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The case for active management from the perspective of Complexity Theory

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Summary

This paper approaches active management of baskets of currencies from the perspective of Complexity theory, where the market is analysed as a Complex Adaptive system. A basket of currencies is constructed using objective probabilities (propensities) and an artificial intelligence optimization technique that allows for implicit learning of correlations dynamics. Statistical tests of the diversification benefit are presented.

Key Words: Portfolio Management; Complexity Theory; Artificial Intelligence.

JEL: G11, G14, G32.

1. Introduction

This paper focuses on active management under the framework of Complexity theory, where the market is analysed as a Complex Adaptive system. The Efficient Market Hypothesis (EMH) is scrutinized from the perspective of the dynamics of the interaction between agents that may have heterogeneous expectations at a microscopic level. The case for the use of propensities as a complement to the subjective probabilities approach is made. An artificial-intelligence-based investment process that searches for the optimal portfolio – given a set of propensities – is designed, so that it will maximize the information ratio under the constraint that the implicit structure of correlations of the resulting portfolio will respond to changes in the environment – factor dynamics and the diversification benefits are significant. An illustration of the benefits of this approach to active management is presented with an investment process for currency baskets, complemented with statistical tests

2. Theoretical support for active management

2.1. EMH: Continuity between the microscopic and macroscopic descriptions.

Traditional financial theory focuses on equilibrium, which in turn establishes an apparent continuity between the macroscopic and the microscopic descriptions of the market by means of the homogeneity of the agents' expectations. The microscopic properties of financial systems are typically represented in terms of agents' expectations, wealth and risk aversion. Of interest, however, are observable macroscopic quantities such as the market's return and risk. The goal is to predict the relationships among these macroscopic variables, starting from knowledge of the equations that rule agents' interaction and the space of the entire system's microscopic state. Two basic assumptions allow current financial theories to connect the microscopic description (collection of microstates and equations of interaction) to the macroscopic behaviour. The first one is homogeneity in agents' expectations homogeneity, which ensures that price changes are random and that agents know the true probability distribution of the stochastic returns. The second is the assumption of "self-averaging" in which the macro-state variables are only expressible in term of themselves. That is,

the macroscopic description does not require knowledge of the detailed statistics of the microscopic variables. For instance, computation of expected return does not require knowledge of the positions held by different investors in the asset, nor their risk aversion.

Standard theory assumes identical agents who are perfectly rational, and given a similar information set, independently arrive at a shared logical conclusion or expectation about the environment they face. From this perspective, single-period asset pricing models of mean-variance maximising investors[2] imply that all market participants will agree on the appropriate risk premium for each asset. Thus, a passive strategy is efficient in this context and no agent has any incentive to create and implement new strategies as prices fully incorporate the expectations and information of all market participants and are completely random and unpredictable. Specifically, no information set should help reduce the forecast error made by an individual. This independence of forecast errors is known as the orthogonality property. It must be noted however that markets will be informationally efficient only if a sufficient proportion of the agents actively gather and trade on information in order to (instantaneously) preclude arbitrage opportunities.

From a microscopic perspective, ex-ante any set of active strategies will be expected to perform as well as any other, as price changes are unpredictable. Two agents with equal risk aversion and wealth will be indifferent to the implementation of any specific active strategy. As with Boltzmann's notion of entropy, the probability is highest when uniformity is achieved. In this case all investors would "know" the true probability distribution of the stochastic returns. The macroscopic characterisation of such a system is easily performed, as it is independent of the microscopic characterisation[3].

The determinism implicit in the supposed ability of agents to accurately assimilate any information relevant to the determination and adjustment of asset prices is only conceivable for an observer situated outside that market. It is a vision of a timeless world, which excludes man from the reality it describes.

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[2] In a "perfect" market where: 1. No taxes on returns are paid; 2. No transaction costs on trades exist; 3. All assets are marketable and perfectly divisible; 4. Unlimited short sales are permitted as well as risk-free borrowing and lending.
[3] An analogy exists between the concept of informational efficiency that results form Samuelson's (1962) Efficient Market Hypothesis (EMH) and frictionless physical systems: an image similar to the world as we perceive it while participating in the markets, must not be sought within the context of these models. In fact, even if the engineer knows that he cannot eliminate friction, he estimates that such a model can nonetheless render the analysis of the complex reality easier (Samuelson, 1962).

However, as Arrow (1988) asserts, economic theory has emphasised in amplitude-reducing behaviour or negative feedbacks. An economy with decentralized knowledge reaches equilibrium by a tâtonnement process where positive excess demand will result in higher prices and supply that ultimately offset the initial perturbation. Arrow (1988) argues that it has been acknowledged that the numerous interrelations within the economy 'might cause instability even though each reaction was stabilizing in its first-order effects'. This is illustrated by the dynamics of the prices of demand-interdependent products: for example, bread and butter. An excess of demand for bread would be expected to have the results described above; however a reduction in the price of butter (to correct an excess of supply) can ultimately increase the demand for bread, amplifying the initial excess of demand.

Nash equilibria (1950a, 1950b) are not necessarily compatible with rational equilibrium results, which presuppose that each player has complete information about the other players' situation. Moreover, if a given game admits more than one Nash equilibrium, the equilibrium selection criteria cannot be used for the prediction of the game's outcome. Modifications to reduce the number of equilibria or avoid unwanted results in economic terms (Selten, 1975), or to account for incomplete information (Harsanyi, 1967) by defining distinct players with different sets of preferences, also fail to account for the evolution (in time) of the economy.

In terms of the dynamic evolution, it is clear that it is the coherent interaction between many dispersed entities acting in parallel that determine the course of the economy, or in particular of financial markets. Coherent in this context means that some characteristic activities – e.g. price formation through trading – require the cooperation of these entities (Prigogine, 1983).

Economic systems do not act only in terms of stimulus and response, the entities also anticipate and may even be structurally modified by their interactions: perpetual novelty and continual adaptation by the acquisition of experience are also present (Arthur et al., 1997). A discussion on the emergence of order, either from fluctuations (Prigogine, 1983, 1991), positive feedbacks (Arthur, 1988; Arrow, 1988; Anderson, 1988) and/or from the dynamical interaction between the process and the environment (Lewin, 1992;

Langton, 1989; Holland, 1998; Arthur et al., 1997; Cariani, 1992; Forrest, 1989; Kaufmann, 1989) and their implications are discussed in Reveiz (2001).

Moreover, as foresight and game-playing have a preponderant role in economics, the predictor is part of the system being predicted. This may yield difficulties of mathematical consistency unless the assumption is made that each player decides on his strategy ignoring what other agents may do. But experience in financial markets shows that participants are also trying to predict what other participants will do. In this context, the interaction between the economic entities may be asymmetric as Francois Perroux already posited in 1964. Going a little further than Perroux's insight, economic trading is not an homogeneous space as the interaction of A with B may not only convey information and exchange but may also lead to the modification of the structure of one of the units. More precisely, from the interaction B may change the model, the information it is using, its risk aversion, etc.

Economies, Markets and Agents are not isolated systems. Exchange of information and models – belief sets – with the external world (markets, economies, etc.) are part of an ongoing process. The problem of establishing the limits of the system, as well as the definition of the units and their interactions becomes critical because, in some way, each is conditioned on the other.

2.2. Deductive and inductive reasoning and homogeneous and heterogeneous expectations.

Over the past few decades, numerous, sometimes complementary, approaches to economic systems have grown from this perspective. An evolutionary framework, in which markets, instruments, institutions, and investors interact and evolve dynamically, and in which constant adaptation and perpetual novelty are integral parts of the development of the system, is common to all of these approaches.

The underpinning factor in all these approaches is the recognition that agents are heterogeneous and dynamically incoherent. Most of them assume either rational or bounded rational agents (Simon, 1982). The implications of the relaxation of this assumption are significant as the continuity between the microscopic and macroscopic

description is broken as the result of the agents' inability to form expectations by deduction through knowledge of other agents' expectations (Arthur, 1995).

Arguments against the assumption of homogeneous expectations assert that traders, the same agents modelled by the theory, devise active strategies to outperform the "market" as they believe that the market offers speculative opportunities. Also, the trade-off between risk and return underlying modern financial theory implies that a reward is demanded by investors in order to hold an asset and bear the subsequent risk. Furthermore, a sufficiently risk-averse investor may be willing to pay to avoid the risk of holding a security. In this context, prices need not be unpredictable even if markets are efficient and agents act rationally (Farmer and Lo, 1999).

Others consider the hypothesis of full rationality as unrealistic because it is conceptually difficult to define what rationality is, and observe that human behaviour is unpredictable. They also make the point that irrational behaviour is present in everyday life. In addition, as information is difficult to interpret, and significant "news" must be selected and its impact determined, agents' resulting expectations may differ. What's more, information gathering is costly and not all investors may be capable or willing to use significant resources for that purpose. Besides, technology is continually changing and the players who use this technology first may acquire, at least temporarily, a competitive advantage[4]. The emergence of derivatives markets is a good example of this phenomenon.

An additional point must be made. If we consider as Rabindranath Tagore did that independently of what we think reality is, we can only access it through mental synthesis, then no information can be discounted into the price without there also being a relevant theoretical framework. And, as we know, distinct models may give rise to different interpretations[5].

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[4] Farmer and Lo (1999) provide an illustration of the absurd implications of the application of the EMH to a biotechnology firm trying to develop a vaccine against AIDS: under that framework "such a vaccine can never be developed – if it could someone would have already done it!" This conclusion is reached because the challenges and gestation lags of biotechnology research are not taken into account. Continuing with their argument, if a company succeeded in developing such a vaccine, profits would be very high. But would these be considered abnormal profits or just appropriate compensation for their efforts, skills and innovation? Finally, they argue that financial markets should not be different.

[5] In this context, we are closer to the German notion of *grund* than the Latin notion of *ratio*, reason. According to Kundera (1990), the latter has two senses: 'before designating the cause it designates the faculty of reasoning. Thus, the reason in terms of cause is always perceived as rational. A reason, for which the corresponding rationality is not transparent, seems unable to cause an effect. However, in German, the reason in terms of cause is expressed as *grund*, a word completely unattached to the Latin *ratio*, which designates the ground first and then a fundamental [translation by the author]' (Kundera, 1990).

As Arthur et al. (1997), Holland (1988) and Prigogine (1983) assert in their own terminology and respective fields, systems with the properties mentioned above operate far from any global or optimum equilibrium (Out-of-Equilibrium dynamics) and together they present difficulties for traditional mathematics (used in economics). For Holland, the economy is an adaptive non-linear network (ANN), as are ecologies, the nervous system and the developmental stages of multi-cellular organisms.

Complexity theory (Gell-Mann, 1995) gives us insight into emergent properties in macroscopic systems that take place (mainly) through feedback mechanisms from the dispersed interaction of the agents at the microscopic level. The implicit use of induction by the entities to adapt to their environment leads to perpetual novelty and keeps the system away from optimal or global equilibrium in the traditional sense.

Historical change is then the result of the interaction of competing microscopic entities that, through their actions, arising from their set of preferences or needs, co-create history by producing events which modify prevailing structures. Intensity and speed of change are related to the amplification and feedback mechanisms. Moreover, a collectively-shared objective, although possible, would result in equilibrium[6] (no change in the structures, therefore no historical change). It seems more likely that heterogeneous objectives, and the actions taken in pursuit of them, shape the engine of the historical process, which by continued adaptation to the environment (changes in structures and hierarchies) is continuously revised and reasserted[7].

If we view the economy or the markets as an adaptive complex system or in Holland's (1988) words as an adaptive nonlinear network, structure matters. It does so in many ways:

(i.) Surfacing of patterns at the microscopic level might result in emergent properties at the macroscopic level by amplification mechanisms such as order by fluctuations, positive feedbacks and/or process-environment dynamics.

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[6] This is in accordance with traditional financial theory where agents share the same information set and the market responds only to the arrival of new information, being otherwise in equilibrium.

[7] This view shares the underlying basic assumption in Sartre's work at the end of his life, 'that all human affairs are conducted under conditions of relative scarcity. For this implies that humans always confront each other as potential competitors, and, according to Sartre, it is this threat which both motivates all social and economic structures, and, in the end unifies history' (Baldwin, 1995).

(ii.) Institutions or conventions (frozen structures) define hierarchy or mitigate positive feedbacks or emergent behaviour.

(iii.) The compression of past information within the structure of the models and procedures used to predict risk and return and to measure the fitness of distinct strategies makes historical narratives significant. More specifically, because agents learn from experience, prevailing procedures at all hierarchical levels are a function of past performance.

(iv.) Networks are important in two ways. Clusters of linked agents may affect the evolution of the market by affecting information transmission or trading practices. On the other hand, if agents modify their expectations (models, information, risk aversion, etc) by interacting with others, then the precise way in which economic units are linked determines the set of possible modifications.

More precisely, by modelling price formation as the result of actions taken by the agents (traders) on their co-evolving set of strategies, in which each agent tries to forecast the other agents' behaviour and trade is not carried out in a neoclassical auction-like process but is the result only of a bid and ask price match (double auction), locally the market may have "open" orders (unfulfilled holding desires). The evolution of these, the surfacing of structure – for example, most of the market wants to be long while the central bank becomes the liquidity provider - or its degradation and prospective return to symmetry, mixed with feedback mechanisms and critical mass thresholds result in emergent, sometimes non-equilibrium, behaviour at the macroscopic level. A similar approach can be taken regarding the dynamics of the actual holdings or the fitness of traders' strategies.

As with any non-linear adaptive system, in financial markets, distinct notions of historical change arise depending upon either the resolution chosen to analyse the system's evolution or the objects of interest. In order to create their price expectation, agents use fundamental, econometric or rule-based models that are chosen only on the strength of their performance. By definition, forecasting with these rules is based on the

assumption that time is deterministic where a set of events gives us a sense of continuity and the future can be described by some combination of occurrence of such events[8].

Agents are only interested in fulfilling their objective of generating returns with no underlying [known or desired] finality. As long as all of the strategies have similar fitness and no persistent correlation emerges between their positions or strategies as a result of their interaction, the market (macro level) is near to equilibrium and in Prigogine's (1991) view, no history is being made. On the other hand, if through an amplification of a fluctuation or a feedback mechanism, structure appears or dramatically changes leading to an out-of-equilibrium situation at the macro level, we can affirm that 'history' *is* being made. In this sense, the creation/destruction of new institutions, conventions or models provides us with a sense of irreversibility in financial markets.

Although closely related to Karl Popper's and Ilya Prigogine's work on an evolutionary approach to objective knowledge and out-of-equilibrium dynamics in thermodynamics, respectively, Complexity Theory seldom refers to these authors[9]. However, concepts such as reflexivity, positive feedbacks and their role in the creation of perpetual novelty, for example, or the observation that microscopic coherence and out-of-equilibrium behaviour at the macroscopic level may occur simultaneously, were posited by these authors since in the early eighties. There is no doubt that, on some fronts, Complexity Theory has outgrown these theories, mainly in the study of *emergence*. However, these concepts, form part of a broader theoretical exposition of the work of these authors, which may in turn enhance our understanding of the repercussions of applying these concepts in, say, economics as well as provide a better theoretical foundation on which to construct our hypotheses.

The attribution of learning abilities to the economic agents (Arthur et al, 1997), combined with a price-formation mechanism that results from an ecology of competing strategies, yields an ill-defined problem where self-referencing exists. Popper (1982) has shown that, when we try to predict a system from within, 'a predictor cannot predict the result of its own predictions – at least not before the 'predicted' event has actually

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[8] Although an error in the prediction is allowed.
[9] The concept of *reflexivity* was applied in finance by George Soros (1998), who references Popper's work.

taken place'. Moreover, he asserts that ecologies of predictors that influence each other can be highly unstable with a loss of efficiency in individual predictors. This is in consistent with the results obtained by the Santa Fe Institute Market and supports the case for active management.

Complexity Theory posits that the dynamics of a process are affected by its geometry (structure), which in turn changes as the process evolves, leading to a feedback mechanism that in turn generates perpetual novelty. This process cannot be completely understood with the use of subjective probabilities where uncertainty is due to our own lack of knowledge. The notion of objective probabilities or propensities (Popper, 1982), an extension of the concept of force, may provide insight to the analysis of evolutionary dynamics as the situation, and not solely the object, changes the possibilities.

Subjective probability is considered to be useful only because we have imperfect knowledge: probability is a property of some given sequence. In the case of objective probability the sequence is determined by its set of generating conditions: probability is then a property of the generating conditions (Popper, 1983). It refers not only to individual events but also to the possibilities inherent in the conditions under which that event occurs. It refers to the possibilities opened up by the conditions. Propensities are conceptually similar to Newtonian forces in that they are a relational property of at least two things. Turning up heads or tails from tossing a coin is not solely a property of the coin but also of the tossing. As Popper explains, if the coin is dropped into sand or mud, we may find lower probabilities as it may come to rest upright[10].

Thus, if price formation results from the interaction of competing strategies and beliefs that may be successful under specific environments and which are based on models that evolve through the interaction, both with other agents and the environment, then a positive feedback between market dynamics and structure exists, as well as an increase in average fitness in the models and technology (quantitative tools). In this context, market participants are confronted with novelty permanently which in turn leads to an inherent *forecasting fallibility* in expected returns.

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[10] The geometry-dynamics duality of complexity theory can be better understood if the notion of propensities is included. Changes in the system geometry (environment) determine the possible paths which the system may follow at each time step which in turn, when a given path is selected, may modify the geometry of the system, opening up a new set of possibilities. Any given path taken by the system – the occurrence of a given set of propensities – is a property of the whole system, a relational property.

Under this framework, it can be argued that active management can lead to excess returns but it is clear that a strategy will outperform only under some environments when the propensities and their interaction with the environment are accounted for in the models and beliefs[11]. This is supported by the empirical observation that consistently obtaining positive excess returns is hardly attainable. In order to obtain excess returns in the long run, an investment process, as well as the metrics and technology implemented, must account for forecasting fallibility by providing support in providing procedures and portfolios that are fit under changing propensities and environments given specific views of the portfolio manager – the investment process must be designed as a complex adaptive system itself.

3. Active Management as a complex adaptive system: an illustration

3.1. The Investment Process

For illustrative purposes, we define the universe of assets allowed for active management as the 10 currencies in table 1.

Table 1 – Universe of Assets

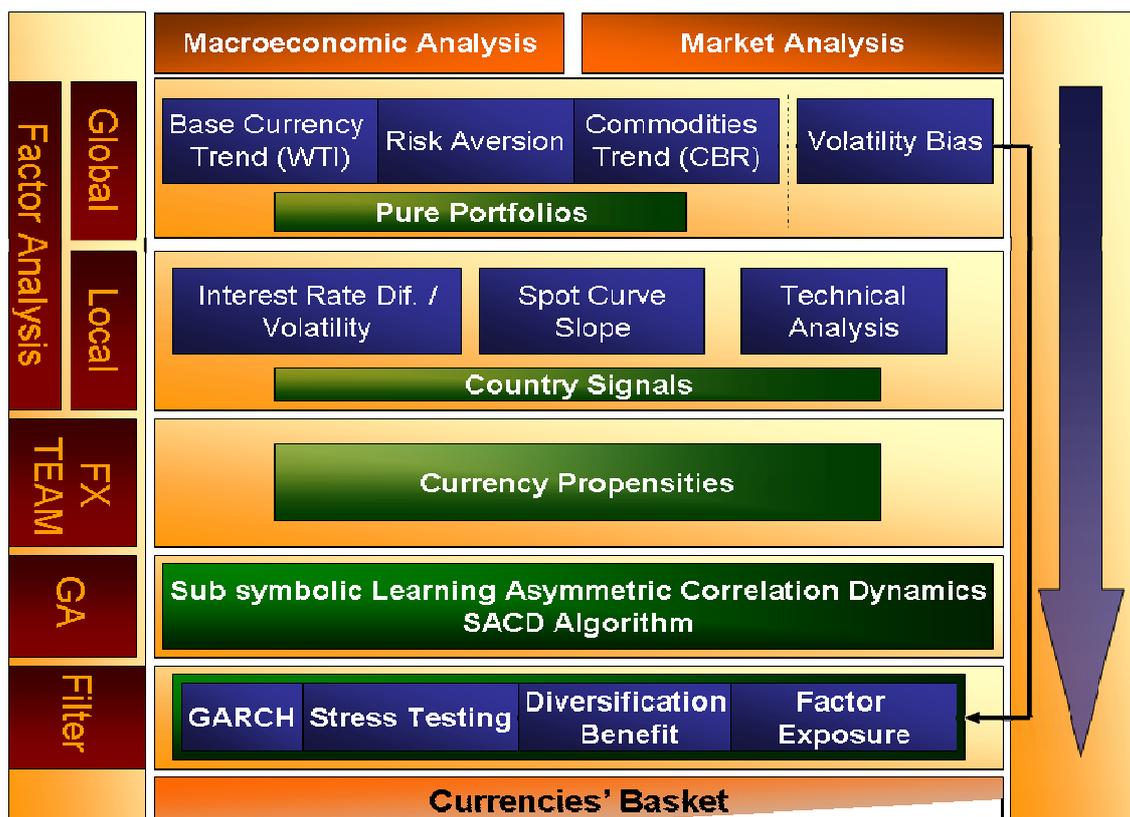| Currency |
| --- |
| EUR |
| JPY |
| CHF |
| GBP |
| AUD |
| CAD |
| NZD |
| SEK |
| NOK |

Our propensities for these currencies' returns are determined in a three step process, allowing for both global and local factors to be taken into account. First, exposures to the following global risk factors are set in a periodic meeting: (i) Pro or anti dollar exposure: (ii) Market risk aversion; (iii) Commodities' Prices and (iv) local or trend volatility. Factors (i) to (iii) are replicated using sets of "pure" portfolios ("baskets" of currencies) estimated using artificial intelligence techniques. Second, separate signals

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[11] For the evolutionary fitness implications of these phenomena see Reveiz (2001).

are produced for each currency from carry and volatility differences, spot curve information and technical analysis. Then, the portfolio manager (FX Team), with the support of the economic team and taking into account market dynamics, determines the specific propensities for each currency by complementing the information embedded in the "factor" portfolios and the specific market signals– see Figure 1.

Figure 1 – Illustration of a FX investment process

The set of signals is optimized using a Genetic Algorithm (SACD) from a population of portfolios that obtained the highest information ratio, while presenting an asymmetric exposure to shocks to the base currency factor. To obtain the optimal portfolio – in the propensities framework – this population is filtered using conditional heterescedasticy and diversification benefits indicators, as well as their exposure to global factors. Finally a stress test is performed to the basket to ensure its fitness to a varying environment.

For illustrative purposes, Figure 2 below shows the information ratio for a 5 year period, the accumulated return and the frequency histogram of the information ratios for the currencies' basket in table 2.

Table 2 – Basket of Currencies

| Currency (Base USD)	Percentage
EUR	-65.0%
JPY	81.7%
CHF	96.2%
GBP	64.7%
AUD	0.1%
CAD	-64.4%
NZD	-44.7%
SEK	-64.7%
NOK	96.2%
USD	100.0%

The information ratio varies between .7 and –0.7, whilst the Information ratio histogram is skewed to the right.

Figure 2 – Information Ratio (IR), Returns and IR histogram
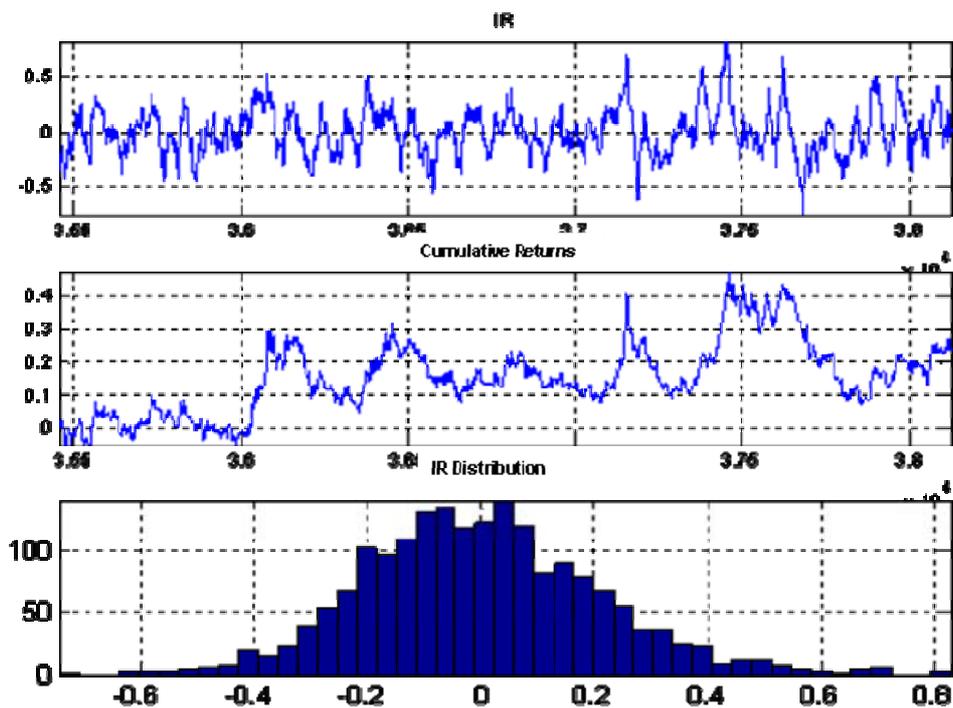
The diversification benefits, the IR using GARCH models and the Value at Risk (GARCH) are presented in figure 3.

Figure 3 – Diversification Benefit, Information Ratio and VaR (GARCH)
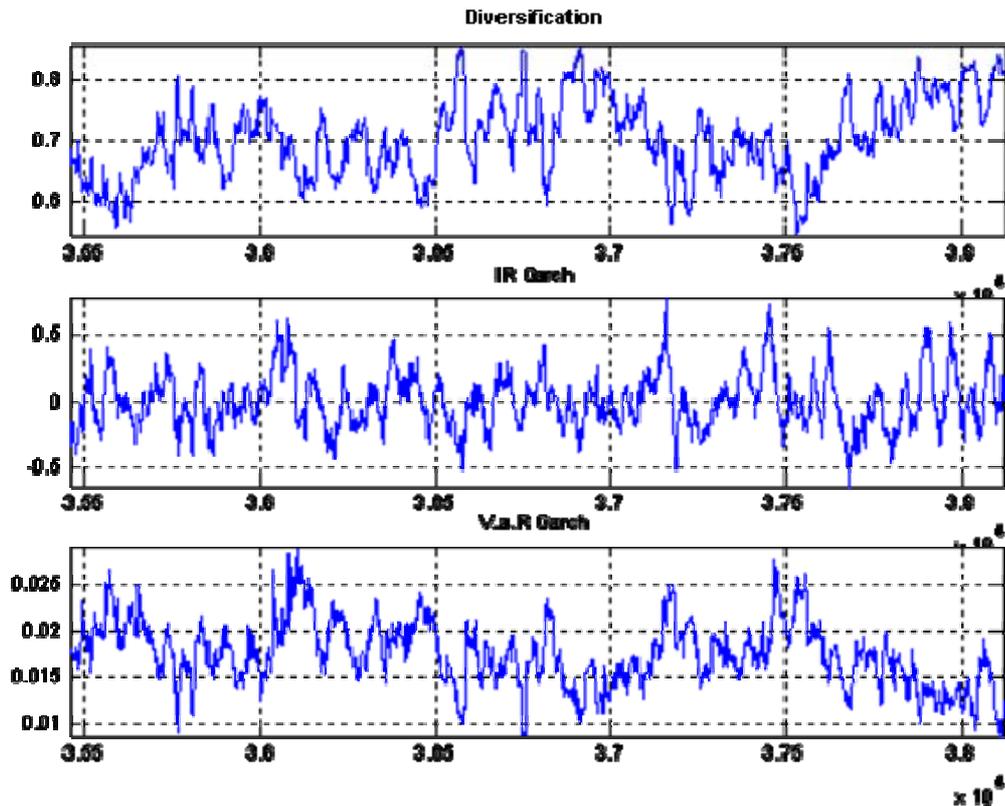
The diversification benefit reaches 85%, slightly over 55%, while the daily Value at Risk is around US$10000 per million of net base currency exposure at the end of the period. The dispersion of the diversification benefit depicts the active dynamics of the implicit correlations – this is what the algorithm learns.

When positive and negative shocks were applied to the basket, as a stress test, using simulations, the resulting information ratios are 0.4409 and -0.1098 respectively.

3.2. Analysis of Results

3.2.1. Diversification benefits

In order to test the ability of the algorithm to obtain high diversification benefits, 31 random sets of signals (currencies' propensities) were optimized to obtain the correspondent currency baskets. The histogram shows that for a million of base currency exposure, the one year volatility is lower than 10% consistently – see figure 4.

Figure 4 – Histogram of Volatilities

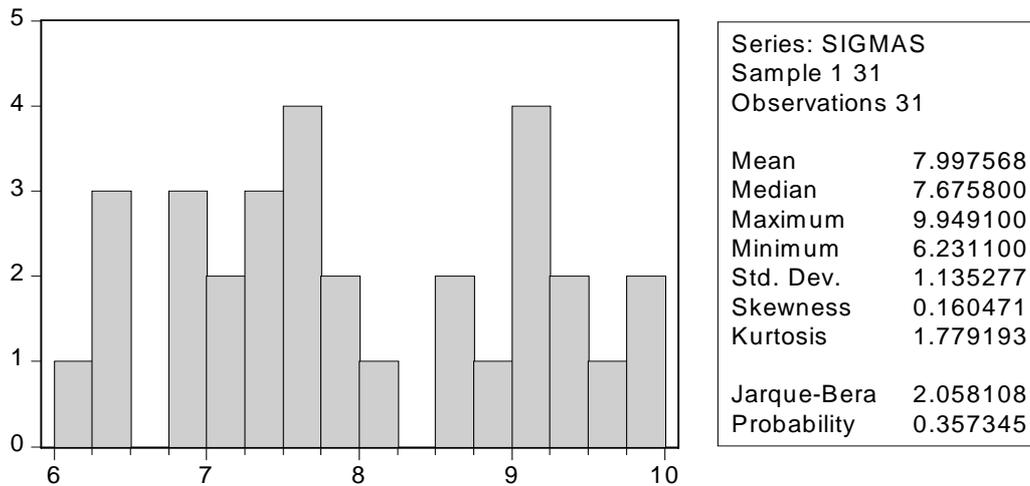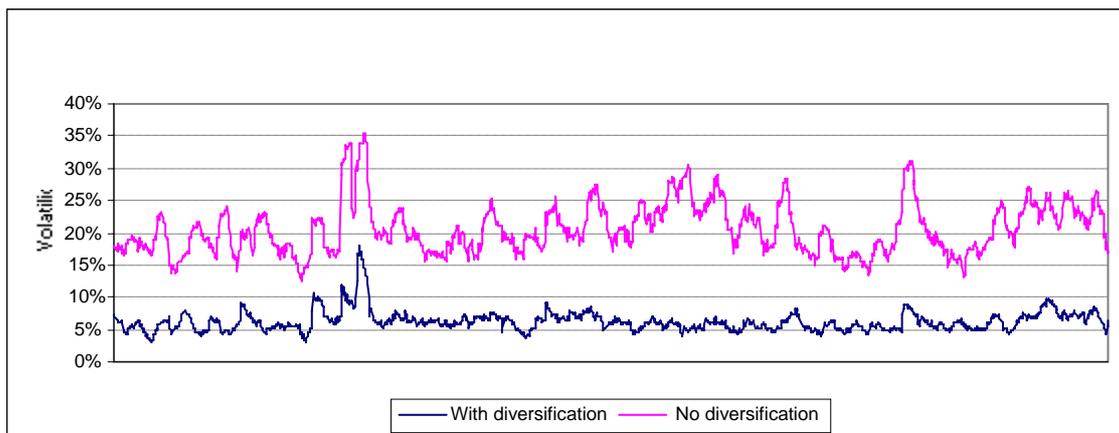| Series: SIGMAS	
Sample 1 31	
Observations 31	
Mean	7.997568
Median	7.675800
Maximum	9.949100
Minimum	6.231100
Std. Dev.	1.135277
Skewness	0.160471
Kurtosis	1.779193
Jarque-Bera	2.058108
Probability	0.357345

Figure 5 presents the evolution of a basket of currencies with and without diversification.

**Figure 5 – Illustration Diversification Benefit**

3.2.2. Asymmetric correlation

An experiment was designed to test for asymmetric response to positive and negative shocks. Random sets of signals are optimized by the Sub-symbolic Learning Asymmetric Correlation Algorithm (SACD). The best 5 baskets of the final "best" population for each run, without final filters, is impacted with positive a negative shocks, from a signal perspective – i.e. is a pro-dollar stance is implicit in the signals periods where both the WTI for the dollar increases or decreases are simulated. The

information ratio is estimated in both cases and the net information ratio is computed[12]. If the latter is significantly different from zero, the algorithm was able to properly learn the dynamics of the correlations between the currencies in differing environments. Results show that consistently, for all the resulting "best populations", the net information ratio is positive – see figure 6.

Figure 6 – Net Information Ratio Histogram
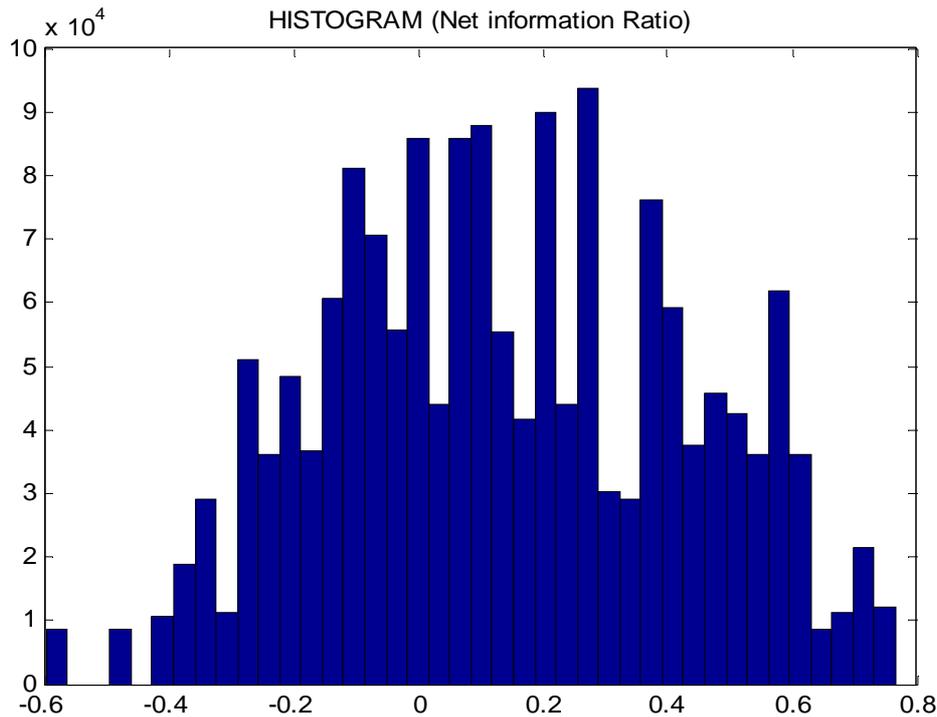
As it is shown in figure 1, the distribution has an asymmetric form with a skewness measure of 0.0524.

4. Conclusions

Viewing market dynamics as a Complex Adaptive System allows, conceptually, for both the emergence of novelty and a positive feedback loop between market structure and dynamics is of utmost importance. The former can be approached by the correlation's structure which is modified with market dynamics. An artificial-intelligence based investment process was presented that searches for the optimal

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[12] This histogram shows the distribution of the sum of the historical IR under a positive and a negative random shock. Shocks were applied to a set of the best 52 historical baskets previously calculated by the GA. Seven "positive" and 7 "negative" shocks were chosen and 5040 simulations were executed in order to obtain 1,834,560 observations.

portfolio – given a set of propensities[13] – that will maximize the information ratio under the constraint that the implicit structure of correlations of the resulting portfolio will respond to changes in the environment – factor dynamics. Statistical tests conducted show that the genetic algorithm is able to learn the combination of assets (risk exposures) that respond asymmetrically to positive and negative shocks, while obtaining a high diversification benefit.

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[13] Propensities understood as objective probabilities where the result of tossing the coin is dependent of the (ever changing) environment and not the subjective probabilities used in risk measurement. See Popper (1990) and Reveiz (2001).

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