Derivative Markets' Impact on Colombian Monetary Policy

Esteban Gomez **Diego Vasquez** **Camilo Zea** *

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Abstract

Derivatives are contingent claims that complete financial markets. Their use allow agents and firms to ameliorate the impact over consumption, production and investment given a change in relative prices induced by an active monetary policy. In this sense, derivatives generate in some cases a loss in the effectiveness of the traditional monetary transmission channels in the short run, and in others, they promote an increase in the speed of transmission itself. Using an investment model, the impact of the use of interest rate and exchange rate derivatives in the dilution of colombian monetary channels is verified. Empirical exercises suggest that monetary policy has lost effectiveness in the short run. In spite of the surprise this result may offer given the relative immatureness of domestic derivative markets, the marginal effect of these instruments appears to be significant, in the face of local financial markets' imperfections. In addition, not only the hedge directly taken by firms with access to this instruments matter; there could be hedging spill-overs whenever commercial banks use derivatives, which allow for a more stable and cheap credit supply for firms with no access to those markets. The natural recommendation deriving from this conclusion suggests an urgent analysis of the derivatives impact over the speed of monetary transmission in Colombia.

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*egomezgo@banrep.gov.co, dvasques@banrep.gov.co, czeagome@banrep.gov.co. Department of Financial Stability, Banco de la Republica. The opinions contained herein correspond to the authors and do not represent those of Banco de la Republica or the Board of Governors. Data provision by Nancy Zamudio and Rocio Mora is gratefully acknowledged, as well as comments by Sergio Clavijo, Hernando Vargas, Juan Pablo Zarate, Ana Fernanda Maiguashca, Mateo Ossa, Sebastian Nieto, the staff at the Department of Financial Stability and seminar participants at Banco de la Republica and Universidad de los Andes. The usual caveat applies.

1 Introduction

The deepening of derivative markets has generated doubts as to whether these may have an impact over prices in goods and capital markets. In particular, derivatives offer hedging possibilities against adverse economic shocks and policy decisions, diluting the traditional monetary policy transmission channels and/or making such transmission faster . In other words, these hedging instruments affect the ways through which changes in monetary policy variables alter production and consumption decisions by firms and agents in the economy.

The aforementioned is founded on the constant development of derivative instruments, which has allowed a fulfilment of financial vacuums, unifying traditionally fractioned markets, facilitating a faster transmission of monetary policy changes, with real effects that may be smaller over time. In a nutshell, financial derivatives may weaken monetary policy channels in certain cases, and in general, allow a faster transmission.

Recent literature counts few papers that study the diluting effect of derivatives over monetary policy transmission channels. Among the most important stand Fender (2000b) for the US and Vrolijk (1997) for the UK.

The present paper analyzes the diluting phenomenon of monetary policy transmission channels due to the availability of derivative markets in Colombia, but leaves aside the question about the effect of the latter over transmission speed. The proposed methodology improves at least in four aspects previous contributions: 1) Firm-level Microdata is used [1], which is novel in the literature, where only aggregate analyzes are found. 2) A model for an optimizing firm is presented. 3) An estimation technique for unbalanced panel data models is used, developed by Biørn (1999a) for the case of one-way random effects over the intercept [2]. 4) A different policy proxy is proposed, instead of the interest rate. In particular, the Monetary Conditions Index (MCI) built at Banco de la Republica is introduced, which evaluates the position of monetary policy with respect to aggregate demand, weighting the impacts of both the interest and exchange rates.

In the second part of the paper a revision of the literature concerning the impact of derivatives over monetary transmission channels is presented. In the third part the main characteristics and functions of derivatives are highlighted. In the fourth, the theory of the effect of derivatives' use over transmission and monetary strategies is outlined. The fifth part presents the

[1] The database is built upon data from Superintendencia de Valores and Superintendencia de Sociedades (Both of which constitute the Colombian Securities Exchange Commission). Both databases present anual figures (end of year) and have info for more than 15,000 firms, from 1995 to 2003.

[2] The case of Swamy-type random coefficients (different intercept and slope for each firm) considered by Biørn (1999b), could not be easily applicable to the present case, taking into account that the historical series have no more than 9 observations in comparison to the total number of firms. This generates computational problems, even in the balanced case.

stylized facts of the colombian derivative markets and in the sixth, the model for investment is described, along with its estimation, data used and results. In the last part, conclusions and policy recommendations are derived.

2 Literature on the Impact of Derivatives over Monetary Policy Transmission Channels

For more than a decade now, a number of papers have been written around the idea that imperfections in financial markets generated by information assymetries, may cause or reinforce real business cycles [3]. In such context, a tight monetary policy can induce an extra cost in addition to the typical cost of capital mentioned in the Keynesian literature, which in turn may lead to a credit crunch.

Oliner and Rudebusch (1996) show that a monetary shock diminishes bank reserves and hence, the credit that they may extend to firms. This phenomenon is known as the credit channel of monetary policy [4].

Fender (2000a), Bernanke, Gertler and Gilchrist (1998) and Bernanke and Gertler (1995) find that a tight monetary policy affects corporate cash flows, lessening the ability of the firm to find credit. This latter effect is known in the literature as the balance sheet effect, wealth effect or financial accelerator.

More recently, Froot et Al. (1993) and Fender (2000a), show that if information asymmetries increase the cost of external finance vis a vis internal financing, incentives are created for firms to manage corporate risk. Businesses that depend upon internal funds need their cash flows to fluctuate the least possible and hence, are interested in using mechanisms that guarantee such stability. Derivatives allow firms to face such financial risk [5], in a way that grants them a shield against changes in the monetary stance, thereby weakening the real effect of monetary policy.

A firm that has access to instruments such as forwards and swaps, is therefore able to hedge against interest or exchange rate risk, loosing sensitivity in the face of changes in monetary conditions, reflecting on real economic activity through its investment decisions. However, several theoretical papers show that in certain circumstances, derivatives could reinforce volatility in financial markets [6], which has shifted the focus of the actual debate to empirical grounds.

Papers that verify the aforementioned hypothesis are scarce, being the

[3] See, for example, Bernanke, Gertler and Gilchrist (1996).

[4] For the credit channel to exist, a reduction in reserves caused by a tight monetary policy must reduce the volume of credit by banks. In addition, it requieres that firms be unable to find other sources of financing without having to incur in greater costs, which induces a lower investment (see, among others, Oliner and Rudebusch, 1996, Romer and Romer, 1990 y Bernanke and Blinder, 1992).

[5] See Von Hagen and Fender (1998) for a detailed explanation.

[6] See Morales (2001) and Van Der Nat (2000).

most important Fender (2000b) for the US and Vrolijk (1997) for the UK. In the former, derivatives are modelled in an implicit manner, using data from the Quarterly Financial Report for Manufacturing Corporations (QFR), with firm aggregations depending on the level of assets. In the latter, derivatives are modelled explicitly and their impact over aggregate variables is verified. Both papers use a Vector Autoregressive Methodology (VAR), although their results differ. While Fender (2000b) finds evidence that the presence of derivative markets has had a structural impact over firms choices in the US, Vrolijk (1997) finds the opposite for the UK and argues that this result may be derived from the fact that the UK financial market may have been sufficiently developed prior to the introduction of derivatives, in such a way that their marginal effect may have been negligible.

3 Characteristics and Functionality of Derivatives

As mentioned before, derivatives allow for an artificial stripping of the risks embedded in a financial asset. As a byproduct, derivatives complete financial markets by allocating risks among investors in an optimal way, given their preferences. The benefits for markets can be grouped in three [7]: 1) Hedging possibilities, 2) Leveraging, given the increase in the volume of transactions while at the same time the cost of capital diminishes, and 3) Substitutability between assets.

On the other hand, derivatives may generate instability, in the sense that: 1) They amplify price movements whenever hedging is dynamic, 2) They may generate systemic risk if margin calls are not carried by a sufficiently collateralized provider, and 3) They may induce adverse capital flows, in the face of erratic policies.

In a nutshell, derivatives may create discipline in economic policy, despite the ambiguity of their final effect. In what follows, the main economic functions of derivatives are presented.

3.1 Economic Functions of Derivatives

Fender and Von Hagen (1998) pinpoint several economic functions of derivatives. First, derivatives reduce transaction costs, which in turn allow for a better exploitation of investors' comparative advantage. For example, two investors may exchange interest payments, by which they can use the others' more efficient cost structure.

Second, derivatives allow a cheaper diversification and hedging. Due to this, portfolios may be less sensitive to interest or exchange rate shocks, making returns more stable.

[7] This section closely follows Vrolijk (1997) and Fender and Von Hagen (1998), and is included for expositional purposes.

Third, by stripping financial assets' risks, greater liquidity is achieved. For example, credit derivatives allow banks to further exploit credit markets, ameliorating the rationing events resulting from information asymmetries.

Fourth, derivatives create bridges between segmented markets, enhancing substitutability among assets and thus, allow further arbitrage opportunities.

Summarizing, derivatives complete financial markets and permit a better allocation of risks. This has induced a wave of financial disintermediation, thereby increasing competition and decreasing margins.

Hence, derivatives could potentially generate both stability and instability in markets, as Fender and Von Hagen (1998) show. By stripping the risks embedded in a financial instrument, different markets are created for each so that new information about a particular risk not only changes its own price, but impacts other prices, although to a lesser extent. Derivatives may also generate instability by allowing traders to take highly leveraged positions, provided they can trade risks related to a particular asset without having to effectively buy it.

However, greater liquidity and substitutability permit a better and smoother shock absorbtion, which implies greater stability. In addition, derivatives facilitate the flow of info to spot prices, through enhanced arbitrage opportunities between the latter and forward prices, diluting the impact of new information in the market.

Although there is no international evidence linking the appearance of derivatives with greater market volatility [8], this question remains unresolved for the case of Colombia.

4 Implications for Monetary Policy of the Existence of Derivative Markets

The effect of derivatives over monetary policy can be described in two spheres[9]: 1) Implications for policy strategies and 2) Effect over monetary transmission channels.

In any case, it is important to take into account that derivatives affect policy transmission in various ways (Vrolijk, 1997): 1) Derivatives are inherently linked to their underlying markets, 2) Derivatives create new financial assets, thereby increasing the available information about prices, and 3) Derivatives may substantially alter the international transmission of shocks, by making arbitrage easier and cheaper. In what follows, the implications for monetary transmission and strategies are outlined.

[8] Fender and Von Hagen (1998) cite the IMF (1997), Cohen (1996) and Board et Al. (1995).

[9] This section closely follows Vrolijk (1997), and is included for expositional purposes.

4.1 Implications for Monetary Strategies

Monetary policy consists of a set of rules to manipulate an instrument (e.g., the quantity of money or the price of an asset such as the interest rate), with the precise goal of altering long-run target variables. In order to achieve this, a medium-run target is established as well, given the difficulty to observe long-run variables.

This said, a first impact of the availability of derivatives for monetary policy is the greater number of prices that may be used as instruments. Moreover, derivatives offer embedded info that may be used to improve monetary strategies. Such is the case of expected volatility implicit in Black and Scholes (1973)-type options. In addition to this, market expectations concerning prices of underlying assets may be extracted from call and put options' volumes. In this way, monetary policy gets a richer view of market expectations (Vrolijk, 1997).

A second impact of derivatives over policy strategies is given in the context of the discussion as to which instrument works best. In rapidly developing derivative markets, the majority of shocks can be expected to arise from the financial system. In such circumstances, Poole (1970) suggests the use of a price as instrument, rather than a monetary aggregate [10]. However, and taking into account that monetary policy works in the low end of the yield curve, derivatives allow for a faster translation of shocks from the low to the high end of this curve (Vrolijk, 1997 and Cottarelli and Kourelis, 1994) such that changes in the economy, independent of their real or nominal nature, turn more rapidly onto price changes. Hence, the development and evolution of derivative instruments which helps markets to discern between real and monetary shocks, will make a monetary aggregate a better instrument in the future (Poole, 1970).

On the other hand, policy associated with the exchange rate is even more affected in the face of derivative markets development. In particular, sterilized interventions to maintain a fixed exchange rate are effective only if domestic and foreign markets are imperfect substitutes. However, a more profound and sophisticated financial system hinders such sterilization, making the coexistence of a fixed exchange rate regime and an active monetary policy impossible, in the context of perfect capital flows.

Finally, Central Banks may use derivatives to conduct monetary policy. For example, the use of exchange rate options may help the CB to reduce volatility in times of turbulence (e.g., Mexico and Colombia), or simply to send policy signals through the implicit depreciation embedded in such instruments.

[10] Moreover, money demand turns virtually impossible to estimate in the presence of multiple financial instruments. On the other hand, the greater substitutability of assets and the enhanced liquidity provided by derivatives make the definition of monetary aggregates increasingly diffuse (Savona and Macario, 1997), especially broader ones.

4.2 Implications for Monetary Transmission Mechanisms

Following Mies et Al. (2003)[11], "the abundant literature about the topic recognizes, at least, five monetary policy transmission channels": 1) Interest rate channel, 2) Asset channel, 3) Exchange rate channel, 4) Credit Channel and 5) Expectations channel.

In what follows a description is provided about the theoretical effect of derivatives over the interest rate, credit and exchange rate channels only, for they are the most common in the literature.

4.2.1 Interest Rate Channel Effect

As Mies et Al. (2003) show, the interest rate channel is the most conventional, and is often used as a general framework to represent the overall effect of all remaining channels.

Vrolijk (1997) asserts that an increase in the interest rate leads agents to substitute savings for credit in order to smooth their consumption intertemporally. The effect of such rise in terms of future investment decisions is negative (capital cost increases) and proves difficult to hedge. However, a possibility could arise in order to hedge the adverse impact of the substitution effect (i.e., an agent could engage in a contract in which she locks-in future interest rates for funding needs). Nevertheless, this is presented as a mere theoretical possibility, for in practice, there exist several unknown variables concerning future credit needs, as Vrolijk (1997) pinpoints. Given these uncertainties and the costs of hedging these risks, derivatives have only a marginal impact in reducing substitution effects arising from interest rate increases. Thus, the majority of agents will face a higher cost of capital, with or without derivatives.

On the other hand, the income effect is given by the change in income or cash flows associated to changes in interest rates. The prevailing effect depends on the profile of net asset holdings by the agent. Vrolijk (1997) shows that the main potential impact of derivatives over the interest rate channel rests in the possibility to hedge the income effect given changes in interest rates. Provided their degree of risk aversion, those who buy these type of instruments have a higher marginal disposition to consume vis a vis unhedged ones, so that the impact over real variables of a change in interest rates, in the presence of derivative markets, is substantially lower. However, the aforementioned fact about the faster transmission of shocks induced by derivatives' use, implies that the effect over unhedged agents will show up earlier. In this way, the real effect of hedging availability in the face of interest rate movements is ambiguous, given the countervailing forces that operate.

[11] These authors provide a detailed description about monetary transmission channels, differentiating the way in which they work domestically and abroad, their composition, instruments and how each manifests.

Finally, the wealth effects are born in the discounted flow of future income in two ways, i.e., through the discount rate or the flows themselves.

Although interest rate hedging is common, illiquid assets such as real estate are rarely covered (Vrolijk, 1997). In a recent book, Shiller (2003) suggests that the hedge against those shocks must come from coordinated international flows. In this sense, the wealth effect is not easily hedged through the use of derivatives either.

4.2.2 Credit Channel Effect

The credit channel is composed of two subchannels, i.e., the bank credit and balance sheet effects[12], and is born from information asymmetries and transaction costs.

With respect to the pure credit channel, Vrolijk (1997) shows that the effect of monetary policy over bank credit is reinforced whenever firms count on intermediaries as the only financing source. Hence, a tight monetary policy reduces both bank reserves and credit.

In such a context, big firms with access to capital markets enjoy the possibility to compensate the credit contraction. However, for smaller firms that do not have such access, derivatives prove important in order to make them less sensitive to the impact monetary policy has on credit.

On the other hand, the balance sheet effect operates over the firm's collateral, which decreases due to an increase in interest rates. As a consequence of this, the firm becomes less credit-worthy [13]. Vrolijk (1997) shows that, contrary to what Bernanke and Gertler (1995) say, financial innovation, especially in the form of derivatives, allows firms to lock-in future interest rates and in this way, maintain the value of their collateral despite facing a higher cost of capital. Due to this, the transmission through the credit channel is weakened.

4.2.3 Exchange Rate Channel Effect

As a particular case of the asset channel, and in line with Mies et Al. (2003), the exchange rate channel allows the transmission of monetary policy over net exports and interest rate parity.

Following Vrolijk (1997), the effect over net exports of exchange rate changes due to monetary policy maneuvers, may be hedged both by exporters and importers through the use of derivatives. However, whenever the shock comes from the real exchange rate, hedging becomes less com-

[12]Vrolijk highlights Bernanke and Gertler (1995) who point out the difficulty in empirically distinguishing among the two.

[13]The descriptions about the operation of this channel given by Mies et Al. (2003) and Vrolijk (1997) coincide in highlighting the importance of financial innovations in affecting the power of monetary policy.

mon[14]. Thus, monetary policy may have short-term real effects through changes in the real exchange rate.

The interest rate parity transmits, in words of Vrolijk (1997), "domestic policy abroad and foreign policy home". The effect of derivatives accelerates the transmission, increases arbitrage and enhances capital flows. In a nutshell, the transmission becomes faster and real impact shows earlier.

5 Stylized Facts of Colombian Derivative Markets

Despite not having a high degree of development, colombian derivative markets have shown an interesting dynamic in the last 5 years[15]. In fact, depending on the type of instrument (i.e., interest or exchange rate), there exist significant differences in terms of development within the derivative markets themselves.

The market which displays less depth is the local interest rate market, where only few instruments are available. For example, there are no forward-rate agreements, nor a great deal of future transactions using bonds. On the other hand, the market does report "simultaneous" operations, or buy-sell backs which consist on selling a bond in the spot market and subsequently purchasing it forward, which financially, is equivalent to a repurchase agreement (repo). The market for this instrument has been developing hand in hand with the TES market in latter years, specifically because TES have been the main underlying assets. Taking the monthly average negotiation reported through Electronic Negotiation System of the Banco de la Republica, this has gone from $3.3 trillion pesos in 2002 to $6.5 trillion pesos in 2004. Although these type of operations offer agents the possibility of maintaining future positions in bonds, they may also be caused by liquidity demand and not to generate a certain exposure in the bonds market.

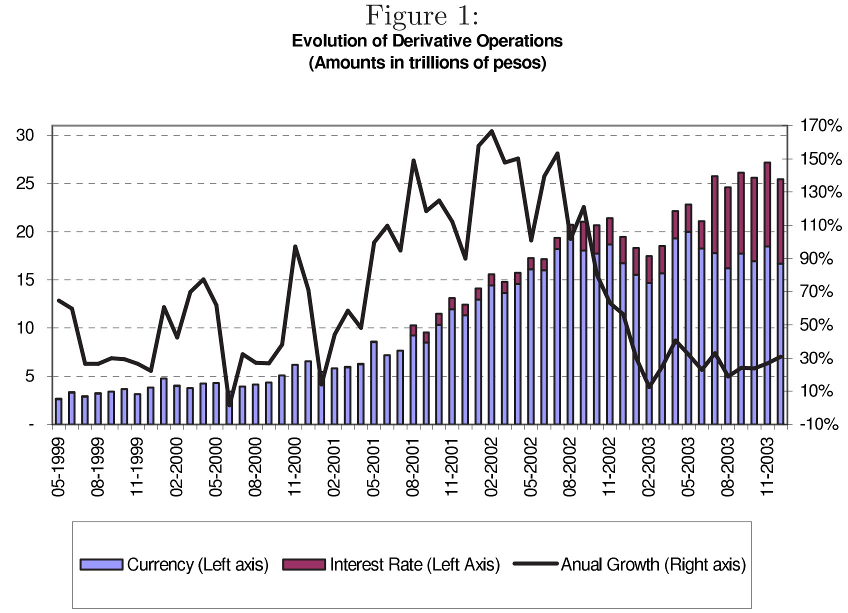
There exist various reasons for the lack of development in this market, which can be summarized in the absence of a healthy environment in which the enhancement of this type of instruments be viable and secure. The lack of reference indexes in the spot market, the absence of liquidity and maturity of the market itself as well as problems in monitoring financial system agents are some of the impediments to the proper development of these type of instruments. Currently, there are various projects in progress which seek to adjust the regulation problems present at the time in addition to generating liquid references in the short-run spot market that would serve as underlying indexes.

The exchange rate derivative market has, on the other hand, shown a much more favorable evolution than that of local interest rate. The exchange rate forward is probably the most liquid of these type of instruments and

[14] Unless a strategy using a combination of nominal exchange rate hedging instruments and indexed bonds is feasible.

[15] We thank Ana Fernanda Maiguashca for the elaboration of this section.

Figure 1:



A word of caution regarding the levels of both interest and exchange rate derivatives presented in Figure 1. Since these figures are obtained from the balance sheet of the financial system, and considering that most transactions are between financial institutions, there may be a problem with the level of notional amounts presented (i.e. double accounting). Nonetheless, the main interest of this study regards the evolution of derivative markets, and not the level itself, and thus, correcting this problem, which may prove extremely troublesome, may be irrelevant for the purpose of this paper.

has grown significantly in the last years, particularly in the second half of the 90's, when local currency suffered speculative attacks which led authorities to replace the exchange rate band with a floating exchange rate regime. The monthly average amount negotiated in this market went from around $US\$450$ million in 1997 to $US\$2,770$ million in 2003. Furthermore, average duration has increased, in that same time period, from 39 to 71 days. There does exist a market for swaps and options [16], but it is not particularly liquid and most instruments are designed for specific necessities of special intermediaries of the financial system, and not as means to guarantee exposure by market agents.

6 Empirical Exercise

6.1 Methodology

There are two main motivations that lead to the election of the particular econometric model and estimation technique chosen in this paper. First, international literature does not yet have a firm-level analysis of the impact derivative markets have on transmission mechanisms. Thus, using panel data to specify the model represents not only a first approximation to the

[16] Particularly since Banco de la Republica had been using them for market intervention.

problem under a new perspective, but also allows the inclusion of a decision variable (investment), at an individual level.

Second, given the database used constitutes an unbalanced panel, the model allows for all the information available to be efficiently used without having to arbitrarily eliminate observations in order to balance the sample. By trying to obtain a balanced panel with the information available, one would incur in selection bias due to the fact that the resulting database would probably be comprised of firms that, either have unlimited access to international and domestic credit markets or else have sufficiently large asset levels as to smoothen the impact of a tight monetary policy. This selection bias would therefore complicate capturing the existing differences between firms, which are at the heart of the financial accelerator.

Having the former in mind, the model is estimated using the methodology developed by Biørn (1999a), from which one obtains the Generalized Least Squares (GLS) parameter estimator for a model of simultaneous equations seemingly unrelated with the error term, using a specification of individual random effects over the intercept, under a structure of unbalanced panel data (see Appendix B).

The random effects specification allows capturing heterogeneity between firms in the sample, which arise fundamentally from idiosyncratic factors, such as differences in organizational structure, varying access to credit and capital markets and, in general, random events and/or circumstances (e.g. productivity shocks generated by an efficient manager) inherent to each firm. Additionally, theory concerning the respective Breusch-Pagan (1980) and Hausmann and Taylor (1981)-type tests to evaluate the presence of individual effects as well as of choice between fixed and random effects, respectively, have yet not been fully developed.

6.2 Data and Variable Description

The database for the empirical model contains information from Superintendencia de Valores and Superintendencia de Sociedades. Both data sets have annual observations for over 15.000 firms from 1995 to 2003. Given that the information is not homogeneous in terms of the number of years each firm reports (i.e. there are firms which begin reporting in years subsequent to 1995 or that stop reporting before the end of the sample either temporally or completely), an arbitrary depuration criteria was used. Only firms with information for all variables for at least 4 consecutive years where taken into account. Thus, the database was split into 6 balanced sub-panels from which the global estimation was generated, with a total of 8.421 firms and 49.862 observations.

To study the effect monetary policy has over the firms' private investment decision under the presence of derivative markets, a model based on "Tobin's q" proposed by Abel and Eberly (2002) is estimated. In the model, the firm chooses its level of investment from the first-order equations resulting from an optimization problem in which capital stock is chosen to maximize the expected present value of profits [17]. Particularly, optimum investment takes the form:

$$\frac{I_{i,t}}{K_{i,t}} = \alpha + \beta_1 q_{i,t} + \beta_2 \rho_{j,t} + \beta_3 r_t + \varepsilon_{i,t}, \tag{6.1}$$

where the sub-index i denotes the $i-th$ firm, j represents the industry the firm belongs to, and t is the moment in time. Investment (I) is defined as the change in property, plant and equipment, while K represents the replacement value of capital. Such replacement value is constructed for each firm using the book value of capital with the following recursion:

$$K_{i,t} = [K_{i,t-1}(1-\delta_j)\frac{P_{k,t}}{P_{k,t-1}} + I_{i,t}], \tag{6.2}$$

where the recursion in period t is initialized using the book value of $K_{i,t-1}$. $P_{k,t}$ is defined as the implicit price deflator of nonresidential investment [18], while δ_j represents the depreciation rate of the $j-th$ industry [19].

With respect to the independent variables, $q_{i,t}$ corresponds to "Tobin's q". Theoretically, q represents the ratio of market value of capital to its book value. However, the pricing of assets is an arduous and extensive task; hence, an impossibility arises as to have a series that allows to define "Tobin's q" in such a way. An alternative for successfully measuring this variable consists in employing a sample which includes only firms that actively participate in the stock market. Those firms would then have a price on their stock, which could be used as indicator of market value. Nonetheless, this approach has a serious selection bias, given the limited number of firms that presently quote in the local stock market and the fact that such firms are generally large, which proves them less sensible to changes in the political stance due to the access they possess to different financing mechanisms.

Conversely, q is also defined as the present value of expected marginal returns on capital. Following Abel and Eberly (2002), it can be shown that under the assumption of a linearly homogeneous operating profit function, marginal and average returns are equal. Thus, one can approximate the value of "Tobin's q" using average future operational profits. However, due

[17] The derivation of the model, as well as its theoretical justification, is developed in detail in Abel, A.B. and J.C. Eberly (2002). "Investment and q with Fixed Costs: An Empirical Analysis".

[18] We would like to thank Gabriel Piraquive, of Departamento Nacional de Planeacion (DNP) for providing us with the series.

[19] In this case there are two different depreciation rates. There is a 5% depreciation rate for machinery and equipment and a 6% rate for depreciation of agricultural capital. Hence, in the estimation the 5% rate is used for depreciating all capital except that which belongs to firms of the Mining, Fishing and Agricultural sectors (calculation of these depreciation rates can be found in: Barrios et. Al., 1993, Orozco, 1977, Harberguer, 1969 and Vasquez, 1998).

to the absence of data concerning these returns, the use of average past returns is proposed as a *proxy*. Specifically, q is defined as:

$$q_{i,t} = \frac{1}{2}[\frac{\pi_{i,t}}{K_{i,t}} + \frac{\pi_{i,t-1}}{K_{i,t-1}}], \tag{6.3}$$

where $\pi_{i,t}$ represents operational profits of the $i - th$ firm at moment t. In equation 6.1, $\rho_{j,t}$ corresponds to the relative price of capital of industry j, and is defined as:

$$\rho_{j,t} = \frac{P_{k,t}}{PPI_{j,t}}, \tag{6.4}$$

where $PPI_{j,t}$ represents the producer price index of industry j at moment t. Finally, r_t in 6.1 corresponds to the real interest rate.

For the specific model used in this paper, three modifications are introduced to the original setup described above. Such changes are justified in the following section.

6.2.1 Monetary Policy Instrument

Contrary to most models found in the literature, monetary policy should not be proxied using the real interest rate alone. A Monetary Conditions Index (MCI) is used instead [20] to capture the effects of monetary policy on all relative prices. In other words, MCI represents the political stance, which affects real variables through all the transmission mechanisms imaginable. In this sense, there is no discrimination of the relative effect a specific monetary transmission channel has; on the contrary, there is an attempt to capture the effect monetary policy has through each and every one of its transmission mechanisms. This channels of monetary transmission are thus not found explicitly in the model, but are instead modelled implicitly. This said, given a particular political stance, the effect over the firms' investment decisions is given trough all the transmission mechanisms possible.

Since MCI jointly considers the combined effects of changes in real interest and exchange rates over the channels of monetary transmission, it constitutes itself as a valuable instrument for policymakers to effectively make better decisions with respect to the degree of tightness or looseness in monetary conditions and the impact this has over aggregate demand.

[20] The Monetary Conditions Index is calculated for Colombia by Rocio Mora and is defined as:

$$MCI^z_t = (r_{t-2} - r_{0-2}) + \frac{\hat{\beta}_q}{\hat{\beta}_r}(q_1 - q_0); \beta_r < 0 \tag{6.5}$$

where MCI combines the effect of the real interest rate, lagged two periods, with the relative effect of changes in the real exchange rate with respect to a base year (1998). See Appendix A for details.

6.2.2 $\widehat{MCI}$

To capture the effect of derivative markets over the firms' investment decision, $\widehat{MCI}$ is constructed. This variable permits the identification of the additional impact monetary policy has on investment decisions once the existence of derivative markets is controlled for.

In order to construct this variable the estimation needed to obtain the value of the interest and exchange rate coefficients of the original MCI must be replicated (see Appendix A), this time including a new variable to proxy the deepening of the derivative market (the notional amount of derivatives held by the financial system is used).

Due to the lack of available data regarding the amount of hedging instruments held by firms, the aggregate notional amount of derivatives held by financial intermediaries is used as an approximation of market size. Despite the fact that this variable does not reflect the total amount of derivatives negotiated in the market, it does replicate the market's trend over time. On the other hand, it is important to point out that the holding of derivatives by financial intermediaries, generates hedging spill-overs to firms financed by them, for that financing will be less volatile in quantity and price than would be otherwise. In this sense, the proxy used for derivative market size reflects two dimensions of hedging through these instruments: 1) Directly, as an approximation of the growth trend derivative markets have shown in recent years, and 2) Indirectly, by means of the positive effect their holding by banks has on firms that have no access to these type of instruments but that do have access to bank financing, in the sense that the credit supply will ultimately be more stable and, probably, less costly.

Therefore, new values are estimated for the coefficients associated with real interest and exchange rates, only now this parameters will implicitly contain information concerning the impact of derivative markets. From this coefficients a new MCI is calculated, such that the difference between this new index and the original generates a variable that captures the specific effect derivatives have on monetary conditions.

A final comment on the construction of $\widehat{MCI}$ is noteworthy. The analysis of the impact of derivatives markets over monetary policy is similar to the problem implied in the evaluation of the effect of a particular public policy over a "control group" (e.g. matching technique, among others). Unfortunately, in the present case, due to the lack of data there is no such control group (say, data of firms that use derivatives vs. those that do not), which obligues the use of the construct $\widehat{MCI}$ as a proxy for the "control group" effect.

6.2.3 Dummy Variable

A dummy variable is introduced in the model, taking the value of 1 in the years 1998 and 1999 to control for the effects of the crudest part of the

Table 1:

Variable	Coefficient	Std Error	"t" Statistic
Intercept	0.31980	0.02994	10.68
MCI(t-2)	-0.00191	0.00062	-3.07
ICMNET(t-2)	0.11912	0.00504	23.63
TOBIN's q	0.00006	0.00003	2.32
RLT PRICE	-0.10696	0.01406	-7.61
DUMMY	-0.09657	0.00587	-16.45

economic downturn, which undoubtedly had substantial consequences over the firms' investment possibilities and decisions.

Hence, the estimated model takes the form:

$$\frac{I_{i,t}}{K_{i,t}} = \alpha + \beta_1 q_{i,t} + \beta_2 \rho_{j,t} + \beta_3 MCI_{t-2} + \beta_4 \widehat{MCI}_{t-2} + \beta_5 D_t + \varepsilon_{i,t}, \quad (6.6)$$

where MCI_{t-2} is the two-period lagged index of monetary conditions, $\widehat{MCI}_{t-2}$ is the additional effect over MCI caused by the inclusion of derivative markets and D_t is the dummy variable [21].

6.3 Results

The results summarized in Table 1, show all coefficients to have the expected sign and to be statistically significant at the 95% level. Since some variables used in the estimation correspond to macro variables while others are firm-level, coefficient interpretation must be careful. Additionally, it is important to keep in mind that both MCI and $\widehat{MCI}$ are stochastic regressors, estimated individually in respective linear regressions. Hence, the total effect of monetary conditions over investment decisions cannot be verified in this setup, since the coefficents associated with MCI and $\widehat{MCI}$ cannot be simply "added". For this reason, the analysis is focused on the sign and statistical significance of the coefficients, from which the relationship between variables predicted by economic theory may be verified, and what is in itself the main interest of this paper [22].

As can be seen, consistent with economic theory, "Tobin's *q*" has a positive sign and hence, a direct relationship with investment. Thus, increases in average operating profits create incentives to invest, possibly by augmenting firms' expected income flows. Increases in profits (measured as the 2 year average of operating profits weighted by the firms' capital stock) partially

[21] Data suggests that monetary policy has real effects only after two periods.

[22] An additional test is performed to verify the statistical difference between the means of the original MCI and the modified MCI which includes the effect of derivatives. See Appendix A for details.

reflect increases in the return to capital and thus serve as investment yield indicators.

The sign of the parameter associated to the relative price of capital is negative, indicating the inverse relationship this variable has with investment, just as theory suggests. The price of capital is a measure of its cost, and the relationship found simply reinforces the notion of a downward sloping demand curve with respect to price.

Additionally, the dummy variable affects investment in a negative way as well. Therefore, the impact of the economic crisis over the real sector is verified, by effectively showing that investment contracts during the recession period. The crisis episode not only affected investment by means of lower returns on capital for firms, but also by generating a strong tightening in credit markets, affecting the supply of resources from financial intermediaries.

The effect of MCI over the investment variable is negative in the model. In other words, when monetary policy is less tight (negative values of MCI), the levels of investment rise. Intuitively, more flexible monetary conditions enhance liquidity, providing a favorable atmosphere for both credit and productive activity.

Finally, a positive sign is observed in the parameter associated to the impact derivative markets have on investment. This is the most relevant variable in this paper, since it establishes the significance of the additional effect that monetary policy has over the firms' individual decisions, once the existence of a market of hedging instruments is accounted for. The positive sign, contrary to that of MCI, evidences the reduced impact of monetary policy measures on the real economy by means of the partial dilution of the main channels of monetary transmission.

There is an important insight arising form this result. As noted earlier, Vrolijk (1997) considers that the absence of evidence in favor of the diluting impact derivatives have on British transmission mechanisms, may be due to the maturity and apparent completeness of the capital market even before the appearance and development of hedging instruments, which makes their marginal contribution insignificant.

The contrary seems to happen in Colombia. The significant impact of derivatives over monetary transmission mechanisms, after their birth, may be due to the fact that such instruments offer important hedging possibilities in a market with significant frictions and fissures inherent to a less developed financial system.

7 Conclusions

In the last few years, derivative markets have undergone a deepening process which has created serious questions concerning their potential impact on prices of traditional goods and capital markets, due to the hedging possibilities they offer to economic agents. Because of these hedging opportunities, derivatives may dilute the main channels of monetary transmission in some cases and/or may promote a faster transmission of changes that monetary policy induces on relative prices in the short-run.

This paper studies the effect derivatives have on Colombia's monetary transmission channels, using a methodology that excels prior research in at least 4 ways. 1) Firm-level data is used, 2) An investment model is estimated, 3) A Generalized Least Squares technique is used to estimate the panel data, and 4) MCI is used as the a proxy for political stance.

The results coincide with what economic theory predicts. Additionally, evidence is given to support that monetary policy has lost some effectiveness to affect real variables in the short-run, due to the partial dilution of the main monetary transmission channels caused by the completion of financial markets that derivative instruments imply. The former occurs even considering the lack of development and depth of the domestic capital market.

Vrolijk (1997) sheds some light on this phenomena in his study of the British market. If the apparent development and depth of the British financial system has made the effect of derivatives but marginal to its completeness, the colombian case offers the opposite evidence: lack of completeness of domestic financial markets make the marginal effect of derivative instruments significant, due to their role in completing such markets.

Additionally, not only does direct hedging by firms with access to these type of instruments matter. There may exist hedging spill-overs that occur when commercial banks buy these derivatives, which translates into a more stable and cheaper credit supply for firms that have no access to derivative markets.

This conclusion, which may seem discouraging since it indicates the loss of effectiveness of monetary policy is, on the contrary, positive. Following Mies et Al.(2003), "...the impact of monetary policy over [economic] activity is produced, in most cases, due to the existence of a market imperfection whose existence may have certain costs in terms of efficiency".

Nonetheless, the results presented herein must be read and interpreted with caution. Even though there exists evidence that indicates a possible weakening in the transmission mechanism in Colombia, it is important to estimate if the transmission itself has speeded up. This is a question left as a policy recommendation for future research.

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Appendix A.

MCI Estimation

The estimated regression models use the following monthly variables (measured between 1993 : 4 and 2003 : 4) (See Mora (2000) for details): i) One lag of the output gap (GAP1) [23], ii) one and two lags of the real interest rate (TIR1, TIR2), iii) the seventh lag of the real exchange rate (TCR7) and iv) The stock of derivatives of the financial system (DERTOT1).

I. Original MCI Parameter Estimates

Dependent Variable: GAP   Method: Ordinary Least Squares
Date: $12-06-04$ Hour: 14:54   Sample: 1993:4 2003:4
Included observations: 41

Variable	Coefficient	Standard Error	Statistic "t"	Significance Level
C	0.919	0.3110	2.9542	0.01
GAP1	1.029	0.0601	17.1289	0.00
TIR1	-0.126	0.0387	-3.2423	0.00
TCR7	0.030	0.0129	2.3233	0.03
R-Squared	0.906	Mean of the Dep. Var.	-1.79	
Adj R-sq	0.898	Std Error of Dep Var	2.94	
Std Err of Estimate	0.939	Akaike inf. Crit.	2.80	
Sum of Sq. Resid	32.601	Schwarz inf. Crit.	2.97	
Log of the Likelihood	-53.477	F Stat.	118.27	
Durbin-Watson Stat.	2.478	Signif Level of the F Stat	0.0000	

Based on the chart, $\beta_r = -0.125$ and $\beta_q = 0.029$, are used to build the monetary conditions index as follows:

$$MCI_t^z = (r_{t-2} - r_{0-2}) - \frac{0.029}{0.125}(q_t - q_0) \qquad \text{(A-1)}$$

II. Stationarity of the Variables Used in the Regressions

[23] The GAP is built using a Hodrick-Prescott filter applied to GDP.

Table 2: **KPSS Stationarity Test**

Model Specification	Variable	Critical Value (5%)	KPSS Test	Ho*
Trend and Intercept	Derivatives	0.146	0.141277	Ho not rejected
	GAP	0.146	0.114117	Ho not rejected
	TIR	0.146	0.078473	Ho not rejected
Intercept	TCR	0.463	0.347988	Ho not rejected

* Ho: variable is stationary

III. MCI Parameter Estimates, Including the Derivative Markets' Effect

Dependent Variable: GAP Method: Ordinary Least Squares
Date: $09 - 27 - 04$ Hour: 16:50 Sample(adjustded): 1998:3 2003:4
Included observations: 22 after the adjustment

Variable	Coefficient	Standard Error	Statistic "t"	Significance Level
C	-1.4979	0.6317	-2.3712	0.030
GAP1	0.6347	0.0861	7.3723	0.000
TIR2	-0.0943	0.0446	-2.1136	0.050
TCR7	0.0160	0.0122	1.3046	0.209
DERTOT1	0.0486	0.0271	1.7944	0.091
R-Squared	0.8862	Mean of the Dep. Var.	-4.0405	
Adj R-sq	0.8594	Std Error of Dep Var	1.7576	
Std Err of Estimate	0.6590	Akaike inf. Crit.	2.2006	
Sum of Sq. Resid	7.3836	Schwarz inf. Crit.	2.4486	
Log of the Likelihood	-19.2070	F Stat.	33.089	
Durbin-Watson Stat.	2.7663	Signif Level of the F Stat	0.0000	

Based on the chart, $\beta_r = -0.094$ and $\beta_q = 0.016$, are used to build the new monetary conditions index as follows:

$$MCI_t^z = (r_{t-2} - r_{0-2}) - \frac{0.016}{0.094}(q_t - q_0) \tag{A-2}$$

IV. Test for Equality of Means Between Original MCI and Modified MCI

Finally, it is important to verify the statistical difference between the means of both the Original MCI and Modified MCI, in order to prove that the effect of derivatives indeed changes the monetary policy stance.

Test for Equality of Means Between Series*			
Date: 04/04/05 Time: 17:47			
Sample: 1990Q1 2004Q2			
Included observations: 58			
Ho: $\mu_0 = \mu_1$			
Method	df	Value	Probability
t-test	114	1.793714	0.0755
Anova F-statistic	(1, 114)	3.217408	0.0755

Appendix B.

G.L.S for an Unbalanced Panel Data Structure, with Random Effects over the Intercept.

The single equation econometric model used in this document is a particular case of the g system-type simultaneous equation SUR originally proposed by Biørn (1999a).

The estimation process is based on an information structure composed by N firms or individuals indexed by $i = 1, 2, \cdots, N$. Each individual i is at least observed once and at most, up to P periods, thus NP denotes the number of observed firms P periods with $P = 1, 2, \cdots, P$, but are not necessarily consecutive. n is defined as the total number of observations, such that $N = \sum_{p=1}^{P} N_p$ y $n = \sum_{p=1}^{P} N_p P$. The accumulated number of firms observed up to p times is $M_p = N_1 + N_2 + \ldots + N_p$ and thus, $M_1 = N_1, M_2 = N_2$ y $M_p = N$. From the above the following subsets of firms observed up to P times denoted by I_p, are defined such that $I_1 = [1, 2, \ldots, M_1]$, $I_2 = [1, 2, \ldots, M_2]$, $\ldots$, $I_p = [1, 2, \ldots, M_p]$. I_1 corresponds to a cross-section and $I_2, I_3, \ldots, I_p$ are balanced sub-panels with $2, 3, \ldots, P$ observations for each individual [24].

Taking into account the information structure described above, it is possible to formulate the *t-th* sample equation for the *i-th* firm, given the H independent variables including the constant term, as follows:

$$y_{it} = x_{it}\beta + \alpha_i + u_{it} \tag{B-1}$$

with $i \in I_p$, $t = 1, 2, \ldots, p$, $p = 1, 2, \ldots, P$, y_{it} corresponds to the dependent variable and x_{it} is a non stochastic vector $(1 \times H)$ containing the *t-th* observation for each of the explanatory variables for the *i-th* firm, including 1 for the intercept. The latent effect (non observed and stochastic) specific to each individual, corresponds to α_i and u_{it} is the stochastic error term containing the changes y_{it} come from changes of variables different from x_{it}. β is the $(H \times 1)$ vector of parameters to be estimated, including the intercept. These parameters are common to firms.

The fundamental assumptions about orthogonality conditions for the stochastic components of the model are given by:

$$E[\alpha_i] = 0, \;\; E[\alpha_i, \alpha_s'] = \delta_{ij}, \;\; E[u_{it}] = 0, \;\; E[u_{it}, u_{js}] = \delta_{ij}, \delta_{ts} \sum\nolimits_u$$

where δ's correspond to the respective delta-Kroneckers, assuming that x_{it}, α_i and U_{it} are uncorrelated. In the case of a single-equation model, the matrices $\sum_\alpha$ y $\sum_u$ become the scalars σ_α and σ_u respectively. The

[24] For the balanced panel date case, for instance with P observations, there is only one nonempty subset I_p.

compound stochastic disturbance term is defined as: $\epsilon_{it} = \alpha_i + u_{it}$ with

$$E[\epsilon_{it}] = 0, \;\; E[\epsilon_{it}, \epsilon_{js}] = 0$$

The specific individual mean of the ϵ's for the $i-th$ individual is defined as:

$$\bar{\epsilon}_{i\cdot} = \begin{cases} \epsilon & \text{para } \; i \subset I_1, \\ 1/2 \sum_{t=1}^{2} \epsilon_{it} & \text{para } \; i \subset I_2 \\ \vdots & \\ 1/p \sum_{t=1}^{p} \epsilon_{it} & \text{para } \; i \subset I_p \end{cases}$$

The global mean is:

$$\bar{\epsilon} = \sum_{p=1}^{P} \sum_{i \subset I_p} \sum_{t=1}^{p} \epsilon_{it} = \frac{1}{n} \sum_{p=1}^{P} \sum_{i \subset I_p} p \bar{\epsilon}_{i\cdot}$$

The covariance matrices for the error components in the model are defined by:

$$\sum\nolimits_u = \frac{W_{\epsilon\epsilon}}{n-N}, \qquad \sum\nolimits_\alpha = \frac{B_{\epsilon\epsilon} - \frac{N-1}{n-N} W_{\epsilon\epsilon}}{n - \frac{\sum_{p=1}^{P} N_p p^2}{n}}$$

where $W_{\epsilon\epsilon} = \sum_{p=1}^{P} \sum_{i \subset I_p} \sum_{t=1}^{p} (\epsilon_{it} - \overline{\epsilon_{i\cdot}})(\epsilon_{it} - \overline{\epsilon_{i\cdot}})'$, $B_{\epsilon\epsilon} = \sum_{p=1}^{P} \sum_{i \subset I_p} (\epsilon_{i\cdot} - \bar{\epsilon})(\epsilon_{i\cdot} - \bar{\epsilon})'$, which are the *within* co-variations for individuals and *between* individuals respectively, being $T_{\epsilon\epsilon} = W_{\epsilon\epsilon} + B_{\epsilon\epsilon}$ the total variation.

The same variation and covariation categories are obtained for the observed balanced sub panels observed up to p times, to define the following estimators:

$$\begin{aligned} \sum\nolimits_{u(p)} &= \frac{W_{\epsilon\epsilon}}{N_p(p-1)} \\ \widehat{\sum\nolimits_{\alpha(p)}} &= \frac{1}{p} \left(\frac{B_{\epsilon\epsilon(p)}}{N_p - 1} - \frac{W_{\epsilon\epsilon(p)}}{N_p(p-1)} \right) \\ \widehat{\sum\nolimits_{(p)}} &= \widehat{\sum\nolimits_{u(p)}} + \widehat{\sum\nolimits_{\alpha(p)}} = \frac{B_{\epsilon\epsilon}}{N_p - 1} \end{aligned}$$

To obtain the G.L.S estimates by groups, which is the fundamental base of the G.L.S estimation, the following matrices are defined:

$$Y_{i(p)} = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{ip} \end{bmatrix} \qquad X_{i(p)} = \begin{bmatrix} X_{i1} \\ \vdots \\ X_{ip} \end{bmatrix} \qquad \epsilon_{i(p)} = \begin{bmatrix} \epsilon_{i1} \\ \vdots \\ \epsilon_{ip} \end{bmatrix}$$

for $i \subset I_p$, $p = 1, 2, \ldots, P$, where, for example, $X_{i(p)} = [x_{i1,\ldots,x_{iH}}]$, is a

$(1 \times H)$ vector containing the first observation for each of the H explanatory variables.

In compact matrix notation for the set of observations, the model for each individual may be written as:

$$Y_{i(p)} = X_{i(p)}\beta + (e_p \otimes \alpha_i) = X_{i(p)}\beta + \epsilon_{i(p)}$$

where e_p is a $(p \times 1)$ vector of ones. For the latter equation it is assumed that:

$$E[\epsilon_{i(p)}] = 0, \qquad E[\epsilon_{i(p)}\epsilon'_{i(p)}] = I_p \otimes \sum\nolimits_u + E_p \otimes \sum\nolimits_\alpha = \Omega_{\epsilon(p)}$$

with I_p a p-dimensional identity matrix, $E_p = e_p e'_p$ a $(p \times p)$ matrix with all of it's elements equal to one and $\Omega_{\epsilon(p)}$ the $(p \times p)$ covariance matrix of the compound error term, which may be rewritten as:

$$\Omega_{\epsilon(p)} = B_p \otimes \sum\nolimits_u^{-1} + A_p \otimes (\sum\nolimits_u + p\sum\nolimits_\alpha)^{-1}$$

where $A_p = (1/p)E_p$ and $B_p = I_p - (1/p)E_p$ are symmetric idempotent matrices, with orthogonal columns.

Given that $\Omega_{\epsilon(p)}$ is a nonscalar identity covariance matrix, the *Aitken*-type Two-Step Generalized Least Squares estimator for β, which uses the entire information set, defined by the subsets, is given by:

$$\widehat{\beta}_{GLS} = \left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\Omega^{-1}_{\epsilon(p)}X_{i(p)}\right)^{-1}\left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\Omega^{-1}_{\epsilon(p)}Y_{i(p)}\right)$$

.

Given that:

$$\Omega^{-1}_{\epsilon(p)} = \left(B_p \otimes \sum\nolimits_u^{-1}\right) + \left(A_p \otimes (\sum\nolimits_u + p\sum\nolimits_\alpha\right) = \left(B_p \otimes \sum\nolimits_u^{-1}\right) + \left(A_p \otimes \sum\nolimits_{(p)}\right)$$

$\widehat{\beta}_{GLS}$ can be re-written as:

$$\begin{aligned}\widehat{\beta}_{GLS} \quad = \quad & \left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(B_p \otimes \sum\nolimits_u^{-1}\right)X_{i(p)} + \sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(A_p \otimes \sum\nolimits_{(p)}^{-1}\right)X_{i(p)}\right)^{-1} \\ & \times\left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(B_p \otimes \sum\nolimits_u^{-1}\right)Y_{i(p)} + \sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(A_p \otimes \sum\nolimits_{(p)}^{-1}\right)Y_{i(p)}\right)\end{aligned}$$

The estimator of the covariance matrix for the estimated parameter vector $\widehat{\beta}_{GLS}$ is:

$$
\begin{aligned}
V(\widehat{\beta}_{GLS}) &= \left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\Omega^{-1}_{\epsilon i(p)}X'_{i(p)}\right)^{-1}\\
&= \left(\sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(B_p\otimes \sum\nolimits_u^{-1}\right)X_{i(p)} + \sum_{p=1}^{P}\sum_{i\subset I_p} X'_{i(p)}\left(A_p\otimes \sum\nolimits_u^{-1}\right)X_{i(p)}\right)
\end{aligned}
$$

The estimators for $\widehat{\sum}_u^{-1}$, $\widehat{\sum}_{(p)}^{-1}$, $(\widehat{\beta}_{GLS})$ and $V(\widehat{\beta}_{GLS})$ are calculated using the following algorithm:

1. Obtain the consistent estimators for the first residuals given by $\epsilon_{it} = Y_{it} - X_{it}\widehat{\beta}_{GLS}$, with $\widehat{\beta}_{GLS}$ taken from a regression with all the observations Y_{it} y X_{it}.
2. Substitute ϵ_{it} with $\widehat{\epsilon_{it}}$ to obtain: $\widehat{W}_{\widehat{\epsilon}\widehat{\epsilon}}$ y $\widehat{\beta}_{\widehat{\epsilon}\widehat{\epsilon}}$.
3. Replace $W_{\epsilon\epsilon}$ and $\beta_{\epsilon\epsilon}$ by $\widehat{W}_{\widehat{\epsilon}\widehat{\epsilon}}$ and $\widehat{\beta}_{\widehat{\epsilon}\widehat{\epsilon}}$ and get the estimators $\widehat{\sum}_u^{-1}$, $\widehat{\sum}_\alpha^{-1}$ and $\widehat{\sum}_{(p)}^{-1}$ of the matrices $\sum_u^{-1}$, $\sum_\alpha^{-1}$ and $\sum_{(p)}^{-1}$.
4. Substitute $\sum_u^{-1}$, $\sum_\alpha^{-1}$ and $\sum_{(p)}^{-1}$ with $\widehat{\sum}_u^{-1}$, $\widehat{\sum}_\alpha^{-1}$ and $\widehat{\sum}_{(p)}^{-1}$, and obtain the estimators $\widehat{\beta}_{GLS}$ and $V(\widehat{\beta}_{GLS})$.