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Short-Term Liquidity Contagion in the Interbank Market¹

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Abstract

We implement a modified version of DebtRank, a measure of systemic impact inspired in feedback centrality, to recursively measure the contagion effects caused by the default of a selected financial institution. In our case contagion is a liquidity issue, measured as the decrease in financial institutions' short-term liquidity position across the Colombian interbank network. Concurrent with related literature, unless contagion dynamics are preceded by a major –but unlikely– drop in the short-term liquidity position of all participants, we consistently find that individual and systemic contagion effects are negligible. We find that negative effects resulting from contagion are concentrated in a few financial institutions. However, as most of their impact is conditional on the occurrence of unlikely major widespread illiquidity events, and due to the subsidiary contribution of the interbank market to the local money market, their overall systemic importance is still to be confirmed.

Keywords: financial networks, contagion, default, liquidity, DebtRank

JEL Codes: G21, L14, C63

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1 Introduction

One lesson from the Global Financial Crisis is that the soundness of each financial institution does not ensure the stability of the financial system, *per se*. Likewise, it has been put forward that financial institutions' liquidity is not ensured by the liquidity position of each single institution, but that their interconnections may determine whether they are able to fulfill their short-term liquidity needs. In this sense, as Allen and Gale (2000) noted, interconnections between financial institutions determine the possibility and extent of financial contagion.

Financial institutions' interconnections comprise direct and indirect linkages (Allen and Babus, 2009). Direct linkages are related to mutual exposures acquired in financial markets (e.g. interbank lending, securities and foreign exchange settlements), whereas indirect linkages correspond to holding similar portfolios (as in fire-sales) or sharing the same mass of depositors (as in deposit runs). We focus on direct-linkage contagion.

Despite differing in their specific features and assumptions, most direct-linkage contagion simulation models focus on how defaults on mutual exposures may erode financial institutions' solvency by affecting their capital buffer.⁵ Theoretical works use artificial networks to investigate how financial systems' structure and capitalization affect systemic risk (see Nier, Yang, Yorulmazer, and Alentorn (2007), and Gai and Kapadia (2010)). They find that contagion decreases with capitalization, but increase with concentration or with the size of interbank liabilities. About connectivity, they find that the relationship with contagion is non-monotonic: when connectivity is low (high), an increase in the number of links increases (decreases) the likelihood of knock-on defaults. Roukny, Bersini, Pirotte, Caldarelli, & Battiston (2013) find that the network topology matters only –but substantially– when financial markets are under stress (e.g. illiquid).

Furfine (2003) is among the first to study contagion by examining actual interbank exposure data. Furfine's main finding is that bilateral interbank exposures in the U.S. are neither large enough nor distributed in a way to cause a great risk of contagion by capital exhaustion, with

⁵ See Upper (2011) for a comprehensive review on interbank contagion simulation methods prior to 2010.

very few cases of knock-on effects arising from a financial institution failing. The number and capitalization of counterparties are identified as key determinants of contagion. As reported by Upper (2011), most extensions and enhancements after Furfine (2003) point out that direct contagion based on actual interbank exposures is likely to be rare, and it can only happen if interbank exposures are large relative to lender's capital.

A recent development on direct-linkage contagion simulation models is DebtRank (Battiston, Puliga, Kaushik, Tasca, & Caldarelli, 2012). Inspired by feedback centrality, DebtRank recursively measures the impact of the default of a selected financial institution on the capital buffer of financial institutions across the entire financial network. DebtRank serves to determine the size of contagion caused by the initial default of a financial institution, besides providing an assessment of the systemic importance of each financial institution based on the severity of its impact over the system. However, unlike previous direct-linkage contagion models based on default cascade dynamics, the impact from default is not limited to those cases in which the capital buffer is exhausted: partial impact on solvency is quantified and accumulated recursively. Some implementations of DebtRank on actual data are available (e.g. Tabak, Souza, and Guerra (2013), Battiston, Caldarelli, D'Errico, and Gurciullo (2015), Poledna, Molina-Borboa, Martínez-Jaramillo, van der Leij, and Thurner (2015)).

Most research on direct-linkage contagion focus on contagion the subsequent failure of other financial institutions by means of capital buffers exhaustion (i.e. a solvency issue). Nevertheless, liquidity is a key factor as well. Furfine (2003) concludes that the liquidity effect, in the form of the unwillingness to lend money due to the inability to borrow, may be greater than the solvency effect in the U.S. interbank markets. Müller (2006) concludes that direct linkages affect solvency and liquidity substantially in the Swiss interbank market, and that both sufficient capital and liquidity buffers are necessary to mitigate spill-overs. Cepeda and Ortega (2015) find that liquidity contagion in the Colombian large-value payment system is mitigated when considering the stock of high-quality assets available as a potential source of liquidity.

We implement a modified version of DebtRank in order to recursively measure the impact of the default of a selected financial institution on the short-term liquidity position of financial institutions across the entire interbank network. We construct the financial network

based on actual interbank (i.e. non-collateralized) data from the Colombian financial market. We use the local version of the Liquidity Coverage Ratio (LCR)⁶, the Liquidity Risk Indicator (*IRL* by its acronym in Spanish), as the initial short-term liquidity position of financial institutions. Our modified version of DebtRank allows for determining the size of short-term liquidity contagion caused by the default of a designated financial institution, and for assessing the systemic importance of each of these institutions based on the severity of its impact over the short-term liquidity of the system.

Consistent with most related literature (e.g. Furfine (2003), Upper (2011), Roukny et al. (2013)) we find that *ceteris paribus*- in the Colombian interbank market contagion effects are not a threat to the stability of the system by themselves. Unless a major –but unlikely– drop in the short-term liquidity position of all participants precedes contagion, we find that contagion effects are rather small. It is most likely that the small size of Colombian interbank market exposures with respect to the short-term liquidity position of financial institutions (about 1.5% of *IRL*), along with the subsidiary contribution of interbank loans to liquidity exchanges between financial institutions (about 9.68%), may explain why contagion effects alone are trivial.

Our results support a salient feature of past financial crisis reported by Upper (2011): the vast majority of banking crisis followed shocks that hit several banks simultaneously rather than domino effects from idiosyncratic failures. Our methodological proposal provides a quantitative assessment of financial institutions' systemic importance based on their potential contagion effect in the short-term liquidity position of the remaining financial institutions across the Colombian interbank network. Moreover, based on the potential effect on the system's liquidity, our results may provide a quantitative assessment of the liquidity that should be obtained from other available sources in case of a default by a financial institution, such as collateralized borrowing (e.g. from other financial institutions or the central bank), selling financial assets or increasing deposits. However, as our results are limited to the local

⁶ The Liquidity Coverage Ratio (LCR) has the purpose of ensuring that each financial institution has an adequate stock of unencumbered high-quality liquid assets that can be easily and immediately convertible into cash, in private markets, so as to meet its liquidity needs for a stress scenario of thirty calendar days (see BCBS (2013)).

interbank market, conclusions are to be weighted according to its contribution to the money market and to the size of the financial system.

2 Methodology

There is a rather recent interest in using network analysis in finance and economics, with great emphasis on systemic risk and financial stability. Under this approach financial institutions are nodes that participate in a system (e.g. large-value payment, securities settlement) or market (e.g. interbank, derivatives), with their exposures or payments as their links. In a formal setting, financial institutions as well as their connections are represented in a network of mutual claims or flows, with elements arranged in a squared and potentially non-symmetric (i.e. non-reciprocal) matrix, with elements in the main diagonal equal to zero due to self-connections' absence or lack of economic interest.

Several methods or measures pertaining to the realm of network analysis have been used to assess the extent to which a default or failure-to-pay by a financial institution may affect others in an interconnected environment. A natural choice is to use centrality measures as proxies for financial institutions' systemic importance, and to use such measures to estimate their contagion potential in the network under analysis.

2.1 From centrality to DebtRank

The simplest measures of centrality, namely *degree centrality* and *strength centrality*, corresponding to the number of links and their weight, are not particularly useful for measuring contagion dynamics. They are local measures of centrality (i.e. non-adjacent nodes are not considered), thus they do not serve the purpose of estimating impact in a network-wide level. Path dependent centrality measures, namely *closeness centrality* and *betweenness centrality*, may take into account non-adjacent nodes by calculating how far nodes are in terms of the number of links that compose the shortest paths between them, and the fraction of those shortest paths that run through each node, respectively. However, measuring centrality based on the shortest path between financial institutions may be difficult to interpret in a financial contagion context (see Soramäki and Cook (2013)).

Feedback centrality refers to all those measures in which the centrality of a node depends recursively on the centrality of the neighbors (Battiston, Puliga, Kaushik, Tasca, & Caldarelli, 2012b). The simplest measure of feedback centrality is eigenvector centrality (Bonacich, 1972), whereby the centrality of a node is proportional to the sum of the centrality of its adjacent nodes. Thus, the eigenvector centrality of a financial institution is the weighted sum of all other financial institutions' centrality at all possible order adjacencies (see Newman (2010)). Eigenvector centrality's analytical value for measuring contagion dynamics is illustrated by Soramäki and Cook (2013), who depict eigenvector centrality as the proportion of time spent visiting each node in an infinite random walk through the network. Other popular feedback centrality measures based on eigenvector centrality are *PageRank* (Brin & Page, 1998), which is the algorithm behind Google's search engine; *hub centrality* and *authority centrality* (Kleinberg, 1998); and *SinkRank* (Soramäki & Cook, 2013).

All feedback centrality measures share a common drawback when applied to contagion dynamics: in presence of a cycle (i.e. a loop) in the network there is an infinite number of reverberations of the impact of a node to the others and back to itself, which impedes simple and measurable economic interpretations (Battiston et al., 2012). That is, despite they are useful by providing relative measures (i.e. scores) of the importance of each node, feedback centrality measures fall short when a monetary value of the size of contagion is required.

DebtRank (Battiston et al., 2012) is a centrality measure inspired in feedback centrality that overcomes this drawback by not allowing such infinite number of reverberations through the network. By excluding walks in which one or more links are repeated it has a measurable economic interpretation (see Appendix 1). As defined by Poledna et al. (2005), it is a quantity that measures the fraction of the total economic value in the financial network that is potentially affected by the distress of an individual node or a set of nodes. Moreover, DebtRank also accounts for the fact that when a default does not propagate in the form of a subsequent default there is still a contagion effect in the form of a reduction in the robustness (i.e. solvency) of those directly affected, and potentially in the robustness of the entire network. These two features allow DebtRank to provide a simple and economically meaningful measure of the size of the contagion dynamics following the default of a designated financial institution, and a forthright measure of its systemic importance.

Our methodological approach to determine the size of contagion caused by the default of a financial institution in an interbank exposures network is closely related to DebtRank. However, our approach does not rely on how the exposure among financial institutions may affect their capital buffer (i.e. a solvency issue) in case of a default by a designated financial institution, but on how it may affect their short-term liquidity. Hence, in our case we measure the depletion of short-term liquidity when financial institutions face the failure-to-pay of a participant of the interbank claims network. A straightforward byproduct is assessing the systemic importance of financial institutions in the local interbank market.

2.2 The inputs

Two main inputs are used in our approach: a proxy for the short-term liquidity of financial institutions participating in the interbank market, and the actual network of interbank financial claims.

The first input, a proxy for the estimated short-term liquidity position of the i -financial institution (\hat{l}_i), is our individual measure of financial robustness –instead of a proxy for solvency. We use the coverage provided by financial institution i 's high-quality liquid assets to meet the estimated net liquidity requirements for a 7-day horizon, as reported by local financial institutions to the Colombian Financial Superintendency.⁷ Hence, $\hat{l}_i(t)$ denotes the estimated short-term liquidity position of financial institution i at time t .

The calculation of the *IRL* involves the estimation of high-quality liquid assets' value and of net liquid requirements; therefore, its calculation is intricate, with several non-linear features.⁸ Nevertheless, for analytical purposes, we use the reported value of the expected short-term liquidity position (\hat{l}_i) as a proxy of the short-term liquidity position of each financial institution, and we affect it in a linear manner: say, not collecting \$1 in interbank loans due to counterparty's default will decrease the short-term liquidity position by \$1. This

⁷ This proxy corresponds to the 7-day *IRL* (*indicador de riesgo de liquidez*). *IRL* resembles the LCR by BCBS (2013). It is an indicator designed by the Colombian Financial Superintendency to gauge the liquidity risk of financial institutions for regulatory purposes. Using the 7-day *IRL* is interesting as it is a rather stringent measure of liquidity, and it is available on a weekly basis. An alternative proxy for the short-term liquidity may be the net liquid assets (i.e. liquid financial assets minus current liabilities), or some other balance-sheet measure of short term liquidity; however, as balance-sheet is a low-frequency source of data (e.g. monthly) our choice appears to be superior in terms of opportunity.

⁸ See Annex 1 of Colombian Financial Superintendency's Circular Externa 017 de 2014.

simplification not only allows designing a generalized version of the algorithm, but also makes changes in liquidity tractable, while preserving the analytical substance of the model.

The second main input in our approach is a directed weighted network in which nodes represent financial institutions participating in the interbank market, with links representing non-collateralized financial claims. Let C be the weighted matrix representing the network of interbank claims, with C_{ij} containing the outstanding amount that financial institution i owes to j .

If financial institution i is unable to refund an interbank loan to j , then j faces an unexpected reduction of its robustness, \hat{l}_i . It is an unexpected reduction because j could not anticipate i 's failure to pay when estimating its short-term liquidity position; that is, j had estimated its short-term liquidity position under the assumption that i would fulfill its commitment to refund.⁹ The unexpected reduction in short-term liquidity faced by i 's counterparties (i.e. the system) is $C_i = \sum_j C_{ij}$.

2.3 The dynamics

Whenever financial institution i fails to pay j the outstanding amount C_{ij} at moment t , the liquidity position of j is affected unexpectedly: $\hat{l}_j(t+1) = \hat{l}_j(t) - C_{ij}$. The aftermath of the updated short-term liquidity position of j depends on the choice of a short-term liquidity threshold that allows considering j as imposing (or not) a significant risk for the system. Let γ be such short-term liquidity threshold, j fails to pay its counterparties as a consequence of the failure of i to refund the outstanding amount C_{ij} whenever $\hat{l}_j(t+1) < \gamma$. In such case j enters into default (i.e. it is unable to pay), and the process continues recursively. On the other hand, if $\hat{l}_j(t+1) \geq \gamma$ j is affected but it does not default: j is able to fulfill its commitments to refund its counterparties, but its short-term liquidity position and that of the entire system have decreased (i.e. the robustness of j and the system has weakened).

⁹ As customary (see Battiston et al. (2012), Tabak et al. (2013)), because bankruptcy procedures may be rather lengthy, we assume that in the short-run there are no losses' recoveries. Likewise, as netting in interbank borrowing is not a common practice in the local market, we also assume that no netting of claims is available; however, netting may be appropriate for examining other types of financial exposures, say derivatives.

A natural choice for the short-term liquidity threshold is $\gamma = 0$. A financial institution i with a short-term liquidity position below zero may be considered in the limit of failing to fulfill its immediate commitments to pay: liquidating the stock of high-quality liquid assets would not suffice to face estimated short-term net liquidity requirements. Technically speaking this does not mean that i is in default or that it is unable to pay; it may still be able to get new funds from other financial institutions or the central bank, to roll-over existing loans or to increase deposits. Nonetheless, $\hat{l}_j < 0$ is a rather clear signal of a substantial exposure to potential liquidity risk, and it should force certain actions from the financial institution. Hence, for analytical purposes, we set $\gamma = 0$ for determining the tipping point of the default cascade, the threshold that determines the transition from undistressed to distressed.¹⁰

Formally, analogous to DebtRank, the dynamics are as follows. As before, \hat{l}_i is the short-term liquidity position of financial institution i , which is a continuous variable with $\hat{l}_i \in [-\infty, \infty]$. s_i is a discrete variable with three possible states, undistressed (U), distressed (D), and inactive (I), corresponding to institutions able, currently unable (i.e. in default), and already unable (i.e. defaulted earlier or with $\hat{l}_i < 0$) to refund their interbank loans, respectively ($s_i \in \{U, D, I\}$). Let $\hat{l}_i(0)$ denote the actual value of \hat{l}_i (i.e. the reported *IRL*), x be the set of financial institutions unable to pay (i.e. distressed or inactive) at $t = 1$, and γ the selected short-term liquidity threshold that determines the ability to pay, the initial conditions ($t = 1$) are:¹¹

$$\begin{aligned} \hat{l}_i(1) &= \hat{l}_i(0) \forall i \notin x & s_i(1) &= U \forall i \notin x \\ \hat{l}_i(1) &= \gamma \forall i \in x & s_i(1) &= D \forall i \in x \end{aligned} \tag{1}$$

¹⁰ Technically, a financial institution with a negative 7-day *IRL* may be able to pay its counterparties, and it may be solvent as well. Likewise, in DebtRank (Battiston et al., 2012) it is arguable that a financial institution may be viable (e.g. able to pay) even after the capital buffer against shocks is exhausted. In fact, as balance sheets are updated on a monthly basis, financial institutions may continue to function for days or weeks before the capital buffer is officially reported as exhausted. Another case is also possible: as in Müller (2006), solvent financial institutions may find themselves in default because they have no liquid assets to refund their borrowing.

¹¹ This means that at $t = 1$ two types of institutions may be unable to pay. Those selected as unable to pay by forcing their state to distressed irrespective of their short-term liquidity position (i.e. designated financial institutions), and those that have already a short-term liquidity position below the selected threshold (i.e. $\hat{l}_i < 0$).

Afterwards (i.e. $t \geq 2$), the dynamics of \hat{l}_i and s_i are determined by the specification below (in [2] and [3]). As usual, the dynamics depend on the initial conditions, namely the initial allocation of robustness ($\hat{l}_i(0)$), the structure of the interbank claims network (C_{ji}), and the initial choice of financial institutions in distress (x). The key in the dynamics is that the sum in [2] (i.e. the liquidity impact) arises from those j financial institutions that entered in distress in the preceding period (i.e. those j that are neither undistressed nor inactive).

$$\hat{l}_i(t) = \max \left\{ \gamma, \hat{l}_i(t-1) - \sum_{j | s_j(t-1)=D} C_{ji} \right\} \quad | \quad t \geq 2 \quad [2]$$

and

$$s_i(t) = \begin{cases} D & \text{if } \hat{l}_i(t) \leq \gamma \text{ and } s_i(t-1) \neq I \\ I & \text{if } s_i(t-1) = D \\ s_i(t-1) & \text{otherwise.} \end{cases} \quad [3]$$

The process continues recursively, and it is repeated for each financial institution with commitments to refund. The process for each i -financial institution stops at time T when all financial institutions are either inactive or undistressed (i.e. no distressed institutions pending to impact the system). The measure of the distress (in [4]) caused by the set x is the change in the overall short-term liquidity position of the system from $t = 1$ to T . If x is a single financial institution, such change is denoted F_i , and it gauges the impact of that i -financial institution in the system's ability to pay as measured by the variation in the short-term liquidity position of its counterparties (i.e. the initial distress in x is not considered). In this case, the nominal value of F_i and its contribution to all financial institutions' impact (\bar{F}_i) are, respectively,

$$F_i = \sum_j \hat{l}_j(T) - \sum_j \hat{l}_j(1) \quad [4]$$

$$\bar{F}_i = F_i / \sum_i F_i \quad [5]$$

As expected, F_i and \bar{F}_i provide a straightforward assessment of the systemic importance of financial institution i in the interbank funds market. The higher the distress caused by a financial institution in the robustness of its counterparties (i.e. their short-term liquidity position), the greater its systemic importance in the interbank funds market.

As pointed out by Tabak et al. (2013), it should be noted that adding the systemic importance of all financial institutions into a single figure ($F = \sum_i F_i$) may not be considered a measure of systemic risk or a measure of financial system's impact. As it is the sum of financial institutions' individual potential stress, it should be considered a proxy for financial system's stress. As usual, a measure of systemic risk would require multiplying the size of the individual potential stress (F_i) by the probability of its occurrence over a determined time horizon (as in Tabak et al. (2013) and Poledna et al. (2015)).

3 The data

Interbank exposures in \mathcal{C} are estimated by means of an implementation of Furfine's algorithm (Furfine, 1999) to data from the Colombian large-value payment system (see León, Cely, and Cadena (2016)).¹² Interbank exposures networks are available with daily frequency for April 1, 2013 – December 30, 2014 (i.e. 428 observations). During this period 33 financial institutions participated in the market¹³. Despite many other types of financial institutions are authorized to borrow and lend in the interbank funds market (e.g. investment funds, broker-dealer firms), actual participants are credit institutions only. As usual in non-collateralized funds markets around the world, most loans have a low time-to-maturity at inception: 78.9% are overnight loans, and the average maturity is about 2.6 calendar days.

Figure 1 exhibits a graph representing \mathcal{C} for a randomly selected date. Nodes represent financial institutions, with their height (width) corresponding to financial institutions' contribution to the total value of claims as a lender (borrower). The direction of the arrows represents the existence of an interbank claim (i.e. from the lender to the borrower), whereas

¹² Contrasting loans identified by implementing Furfine's method on Colombian large-value payment system data with those consolidated from financial institutions' reported data suggests that the algorithm performs well, and it is robust to changes in its setup (León et al., 2016).

¹³ In some days the number of participating financial institutions is lower.

their width represents its contribution to the total value of claims in the system. Interbank exposures in C let us follow the path of direct linkages considered by the algorithm.

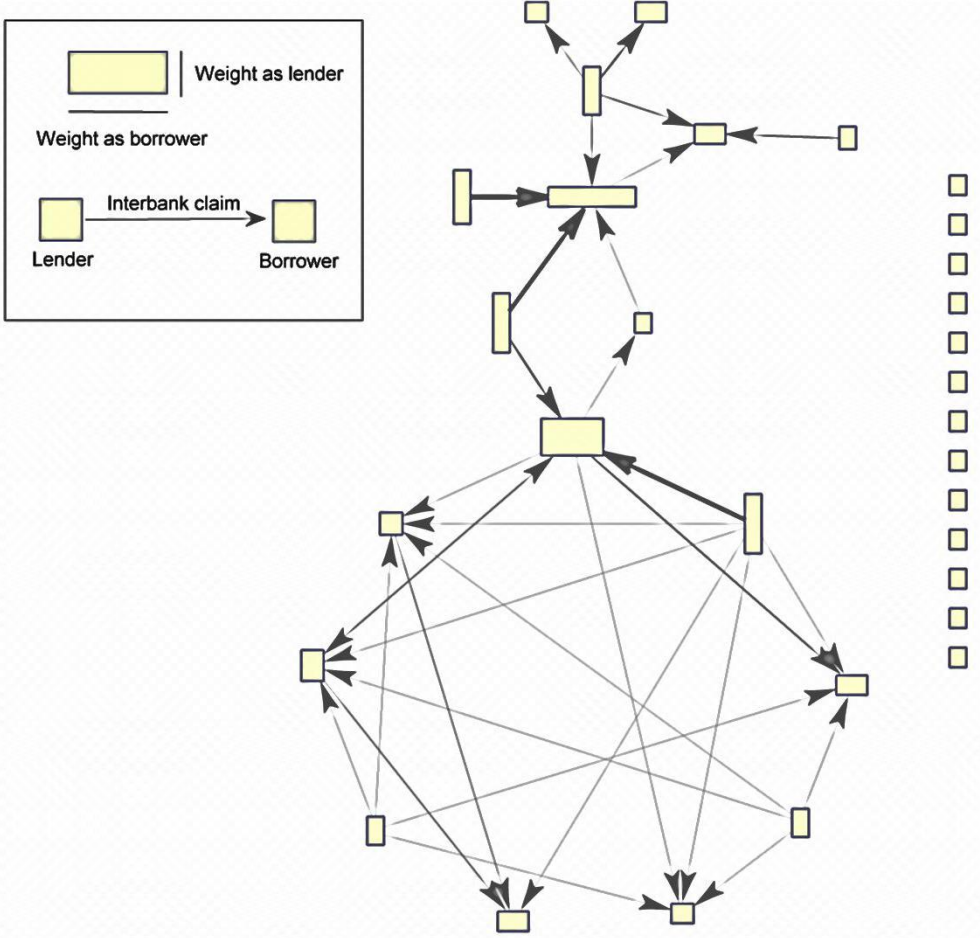


Figure 1. Interbank claims network for a randomly selected date. Nodes (in rectangles) correspond to participating financial institutions. The height (width) of each node corresponds to its contribution to the total claims of the market as a lender (borrower). The direction of the arrows represents the existence of an interbank claim (i.e. from the lender to the borrower), whereas their width represents its contribution to the total value of claims in the system. Non-connected nodes (in the right border of the graph) correspond to financial institutions without outstanding lent or borrowed amounts in the selected date. (Source: León et al. (2016)).

The proxy variable we use for the short-term liquidity position is the 7-day *IRL* calculated by the Colombian Financial Superintendency based on financial institutions' reports. This

indicator is available at a weekly frequency (each Friday) from January 4, 2013 to December 26, 2014 (i.e. 104 observations). As the proxy for the short-term liquidity position has the lowest frequency (i.e. weekly) and the least number of observations, this variable determines the period and the frequency of data used in the exercise. Thus, the sample period goes from April 5, 2013 to December 26, 2014, which corresponds to 90 weekly observations ($n = 90$).

In Colombia the short-term liquidity position (7-day *IRL*) exceeds the interbank (i.e. non-collateralized) exposures by two orders of magnitude (see Table 1). The mean (and maximum) interbank exposure is about 1.5% the mean (and maximum) short-term liquidity position. This is expected because the size of the local interbank funds market is rather small. Most liquidity exchanges between financial institutions in the money market consists of collateralized lending in the form of sell/buy backs (i.e. *simultáneas*), with the interbank (i.e. non-collateralized) market contributing with about 9.68% of the total.¹⁴ Despite the size of the interbank exposures appears to be negligible and incapable of resulting in sizeable liquidity contagion, examining how the short-term liquidity position is affected is relevant for analytical purposes.

<i>In Million COP (on daily data)</i>	Interbank exposures	Short-term liquidity position
Mean	16,299	1,098,813
Standard deviation	2,731	2,195,172
Maximum	215,500	14,391,923
Minimum	50	156

Table 1. Descriptive statistics for interbank exposures and short-term liquidity position datasets. In millions of Colombian pesos (COP), based on daily data for the 90 days under analysis. The short-term liquidity position exceeds the interbank exposures by two orders of magnitude. Only data with values greater than zero were used for the estimation of the statistics.

¹⁴ Based on 2014's figures (see Banco de la República (2015)), collateralized lending between financial institutions (i.e. sell/buy backs and repos) account for about 90.32% of money market transactions. Interbank (i.e. non collateralized) lending accounts for the residual (9.68%). Intraday interbank lending is not considered because it does not entail a financial exposure at the end of the day.

Other research works do not limit their analysis to non-collateralized borrowing –like we do. For instance, it is unclear whether Battiston et al. (2012) distinguishes between collateralized or non-collateralized investments (i.e. funding) among financial institutions. However, in the case of collateralized funding (e.g. repos, sell/buy backs) the default would be followed by a rather swift process of liquidating and collecting the cash value of the pledged collateral, thus rendering direct contagion as an unlikely outcome. Consequently, despite including collateralized borrowing could make contagion effects sizeable, they should have a negligible impact in our examination of direct contagion: the main impact arising from a default (i.e. principal risk) is minimized by pledged collateral.¹⁵ The same argument applies for foreign exchange and securities transactions that are settled under exchange-for-value arrangements (e.g. delivery-versus-payment).

Accordingly, instead of including collateralized funding or exchange-for-value transactions in order to magnify and examine the dynamics of liquidity contagion under debatable assumptions, we consider short-term illiquidity scenarios. We choose to examine the dynamics of liquidity contagion following an *ex-ante* generalized reduction in the short-term liquidity position equivalent to a fraction of observed short-term liquidity position (*IRL*). Let π be a fraction ($\pi \in [0, 0.99]$), \hat{l}^π is the short-term liquidity scenario after a drop of $\pi \times \hat{l}$, with $\hat{l}^\pi = (1 - \pi)\hat{l}$. We expect that illiquidity scenarios, consisting of reducing the initial short-term liquidity position of financial institutions, will reveal how the dynamics of liquidity contagion may occur in a hypothetical stress setup.¹⁶

¹⁵ Yet, other risks related to collateralized lending –not considered here- would remain, such as replacement cost risk arising from a collateral with market value below the refund value, and the potential fire-sale risk arising from the widespread liquidation of collaterals to face the default.

¹⁶ Moreover, the illiquidity scenarios considered, from 100% to 1% of 7-day *IRL*, allow for implicitly evaluating particularly interesting short-term liquidity levels. For instance, as reserve requirements are representative for the calculation of the *IRL* (i.e. the mean ratio of reserve requirements to *IRL* is about 24% for the selected sample), illiquidity scenarios corresponding to about 76% of the short-term liquidity are interesting to examine.

4 Main results

We choose to examine the dynamics of liquidity contagion following an *ex-ante* generalized reduction in the short-term liquidity position. 100 scenarios are selected, starting with a base scenario consisting of a null reduction ($\hat{l}^{\pi=0} = 1.00\hat{l}$), throughout a scenario consisting of short-term liquidity reduction equivalent to 99% of observed *IRL* ($\hat{l}^{\pi=.99} = 0.01\hat{l}$), with 1% increases ($\pi = 0, 0.01, 0.02, \dots, 0.99$). We expect the first scenario ($\pi = 0$) to show slight contagion effects –if any. Regarding the other 99 scenarios, we expect results to be monotonically increasing in the size of the reduction in short-term liquidity: the higher π (i.e. the size of *ex-ante* liquidity reduction), the higher the contagion effects.

First, we report the effect of contagion. For each day and illiquidity scenario, we examine the average and maximum change in the short-term liquidity position of the system, and the number of financial institutions entering into default as a result of contagion. Second, concerned about financial institutions' systemic importance, we report how designated individual financial institutions contribute to the contagion effect estimated for each day and illiquidity scenario.

4.1 Contagion effects

Figure 2 shows the mean contagion effects. Each (blue) line in Figure 2 corresponds to one of the 90 n -day estimated average contagion effects initiated by all financial institutions with outstanding claims in the interbank market. That is, lines display the average percent drop in financial system's short-term liquidity (y-axis) as a function of the selected illiquidity scenario ($\hat{l}^{\pi=0,0.01,0.02,\dots,0.99}$). The bold (red) line is the mean of the 90 lines.

As expected, the average contagion effect increases monotonically. Concerning the average contagion effect for the base case scenario ($\hat{l}^{\pi=0} = 1.00\hat{l}$), effects are bounded to a rather negligible reduction in short-term liquidity, between 0.00% and 0.11%. The greatest n -day average contagion effect in our sample is equivalent to a reduction of about 5.90% in short-term liquidity, but it occurs in the worst scenario ($\hat{l}^{\pi=.99} = 0.01\hat{l}$). It is straightforward that average contagion effects in short-term liquidity become relevant only after extreme illiquidity scenarios are considered (e.g. $\hat{l}^{\pi>.80}$).

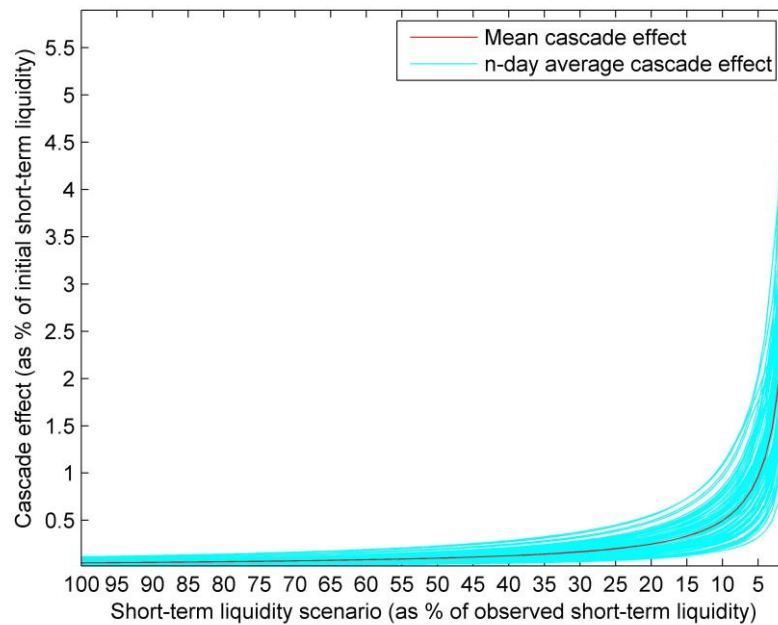


Figure 2. Average contagion effects. Each line corresponds to one of the 90 n -day estimated average contagion effects caused by all financial institutions with outstanding claims in the interbank market (y-axis), as a function of the selected scenario ($\hat{l}^\pi=0,0.01,0.02,\dots,0.99$). The bold line is the mean of the 90 lines.

Studying the average contagion may hinder interesting effects in networks that are characterized by an inhomogeneous connective structure. By focusing on the average effect we are implicitly relying on the existence of a typical financial institution, a misleading approach due to the well-documented heterogeneous distribution of linkages and their weights among institutions participating in financial networks.¹⁷ Therefore, as it is advisable

¹⁷ It is well-documented that most real-world networks are inhomogeneous, with particularly skewed distributions of their connections (i.e. degree) and their weights, allegedly following a power law distribution in the form of a scale-free network. Actual financial networks have also been characterized as particularly skewed, either following a power-law distribution of linkages (see Boss, Elsinger, Summer, and Thurner (2004), Inaoka, Ninomiya, Tanigushi, Shimizu, and Takayasu (2004), Sorämaki, Bech, Arnold, Glass, and Beyeler (2007), Bech and Atalay (2010)) or some other type of skewed distribution (see Martínez-Jaramillo, Alexandrova-Kabadjova, Bravo-Benítez, and Solórzano-Margain (2012), Craig and von Peter (2014), Fricke and Lux (2014)). In the Colombian case actual financial networks have been characterized as approximately following a power-law distribution of linkages and their weights, including interbank networks (see Cepeda (2008), León, Machado, and Sarmiento (2014), and León and Berndsen (2014)).

to study extreme cases in particularly heterogeneous systems –such as financial systems-, Figure 3 exhibits the maximum contagion effects.

Maximum contagion effect increases monotonically as well. The maximum contagion effect for the base case scenario ($\hat{l}^{\pi=0} = 1.00\hat{l}$) is bounded to a reduction in short-term liquidity between 0.00% and 1.21%, which is –once more- rather negligible. The greatest n -day maximum contagion effect in our sample is equivalent to a short-term liquidity reduction of about 45.78%, but it occurs –again- only after a rather extreme and very unlikely illiquidity scenario ($\hat{l}^{\pi=.99} = 0.01\hat{l}$).

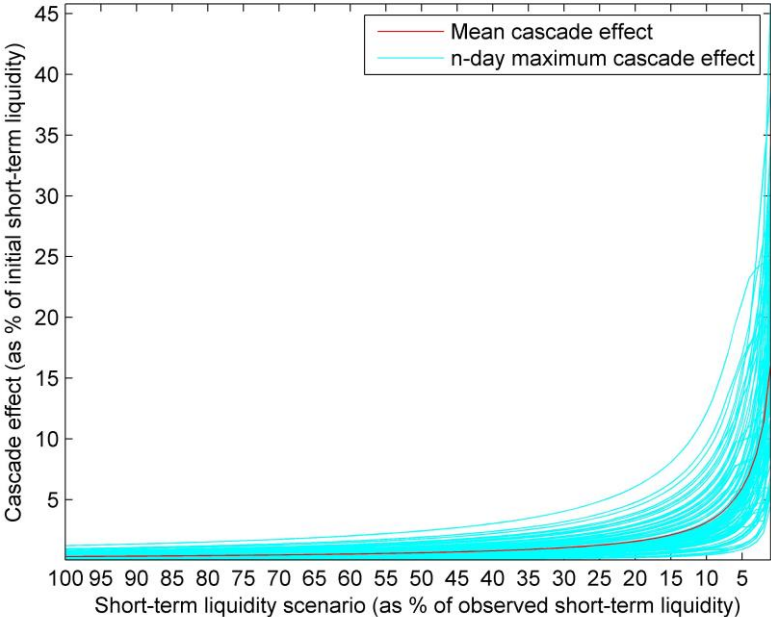


Figure 3. Maximum contagion effects. Each line corresponds to one of the 90 n -day estimated maximum contagion effects caused by all financial institutions with outstanding claims in the interbank market (y-axis), as a function of the selected scenario ($\hat{l}^{\pi=0,0.01,0.02,\dots,0.99}$). The bold line is the mean of the 90 lines.

Figure 4 compares the distribution of the average and maximum contagion effects for all financial institutions, and all illiquidity scenarios. As before, the average contagion effect is negligible, below 6% of the initial short-term liquidity for any financial institution or illiquidity scenario. The distribution of the maximum contagion effects displays sizeable

reductions in short-term liquidity, but they correspond to extreme illiquidity scenarios that appear to be implausible at best.¹⁸

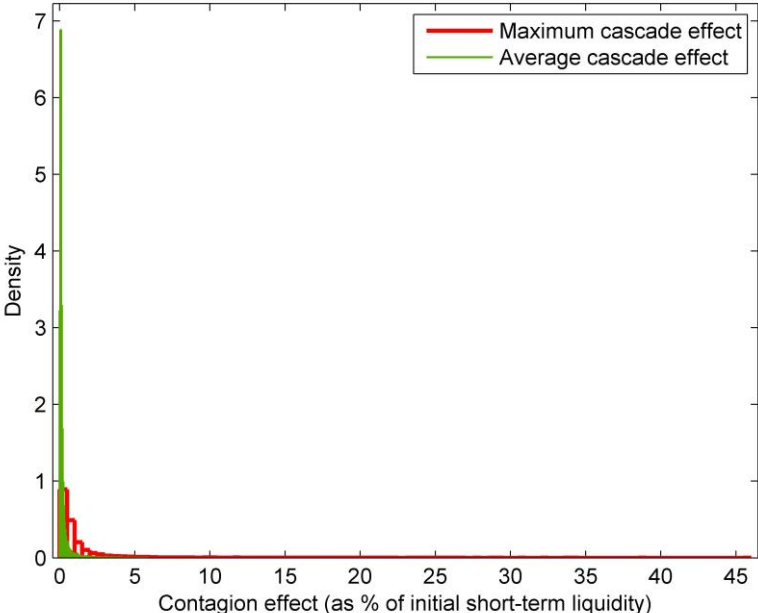


Figure 4. Distribution of average and maximum contagion effects. The average contagion effect is negligible. The distribution of the maximum contagion effects displays sizeable reductions in short-term liquidity, but they correspond to extreme illiquidity scenarios that appear to be implausible at best.

The time-series dynamics of potential contagion effects may be illustrative for monitoring purposes by financial authorities. For instance, tracking the dynamics of the average and maximum contagion effect for the base scenario ($\hat{l}^{\pi=0} = 1.00\hat{l}$) may help to identify changes in the potential outcomes of a default for the interbank market, and the potential liquidity needs that the system may face in such event. Correspondingly, Figure 5 presents the dynamics of the estimated average and maximum contagion effects throughout the sample in the absence *ex-ante* liquidity reductions. Consistent with previous results, in the base case scenario the interbank market would face an average drop in short-term liquidity in the 0.00%-0.11% range, whereas the maximum drop would be in the 0.00%-1.21% range.

¹⁸ It is quite likely that financial authorities will avoid these extreme scenarios by any means necessary (e.g. last-resort lending facilities, emergency acquisitions or bail outs, etc.).

Once again, contagion in this type of base case scenario appears to be minor, but their time-series dynamics may be worth monitoring by financial authorities.

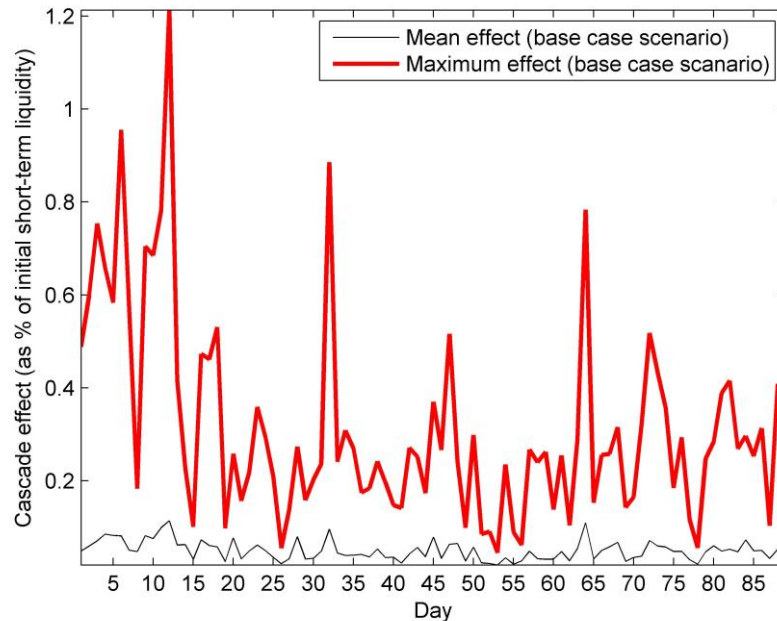


Figure 5. Contagion effects throughout the sample. This figure displays the average and maximum contagion effect arising from the default of a financial institution for each day in the sample in the base case scenario ($\hat{l}^{\pi=0} = 1.00\hat{l}$). Consistent with previous results, in this scenario the interbank market would face an average drop in short-term liquidity in the 0.00%-0.11% range, whereas the maximum drop would be in the 0.00%-1.21% range.

Estimating the effects caused by each financial institution defaulting under each illiquidity scenario for each of the 90 days in the sample yields 138,900 observations,¹⁹ of which 98.97% correspond to dynamics not leading to any default. That is, irrespective of the designated default or the illiquidity scenario, subsequent defaults caused by contagion are particularly rare. As exhibited in Figure 6, 1,197 (0.86%) observations correspond to one financial institution defaulting. Cascades consisting of two, three, four, five and six defaulting institutions are rare as well, and they are observed in 172 (0.12%), 44 (0.03%), 17

¹⁹ Observations result from multiplying the number of days (90) by the scenarios (100) by the number of financial institutions with outstanding borrowing in the interbank market in each day.

(0.01%), 3 (0.00%) and 1 (0.00%) occasions, respectively. Consequently, as expected from the size of the Colombian interbank market, contagion effects are rather minor, and they tend to occur as the illiquidity scenario becomes tougher (i.e. $\hat{l}^{\pi > .80}$).

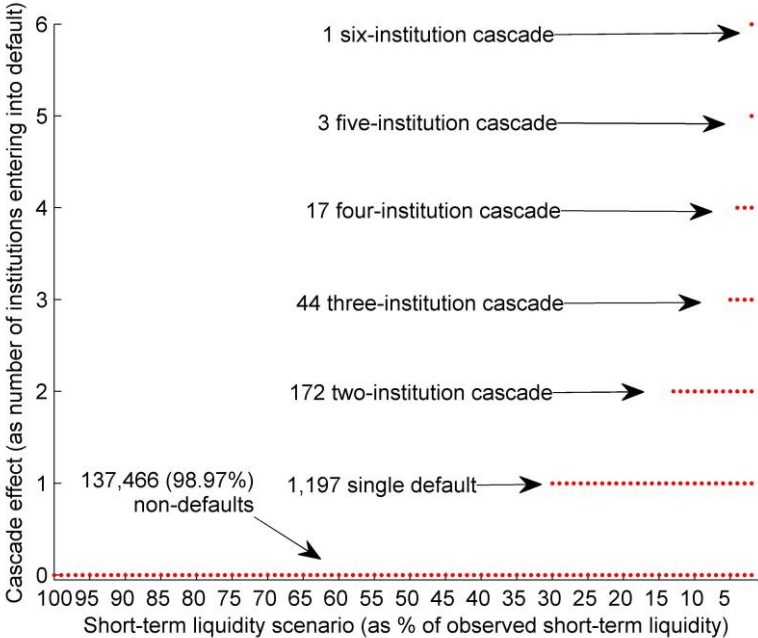


Figure 6. Number of financial institutions entering into default as a result of contagion. This figure displays the number of financial institutions that entered into default as a result of contagion dynamics (y-axis) for each one of the illiquidity scenarios (x-axis) for each of the 90 days in the sample. Each dot may represent more than one observation. Most of the observations (98.97%) correspond to no defaults.

All in all, it is rather evident that contagion effects by themselves are not a threat to the stability of the system under analysis. Irrespective of the metric employed (i.e. the reduction in short-term liquidity or the number of institutions entering into default), results tend to display negligible or non-substantial contagion effects. Unless a major drop in the short-term liquidity of all participants precedes the contagion dynamics, we consistently find that the interbank network is rather robust to average events (i.e. the default of an average financial institution). Likewise, most maximum contagion events are far from substantial, whereas a

major –but unlikely- drop precedes those that may be important for the short-term liquidity of all participants as well.

This result may be related to the size of the interbank market and its corresponding claims network. This lack of substantial contagion effects in the Colombian financial market is not limited to this study. Cepeda and Ortega (2015) also find that contagion in the Colombian large-value payment system is mitigated once high-quality assets are considered as potential sources of liquidity. Upper (2011) suggests that contagion due to exposures in the interbank loan market is an unlikely event in the sense that it happens in only a small fraction of the scenarios considered. In this vein, Roukny et al. (2013) report that contagion effects in financial networks are not substantial if no additional sources of distress (e.g. deposit runs, fire-sales, credit runs) are considered. Battiston et al. (2015) suggest that as financial regulation recommends financial institutions to keep individual credit exposures to a manageable limit (e.g. with respect to equity or total credit exposure), it is very unlikely that a single initial financial institution’s default triggers any other default. Therefore, our results regarding the limited impact of contagion effects in the local interbank market is an already documented trait of other financial markets.

4.2 Systemic importance of financial institutions

The previous section concluded that contagion effects are non-substantial. The number of financial institutions entering into default as a consequence of contagion dynamics is low, and it is rather an exceptional outcome that involves unlikely extreme illiquidity scenarios. Also, most reductions in short-term liquidity caused by contagion are non-substantial, and those that are non-negligible involve implausible extreme illiquidity scenarios as well. However, examining how individual financial institutions contribute to the occurrence of defaults and to the reduction in short-term liquidity may be illustrative about their systemic importance. The higher the contribution of financial institution i to contagion-related total short-term liquidity drops and defaults, the higher its systemic importance.

Figure 7 displays to what extent each financial institution (y-axis) contributes to the contagion-related total short-term liquidity reduction for all illiquidity scenarios. It is evident that the default of financial institution #26 contributes the most to reductions in system’s

short-term liquidity, about 14.2%. Accordingly, financial institution #26 may be easily deemed as the most systemically important for the interbank network under analysis in terms of its short-term liquidity effects. Financial institutions #24, 28, and 20 belong to a second tier of systemically important financial institutions contributing with about 8%-9% each, whereas those remaining contribute with less than 7% each.

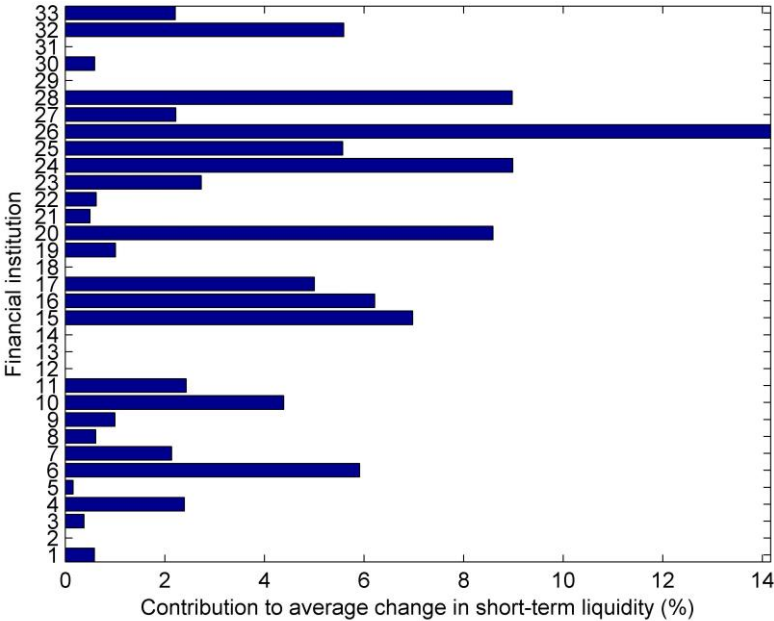


Figure 7. Financial institutions’ individual contribution to system’s short-term liquidity reduction for all illiquidity scenarios. The default of financial institution #26 contributes the most to reductions in system’s short-term liquidity, about 14.2%.

About the contribution to the total number of defaults caused by contagion effects, Figure 8 shows that financial institution #24 is the most representative (21.2%), and –hence- it may be considered the most systemically important financial institution in the Colombian interbank market in terms of subsequent defaults. The second financial institution is #11 (17.6%). Financial institutions #17 and 26 belong to a third tier of systemic importance, contributing with about 11% and 10%, respectively. The remaining financial institutions contribute with less than 6% each.

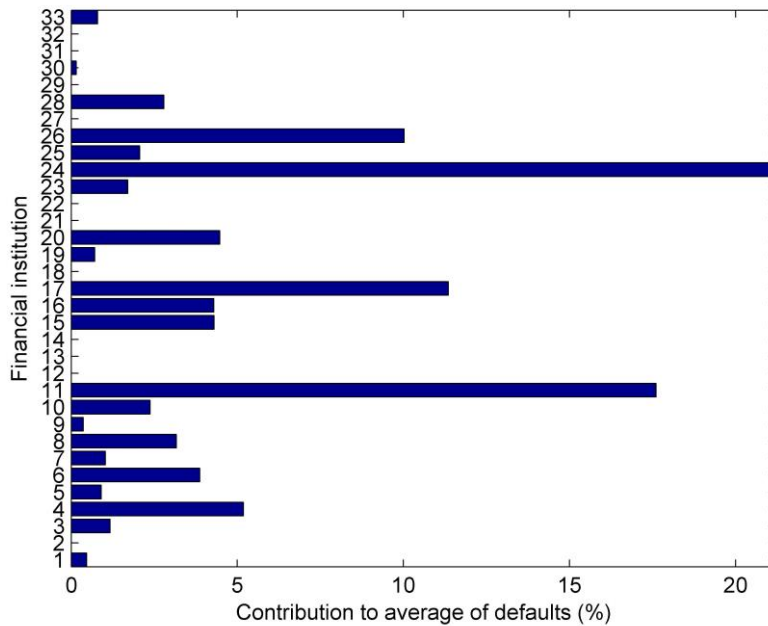


Figure 8. Financial institutions’ individual contribution to system’s total defaults for all illiquidity scenarios. The default of financial institution #24 contributes with 21.2% of the defaults.

As expected when assessing financial institutions’ systemic importance, we find that the negative effects resulting from contagion are decidedly concentrated in a few of them, namely in financial institutions #26, 24, and 11. However, as most contagion effects here portrayed are conditional on the occurrence of major –but very unlikely- scenarios of generalized illiquidity, conclusions about the systemic importance of these financial institutions for the entire financial system may be unjustified. Furthermore, their systemic importance is bounded to the local interbank network, which is not particularly representative of the whole financial system in the Colombian case.

5 Final remarks

We took advantage of the DebtRank methodology (Battiston et al., 2012) in order to examine how the default of a selected financial institution in the Colombian interbank network impacts the short-term liquidity position of its counterparties and the system as a whole. Instead of

focusing on the impact of default on financial institutions' capital buffer (i.e. their solvency), we focused on how an initial default eroded their ability to refund interbank loans (i.e. their short-term liquidity) and eventually forced them into default.

Consistent with literature on direct-linkage financial contagion (Furfine, 2003; Upper, 2011; Roukny et al., 2013; Cepeda & Ortega, 2015), contagion effects resulting from an initial default in the interbank market are non-substantial. Unless contagion dynamics are preceded by a major –but unlikely– drop in the short-term liquidity position of all participants, we find that contagion effects on individual and system's short-term liquidity are negligible. Our results are consistent with reported features of banking crisis, which tend to be caused by shocks that hit several banks simultaneously rather than domino effects from idiosyncratic failures (see Upper (2011)). Likewise, our results concur with those reported by Roukny et al. (2013), who find that network topology matters only when financial markets are under stress (e.g. illiquid).

The methodological contribution of our work is relevant. By modifying DebtRank to recursively measure contagion effects in the short-term liquidity position of financial institutions we supplement financial authorities' monitoring tools. In this sense, we capture the advantages of DebtRank to conveniently measure how contagion may affect financial institutions' ability to refund interbank loans in the short-term.

Despite the lack of systemic impact of contagion effects in the base case scenario, our results are valuable for financial authorities as well. The numerical outcomes provide an economically meaningful quantitative assessment of the systemic importance of financial institutions based on their potential effect in financial institutions' short-term liquidity. Moreover, based on the potential effect on the system's liquidity, our results provide a quantitative assessment of the liquidity that should be obtained from other available sources in case of a default by a financial institution, such as collateralized borrowing (e.g. from other financial institutions or the central bank), selling financial assets or increasing deposits. Nevertheless, as most contagion effects here portrayed are conditional on the occurrence of major –but unlikely– scenarios of generalized illiquidity, conclusions about the systemic importance may be unjustified. Consequently, it is important to emphasize that systemic

importance resulting from this exercise is bounded to the local interbank network, which may not be particularly representative of the whole financial system in the Colombian case.

Due to the aim and scope of our research work there are several issues that should be addressed in order to enhance the examination of financial contagion in the Colombian case. For instance, as in Müller (2006), it is advisable to simultaneously examine the impact of default contagion on solvency and liquidity. Estimating how financial institutions react to their counterparties' defaults (see Martínez and Cepeda (2015)) and incorporating such reactions in the contagion dynamics may enrich the analytical reach of the model as well; reactions by financial authorities should be interesting to consider too. Additionally, as in Tabak et al. (2013) and Poledna et al. (2015), it is imperative to articulate this type of systemic importance assessment with the estimation of default probabilities to assess systemic risk as financial systems' expected impact over a determined time horizon. Furthermore, as illustrated in the multi-layer financial exposures network model by Poledna et al. (2015), it is convenient to link different sources of exposures among financial institutions (e.g. derivatives, security cross-holdings) in order to have a comprehensive measure of direct-linkage contagion; in this vein, it is likely that the non-substantial contagion effects here reported may be due to the underestimation of systemic impact that results from focusing on the interbank market only. Finally, it is also convenient to couple direct- (e.g. mutual exposures) and indirect-linkage (e.g. fire-sales, deposit runs, credit runs) contagion models with the aim of attaining a comprehensive measure of financial contagion.

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7 Appendix 1: DebtRank

As noted by Battiston et al. (2012), there are two variables associated to each node in a financial exposures network. One that measures each financial institution's level of distress (h_i) and another (S_i) that denotes three possible states that this financial institution may take: undistressed (U), distressed (D) and inactive (I). The individual level of distress (h_i) is a continuous variable that takes a value in the zero-one closed interval $[0, 1]$. Thus, $h_i(t) = 0$ corresponds to an undistressed financial institution whereas $h_i(t) = 1$ belongs to a financial institution in default:

$$h_i(t) = \min \left\{ 1, \quad h_i(t-1) + \sum_{j|S_j(t-1)=D} W_{ij} h_j(t-1) \right\} \quad [6]$$

For a given point in time t , the dynamics for the $i - th$ node (financial institution) are given by the minimum value between one and its updated level of distress. This updated level depends on its own level of distress registered in the prior period ($h_i(t-1)$) and the distress level that financial institution i received from its counterparties (represented by the summation of the impacts caused by all the $j - th$ institutions that entered into distress in the former period ($h_j(t-1)$)).

The weights matrix (W) required to compute the individual level of distress ($h_i(t)$) contains impacts measured as the minimum value between one and the ratio of the total amount invested by a financial institution i in the funding of j (A_{ij}) to the level of capital of that financial institution (E_i): $W_{ij} = \min \left\{ 1, \frac{A_{ij}}{E_i} \right\}$. If node j defaults, node i suffers a loss equal to A_{ij} . As long as its level of capital overpass that loss ($E_i > A_{ij}$) the impact of node j on node i is given by the liabilities-to-capital ratio, otherwise, that impact is equal to one (indicating that node i entered into default).

The individual level of distress (given by [6]) can be computed only for $t \geq 2$. For $t = 1$ an initial condition should be imposed in order to make this expression mathematically possible. This initial condition consists of setting $h_i(1) = \psi$, $\forall i \in S_f$, where the (assumed) initial

level of distress is ψ , and S_f is the set of distressed nodes at $t = 1$. It is also assumed that $\psi \in [0, 1]$, and that $\psi = 1$ represents the distressed node (Battiston et al., 2012). Therefore, for $t \geq 2$ equation [6] determines the DebtRank dynamics, understood as the cases based on impacts that affect the nodes irrespective of whether default occurred (Battiston et al., 2015). The procedure continues computing impacts until all nodes in the network are either undistressed (U) or inactive (I). At that point the dynamics stop and the DebtRank (DR) measure can be calculated as:

$$DR = \sum_j h_j(T)v_j - \sum_j h_j(1)v_j \quad [7]$$

In equation [7] the economic value of a node is given by v_j , and is measured by financial institution's assets invested as a fraction of the total assets invested in the market ($v_j = A_j / \sum_j A_j$). Hence, DebtRank measures the distress of the entire system excluding the initial (assumed) level of distress (second term in equation [7]). In economic terms, this measure computes the total loss in the system (measured in monetary terms) generated by the assumed initial default (Battiston et al., 2012).

Several authors have remarked the advantages of DebtRank, in contrast to other measures of systemic distress in a network (Battiston et al., 2012, Thurner & Poledna 2013, and Tabak et al., 2013). In particular, the DebtRank measure has an economic interpretation in monetary terms and, also, it is considered a good early-warning indicator candidate. Likewise, the computation of distress by means of DebtRank excludes the possibility of double-counting the impacts of a default. In other words, once a shocked financial institution has affected its counterparties it enters into an inactive state (I), which permits that this institution be impacted by shocks coming from other participants in the market but blocks the re-transmission of these shocks. For this reason, unlike eigenvector centrality or *PageRank*, it is recognized that under the DR measure cycles have finite reverberation (Battiston et al., 2012).



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