real Rate on Net Discretionary Expenditure ( $\frac{\bar{w}}{\tau} (\frac{\bar{w}}{\tau \bar{w}})^{-\delta} - G_{wr}$ )	1.90	1.30	7.74	-0.47	4.21
Rate of Return Volatility ( $\sigma_{33}$ )	3.4%	5.7	1.1	3.6	3.6

A different aspect of financial development is pro- versus counter-cyclicality. If financial development is poor, rates of real return on net assets will interfere with the ability of net assets to act as a cushion — discouraging saving in good times and spending in bad times. The most direct way that this could be operationalised is to set a rule to make the real return on net assets positively conditionally linearly dependent on surprises to resource revenues as in equation 6. In the baseline, real rates of return on the net reproducible assets of the states do not covary with non-discretionary revenues. A more procyclical policy would make the return on net assets covary positively with revenues while a countercyclical bias would imply a negative covariance. We would expect activism, whatever the orientation, raises the unpredictability of real rates.<sup>20</sup> Apart from that, the effect of procyclicality is analytically complex, as for welfare it is not the just the predictability of next period's

<sup>20</sup>In the model, a procyclical policy is induced by making the shocks to interest rates  $u_{r,t+1}$  a positive function of shocks to non-discretionary revenue  $u_{1,t+1}$ , such that interest rates will rise when revenue shocks.

discretionary expenditures, but all future expenditures that matter. In the face of such theoretical ambivalence, numerical calculations are needed to establish the implications.

I compared the effect of swinging between pro- and counter-cyclical policy. At one extreme, there is a countercyclical policy which lowers real returns by 0.8pp when non-discretionary revenue is shocked by 1pp of GDP. At the other, a strong procyclical bias raises real returns by the same magnitude. The baseline lies at the midpoint. As I argue below, we are likely to observe a narrower range of policies in practice. The results are also in Table 4.

Both pro- or countercyclical policies raise the volatility in asset returns relative to the benchmark simply through more activism as we expected. This effect may seem slight, but, as we shall see in a later section, even a small rise in return volatility can have important welfare consequences. The countercyclical policy differentially raises the short-run unpredictability of discretionary expenditure, but this effect is relatively small. The most important differential effect of moving on the scale from pro- to counter-cyclical policy is on the size of government's net balance sheet: net assets rise to 32.7% of GDP with extreme countercyclical policy. This is because a countercyclical policy improves the functioning of the portfolio by making the covariance of revenue and returns more negative and thus lowering the risk premium. The greater level of net assets lowers the variance of discretionary expenditure discounted optimally over the future offsetting the rise in short-run unpredictability. Conversely under an extremely procyclical rate scenario the amount of net assets would fall, but to about 23% — which is still 10pp above what I estimate is observed in reality.

## 7 The effect of political impatience

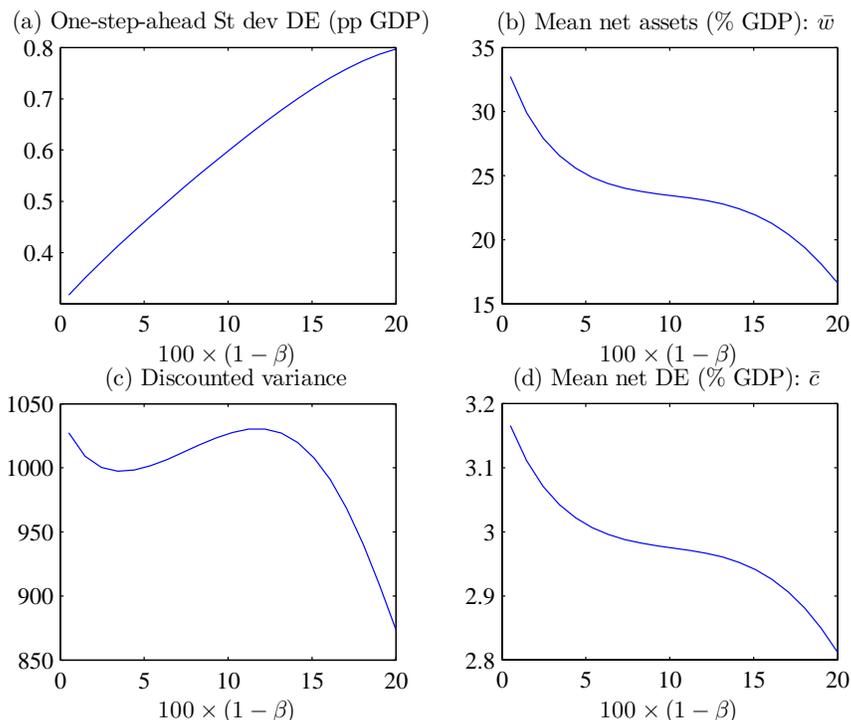
Up until this point, I have assumed that the future is discounted at 2% by policymakers and that this is the social discount rate. In the introduction, I cited many studies which argue that policymakers receiving resources revenue discount the future at a suboptimally high rate.

In Chart 1, I describe the effect of more policy short-termism. In panel (a), we see that the one-step ahead conditional variance of discretionary expenditure increases, doubling in scale as the discount rate rises from 2% to 20%. More impatient policymakers hold much lower net assets: the mean value of net assets falls to 16% (panel b). As it was difficult to obtain such low levels of net assets by adjusting revenue or return volatility, this would suggest that, *prima facie*, policy impatience is the most likely cause of low buffers.

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See equation 6. Naturally the volatility of rates will rise, implying more activism. This is intuitively what one would expect if there were, for example, an accommodating monetary policy rule in place. A countercyclical policy is created by making interest rates a negative function of shocks to revenue. Policy pro- or counter-cyclical policy in this sense is not determined by the interest-rate sensitivity to net assets (see Appendix A).

**Chart 1: Effect on risky steady state of political impatience<sup>a</sup>**

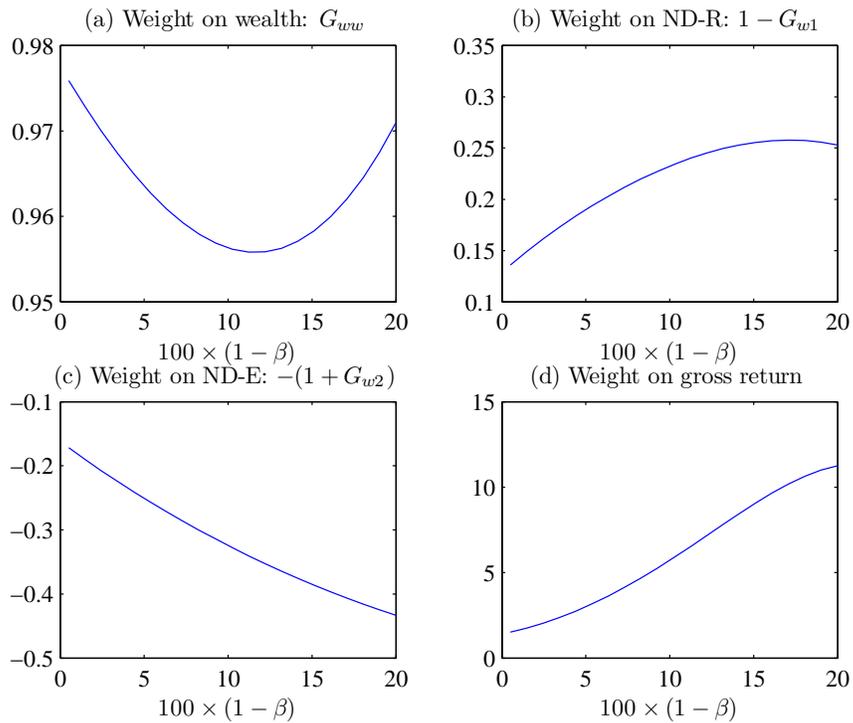



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<sup>a</sup>x-axis describes the discount rate used in formulating policy. Discounted variance in welfare is defined as  $\sum_{s=t}^{\infty} \hat{\beta}^{s-t} (1+n)^{s-t} \tau^{(s-t)(1-\gamma)} Var_t[\hat{c}_s]$  in equation 19.

Chart 2 describes how the parameters in the savings rule are also affected. Panel (b) shows that with more impatience, there is a significantly greater propensity to adjust spending on accrual. The multiplier of non-discretionary revenues on net discretionary spending rises from 0.16 to 0.26 in the extreme case of a 20% discount rate. Similarly, an immediate liability is met by simultaneously cutting discretionary expenditures by 0.44pp (panel c). Moreover, more is immediately spent from unexpected rises in the real return on assets: a 1pp rise in the return leads to 0.12pp rise in discretionary expenditure in the extreme case — panel (d).

**Chart 2: Effect on parameters in savings rule of political impatience<sup>a</sup>**



<sup>a</sup>x-axis describes the policy discount rate ( $\beta$ ). The weight on the gross return in panel (d) is  $(\frac{\bar{w}}{\tau}(\frac{\bar{w}}{\tau\tilde{w}}))^{-\delta} - G_{wr}$ .

## 8 Welfare effects

In this section I compare the different causes against a common metric. Table 5 presents the effects of deviations from the baseline case along the dimensions of political impatience and financial shallowness, separately.

**Table 5: Comparing explanations**

	Welfare <sup>a</sup>	Net Assets <sup>b</sup>	Weight N-DE <sup>c</sup>	Weight Returns <sup>d</sup>	Smoothness <sup>e</sup>
<hr/>					
Political Impatience					
discount rate (%)					
2	0	29	0.16	0.02	0.23
20	11	17	0.25	0.11	0.50
<hr/>					
Limited Financial Development					
<hr/>					
Asset Return Volatility					
pp					
3.4	0	29	0.16	0.02	0.23
8.2	11	29	0.16	0.01	0.24
<hr/>					
Asset return pro-cyclicalty					
$100 \times \frac{\sigma_{1r,t}}{\sigma_{1,t}^2}$					
0.0	0	29	0.16	0.02	0.23
2.5	11	16	0.16	-0.02	0.20
<hr/>					

<sup>a</sup>% increase in consumption in perpetuity needed to make welfare equivalent to baseline case. See equation 20.

<sup>b</sup>% GDP

<sup>c</sup> $1 - G_{w1}$

<sup>d</sup> $(\frac{\bar{w}}{\tau} (\frac{\bar{w}}{\tau \bar{w}})^{-\delta} - G_{wr})$

<sup>e</sup> $Var_t[c]^{0.5} \div \sigma_{1,t}$

Our first experiment is over a 20% discount rate for policy decisions compared to a 2% rate for welfare decisions. 20% is designed to approximate the political life span: discounting a fixed cash-flow at this rate would imply a Macaulay duration of 4.2 years — or that politicians' utility is as if they were receiving a zero-coupon bond that matures in 4.2 years, compared to nearly 50 years under the optimal discount rate.

Table 5 shows that the welfare losses of this degree of political impatience are huge: citizens whose politicians operate with a discount rate of 20%, 18pp below the social rate of 2%, would pay 11% of the state's contribution to consumption in perpetuity to have a policymaker that discounts at the social rate! The main reason is that the costs of volatility are here attributed to a fundamental characteristic (political impatience), which can also affect the ergodic mean level of utility.

Only very volatile asset returns — which as I argued would have to be due to enormous domestic financial frictions — could be as costly for welfare. Table 5 shows that a rise in the standard deviation of asset returns by 4.8 pp from the baseline has the same effect as politicians discounting at 20%. A rise of this size would mean that the 95 % range for next-period returns broaden from 4% - 17 % to -4% - 27%.

Equivalent welfare losses can similarly only be generated by incredibly procyclical interest rates, such that a 1pp of GDP rise in non-discretionary revenue next period triggers a 2.5pp rise in the real return on net assets. In the absence of further evidence, I would judge this degree of procyclicalty to be very unlikely to be observed in practice; Kaminsky, Reinhart, and Végh (2004) estimated a significant procyclical linear dependence of the order of 0.27

(using the Treasury Bill rate for lower middle income countries).

The columns further to the right in Table 5 provide further discrimination. They show that a greater impatience does generate startling different predictions compared to either more volatile asset returns or greater procyclicality, for the equivalent welfare loss. Most starkly of all, net assets will be lower — a whole 10pp of GDP lower — under political impatience. Under all of the other explanations, there are only slightly less net assets held than the baseline. The only exception is when there is a this very implausible degree of procyclicality. Considering that the level of net assets in Colombia was estimated to be well below the benchmark, it would seem that political impatience is the most likely channel through which resource dependence affects fiscal service, at least in Colombia.

Political impatience also makes discretionary expenditure exceptionally sensitive to resource returns. While in the baseline, a 1pp rise in revenue leads to a 0.16 pp of GDP rise in expenditure, with political impatience this jumps to 0.25pp. Similarly, only because of impatience can we expect to see a much greater proportion of asset returns spent rather than absorbed back into saving: over five times as much compared to the socially patient level! Finally political impatience predicts that discretionary expenditures will be half as volatile as the resource revenue. Even under very limited financial development, volatility remains at the baseline ratio of one fifth or one quarter.

## 9 Conclusion and discussion

In this paper, I aimed to test political impatience against limited financial development as key frictions in the fiscal problem of volatile resources. As part of the process I introduced two new elements. Firstly, that revenues, expenditures and rates of returns on assets are uncertain: they should be treated as known unknowns in the natural resource fiscal problem. Second that public capital should take its rightful place in the government's economic balance sheet, providing that we acknowledge that returns to public investment are of high risk and high social yield. The purpose is to discriminate between political impatience and a lack of financial depth as explanations of why some countries fail to successfully exploit the boon of natural resources.

After calibrating and solving a model for Colombia, I found that political impatience as opposed to financial frictions is the most likely explanation. While both imply large welfare costs when the full effect of uncertainty is incorporated, only political impatience generates the plausible prediction that the holdings of effective net assets will be suboptimally small and that current spending decisions are oversensitive to current revenues. Though the explanations tested for are not mutually exclusive, given the dominant effect of political impatience, this factor must play the key role in understanding the policy response of Colombia to resource revenues.

Thus the results of this paper justify a focus on institutions to remove policy impatience in the first instance rather than on alleviating financial frictions if the country is to escape the clutches of the resource curse. Frankel (2010) offers some practical suggestions along these lines. In particular, he argues that forecasts for fiscal planning should be done independently

of the political decision. I would only add that these forecasts should take account of uncertainty. My findings suggest that the consequences of underpredicting volatility can be more severe than optimism about growth.

Naturally, I have excluded other important aspects of the resource problem. One is the possibility of hedging through futures and other derivative contracts. Second that the private physical and human capital stock should also matter (Canuto and Cavallari (2012)). One hurdle to overcome in incorporating the private sector is to clarify how the government can influence private investment in these productive capitals. A third extension is to allow for data uncertainty, explicitly acknowledging that resource revenue forecasts are in effect noisy data about the future. In this way, we could tackle head on the problematics of separating forecasts of prices of the resource from those of volumes and, within each, separating cycle from trend.

# Appendices

## A Interpretation of the real return on net assets

I have assumed that there is such a thing as the real return on net assets, that part of this return is stochastic and exogenous and that the other part depends on net worth with a negative elasticity. The reader might welcome some more detail and clarification on these points.

Let us assume that the public capital stock  $A_{t-1}$  produces a service flow  $Y_t$  according to a Cobb-Douglas production function:

$$Y_t = r_{a,t}(A_{t-1})^\zeta(\Lambda_t L_t)^{1-\zeta-\mu}(W_{t-1})^\mu. \quad (21)$$

where there are constant returns in physical capital,  $A_{t-1}$ , and labour,  $L_t$ .  $r_{a,t}$  is an exogenous productivity shock (to be fully specified below). An unusual feature is that previous net worth is an input into production as working capital because the greater the net worth, the easier it is to deal with unanticipated expenditures. For example, under duress, while a government might contemplate raising finance through privatisations, that might only translate into retired liabilities at a poor conversion rate (because of firesaling). For simplicity, I assume that this is an externality.<sup>21</sup>

The marginal product of capital using the production function 21 is

$$\begin{aligned} r_{k,t+1} &\equiv r_{a,t+1}\zeta(A_t)^{\zeta-1}(\Lambda_{t+1}L_{t+1})^{1-\zeta-\mu}(W_t)^\mu \\ &= r_{a,t+1}\zeta(\vartheta_{1,t})^{\zeta-1}(\Lambda_{t+1}L_{t+1})^{1-\zeta-\mu}(W_t)^{\mu+\zeta-1} \\ &= r_{a,t+1}\zeta(\vartheta_{1,t})^{\zeta-1}\left(\frac{w_t}{\tau}\right)^{\mu+\zeta-1} \end{aligned} \quad (22)$$

where  $\vartheta_{1,t}$  is the share of time  $t$  net wealth held in physical capital.

To finance production the government invests in net risky financial assets paying a gross interest rate ( $r_{d,t+1}$ ) and an asset with a risk-free gross return ( $r_f$ ), fixed for simplicity. A negative investment in the risky liquid asset is interpreted as debt being greater than the total of assets such as foreign reserves and wealth funds.

$r_{a,t+1}$  and  $r_{d,t+1}$  follow jointly distributed autocorrelated log-normal process :

$$\begin{aligned} lr_{a,t+1} &= \kappa_a lr_{a,t} + u_{a,t+1} \\ lr_{d,t+1} &= (1 - \kappa_d)lr_f + \kappa_d lr_{d,t} + u_{d,t+1} \end{aligned} \quad (23)$$

with  $lr_{x,t+1} \equiv \ln(r_{x,t+1})$  for  $x = (a, d, f)$ .  $u_{a,t+1}$  and  $u_{d,t+1}$  are normally distributed variables with means of zero, respective variances of  $d_{aa}$  and  $d_{dd}$  and a covariance  $d_{ad}$ . The excess log returns to net liquid assets and capital are on average equal to the risk-free rate, by arbitrage.

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<sup>21</sup>In assessing the social optimal, I continue to take this to be an externality.

The budget constraint of the state is as in equation 3 but with net worth now explicitly disaggregated into net financial assets and public capital:

$$\begin{aligned} C_{t+1} &= ((1 - \vartheta_{1,t} - \vartheta_{2,t})r_f + \vartheta_{1,t}(r_{k,t+1} - \delta_k) + \vartheta_{2,t}r_{d,t+1})W_t - W_{t+1} + X_{1,t+1} - X_{2,t+1} \\ \Rightarrow C_{t+1} &= r_{p,t+1}W_t - W_{t+1} + X_{1,t+1} - X_{2,t+1} \end{aligned} \quad (24)$$

where  $\vartheta_{2,t}$  is the share of time  $t$  net wealth represented by net financial assets. This will be negative if debt is greater in amount than financial assets.  $r_{p,t+1}$  is the gross return on the government's portfolio defined as

$$\begin{aligned} \frac{r_{p,t+1}}{r_{f,t+1}} &= 1 + \vartheta_{1,t}\left(\frac{r_{k,t+1} - \delta_k}{r_f} - 1\right) + \vartheta_{2,t}\left(\frac{r_{d,t+1}}{r_f} - 1\right) \\ \frac{r_{p,t+1}}{r_{f,t+1}} &= 1 + \vartheta_{1,t}(e^{lr_{k,t+1} - \delta_k - lr_f} - 1) - \vartheta_{2,t}(e^{lr_{d,t+1} - lr_f} - 1). \end{aligned} \quad (25)$$

Taking logs of the above,

$$lr_{p,t+1} - lr_f = \ln(1 + \vartheta_{1,t}(e^{lr_{k,t+1} - \delta_k - lr_f} - 1) - \vartheta_{2,t}(e^{lr_{d,t+1} - lr_f} - 1)). \quad (26)$$

Define a vector of excess returns as

$$\mathbf{lr}_{s,t+1} \equiv \begin{bmatrix} lr_{k,t+1} - \delta_k - lr_f \\ lr_{d,t+1} - lr_f \end{bmatrix}. \quad (27)$$

Then

$$\begin{aligned} \mathbb{E}_t[\mathbf{lr}_{s,t+1}] &= \begin{bmatrix} \mathbb{E}_t[lr_{k,t+1}] - \delta_k - lr_f \\ \mathbb{E}_t[lr_{d,t+1}] - lr_f \end{bmatrix} \\ &= \begin{bmatrix} \kappa_a lr_{a,t} + \ln(\zeta(\vartheta_{1,t})^{\zeta-1}(\frac{w_t}{\tau})^{\mu+\zeta-1} - \delta_k - lr_f) \\ \kappa_d(lr_{d,t} - lr_f) \end{bmatrix} \end{aligned} \quad (28)$$

and

$$\mathbf{Var}_t[\mathbf{lr}_{s,t+1}] = \begin{bmatrix} \mathit{Var}_t[lr_{k,t+1}] & \mathit{Cov}_t[lr_{k,t+1}, lr_{d,t+1}] \\ \mathit{Cov}_t[lr_{k,t+1}, lr_{d,t+1}] & \mathit{Var}_t[lr_{d,t+1}] \end{bmatrix} \quad (29)$$

$$= \begin{bmatrix} d_{aa} & d_{ad} \\ d_{ad} & d_{dd} \end{bmatrix}. \quad (30)$$

Consider a first-order approximation of  $lr_{p,t+1}$  with respect to  $\mathbf{lr}_{s,t+1}$  about  $\mathbf{0}$

$$lr_{p,t+1} \approx (1 - \vartheta_{1,t} - \vartheta_{2,t})lr_f + \vartheta_{1,t}(lr_{a,t+1} + \ln(\zeta(\vartheta_{1,t})^{\zeta-1}) + (\mu + \zeta - 1)\ln(\frac{w_t}{\tau}) - \delta_k) + \vartheta_{2,t}lr_{d,t+1} \quad (31)$$

such that the gross portfolio return is approximately the product of the exogenous return and an endogenous component, just as in equation 3:<sup>22</sup>

$$\begin{aligned}
r_{p,t+1} &\approx r_{r,t+1} \left(\frac{w_t}{\bar{w}}\right)^{-\delta}. \\
\text{where } r_{r,t+1} &= r_f^{1-\vartheta_{1,t}-\vartheta_{2,t}} r_{a,t+1}^{\vartheta_{1,t}} r_{d,t+1}^{\vartheta_{2,t}} e^{-\delta_k \vartheta_{1,t}} \zeta^{\vartheta_{1,t}} (\vartheta_{1,t})^{\vartheta_{1,t}(\zeta-1)}, \\
\delta &= \vartheta_{1,t}(1 - \zeta - \mu) \\
\text{and } \bar{w} &= \tau.
\end{aligned} \tag{32}$$

If debt dominates financial assets ( $\vartheta_{2,t} < 0$ ) and there is a rise in rate of return on risky financial claims (unrelated to the productivity shock  $r_{a,t}$ ) then the exogenous component of the rate of return on net assets ( $r_{r,t}$ ) will fall. Under these circumstances, a countercyclical policy that raises rates on risky financial claims when windfall revenues are high connotes a *negative* conditional dependence between the real rate on net assets and revenue. Conversely, procyclicality implies a positive conditional dependence between the real rate on net assets and non-discretionary revenue. This is the interpretation I follow in the rest of the text.

Taking expectations conditional on period  $t$  information,

$$\text{Var}_t[lr_{r,t+1}] \approx \vartheta_{1,t}^2 d_{aa} + \vartheta_{2,t}^2 d_{dd} + 2\vartheta_{1,t}\vartheta_{2,t}d_{ad}. \tag{33}$$

Equation 33 links limited diversification to risk, just as in standard portfolio theory. Poor diversification in this context is equivalent to a more negative covariance between lending costs and investment returns such that for example a lower rate on investments is more likely to be associated with creditors raising their offered lending rates. According to equation 33, the more negative  $d_{ad}$ , the larger conditional variance of the log returns on net assets providing that debt dominates liquid financial assets ( $\vartheta_{1,t}$ ).

Turning now to the interpretation of the endogenous component in equation 32, the elasticity of the quantity of net assets on the endogenous component of the return on net assets,  $\delta$ , is a combination of two opposing forces of diminishing marginal returns to capital and the beneficial effect of having higher net worth on production.

But how can this parameter be calibrated? The first influence is equal to one minus the share of capital in nominal public output ( $1-\zeta$ ) multiplied by the share of public capital in net worth  $\vartheta_{1,t}$ . As the share of government spending on GDP is about 40% and the factor share of public capital in total GDP was estimated by Gupta, Kangur, Papageorgiou, and Wane (2011) to be about 20%,  $(1 - \zeta)$  should be about  $0.5 = \frac{20}{40}$ . But  $\vartheta_1$  is an optimal share and is therefore difficult to estimate given political impatience. And neither are there, as far as I know, direct estimates of the importance of net worth to production,  $\mu$ , although Lipschitz, Messmacher, and Mourmouras (2006)'s estimates of the beneficial impact of reserves on debt financing costs suggest that it could be substantial. In what follows I take the view that that  $\delta$  is positive, but only just.

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<sup>22</sup>This interpretation depends on the accuracy of the first-order approximation and portfolio shares being updated infrequently. Campbell, Chan, and Viceira (2003) implement a second-order approximation in a version of this problem and solve for the portfolio shares, but only in the absence of stochastic income.

## B Derivation of solution

Differentiating  $f_1(\cdot)$  from 9 with respect to  $c_{t+1}$  and  $r_{t+1}$ , we have

$$\frac{\partial^2 f_1}{\partial c_{t+1}^2} = \frac{\gamma(\gamma+1)(1-\delta)}{c_{t+1}^2} (\beta(\frac{c_{t+1}\tau}{c_t})^{-\gamma} r_{t+1}) (\frac{w_t}{\tau\tilde{w}})^{-\delta}, \quad (34)$$

$$\frac{\partial^2 f_1}{\partial r_{t+1}^2} = 0, \quad (35)$$

and

$$\frac{\partial^2 f_1}{\partial c_{t+1} \partial r_{t+1}} = \frac{d^2 f_1}{dr_{t+1} dc_{t+1}} = \frac{-\gamma(1-\delta)}{c_{t+1} r_{t+1}} (\beta(\frac{c_{t+1}\tau}{c_t})^{-\gamma} r_{t+1}) (\frac{w_t}{\tau\tilde{w}})^{-\delta}. \quad (36)$$

Substituting 34, 35 and 36 into 9 (and assuming that the expression  $\beta(1-\delta)(\frac{\mathbb{E}_t[c_{t+1}]\tau}{c_t})^{-\gamma} \times \mathbb{E}_t[r_{t+1}](\frac{w_t}{\tau\tilde{w}})^{-\delta}$  is always non-zero) leads to equation 12 in the main text. Pushing the expectations rule 16 one period forward, taking expectations and substituting in from equation 17, yields expressions for the conditional mean and variance of  $c_{t+1}$  as well as its covariance with the rate of return. These are substituted into 12 prior to solution.

The risky steady state is defined by the values of  $G_{ww}$ ,  $G_{wr}$ ,  $G_{wi}$ ,  $\bar{w}$  and  $\bar{c}$  that constitute the joint solution of the second-order approximation of the first-order condition (12), the budget constraint as well as the condition that the total derivatives of  $\hat{\Phi}$  with respect to the states  $w_{t-1}$ ,  $r_t$ ,  $x_{1,t}$  and  $x_{2,t}$  are zero (all evaluated at the steady state). The solutions are a function of the underlying parameters  $\beta$ ,  $\tau$ ,  $\delta$  and  $\gamma$  and the parameters describing the exogenous processes —  $\bar{l}x_1$ ,  $\bar{l}x_2$ ,  $\kappa_1$ ,  $\kappa_2$ ,  $\bar{l}r$ ,  $\kappa_r$ ,  $\mathbf{D}$ .

## C Approximation to the conditional variance of future consumption

We work with the following linearised version of the state system and the budget constraint

$$\begin{aligned} w_{s+1} &= \bar{w} + G_{ww}(w_s - \bar{w}) + G_{wr}(r_{s+1} - \bar{r}) + G_{w1}(x_{1,s+1} - \bar{x}_1) + G_{w2}(x_{2,s+1} - \bar{x}_2); \\ x_{1,s+1} &\approx (1 - \kappa_1)\bar{x}_1 + \kappa_1 x_{1,t} + \bar{x}_1 u_{1,s+1}; \\ x_{2,s+1} &\approx (1 - \kappa_2)\bar{x}_2 + \kappa_2 x_{2,t} + \bar{x}_2 u_{2,s+1}; \\ r_{s+1} &\approx (1 - \kappa_r)\bar{r} + \kappa_r r_s + \bar{r} u_{s+1}; \\ \hat{c}_{s+1} &\approx ((1 - \delta)\frac{\bar{r}}{\tau}(\frac{\bar{w}}{\tilde{w}\tau})^{-\delta} - G_{ww})w_s + ((\frac{\bar{w}}{\tau})^{1-\delta}\tilde{w}^\delta - G_{wr})r_{s+1} + (1 - G_{w1})x_{1,s+1} - (1 + G_{w2})x_{2,s+1} \\ &\quad - (\frac{\bar{w}}{\tau})^{1-\delta}\tilde{w}^\delta\bar{r} + (G_{ww} + 1)\bar{w} + G_{wr}\bar{r} + G_{w1}\bar{x}_1 + G_{w2}\bar{x}_2. \end{aligned} \quad (37)$$

Define  $\mathbf{z}_n \equiv (w_n, x_{1,n}, x_{2,n}, r_n)^T$ . Then

$$\mathbf{z}_{n+1} = \Omega_1^{n+1-t} \mathbf{z}_t + \sum_{k=1}^{n+1-t} \Omega_1^{n+1-t-k} \Omega_2 \mathbf{D}^{\frac{1}{2}} \mathbf{v}_{t+k} + \Omega_3 \quad (38)$$

for  $n \geq t$ . Here

$$\mathbf{\Omega}_1 \equiv \begin{pmatrix} G_{ww} & G_{w1}\kappa_1 & G_{w2}\kappa_2 & G_{wr}\kappa_r \\ 0 & \kappa_1 & 0 & 0 \\ 0 & 0 & \kappa_2 & 0 \\ 0 & 0 & 0 & \kappa_r \end{pmatrix},$$

$$\mathbf{\Omega}_2 \equiv \begin{pmatrix} G_{w1}\bar{x}_1 & G_{w2}\bar{x}_2 & G_{wr}\bar{r} \\ \bar{x}_1 & 0 & 0 \\ 0 & \bar{x}_2 & 0 \\ 0 & 0 & \bar{r} \end{pmatrix},$$

$\mathbf{D}^{\frac{1}{2}}$  is the Cholesky decomposition of the matrix  $\mathbf{D}$ , defined in equation 7,

$$\mathbf{\Omega}_3 \equiv \begin{pmatrix} (1 - G_{ww})\bar{w} - \kappa_r G_{wr}\bar{r} - \kappa_1 G_{w1}\bar{x}_1 - \kappa_2 G_{w2}\bar{x}_2 \\ (1 - \kappa_1)\bar{x}_1 \\ (1 - \kappa_2)\bar{x}_2 \\ (1 - \kappa_r)\bar{r} \end{pmatrix}$$

and  $\mathbf{v}_{t+1}$  is a vector of three mean zero, unit variance, independent normally distributed shocks and  $\mathbf{I}_N$  is a  $N \times N$  identity matrix. Using equation 38 we can calculate  $E_t[\mathbf{z}_{s+1}^2 - E_t[\mathbf{z}_{s+1}]^2]$  and  $E_t[\mathbf{z}_{s+1}\mathbf{z}_s^T - E_t[\mathbf{z}_{s+1}]E_t[\mathbf{z}_s]]$  for all  $s > t$ . The terms in these matrices give us the necessary expressions to calculate  $Var_t[\hat{c}_s]$ , using the linear approximation to the budget constraint in the last row of equation 38. Our approximation to utility follows from inserting the terms  $Var_t[\hat{c}_s]$ ,  $s > t$  into the expression 19 in the main text.

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