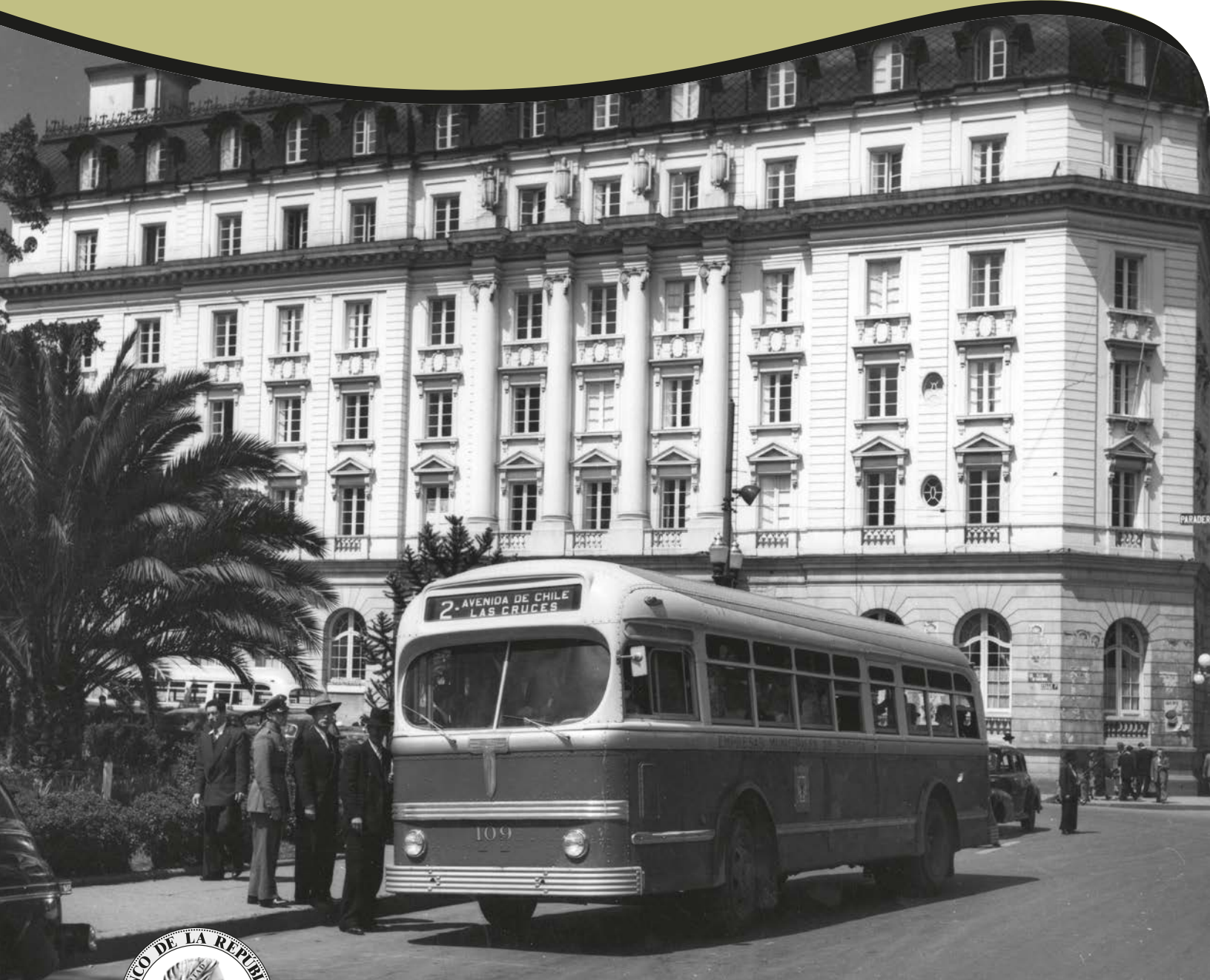


State-dependent Forward Guidance
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Announcements

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State-dependent Forward Guidance and the Problem of Inconsistent Announcements

Julian A. Parra-Polania*

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Abstract

Florez-Jimenez and Parra-Polania (2016) show that unconditional forward guidance (FG) performs poorly except in the most extreme zero lower-bound (ZLB) events and that for any ZLB situation it is better to resort to state-dependent (threshold-based) FG. The model of that paper is solved under the assumption that the threshold (to revert to the optimal discretionary policy) is announced in terms of an exogenous variable (the demand shock). The present paper shows that, taking full credibility as given, the solution does not change when the threshold is announced in terms of an endogenous variable (output or inflation). However, the paper also illustrates the fact that an endogenous-variable threshold gives rise to inconsistency: the action taken may not conform to the central bank announcement. Consistency imposes limits on the policy rate that can be set since reverting to the optimal discretionary rate can be incompatible with exceeding the threshold.

Keywords: forward guidance; zero-lower bound; central bank announcements; monetary policy; threshold

JEL Classification: E52, E58, E37

* Contact email: jparrapo@banrep.gov.co. I am grateful to Carmiña Vargas for her valuable comments. All remaining errors are my own.

1 Introduction

As an alternative way to respond to the recent global financial crisis, some major central banks committed to deliver a relatively low future policy rate. This strategy -usually referred to as *Odyssean* forward guidance (FG)- intends to provide stimulus to the economy through the expectations channel. Such unconventional policy, if credible, might be particularly useful when facing zero lower-bound (ZLB) events, that is to say, situations where the monetary authority is not able to further reduce its conventional policy instrument, the short-term interest rate.

FG can be provided as an unconditional promise such that the announced rate will be set under any circumstance or, alternatively, the promise may include an escape clause, i.e. a particular state of the economy under which the central bank would not deliver such rate and, instead, it would revert to setting policy under discretion. The escape clause can be expressed as a threshold in terms of a specific variable (inflation, employment, output...).

Florez-Jimenez and Parra-Polania (2016) show that unconditional FG performs poorly except in the most extreme ZLB events and that for any ZLB situation it is better to resort to state-dependent (threshold-based) FG.¹ The model of that paper is solved under the assumption that the announced threshold is expressed in terms of an exogenous variable (the demand shock). The first part of the present paper shows that, taking full credibility as given, both the level of the threshold and the promised rate that minimise macroeconomic volatility remain unchanged when the threshold is announced in terms of an endogenous variable (output gap or inflation).

Based on the abovementioned result only, one would conclude that expressing the optimal threshold in terms of an exogenous variable is equivalent to expressing it in terms of an exogenous one. However, as the second part of the present paper illustrates, an endogenous-variable threshold gives rise to inconsistency since the action taken may not conform to the central bank announcement.² This can be explained as follows. The central bank announces, say, an output threshold and promises that if it is not exceeded the policy rate will be relatively low (at a specific value). If instead output exceeds the threshold, the policy rate will be at the optimal discretionary level. However, the fact that output is affected by the policy rate causes the inconsistency with the announcement: in some cases delivering the promised rate would stimulate the economy up to a level in which the threshold would be exceeded but increasing the policy rate up to the optimal discretionary level would not provide sufficient stimulus for the threshold to be exceeded.

To my knowledge, the possibility of inconsistent announcements in (endogenous) threshold-based FG was firstly mentioned by Boneva et al. (2017, footnote 6). These authors, nevertheless, avoid the problem by modifying the standard IS curve so that the output gap (i.e. the endogenous variable) is affected by the expected value rather than the observed value of the

¹Feroli et al. (2017) also highlight the fact that time-based FG should be used in only very unusual circumstances. Coenen et al. (2017) show that FG reduces uncertainty more effectively when it is state-dependent (or when it provides guidance about a long horizon). Boneva et al. (2017) show that, with the appropriate choice of thresholds, state-dependent FG outperforms time-dependent FG.

²This is in addition to the time-inconsistency problem faced, by definition, by any kind of *Odyssean* FG, as widely acknowledged by the related literature.

policy rate. In that case, like in that of an exogenous-variable threshold, the announcement is always consistent since whether or not the threshold is exceeded is a fact that is determined before setting the policy rate. In contrast, I preserve the standard IS curve and formally analyse the problem of inconsistent announcements.

Consistency imposes limits on the policy rate that can be set since reverting to the optimal discretionary rate can be incompatible with exceeding the threshold. To avoid inconsistency the central bank has to announce that if the threshold is breached the policy rate will increase but may not be as high as the optimal discretionary rate (as mentioned above, at such level there could be insufficient stimulus for the threshold to be exceeded).

2 Equivalence of the exogenous- and the endogenous-threshold solutions

In this section, the model is the same as that described by Florez-Jimenez and Parra-Polania (2016) with the only change that I will also analyse the case where the central bank chooses the optimal threshold in terms of an endogenous variable.

It is a three-equation two-period New Keynesian model.³ The period loss function is

$$L_t = \pi_t^2 + \lambda y_t^2, \quad (1)$$

where π is inflation and y denotes the output gap. The Phillips curve and the IS curve, respectively, are

$$\pi_t = \beta E_t \pi_{t+1} + k y_t, \quad (2)$$

$$y_t = E_t y_{t+1} - \frac{1}{\delta} (i_t - E_t \pi_{t+1}) + \varepsilon_t, \quad (3)$$

where E_t is the expectations operator conditional on information available at time t , i_t is the nominal interest rate, and ε_t is the demand shock (which is assumed to be independently distributed over time).

There is a lower bound for the nominal interest rate ($i_t \geq i_l$), and hence in rare but possible cases (when large negative shocks are realised) the central bank is not able to set i_t at its optimal level ($i_t^* = \delta \varepsilon_t < i_l$). It is assumed that there is a large negative demand shock in period t such that the interest-rate lower bound is reached ($i_t = i_l$). The economic recovery is expected in $t + 1$ (i.e. the economy will no longer be under a lower-bound situation).

Under discretion there is a positive loss in period t since the interest rate is different from the optimal one ($i_t = i_l > \delta \varepsilon_t$) and, as the economy recovers in $t + 1$, L_{t+1} would be zero. Alternatively, the central bank may reduce the loss in period t (through the expectations channel) by credibly committing to a specific value for the interest rate in $t + 1$ (θ_{t+1}), i.e. the monetary authority may provide Odyssean FG. In doing so, the central bank mitigates the loss of period t by reducing monetary policy flexibility in $t + 1$ (and therefore at the cost of increasing the expected loss of that period).

If there is a large positive demand shock in period $t + 1$ (ε_{t+1}) the central bank will face a high cost of fulfilling its promise because there will be a significant deviation of the announced interest rate from the optimal one. As a consequence, the central bank might find it useful to

³ A third period ($t + 2$) is assumed in which all shocks and variables return to their long-run expected values.

announce as well an escape clause, i.e. a condition under which it will renege on commitment to the announced interest rate and will set such rate in a discretionary manner.

More details on the model can be found in Florez-Jimenez and Parra-Polania (2016).

2.1 Exogenous threshold

In period t the central bank announces θ_{t+1} , i.e. a specific value for the interest rate in $t + 1$, together with an escape clause. In Florez-Jimenez and Parra-Polania (2016) this condition is chosen in terms of the demand shock in $t + 1$: if it exceeds the threshold (i.e. $\varepsilon_{t+1} \geq \varepsilon_h$) the interest rate will be that of the discretionary case, $i_{t+1}^* = \delta\varepsilon_{t+1}$.

In period t , the central bank picks both the announced interest rate θ_{t+1} and the threshold ε_h so as to minimise the following function⁴

$$G(\varepsilon_h, \theta_{t+1}) \equiv L_t(i_t, \theta_{t+1}) + \beta \int_{-\infty}^{\varepsilon_h} f(\varepsilon_{t+1}) L_{t+1}(\theta_{t+1}, \varepsilon_{t+1}) d\varepsilon_{t+1}, \quad (4)$$

where $f(\cdot)$ corresponds to the pdf of ε_{t+1} and it has been taken into account that when the interest rate is set at its optimal level i_{t+1}^* (when $\varepsilon_{t+1} \geq \varepsilon_h$) the loss of period $t + 1$ is zero, i.e. $L_{t+1}(i_{t+1}^*) = 0$. Using Equations (2) and (3), the central bank objective function $G(\varepsilon_h, \theta_{t+1})$ can be written as⁵

$$\begin{aligned} & k^2 \left[\left(1 + \beta + \frac{k}{\delta} \right) E_t y_{t+1} - \frac{1}{\delta} i_t + \varepsilon_t \right]^2 \\ & + \lambda \left[\left(1 + \frac{k}{\delta} \right) E_t y_{t+1} - \frac{1}{\delta} i_t + \varepsilon_t \right]^2 \\ & + \beta (k^2 + \lambda) E_t y_{t+1}^2, \end{aligned} \quad (5)$$

$$\begin{aligned} \text{where } E_t y_{t+1} &= \int_{-\infty}^{\varepsilon_h} f(\varepsilon_{t+1}) \left(\varepsilon_{t+1} - \frac{\theta_{t+1}}{\delta} \right) d\varepsilon_{t+1} \\ \text{and } E_t y_{t+1}^2 &= \int_{-\infty}^{\varepsilon_h} f(\varepsilon_{t+1}) \left(\varepsilon_{t+1} - \frac{\theta_{t+1}}{\delta} \right)^2 d\varepsilon_{t+1}. \end{aligned}$$

2.2 Endogenous threshold

In this case the central bank announces a threshold in terms of an endogenous variable. Details are shown for a threshold expressed in terms of the output gap; however, the case of a threshold in terms of inflation is analogous.

If the output gap exceeds the threshold (i.e. $y_{t+1} \geq y_h$), the interest rate will be that of the discretionary case, $i_{t+1}^* = \delta\varepsilon_{t+1}$. If, instead, the output gap is below the threshold, the interest rate will be the promised rate θ_{t+1} .

⁴In Florez-Jimenez and Parra-Polania (2016) the lower limit of the integral is zero since in that paper it is assumed that ε_{t+1} takes positive values only. This assumption is not necessary for the purposes of the present paper.

⁵The first two terms of (5) correspond to L_t and the third one to $\beta E_t L_{t+1}$. It has been taken into consideration that: $E_t \pi_{t+1} = k E_t y_{t+1}$, $E_{t+1} \pi_{t+2} = E_{t+1} y_{t+2} = 0$.

The central bank picks both θ_{t+1} and y_h so as to minimise the total loss. Since $y_{t+1} = \varepsilon_{t+1} - \theta_{t+1}/\delta$, the condition $y_{t+1} \geq y_h$ is equivalent to $\varepsilon_{t+1} \geq y_h + \theta_{t+1}/\delta$.⁶ Then, the objective function takes the same form as that in Equation (5) but with the following expected values:

$$E_t y_{t+1} = \int_{-\infty}^{y_h + \theta_{t+1}/\delta} f(\varepsilon_{t+1}) \left(\varepsilon_{t+1} - \frac{\theta_{t+1}}{\delta} \right) d\varepsilon_{t+1}$$

and $E_t y_{t+1}^2 = \int_{-\infty}^{y_h + \theta_{t+1}/\delta} f(\varepsilon_{t+1}) \left(\varepsilon_{t+1} - \frac{\theta_{t+1}}{\delta} \right)^2 d\varepsilon_{t+1}$.

Notice that we can define $\varepsilon_h(\theta_{t+1}, y_h) \equiv y_h + \theta_{t+1}/\delta$ and thus the objective function for the endogenous-threshold case can be represented as $G(\varepsilon_h(\theta_{t+1}, y_h), \theta_{t+1})$. We can then use the following proposition:

Proposition 1 *Suppose there is a function $\varepsilon_h(\theta_{t+1}, y_h)$ and $\partial\varepsilon_h/\partial y_h \neq 0$. The point $(\theta_{t+1}^*, \varepsilon_h^*)$ is a solution to the problem of minimising $G(\varepsilon_h, \theta_{t+1})$ with respect to θ_{t+1} and ε_h if and only if it is also a solution to minimising $G(\varepsilon_h(\theta_{t+1}, y_h), \theta_{t+1})$ with respect to θ_{t+1} and y_h .*

Proof. See Appendix A.1 ■

Since the model of the present paper represents a particular case of Proposition 1, in which $\partial\varepsilon_h/\partial y_h = 1$, we can state that the solutions obtained for the model described by Florez-Jimenez and Parra-Polania (2016), in which the optimal threshold is chosen in terms of an exogenous variable (i.e. the demand shock), are also the solutions for the same model when the optimal threshold is chosen in terms of an endogenous variable (i.e. output gap or inflation).

3 The problem of inconsistent announcements

The crucial element in this section is the coherence between the central bank announcement and the action taken. More specifically, when the threshold is exceeded the central bank should have indeed set, as announced, the optimal discretionary rate and, instead, when it is not exceeded the promised rate should have been delivered.

Since the promised rate and the threshold are announced in period t (before the demand and the supply shocks of $t+1$ are known with certainty), it is important to distinguish between ex-ante and ex-post consistency. The latter requires coherence between the announcement (in t) and the action taken (in $t+1$), given the specific shocks realised in $t+1$. The former, a stronger condition, requires coherence between the announcement and the action taken, for any possible shock that may occur in $t+1$. Although ex-post consistency is the one that agents observe (in $t+1$), ex-ante consistency is essential: if it is not assured, there will be a positive probability of facing ex-post inconsistency as the central bank cannot control the occurrence of shocks. This is why I mainly focus on analysing whether or not central bank announcements satisfy ex-ante consistency.

⁶As $\pi_{t+1} = ky_{t+1}$, it is straightforward to deduce the results for the inflation-threshold case from those for the output-threshold case.

To illustrate the problem of inconsistent announcements I use a more general model (than that of the previous section), which includes persistent supply and demand shocks. The IS curve does not change so it is equal to Equation (3). The Phillips curve is

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + s_t, \quad (6)$$

and the supply and demand shocks (s_t and ε_t) follow independent autoregressive processes:

$$\begin{aligned} s_t &= \rho_s s_{t-1} + v_{s,t}, \\ \varepsilon_t &= \rho_\varepsilon \varepsilon_{t-1} + v_{\varepsilon,t}, \end{aligned}$$

where $v_{s,t}$ and $v_{\varepsilon,t}$ are white-noise disturbances.

Unlike the model in the previous section, this is an infinite-horizon model. The central bank sets the policy rate path so as to minimise the expected value of the discounted loss:

$$E_t \sum_{i=0}^{\infty} \beta^i (\pi_{t+i}^2 + \lambda y_{t+i}^2).$$

Appendix A.2 solves this problem for the case in which there are no ZLB events.

Like in the model of the previous section, I will assume that the economy experiences a ZLB situation in period t and then recovers in the following period (i.e. it will not be under a lower-bound situation in $t+1$).⁷ For simplicity I also assume that no more ZLB events are expected.

It is easier to start with the case in which there are demand shocks only (i.e. $s_t = 0, \forall t$). Supply shocks will be included afterwards.

3.1 Demand shocks only

In period t , when facing a ZLB event, the central bank announces a specific value for the interest rate in $t+1$ (θ_{t+1}) together with an escape clause, i.e. a threshold (y_h). If the output gap breaches the threshold in period $t+1$ (i.e. $y_{t+1} \geq y_h$), the interest rate will be that of the discretionary case (i_{t+1}^*). Since no more ZLB events are expected, from Appendix A.2 it is easy to see that, in the absence of supply shocks, $i_{t+1}^* = \delta \varepsilon_{t+1}$ and $y_{t+1}^* = 0$. Since this occurs when the threshold is breached, the consistency of the escape-clause announcement requires that $y_h \leq 0$.⁸

If, instead, the threshold is not breached, the promised rate θ_{t+1} applies and it must be the case that⁹

$$y_{t+1} = -\frac{1}{\delta} \theta_{t+1} + \varepsilon_{t+1} < y_h \leq 0$$

⁷Removing this assumption does not eliminate the problem described in the present paper and actually further inconsistencies may arise. Boneva et al. (2017, Appendix A.1) allow for the occurrence of ZLB during several periods and show that promising to exit from the FG policy with certainty when the threshold is exceeded may be inconsistent: in some cases exit at time T implies insufficient stimulus for the threshold to be exceeded at that time but exit at time $T+1$ implies enough stimulus to exceed the threshold at an earlier date. The authors solve this problem by assuming probabilistic exit i.e. if the threshold is breached the probability of exiting from FG becomes positive but less than 1.

⁸According to the results obtained by Florez-Jimenez and Parra-Polania (2016) - in a two-period model with demands shocks only- the solution, for all the parameter values considered, always implied $y_h > 0$. Consequently, in all of those cases, expressing the threshold in terms of an endogenous variable would imply an inconsistent solution.

⁹ $E_{t+1} y_{t+2}$ and $E_{t+1} \pi_{t+2}$ are zero as no more ZLB events are expected and there are no supply shocks.

and this condition implies that $\delta\varepsilon_{t+1} = i_{t+1}^* < \theta_{t+1}$, that is to say, consistency requires that when the announced rate θ_{t+1} is delivered it must be higher than the optimal discretionary rate that would be set when facing the same shock ε_{t+1} .¹⁰

The abovementioned result illustrates the problem of inconsistent announcements (when only demand shocks occur): the central bank resorts to forward guidance to stimulate the economy, in t , by promising to keep the future policy rate lower than the one that would be expected under normal conditions; however, consistency requires the opposite, i.e. to promise a higher policy rate. In other words, to provide FG so as to stimulate the economy in t the central banks needs to make an inconsistent promise.

3.2 Demand and Supply shocks

This subsection illustrates the problem of inconsistency when supply shocks are included.

If the output gap exceeds the threshold in period $t + 1$ (i.e. $y_{t+1} \geq y_h$) the interest rate will be the optimal one under discretion. From Appendix A.2, the optimal interest rate and the output gap, respectively, are

$$i_{t+1}^* = \frac{k\delta(1 - \rho_s) + \lambda\rho_s}{k^2 + \lambda(1 - \beta\rho_s)}s_{t+1} + \delta\varepsilon_{t+1}, \quad (7)$$

and

$$y_{t+1}^* = -\frac{k}{k^2 + \lambda(1 - \beta\rho_s)}s_{t+1}. \quad (8)$$

Since $\partial y_{t+1}(s_{t+1})/\partial s_{t+1} < 0$, there must be a value s_{t+1}^h such that $y_{t+1}(s_{t+1}^h) = y_h$ and $y_{t+1}(s_{t+1}) \geq y_h$ for all $s_{t+1} \leq s_{t+1}^h$. If, instead, the threshold is not exceeded ($s_{t+1} > s_{t+1}^h$), the promised rate applies and it must be the case that

$$y_{t+1} = E_{t+1}y_{t+2} - \frac{1}{\delta}(\theta_{t+1} - E_{t+1}\pi_{t+2}) + \varepsilon_{t+1} < y_h = -\frac{k}{k^2 + \lambda(1 - \beta\rho_s)}s_{t+1}^h. \quad (9)$$

Since no more ZLB events are expected, from the results presented in Appendix A.2, it is easy to see that

$$E_{t+1}y_{t+2} = -\frac{k}{k^2 + \lambda(1 - \beta\rho_s)}\rho_s s_{t+1}, \quad (10)$$

$$E_{t+1}\pi_{t+2} = \frac{\lambda}{k^2 + \lambda(1 - \beta\rho_s)}\rho_s s_{t+1}, \quad (11)$$

and thus, after some algebra, condition (9) becomes

$$\frac{k\delta(1 - \rho_s) + \lambda\rho_s}{k^2 + \lambda(1 - \beta\rho_s)}s_{t+1} + \delta\varepsilon_{t+1} + \frac{\delta k}{k^2 + \lambda(1 - \beta\rho_s)}(s_{t+1}^h - s_{t+1}) < \theta_{t+1}. \quad (12)$$

The following is an easy way to see that the central bank announcements are inconsistent (when both supply and demand shocks are present). Notice that if both the supply and demand shocks in $t + 1$ are sufficiently large (such that $s_{t+1} > s_{t+1}^h$ and condition (12) does

¹⁰It is straightforward to see that the same happens with a threshold expressed in terms of inflation: $\pi_{t+1} = k(-\frac{1}{\delta}\theta_{t+1} + \varepsilon_{t+1}) < \pi_h \leq 0$

not hold) there will be no equilibria with ex-post consistent announcements¹¹: the threshold will not be exceeded when the central bank sets the optimal discretionary rate and, instead, it will be exceeded when the promised rate is delivered.¹²

3.3 Avoiding Inconsistency

Consistency imposes limits on the policy rate that can be set since reverting to the optimal discretionary rate can be incompatible with exceeding the threshold.

Since, with an endogenous-variable threshold, delivering the promised rate θ_{t+1} may imply that the threshold is exceeded, then the consistent option for the central bank is announcing that when the threshold is breached the policy rate will increase but may not be as high as the optimal discretionary rate. In that case the policy rate should be within the set of the ones that do not make the announcement inconsistent (i.e. the set of policy rates that do not violate the condition $y_{t+1} \geq y_h$) and hence the central bank will set the rate according to¹³

$$i_{t+1} = \min \left\{ \frac{\lambda - k\delta}{k^2 + \lambda(1 - \beta\rho_s)} \rho_s s_{t+1} + \delta\varepsilon_{t+1} - y_h, i_{t+1}^* \right\},$$

which may be well below the optimal discretionary rate (cf. (7)). Increasing the policy rate up to the optimal discretionary level might not provide sufficient stimulus for the threshold to be exceeded, and hence it may be an inconsistent option.

As mentioned in the introduction, Boneva et al. (2017) refer to the possibility of inconsistent announcements in (endogenous) threshold-based FG but avoid this problem by modifying the standard IS curve so that the output gap is determined before the policy rate of the same period is set. In practice, the escape clause has been activated once the corresponding variable has been realised (and observed), and therefore the problem of inconsistent announcements seems to be just a theoretical one. However, as can be easily deduced from the type of models considered in this paper, waiting to act only after the (endogenous) variable has been observed (and hence keeping the promised rate during one more period) may imply a high cost due to the size of the deviation of the promised policy rate from the optimal one. It would be ideal for the central bank to try to anticipate, as long as possible, such circumstance so as to minimise the length of situations in which the cost of keeping the promised rate is significantly high. Nonetheless, it is important to point out that this might imply announcing the threshold in terms of a forecast, which might affect credibility.

¹¹The fact that both the supply and the demand shock have to be large enough is a sufficient but not a necessary condition for ex-post inconsistency. Notice that if $\lambda - k\delta > 0$, the LHS of (12) is increasing in s_{t+1} , and therefore for given values of θ_{t+1} , y_h and ε_{t+1} we can find a sufficiently large value of $s_{t+1} > s_{t+1}^h$ such that condition (12) does not hold.

¹²If the threshold is expressed in terms of inflation (π_h) it must be taken into consideration that, from (22) in Appendix A.2, $\partial\pi_{t+1}(s_{t+1})/\partial s_{t+1} > 0$. Following an analogous procedure to that of the present subsection there must be a value s_{t+1}^h such that $\pi_{t+1}(s_{t+1}^h) = \pi_h$ and $\pi_{t+1}(s_{t+1}) \geq \pi_h$ for all $s_{t+1} \geq s_{t+1}^h$. If, instead, the threshold is not exceeded ($s_{t+1} < s_{t+1}^h$) it must be the case that $\frac{k\delta(1-\rho_s)+\lambda\rho_s}{k^2+\lambda(1-\beta\rho_s)} s_{t+1} + \delta\varepsilon_{t+1} + \frac{\lambda}{k^2+\lambda(1-\beta\rho_s)} \frac{\delta}{k} (s_{t+1} - s_{t+1}^h) < \theta_{t+1}$. If the supply shock is small ($s_{t+1} < s_{t+1}^h$) and the demand shock sufficiently large, there will be no equilibria with ex-post consistent announcements.

¹³Equations (3), (10) and (11) have been taken into account so as to obtain this condition. For consistency, it is required that $y_{t+1} \geq y_h$. When the central bank cannot increase the policy rate up to the optimal level, it sets the highest possible rate that satisfies consistency, and hence $y_{t+1} = y_h$ (since $\partial y_{t+1}/\partial i_{t+1} < 0$).

4 Conclusion

Using a two-period three-equation New Keynesian model, Florez-Jimenez and Parra-Polania (2016) show that, for standard values of the underlying parameters, unconditional forward guidance (FG) performs poorly except in the most extreme zero-lower bound (ZLB) events and that for any ZLB situation it is better to resort to state-dependent (threshold-based) FG, i.e. to include an escape clause. The model of that paper is solved under the assumption that the threshold is announced in terms of an exogenous variable (the demand shock). The first part of the present paper extends that article to show that the solution (i.e. the threshold and the promised policy rate that minimise macroeconomic volatility, taking full credibility as given) does not change when the threshold is announced in terms of an endogenous variable (output gap or inflation).

Based on the abovementioned result only, one would conclude that it is equivalent to express the optimal threshold in terms of either an exogenous variable or an endogenous one. However, the second part of the present paper shows that an endogenous-variable threshold gives rise to inconsistency since the action taken may not conform to the announcement. The central bank announces that it will revert to the optimal discretionary rate if, and only if, the threshold is breached; however, for some cases, the endogenous variable would not exceed the threshold if the central bank set the optimal discretionary rate and it would exceed the threshold if the promised rate were delivered. To avoid inconsistency the central bank has to announce that if the threshold is breached the policy rate will increase but may not be as high as the optimal discretionary rate.

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A Appendix

A.1 Proof of Proposition 1

Proof. The objective function is $G(\varepsilon_h(\theta_{t+1}, y_h), \theta_{t+1})$ and the first order conditions are

$$\frac{\partial G}{\partial \varepsilon_h} \frac{\partial \varepsilon_h}{\partial y_h} = 0, \quad (13)$$

$$\frac{\partial G}{\partial \varepsilon_h} \frac{\partial \varepsilon_h}{\partial \theta_{t+1}} + \frac{\partial G}{\partial \theta_{t+1}} = 0. \quad (14)$$

If and only if a solution satisfies the first order conditions (FOC) for the minimisation of $G(\varepsilon_h, \theta_{t+1})$ then it satisfies the FOC for the minimisation of $G(\varepsilon_h(\theta_{t+1}, y_h), \theta_{t+1})$: (if) for $G(\varepsilon_h, \theta_{t+1})$, at the optimal values ($\theta_{t+1} = \theta_{t+1}^*$ and $\varepsilon_h = \varepsilon_h^*$) $\partial G / \partial \varepsilon_h = 0$ and $\partial G / \partial \theta_{t+1} = 0$, and thus Equations (13) and (14) hold. (only if) since $\partial \varepsilon_h / \partial y_h \neq 0$, Equations (13) and (14) hold only if $\partial G / \partial \varepsilon_h = 0$ and $\partial G / \partial \theta_{t+1} = 0$.

With regard to the second order (sufficient) conditions (SOC), for the minimisation of $G(\varepsilon_h(\theta_{t+1}, y_h), \theta_{t+1})$ (we define $\Theta \equiv \{\theta_{t+1}, \varepsilon_h\}$ and write $\Theta = \Theta^*$ for short):

$$\left[\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 + \frac{\partial G}{\partial \varepsilon_h} \frac{\partial^2 \varepsilon_h}{\partial y_h^2} \right]_{\Theta = \Theta^*} > 0,$$

$$\left[\begin{array}{c} \left(\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 \right) \left(\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right)^2 + 2 \frac{\partial^2 G}{\partial \varepsilon_h \partial \theta_{t+1}} \frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right) \\ + \frac{\partial G}{\partial \varepsilon_h} \frac{\partial^2 \varepsilon_h}{\partial y_h^2} + \frac{\partial^2 G}{\partial \theta_{t+1}^2} + \frac{\partial G}{\partial \varepsilon_h} \frac{\partial^2 \varepsilon_h}{\partial \theta_{t+1}^2} \\ - \left(\frac{\partial^2 G}{\partial \varepsilon_h \partial \theta_{t+1}} \frac{\partial \varepsilon_h}{\partial y_h} + \frac{\partial G}{\partial \varepsilon_h} \frac{\partial^2 \varepsilon_h}{\partial y_h \partial \theta_{t+1}} + \frac{\partial^2 G}{\partial \varepsilon_h^2} \frac{\partial \varepsilon_h}{\partial y_h} \frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right)^2 \end{array} \right]_{\Theta = \Theta^*} > 0,$$

taking into consideration the FOC (i.e. $\partial G / \partial \varepsilon_h = 0$ and $\partial G / \partial \theta_{t+1} = 0$, at $\Theta = \Theta^*$) these SOC reduce to

$$\left[\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 \right]_{\Theta = \Theta^*} > 0,$$

$$\left[\begin{array}{c} \left(\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 \right) \left(\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right)^2 + 2 \frac{\partial^2 G}{\partial \varepsilon_h \partial \theta_{t+1}} \frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right) \\ + \frac{\partial^2 G}{\partial \theta_{t+1}^2} \\ - \left(\frac{\partial^2 G}{\partial \varepsilon_h \partial \theta_{t+1}} \frac{\partial \varepsilon_h}{\partial y_h} + \frac{\partial^2 G}{\partial \varepsilon_h^2} \frac{\partial \varepsilon_h}{\partial y_h} \frac{\partial \varepsilon_h}{\partial \theta_{t+1}} \right)^2 \end{array} \right]_{\Theta = \Theta^*} > 0,$$

which after some algebra become:

$$\left[\frac{\partial^2 G}{\partial \varepsilon_h^2} \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 \right]_{\Theta = \Theta^*} > 0,$$

$$\left[\left(\frac{\partial^2 G}{\partial \varepsilon_h^2} \frac{\partial^2 G}{\partial \theta_{t+1}^2} - \left(\frac{\partial^2 G}{\partial \varepsilon_h \partial \theta_{t+1}} \right)^2 \right) \left(\frac{\partial \varepsilon_h}{\partial y_h} \right)^2 \right]_{\Theta = \Theta^*} > 0.$$

These conditions are the same as those that are satisfied by the minimisation of $G(\varepsilon_h, \theta_{t+1})$ multiplied by a positive term: $(\partial \varepsilon_h / \partial y_h)^2 > 0$. ■

A.2 Solution for the model with persistent shocks, in the absence of ZLB events

This appendix shows a simple way to solve the model described at the beginning of Section 3.

We can start by consistently (as can be verified below) postulating the following forms for the output gap and inflation:

$$y_t = \frac{A_y}{\rho_s} s_t, \quad (15)$$

$$\pi_t = \frac{A_\pi}{\rho_s} s_t, \quad (16)$$

where A_π and A_y are coefficients to be determined. These expressions imply that $E_t \pi_{t+1} = A_\pi s_t$, $E_t y_{t+1} = A_y s_t$ and the fact that the losses of future periods do not depend on the current interest rate, and hence to obtain a solution for the optimal rate it will be enough to minimise the corresponding period loss function:

$$L_t = \pi_t^2 + \lambda y_t^2. \quad (17)$$

Substituting the expressions for expectations into (6) and (3) gives us

$$\pi_t = (\beta A_\pi + 1) s_t + k y_t, \quad (18)$$

$$y_t = \left(A_y + \frac{A_\pi}{\delta} \right) s_t - \frac{1}{\delta} i_t + \varepsilon_t. \quad (19)$$

By substituting these expressions into (17) and deriving with respect to i_t we can obtain:

$$i_t = \left(A_y + \frac{A_\pi}{\delta} \right) \delta s_t + \frac{(1 + \beta A_\pi) k}{k^2 + \lambda} \delta s_t + \delta \varepsilon_t,$$

which together with (18) and (19) yield

$$y_t = -\frac{(1 + \beta A_\pi) k}{k^2 + \lambda} s_t, \quad (20)$$

$$\pi_t = \frac{(1 + \beta A_\pi) \lambda}{k^2 + \lambda} s_t, \quad (21)$$

(notice that from these two results and the postulated forms we can state that $A_y = -k A_\pi / \lambda$). Then, by comparing (20) and (21) to (15) and (16) we can determine coefficients A_π and A_y :

$$A_\pi = \frac{\lambda \rho_s}{k^2 + \lambda (1 - \beta \rho_s)},$$

$$A_y = -\frac{k \rho_s}{k^2 + \lambda (1 - \beta \rho_s)},$$

then the optimal interest rate is

$$i_t = \frac{k \delta (1 - \rho_s) + \lambda \rho_s}{k^2 + \lambda (1 - \beta \rho_s)} s_t + \delta \varepsilon_t,$$

and the final solutions for inflation and the output gap, respectively, are

$$\pi_t = \frac{\lambda}{k^2 + \lambda(1 - \beta\rho_s)} s_t, \quad (22)$$

$$y_t = -\frac{k}{k^2 + \lambda(1 - \beta\rho_s)} s_t. \quad (23)$$

