Borradores de ECONOMÍA

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A Macro CGE Model
for the Colombian Economy[*]

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Abstract

This paper presents the construction of a tailor-made Macro Computable General Equilibrium Model for the Colombian economy that satisfies Banco de la República's macroeconomic programming and forecasting interests. Using information on the national accounts divulged by the National Statistics Department (DANE), we set an easily updatable Macro Social Accounting Matrix that serves as a starting point for the model parameters calibration and estimation.

Keywords: Social Accounting Matrix, Computable General Equilibrium Models, Colombian Economy, Macroeconomic Programming.

JEL: C67, C68, D57, D58

1. Introduction

Applied economic policy analyses need both a theoretically consistent framework and a solid dataset that represents the current state of an economy. Applied Computable General Equilibrium Models (CGEM) provide the tools to achieve such goals. These models have been found useful in a wide range of policy-assessment oriented applications, including, trading policies, environmental impacts, taxation and fiscal effects, productivity shocks and economic growth, income distribution, among other important topics (Dixon and Jorgenson, 2013).

Any CGEM consists of two main parts: The first one, a square table known as the Social Accounting Matrix (SAM), which is a detailed and coherent summary of the transactions

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and flows of goods and services and the cost structure of a given economy in a certain (fixed) period of time. The second one, a whole set of equations that analytically model these transactions and flows, consistent in a microeconomic sense and in accordance to a (often competitive) general equilibrium framework. Some examples of CGEM for Colombia are those in Bussolo et al. (1998) and Rutherford and Light (2002). A much broader compilation of pioneering works on CGE modeling for the Colombian economy can be found in López et al. (1994). Although possible, a deeper literature review on this topic is beyond the scope of this paper.

This paper is intended to present a tailor-made Macro-CGEM that meets the needs at the Central Bank of Colombia for incorporating the Balance of Payments macroeconomic programming results alongside with other projections made for relevant macroeconomic variables for a small and open economy. The Macro-SAM built for this model is presented in section 2, whilst the key equations for this model are presented in section 3. Parameter calibration is explained in section 4, while section 5 and 6 document a sensibility analysis related to the choice of the elasticities in the model. Section 7 concludes and poses further relevant extensions to this model.

2.   A Macro Social Accounting Matrix for Colombia

A Social Accounting Matrix (SAM) is a compact representation of the economic structure of a country. Although summarized, a SAM contains the relevant transactions that occur between agents in the economy, the existing flows of goods and services and the cost structure of the production (supply) side of the economy. More detailed SAMs often include a description of the taxation structure, welfare distribution by quantiles of income, international trade partners, etc. Its size and details depend largely on the available information and its final purpose. A very comprehensive manual for SAMs construction and their applications is that of Pyatt and Round (1985). Some examples of detailed SAMs built for the Colombian economy are those in Valderrama and Gutiérrez (1995), Bussolo and Correa (1998), Prada and Guzmán (2002), Ramírez and Prada (2000), Karl (2004), Jensen and Karl (2004), Hernández (2003), Corredor and Pardo (2008) and Céspedes (2011). These SAMs served as basis for the implementation of specialized CGE models with financial, health policies and taxation impact aims.

The construction of the SAM supporting the Macro-CGE model presented in this paper does not involve the detail available for the other matrices mentioned above, since the main objectives of this model do not require it. In this section we describe in detail the construction of a tailor-made Macro Social Accounting Matrix (Macro-SAM), benchmark year 2011 (extended to 2012 as well). This Macro-SAM is an aggregation of an actual SAM and is also largely based on the guidelines presented in Lora (2008, chapter 13). We recall that the main advantage of this Macro-SAM structure is that it can be updated every year when DANE publishes the annual national account results (i.e. the Supply-Demand Balance and the Integrated Economic Accounts matrices). Therefore, the counterfactual results derived from the model's application are compared to a up-to-date baseline scenario.

The Macro-SAM we present in this section aggregates the economy over six different dimensions, as seen in Figure 1:

1. Factors (F),

2. Production and Products (P),

3. Distribution (D),

4. Agents (A),

5. Rents, Taxes and Transfers (T), and

6. Savings-Investment Balance (B).

The Macro-SAM is read as it follows: every non-empty sub-matrix arising from the intersection of each of the dimensions above represents a set of transactions in the economy. Columns represent purchases or payments and rows represent sales or recipients, e.g, the cell FAC (in P, column) - L (in F, row) represents the total payment from the account factor aggregation (FAC) to labor supply (L) for its marginal product in the aggregate value production process. The sub-matrix P-FAC accounts for the marginal product paid to each production factor (labor, capital and mixed income).

Every non-empty slot in the Macro-SAM shown in Figure 1 represents a transaction between agents in a certain stage of the production or income distribution or demand blocks of the economy, as explained in section 3.1. These transactions are also modeled explicitly through equations that are presented in section 3.2.

3. A Macro CGE Model

3.1. The Economy

The structure of the economy, which is based on the National Accounts and the SAM results is summarized in the scheme shown in Figure 2. In this economy there is an aggregated good produced by a representative activity in the supply block that demands three different production factors: Capital (K), Labor (L), and Mixed Income (Z), the latter arising from the impossibility of successfully classifying this factor's remuneration into either Capital or Labor. These three factors are aggregated into a single production factor, FAC, which is the base for added value (AV) formation. Jointly with indirect taxes net of subsidies, AV forms Gross Domestic Products (Y), which added to intermediate consumption (IC) yields domestic supply (OUT). Total supply (ACT) results from adding up OUT and imports, M.

Total supply is then distributed between IC (meeting the former IC demand) and final consumption (FC), which in turn is further divided into household's Consumption (C), Investment (I), Government's expenditure (G), and Traditional (X_T) and Non-traditional Exports (X_{NT}). These types of FC are demanded by four institutional agents, namely: Households (HH), Firms (FR), the Government (GV) and the Rest of the World (RW). Transactions that

Figure 1 — Macro Social Accounting Matrix, 2011 (values in the cells; row labels with their codes on the left, column codes across the top).

Account	L	K	Z	FAC	AV	Y	iCD	OUT	M	ACT	ICS	FC	C	I	ihh	Ifr	Igv	Ipr	G	X	Xt	Xn	HH	FR	GV	RW	R	TXv	TXyy	TRf	Thh	Tac	SC	SB	CT	PT	SI	TOT.
	Factors			Production and products										Distribution									Agents				Rents taxes and transfers										Bal.	
LABOUR (L)				196																																		196
CAPITAL (K)				223																																		223
MIXED (Z)				135																																		135
PRODUCTION (FAC)					554																																	554
ADDED VALUE (AV)						567																																567
GROSS DOMESTIC PRODUCT (Y)								622																														622
DEMAND OF INT. CONSUMP. (ICD)								479																														479
OUTPUT (OUT)										1100																												1100
IMPORTS (M)										123																												123
ACTIVITY (ACT)											479	745																										1223
SUPPLY OF INT. CONSUMP. (ICS)							479																															479
FINAL CONSUMPTION (FC)													423	148					56		81	37																745
PRIVATE CONSUMPTION (C)																							423															423
INVESTMENT (I)															35	92	21																					148
HOUSEHOLDS INVESTMENT (ihh)																		35																				35
FIRMS INVESTMENT (Ifr)																		92																				92
GOVERNMENT INVESTMENT (Igv)																																					21	21
PRIVATE INVESTMENT (Ipr)																																					126	126
GOVERNMENT EXPENDITURE (G)																									56													56
EXPORTS (X)																										118												118
TRADITIONAL EXPORTS (Xt)																				81																		81
NON-TRADITIONAL EXPORTS (Xn)																				37																		37
HOUSEHOLDS (HH)	198	26	135																								59							51	25	43		538
FIRMS (FR)		191																									46						18					254
GOVERNMENT (GV)		6																									25	13	49	5	7	31	36		3			176
REST OF THE WORLD (RW)	-2								123																		27											148
RENTS (R)																							11	130	16													157
PRODUCTION TAXES (TXva)					13																																	13
PRODUCT TAXES (TXyy)						49																																49
IMPORT TARIFFS (TRff)						5																																5
DIRECT TAXES TO HH (Thh)																							7															7
DIR TAXES TO FR & GV (Tac)																								31														31
SOCIAL CONTRIBUTIONS (SC)																							53															53
SOCIAL BENEFITS (SB)																								5	46													51
CURRENT TRANSFERS (CT)																								18		10												28
PRODUCT TRANSFERS (PT)																									43													43
SAVINGS-INVESTMENT BAL. (SI)																							43	70	15	20												148
TOTAL	196	223	135	554	567	622	479	1100	123	1223	479	745	423	148	35	92	21	126	56	118	81	37	538	254	176	148	157	13	49	5	7	31	53	51	28	43	148	

Figure 1: Macro Social Accounting Matrix, 2011.
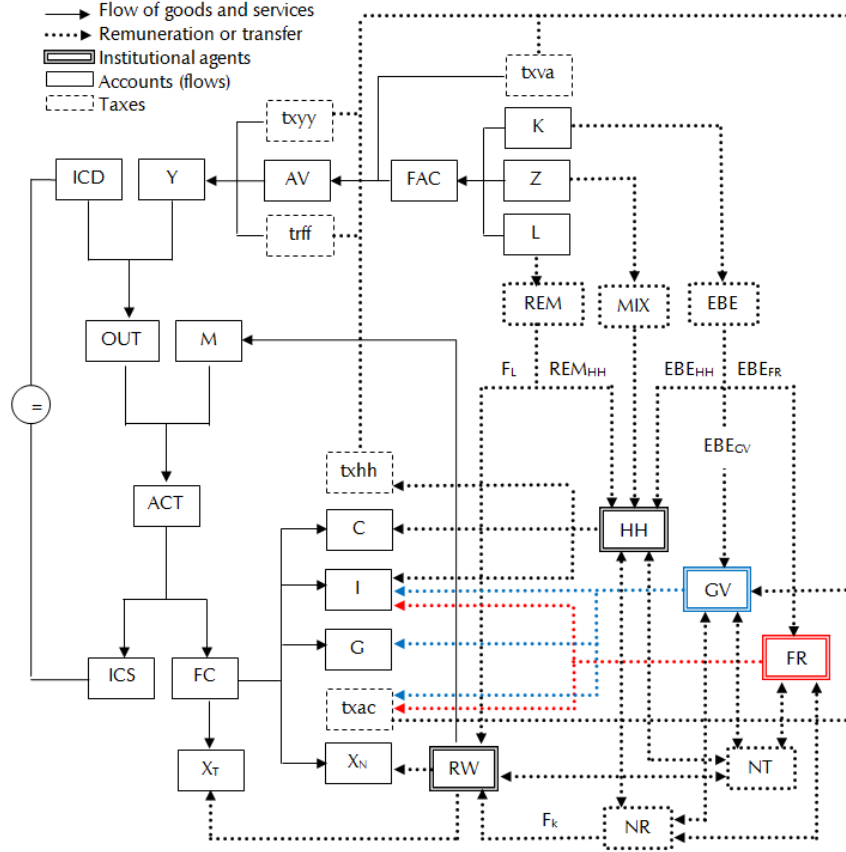Source: Author's calculations using national accounts information published by DANE.

Figure 2: The Economy.

involve FC demand from any of the institutional agents are framed in the demand block of the model.

The third block of this model is the income generation block. Institutional agents inelastically supply factors of production, and therefore receive optimal factor remuneration (i.e. wages (REM) to HH and RW, capital cost (EBE) to HH, FR and GV, and mixed income payments (MIX) to HH) which results from matching demand needs in the production block and supply conditions in this block, as shown in Figure 2. Also, due to natural economic transactions between agents, primary and secondary income distribution is achieved after accounting for Net Transfers (NT) and Net Rents (NR).

3.2. The Model

The economic transactions briefly described above are explicitly modeled through a large set of equations that aim to set an economic problem in every stage of the supply, demand and income generation sides of the economy. These equations are presented in the following subsections.

3.2.1. The Supply Side

In the supply side of the economy production factors, indirect taxes, intermediate consumption and imports are combined to create total supply of a single representative (aggregated) good. This process involves solving three different cost minimization problems: First, the firm solves for factor demands in a third-level-nested cost minimization problem. Second, it solves for combination of GDP and intermediate consumption in a second-level cost minimization problem. Lastly, the firm solves for combination of domestic output (supply) and imports in the first-level minimization cost problem. As a result, total supply, ACT, is obtained.

On the second stage of the supply side, firm maximizes its revenue by optimally solving a second-level-nested profit maximization problem which involves solving for the FC and IC distribution problem on the first part and then the FC distribution problem, which is the actual link to demand side of the economy.

Factor Demand Problem, Added Value and Gross Domestic Product

Suppose the firm is able to merge the three production factors in our economy into a single factor (FAC). The latter, along with production taxes, yields added value. Therefore, firm optimally minimizes its expenditure in production factors (1), subject to a Constant Elasticity of Substitution (CES from now on) technology for factor aggregation (2), as in:

$$\min_{\{L,K,Z\}} p_L L + p_K K + p_Z Z, \tag{1}$$

$$FAC = \theta_F \left(\pi_L L^{\frac{\sigma_F-1}{\sigma_F}} + \pi_K K^{\frac{\sigma_F-1}{\sigma_F}} + \pi_Z Z^{\frac{\sigma_F-1}{\sigma_F}} \right)^{\frac{\sigma_F}{\sigma_F-1}} \tag{2}$$

Recall that elasticity of substitution among factors satisfies $\sigma_F > 0$. From the first order conditions (FOC) we derive the optimal demand of factors:

$$L = \left(\theta_F \pi_L \frac{p_F}{p_L} \right)^{\sigma_F} \frac{FAC}{\theta_F} \tag{3}$$

$$K = \left(\theta_F \pi_K \frac{p_F}{p_K} \right)^{\sigma_F} \frac{FAC}{\theta_F} \tag{4}$$

$$Z = \left(\theta_F \pi_Z \frac{p_F}{p_Z} \right)^{\sigma_F} \frac{FAC}{\theta_F} \tag{5}$$

where the aggregated price of factors p_F is expressed as

$$p_F = \frac{1}{\theta_F} \left(\pi_L^{\sigma_F} p_L^{1-\sigma_F} + \pi_K^{\sigma_F} p_K^{1-\sigma_F} + \pi_Z^{\sigma_F} p_Z^{1-\sigma_F} \right)^{\frac{1}{1-\sigma_F}} \tag{6}$$

Added Value (in nominal terms), AV, is completed once indirect production taxes are acknowledged, therefore we have that:

$$p_{AV} AV = p_F FAC + TX_{va} \tag{7}$$

where tax revenue in production is given by:

$$TX_{va} = tx_{va} p_F FAC \tag{8}$$

Note that in the base year prices are all set equal to one, therefore combination of Equations (7) and Equation (8) yields

$$AV = FAC \ (1 + tx_{va}) \tag{9}$$

GDP (Y) supply is obtained by adding up nominal AV, indirect taxes and import tariffs, as it follows:

$$p_Y Y = p_{AV} AV + TX_{yy} + TR_{ff} \tag{10}$$

where indirect (net) taxes over products (TX_{yy}) and import tariffs (TR_{ff}) are given by a share of nominal AV and imports:

$$TX_{yy} = tx_{yy} p_{AV} AV, \text{ and } TR_{ff} = tr_{ff} p_M M \tag{11}$$

Domestic Supply Problem

GDP and IC combined yield total domestic supply of the representative good., therefore the firm chooses optimal combination of GDP and intermediate consumption in the domestic output's second-level cost minimization problem, which involves solving for the following optimization problem, subject to a CES technology of aggregation, with elasticity of substitution between GDP and intermediate consumption satisfying $\sigma_O > 0$,

$$\min_{\{Y, IC\}} p_Y Y + p_{IC} ICD \tag{12}$$

$$OUT = \theta_O \left(\pi_Y Y^{\frac{\sigma_O - 1}{\sigma_O}} + \pi_{ICD} ICD^{\frac{\sigma_O - 1}{\sigma_O}} \right)^{\frac{\sigma_O}{\sigma_O - 1}} \tag{13}$$

As above, from the FOC of this problem optimal GDP and Intermediate Consumption demands are derived and expressed as

$$Y = \left(\theta_O \pi_Y \frac{p_O}{p_Y} \right)^{\sigma_o} \frac{OUT}{\theta_O} \tag{14}$$

$$ICD = \left(\theta_O \pi_{ICD} \frac{p_O}{p_{IC}} \right)^{\sigma_o} \frac{OUT}{\theta_O} \tag{15}$$

where the aggregated price of domestic output p_O is given by

$$p_O = \frac{1}{\theta_O} \left(\pi_Y^{\sigma_O} p_Y^{1-\sigma_O} + \pi_{ICD}^{\sigma_O} p_{IC}^{1-\sigma_O} \right)^{\frac{1}{1-\sigma_O}}. \tag{16}$$

Recall that with GDP supply in Equation (10) and demand in Equation (14) one can easily derive the price at which the market clears, p_Y, as:

$$p_Y = \left[\frac{p_{AV} AV + TX_{yy} + TR_{ff}}{(\theta_O \pi_Y p_O)^{\sigma_o} \frac{OUT}{\theta_O}} \right]^{\frac{1}{1-\sigma_O}} \tag{17}$$

Aggregated Supply Problem

Total supply is the result of the sum of domestic output, Y, and imports, M. Optimal demands from both inputs are estimated from the first-level cost minimization problem given by

$$\min_{\{OUT,M\}} p_O OUT + p_M M \tag{18}$$

subject to a CES technology of aggregation, with elasticity of substitution between output and imports of $\sigma_A > 0$,

$$ACT = \theta_A \left(\pi_O OUT^{\frac{\sigma_A-1}{\sigma_A}} + \pi_M M^{\frac{\sigma_A-1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A-1}} \tag{19}$$

From FOC derived from the latter, optimal domestic output and imports demands are, respectively

$$OUT = \left(\theta_A \pi_O \frac{p_A}{p_O} \right)^{\sigma_A} \frac{ACT}{\theta_A} \tag{20}$$

$$M = \left(\theta_A \pi_M \frac{p_A}{p_M} \right)^{\sigma_A} \frac{ACT}{\theta_A} \tag{21}$$

with aggregated price of the activity, ACT, p_A given by

$$p_A = \frac{1}{\theta_A} \left(\pi_O^{\sigma_A} p_O^{1-\sigma_A} + \pi_M^{\sigma_A} p_M^{1-\sigma_A} \right)^{\frac{1}{1-\sigma_A}} . \tag{22}$$

Again, from OUT supply determined in Equation (13) and OUT demand in Equation (20) we solve for the price p_O that fulfills the market clearing condition:

$$p_O = \theta_A \pi_O \left[\frac{\frac{ACT}{\theta_A}}{\theta_O \left(\pi_Y Y^{\frac{\sigma_O-1}{\sigma_O}} + \pi_{ICD} ICD^{\frac{\sigma_O-1}{\sigma_O}} \right)^{\frac{\sigma_O}{\sigma_O-1}}} \right]^{\frac{1}{\sigma_A}} p_A \tag{23}$$

Additional considerations are imposed upon ACT formation. First one follows from the clearing market condition, which assures that total supply ACT in nominal terms equals the sum of total nominal domestic supply and total external nominal supply:

$$p_A ACT = p_O OUT + p_M M$$

$$ACT = \frac{p_O OUT + p_M M}{p_A} \tag{24}$$

Also, we assume that RW elastically supplies imports at an international price p_M^*. Therefore, domestic import price is given by the expression

$$p_M = e \cdot p_M^* , \tag{25}$$

where e is the nominal exchange rate.

Supply Distribution Problem: First Stage

On a first stage of supply distribution, ACT is distributed between intermediate (IC) and final consumption (FC). The firm determines optimal amounts of IC and FC by maximizing revenue from sales:

$$\max_{\{IC,FC\}} p_{IC}ICS + p_{FC}FC \tag{26}$$

subject to a CET technology of distribution with elasticity of transformation between intermediate and final consumption is $\tau_A < 0$,

$$ACT = \theta_{AD}\left(\pi_{ICS}ICS^{\frac{\tau_A-1}{\tau_A}} + \pi_{FC}FC^{\frac{\tau_A-1}{\tau_A}}\right)^{\frac{\tau_A}{\tau_A-1}} \tag{27}$$

Yet again, from the FOC derived for this problem, optimal supply of intermediate and final consumption are, respectively:

$$ICS = \left(\theta_{AD}\pi_{ICS}\frac{p_A}{p_{IC}}\right)^{\tau_A}\frac{ACT}{\theta_{AD}} \tag{28}$$

$$FC = \left(\theta_{AD}\pi_{FC}\frac{p_A}{p_{FC}}\right)^{\tau_A}\frac{ACT}{\theta_{AD}} \tag{29}$$

where activity price, p_A, is given by the expression

$$p_A = \frac{1}{\theta_{AD}}\left(\pi_{ICS}^{\tau_A}p_{IC}^{1-\tau_A} + \pi_{FC}^{\tau_A}p_{FC}^{1-\tau_A}\right)^{\frac{1}{1-\tau_A}} \tag{30}$$

Nevertheless, p_A is determined through the market clearing condition in the distribution of ACT between ICS and FC:

$$p_A ACT = p_{IC}ICS + p_{FC}FC$$
$$p_A = \frac{p_{IC}ICS + p_{FC}FC}{ACT} \tag{31}$$

With ICD in (15) and ICS (28) we solve for the intermediate consumption price, p_{IC}, as:

$$p_{IC} = \left[\frac{(\theta_O\pi_{ICD}p_O)^{\sigma_o}\frac{OUT}{\theta_O}}{(\theta_{AD}\pi_{ICS}p_A)^{\tau_A}\frac{ACT}{\theta_{AD}}}\right]^{\frac{1}{\sigma_o-\tau_A}} \tag{32}$$

Supply Distribution Problem: Second Stage

The firm determines distribution of FC supply between Consumption (C), Investment (I), Government Expenditure (G) and Exports (X). X are classified between traditional, $\bar{X}_T$ (which are assumed as exogenous); and non-traditional, X_{NT}, which in turn are considered as endogenous. The firm maximizes its revenue for selling final consumption by optimally solving for the following problem:

$$\max_{\{C,I,G,X_N\}} p_C C + p_I I + p_G G + p_{X_T}\bar{X}_T + p_{X_{NT}}X_{NT} \tag{33}$$

subject to a linear-CET technology for distribution, with an elasticity of transformation between different types of FC that satisfies $\tau_{FC} < 0$:

$$FC = \bar{X}_T + \theta_{FC} \left(\pi_C C^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_I I^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_G G^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_{X_{NT}} X_{NT}^{\frac{\tau_{FC}-1}{\tau_{FC}}} \right)^{\frac{\tau_{FC}}{\tau_{FC}-1}} \qquad (34)$$

After solving for the optimal quantities of C, G, I and X_{NT}, from the FOC we have that the firm distributes FC according to the following expressions:

$$C = \left[\theta_{FC} \pi_C \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{p_C \left(FC - \bar{X}_T \right)} \right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}} \qquad (35)$$

$$I = \left[\theta_{FC} \pi_I \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{p_I \left(FC - \bar{X}_T \right)} \right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}} \qquad (36)$$

$$G = \left[\theta_{FC} \pi_G \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{p_G \left(FC - \bar{X}_T \right)} \right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}} \qquad (37)$$

$$X_{NT} = \left[\theta_{FC} \pi_{X_{NT}} \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{p_{X_{NT}} \left(FC - \bar{X}_T \right)} \right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}} \qquad (38)$$

where the nominal value of FC, $p_{FC} FC$, is given by:

$$p_{FC} FC = \frac{1}{\theta_{FC}} \left(\sum_{i \in \{C,G,I,X_{NT}\}} \pi_i^{\tau_{FC}} p_i^{1-\tau_{FC}} \right)^{\frac{1}{1-\tau_{FC}}} \left(FC - \bar{X}_T \right) + p_{X_T} \bar{X}_T \qquad (39)$$

From optimal supply of FC in (29) and its distribution technology (demand) in (34) we can solve for the optimal price of FC, p_{FC} as

$$p_{FC} = \left[\frac{(\theta_{AD} \pi_{FC} p_A)^{\tau_A} \frac{ACT}{\theta_{AD}}}{\bar{X}_T + \theta_{FC} \left(\sum \pi_i i^{\frac{\tau_{FC}-1}{\tau_{FC}}} \right)^{\frac{\tau_{FC}}{\tau_{FC}-1}}} \right]^{\frac{1}{\tau_A}} \quad \text{for } i \in \{C, G, I, X_{NT}\} \qquad (40)$$

3.2.2. Income Distribution

We now describe the second block of the model: the income distribution block. In this Macro CGE model, three out of four institutional agents take part in the production process and in consequence, receive factor remuneration. Households, firms and the Government are paid their marginal product from labor (REM), capital (EBE) and mixed income factors (MIX). Also, institutional agents exchange net transfers and rents between themselves (NT and NR). Taxes caused during production and product formation are transfered to the Government, as well as direct taxes paid by households, firms, and Government-owned firms (txva, txyy, trff, txhh and txac).

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Factor supply, remuneration and distribution

We recall that this model assumes inelastic supply of all three production factors, $\bar{L}$, $\bar{K}$ and $\bar{Z}$. Hence, factor remuneration prices are entirely determined by endogenous demand conditions: Given the optimal demands of factors in (3) to (5), and the inelastic supplies, we derive the factor prices:

$$p_L = \theta_F \pi_L \left(\frac{FAC}{\theta_F \bar{L}}\right)^{\frac{1}{\sigma_F}} p_F \tag{41}$$

$$p_K = \theta_F \pi_K \left(\frac{FAC}{\theta_F \bar{K}}\right)^{\frac{1}{\sigma_F}} p_F \tag{42}$$

$$p_Z = \theta_F \pi_Z \left(\frac{FAC}{\theta_F \bar{Z}}\right)^{\frac{1}{\sigma_F}} p_F \tag{43}$$

Therefore the following equalities must hold:

$$p_L \bar{L} = REM = REM_{HH} + F_L \tag{44}$$

$$p_K \bar{K} = EBE = EBE_{HH} + EBE_{FR} + EBE_{GV} \tag{45}$$

$$p_Z \bar{Z} = MIX = MIX_{HH} \tag{46}$$

where, in (44), REM stands for labor remuneration and sub-index HH implies that the households are the recipient; F_L are net labor remuneration to RW; in (45), EBE stands for capital remuneration and sub-indexes FR and GV imply firms and the Government are the recipients, respectively; and in (46), MIX stands for mixed income rents, a third factor paid only to HH.

Distribution of factors remuneration and rents among institutional agents (RHS parts of equations (44) to (46)) are paid according to fixed coefficients:

$$REM_{HH} = \pi_{HH}^{REM} REM \text{ and } F_L = \pi_{RW}^{REM} REM \tag{47}$$

$$EBE_{HH} = \pi_{HH}^{EBE} EBE \text{ , } EBE_{FR} = \pi_{FR}^{EBE} EBE \text{ and } EBE_{GV} = \pi_{GV}^{EBE} EBE \tag{48}$$

where upper-indexes indicate the account (or institutional agent) from which flows of resources are coming and, as before, sub-indexes denote which agent is the recipient, e.g. π_{HH}^{REM} is the share of REM (account) that is paid to HH (recipient).

Rents

Institutional agents pay rents, R, according to a fixed share of their capital remuneration income, EBE.

$$R^{HH} = \pi_R^{HH} EBE_{HH}, \ R^{FR} = \pi_R^{FR} EBE_{FR}, \text{ and } R^{GV} = \pi_R^{GV} EBE_{GV} \tag{49}$$

Likewise, rents are distributed amongst institutional agents according to constant coefficients:

$$R_{HH} = \pi_{HH}^R R, \ R_{FR} = \pi_{FR}^R R, \ R_{GV} = \pi_{GV}^R R \text{ , and } F_K = \pi_{RW}^R R \tag{50}$$

From (49) and (50), we have that

$$R \equiv R^{HH} + R^{FR} + R^{GV} \equiv R_{HH} + R_{FR} + R_{GV} + F_K \tag{51}$$

Direct Taxes

Domestic institutional agents pay direct taxes according to a fixed portion of their income, defined for each one as

$$Y_{HH} = REM_{HH} + EBE_{HH} + MIX_{HH} + \left(R_{HH} - R^{HH}\right) \tag{52}$$

$$Y_{FR} = EBE_{FR} + \left(R_{FR} - R^{FR}\right) \tag{53}$$

$$Y_{GV} = EBE_{GV} + \left(R_{GV} - R^{GV}\right) \tag{54}$$

which are the equations for household's income, Y_{HH}, firm's income, Y_{FR}, and Government's income, Y_{GV}, respectively. Net rents (NR) are in parenthesis for each institutional agent. Assuming no tax evasion and perfect fiscal compliance, institutional agents pay direct as it follows:

$$TXhh = txhh \cdot Y_{HH} \tag{55}$$

$$TXac_{FR} = txac_{FR} \cdot Y_{FR} \tag{56}$$

$$TXac_{GV} = txac_{GV} \cdot Y_{GV} \tag{57}$$

As in (51), from equations (55) to (56), we have that total direct taxes are given by

$$T = TXhh + TXac_{FR} + TXac_{GV} \tag{58}$$

Transfers

There are four types of transfers: social contributions (SC), social benefits (SB), other current transfers (CT), and product transfers (PT). We assume exogenous payments of social contributions from households, $\bar{SC}^{HH}$, which are distributed among FR and GV according to fixed coefficients:

$$SC_{FR}^{HH} = \pi_{FR}^{SC} \bar{SC}^{HH} \tag{59}$$

$$SC_{GV}^{HH} = \pi_{GV}^{SC} \bar{SC}^{HH} \tag{60}$$

Also, households receive exogenously assumed social benefits, $\bar{SB}_{HH}$, paid by firms and the Government, according to a fixed share:

$$SB_{HH}^{FR} = \pi_{FR}^{SB} \bar{SB}_{HH} \tag{61}$$

$$SB_{HH}^{GV} = \pi_{GV}^{SB} \bar{SB}_{HH} \tag{62}$$

Other current transfers (CT) are exogenously transfered from RW and FR

$$\bar{CT}^{RW} + \bar{CT}^{FR} = CT \tag{63}$$

and distributed to HH and GV according to fixed coefficients:

$$CT_{HH} = \pi_{HH}^{CT} CT \tag{64}$$

$$CT_{GV} = \pi_{GV}^{CT} CT \tag{65}$$

We also assume exogenous product transfers, $\bar{PT}$, paid by GV to HH,

$$\bar{PT}^{GV} = \bar{PT}_{HH} \tag{66}$$

Given equations (59) to (66), net transfers, NT, for each institutional agent are summarized by the following equations:

$$NT_{HH} = -\bar{SC}^{HH} + \bar{SB}_{HH} + CT_{HH} + \bar{PT}_{HH} \tag{67}$$

$$NT_{FR} = SC_{FR}^{HH} - SB_{HH}^{FR} - \bar{CT}^{FR} \tag{68}$$

$$NT_{GV} = SC_{GV}^{HH} - SB_{HH}^{GV} + CT_{GV} - \bar{PT}^{GV} \tag{69}$$

$$NT_{RW} = -\bar{CT}_{RW} \tag{70}$$

for net transfers for HH, FR, GV and RW, respectively.

3.2.3.   The Demand Side

We now describe the third and final block of this model: the demand side. Given the supply of different types of FC, as developed in the first block, and subject to the budget restrictions for each agent that arise from income distribution described in the second block, institutional agents demand a representative good labeled generically as Consumption (C), Investment (I), Public Consumption (G) and non-traditional Exports (X_{NT}). The demand block can be divided into domestic (from HH, FR and GV) and external (from RW) sub-blocks, which we readily describe:

Domestic Demand

We assume that households face Cobb-Douglas preferences with savings in the utility function, which produces consumption demand (C) and HH's savings, S_{HH}, of the form

$$C = \alpha \frac{DY_{HH}}{p_C} \tag{71}$$

$$S_{HH} = DY_{HH} - p_C C \tag{72}$$

where DY_{HH} stands for HH's disposable income, defined using (52), (55) and (67) as:

$$DY_{HH} = Y_{HH} + NT_{HH} - TXhh \tag{73}$$

Both households' and firms' investment, I_{HH} and I_{FR} respectively, are defined by private investment, I_{PR}. In this model, I_{HH} and I_{FR} are determined as a fixed share β of I_{PR}:

$$I_{HH} = \beta \cdot I_{PR} \tag{74}$$

$$I_{FR} = (1 - \beta) \cdot I_{PR} \tag{75}$$

Using (53), (56) and (68) we define firms' savings, S_{FR} as

$$S_{FR} = Y_{FR} + NT_{FR} - TXac_{FR} \tag{76}$$

We assume the Government expenditure and investment to be exogenous:

$$G = \bar{G} \tag{77}$$

$$I_{GV} = \bar{I}_{GV} \tag{78}$$

With public investment given in (78) and private investment resulting from the sum of (74) and (75), total demand for investment, I, is defined as the aggregate

$$I = I_{PR} + I_{GV} \tag{79}$$

Given Investment demand in (79), and bearing in mind that Investment supply is given by (36), we solve for the optimal price for investment:

$$p_I = \theta_{FC}\pi_I \frac{p_{FC}FC - p_{X_T}\bar{X}_T}{(FC - \bar{X}_T)} \left[\frac{FC - \bar{X}_T}{\theta_{FC}\left(I_{PR} + \bar{I}_{GV}\right)} \right]^{\frac{1}{\tau_{FC}}} \tag{80}$$

Following the same fashion, Government expenditure supply in (37) and exogenous assumed demand in (77), jointly determine the optimal price for Government expenditure:

$$p_G = \theta_{FC}\pi_G \frac{p_{FC}FC - p_{XT}\bar{X}_T}{(FC - \bar{X}_T)} \left(\frac{FC - \bar{X}_T}{\theta_{FC}\bar{G}} \right)^{\frac{1}{\tau_{FC}}} \tag{81}$$

Again, using definitions in (8), (11), (54), (58) and (69), we obtain an expression for public (Government) savings:

$$S_{GV} = Y_{GV} + NT_{GV} + Tx + T - TXac_{GV} - p_G\bar{G} \tag{82}$$

where Tx groups indirect taxes, i.e.

$$Tx = TX_{va} + TX_{yy} + TR_{ff} \tag{83}$$

External demand

Optimal RW demand for non-traditional exports coming from Colombian markets is obtained from the FOC derived from the RW's imports (exports for their trading counterparts) demand function. We assume a CES-type function that aggregates imports from all possible origins. Therefore, demand for Colombian X_{NT} is given by

$$X_{NT} = \left(\theta_{\bar{M}^*} \cdot \pi_{COL} \frac{e \cdot \bar{p}_{\bar{M}^*}^*}{p_{X_{NT}}} \right)^{\sigma_{\bar{M}}^*} \frac{\bar{M}^*}{\theta_{\bar{M}^*}} \tag{84}$$

where $\theta_{\bar{M}^*}$ an π_{COL} are scale and Colombia's share parameter in the aggregation of RW imports. Note since we assumed a CES-type function, $\sigma_{\bar{M}}^* > 0$ is supposed to hold. RW imports, $\bar{M}^*$, and their price, $\bar{p}_{\bar{M}^*}^*$, are assumed to follow exogenous dynamics.

Given the exogenous demand for traditional exports X_T and the endogenous demand for non-traditional exports X_{NT} in (84), total exports are computed as the sum of the traditional and non-traditional components

$$X = X_{NT} + \bar{X}_T \tag{85}$$

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In addition, price of total exports must satisfy

$$p_X = \frac{p_{X_T}\bar{X}_T + p_{X_{NT}}X_{NT}}{X} \tag{86}$$

and price of non-traditional exports is determined jointly by their supply in (38) and their demand in (84)

$$p_{X_{NT}} = \left[\frac{\left(\theta_{\bar{M}^*} \cdot \pi_{COL} \cdot e \cdot \bar{p}_{\bar{M}^*}^*\right)^{\sigma_M^*} \frac{\bar{M}^*}{\theta_{\bar{M}^*}}}{\left(\theta_{FC} \cdot \pi_{X_{NT}} \cdot \frac{p_{FC}FC - p_{X_T}\bar{X}_T}{\left(FC - \bar{X}_T\right)}\right)^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}}}\right]^{\frac{1}{\sigma_M^* - \tau_{FC}}} \tag{87}$$

Finally, we assume that RW demands traditional exports at an exogenous international price $p_{X_T}^*$, which implies that internal price for X_T is given by

$$p_{X_T} = e \cdot p_{X_T}^* \tag{88}$$

3.2.4. Closure of the Model

The closure of a CGE model is defined as a set of equations (exogenous variables) that assure that markets equilibrium is achieved in a Walrasian sense. In other words, a model is *determined* (closed) when the number of endogenous variables equals the number of equations, and therefore, there exists a unique solution to the system of equations. Closing a model is akin to drop a specific assumption from the original model (Sen, 1963). Several alternatives of closure equations have been explored over the past few decades, each one depending on the particular way the researcher understands the macroeconomic mechanisms that rule the modeled economy. In this paper we present two alternative closures: a Kaldorian-type one, called the *investment* closure; and a Classic-type one, known as the *external savings* closure (Decaluwé et al., 1987)[1].

In the former, total investment is assumed to be fixed at a certain level, and total savings adjust to satisfy such demand for investment requirements; whilst in the latter, total savings levels are assumed to be fixed and investment adjusts itself given the available amount of resources in the economy. In this CGEM we have four variables that are not yet determined: two nominal prices, the nominal exchange rate, e, and the price of consumption, p_C; and two flows: external savings, S_{RW}, and private investment, I_{PR}. Each closure assumes fixed values for a nominal price and a flow, as we shall see promptly.

Also, from equations (47), (50) and (70), we set RW bilateral income as

$$Y_{RW} = F_L + F_K - \bar{C}T^{RW} \tag{89}$$

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[1]Authors recall that names are not necessarily related to the author's main ideas, e.g. Kaldor, Keynes, etc.

Investment Closure

This closure assumes exogenous nominal exchange rate and fixed private investment levels:

$$e = \bar{e} \tag{90}$$

$$I_{PR} = \bar{I}_{PR} \tag{91}$$

Given (90) and (91), RW savings in the country, S_{RW}, is left to be an endogenous variable, given by:

$$S_{RW} = Y_{RW} + p_M M - p_X X \tag{92}$$

Also, given S_{RW} in (92), we complete aggregate savings $S = S_{HH} + S_{FR} + S_{GV} + S_{RW}$. This leaves the Savings-Investment (S-I) balance depending on the price of consumption, by replacing (72) in the latter expression and solving for p_C. Therefore we obtain

$$p_C = \frac{DY_{HH} + S_{FR} + S_{GV} + S_{RW} - p_I \bar{I}}{C} \tag{93}$$

External Savings Closure

This closure assumes exogenous consumption price and RW savings:

$$p_C = \bar{p}_C \tag{94}$$

$$\bar{S}_{RW} = \bar{S}_{RW} \tag{95}$$

We use the definition of external savings proposed in (92) to solve for the nominal exchange rate, e:

$$\bar{S}_{RW} = Y_{RW} + p_M(e)M(e) - p_X(e)X(e) \tag{96}$$

where (e) denotes a non-linear function of the nominal exchange rate, e. Once determined e, private investment I_{PR} adjusts its levels to match the S-I balance, and therefore, is given by:

$$I_{PR} = \frac{S}{p_I} - \bar{I}_{GV} = \frac{S_{HH} + S_{FR} + S_{GV} + \bar{S}_{RW}}{p_I} - \bar{I}_{GV} \tag{97}$$

4. Parameter calibration

The scale and share parameters in each one of the CES and CET structures used throughout the three blocks of the model discussed in section 3 can be calibrated using information from the tailor made Macro-SAM constructed for this model, presented in section 2. In this section we present an example of how share and scale parameters in a CES function are calibrated in this model. All other scale and share parameters can be calibrated analogously.

Share parameters

From (3) we have that share parameters π_i can be expressed as:

$$\pi_K = \pi_L \frac{p_K}{p_L} \left(\frac{K}{L}\right)^{\frac{1}{\sigma_F}}$$

$$\pi_Z = \pi_L \frac{p_Z}{p_L} \left(\frac{Z}{L}\right)^{\frac{1}{\sigma_F}}$$

Using the fact that share parameters sum up to unity:

$$\pi_L + \pi_K + \pi_Z = 1 \ ,$$

we obtain:

$$\pi_L = \frac{p_L L^{\frac{1}{\sigma_F}}}{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}$$

Following the same steps for K and Z we have that their share parameters are given by

$$\pi_K = \frac{p_K K^{\frac{1}{\sigma_F}}}{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}$$

$$\pi_Z = \frac{p_Z Z^{\frac{1}{\sigma_F}}}{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}$$

Scale parameter

Scale parameter θ_F is calibrated using the previously calibrated share parameters and (2):

$$\theta_F = FAC \left(\frac{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}{p_L L + p_K K + p_Z Z} \right)^{\frac{\sigma_F}{\sigma_F - 1}}$$

All other scale parameters in the model are calibrated analogously.

Scale Parameters for RW imports

In order to calibrate the scale parameter in RW demand for imports function, $\theta_{\bar{M}^*}$, all (domestic and foreign) prices equal are set to equal unity in the baseline scenario. After controlling for that, from equation (87), it must hold that:

$$\theta_{\bar{M}^*} = \left[\frac{\pi_{X_{NT}}^{\tau_{FC}} \left(FC - \bar{X}_T \right)}{\theta_{FC}^{1-\tau_{FC}} \cdot \pi_{COL}^{\sigma_{\bar{M}}^*} \cdot \bar{M}^*} \right]^{\frac{1}{\sigma_{\bar{M}}^* - 1}}$$
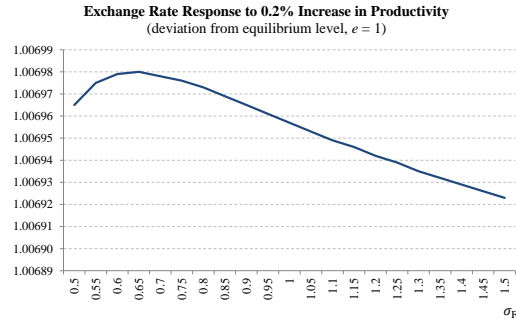
5.   Elasticities: A Sensibility Analysis

As stated before, scale and share parameters are calibrated using observed data on the Macro-SAM built for the benchmark year presented in section 2. However, elasticities are not easily calibrated and are either taken as given from the relevant existing literature or usually estimated through econometric methods (as in Hillberry and Hummels, 2013). In this paper we adopt the first approach and we do not present an econometric estimation of the elasticities in the model. However, we do present sensibility analyses for each elasticity to be considered.

One advantage of the CES and CET functional forms is that they can be considered as a general case of a linear, a Cobb-Douglas, or a Leontieff type of isoquant curves (both on the consumer's and producer's optimization problem), depending on the value of the parameter σ or τ. We have three cases:
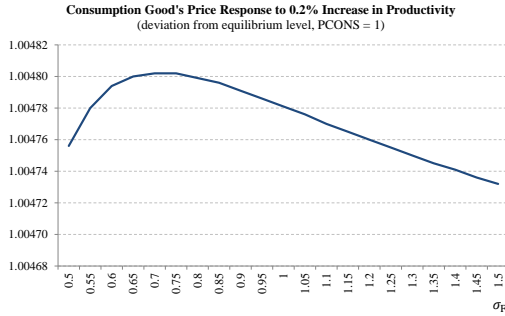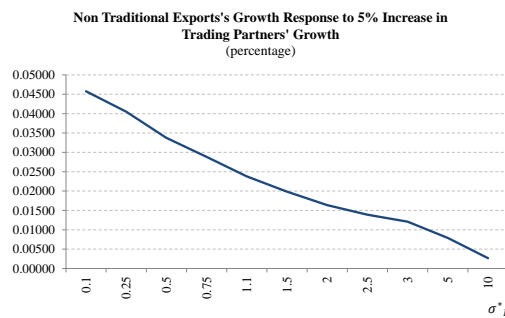
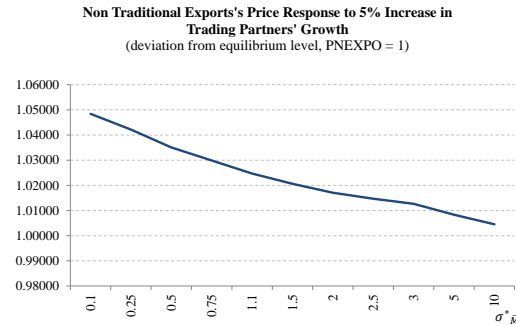(a) PCONS response. Savings Closure. (b) e response. Investment Closure.

Figure 3: Sensitivity analysis for σ_F.

- When $\frac{\sigma-1}{\sigma} \to 1$, i.e. $\sigma \to \infty$ $\left(\frac{\tau-1}{\tau} \to 1,\text{ i.e. } \tau \to -\infty\right)$, then goods are perfect substitutes (linear isoquant);

- When $\frac{\sigma-1}{\sigma} \to 0$, i.e. $\sigma \to 1$ $\left(\frac{\tau-1}{\tau} \to 2,\text{ i.e. } \tau \to -1\right)$, then there exists some degree of complementariness, as in a Cobb-Douglas type function; and

- When $\frac{\sigma-1}{\sigma} \to -\infty$, i.e. $\sigma \to 0$ $\left(\frac{\tau-1}{\tau} \to -\infty,\text{ i.e. } \tau \to 0\right)$, then goods are perfect complements (max or min isoquant).

We conduct several sensitivity analyses for different values on each of the σ and τ parameters in the model. Some of them are presented in Figure 3 and Figure 4. We set these parameters to the values that yield coherent results according to the existent literature when applying certain shocks to exogenous variables that are considered to be relevant across the different blocks of the economy.

(a) NEXPO response. Savings Closure. (b) PNEXPO response. Investment Closure.

Figure 4: Sensitivity analysis for $\sigma_{\bar{M}}^*$.

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6. Comparative Analysis

After calibrating the parameters and fixing the elasticities, in this section we present some comparative results from both closures of the model. Our aim is to replicate 2012's economy by applying observed shocks to the exogenous variables in our model using information from BOP, National Accounts and other relevant sources. We replicate 2012 SAM since it is built with the latest information available published by DANE. Results are shown in Table 1:

| | Main Results - 2012 | | | | |
| | CGEM | | | Observed | Difference (pp) |
Variable	Investment	Savings	Average		
GDP	4.2	4.1	4.1	4.0	0.1
Consumption	4.5	3.5	4.0	4.4	-0.4
Gov. Spending	5.7	5.7	5.7	5.7	0.0
Investment	5.0	4.0	4.5	4.6	-0.1
I_{PR}	4.9	3.8	4.4	4.9	-0.5
I_{GV}	5.3	5.3	5.3	5.3	0.0
Exports	4.6	5.3	5.0	6.1	-0.9
Imports	7.2	3.5	5.4	8.9	-3.5
CAD (%GDP)	4.0	3.1	3.6	3.1	0.5

Table 1: Comparative Analysis Exercise. 2012 SAM replication.

Despite of the differences, the model captures the main tendencies observed for the macroeconomic variables in the Colombian economy. The difference between observed GDP annual growth in 2012 versus that obtained from the model is only about 0.1 percentage points (pp). The most notorious differences are observed in the international trade accounts, greater for the imports that for the exports. These discrepancies follow from the simple structure of the model, since we have not opened trade partners into regions or countries and we suppose an unique composite final good to be imported. Greater accuracy will be achieved once the model structure is expanded, both on the supply side (sectors) and on the international trade block (differentiating trading partners); and once the elasticities are estimated through econometric methods.

7. Conclusions and Further Extensions

Computable General Equilibrium Models (CGEM) have been found useful when conducting fiscal, trading, environmental policies analyses, as well as welfare and distribution impacts following different shocks in the models variables. In this paper we have presented a tailor-made Macro Computable General Equilibrium Model (Macro-CGEM) that meets the needs and interests related to macroeconomic modeling at the Central Bank of Colombia, which are mainly concerned about incorporating staff's Balance of Payment's projections many other macroeconomic variables forecasts relevant for long run GDP growth analysis in a small and open economy. This Macro-CGEM is a simple model and it is not intended to address economic phenomena such as welfare distribution, taxation impacts and trade policies. However,

once the SAM is configured properly, it will serve as a starting point for conducting fiscal and trading policies analyses.

This model consists of three main blocks: production (supply), distribution and demand blocks. In each one the agents face optimization problems concerning maximizing certain quantity subject to a technology of aggregation or transformation (CES and CET, respectively). Two alternative closures are presented: i) one in which private investment is assumed to be exogenous, as well as exchange rate. This leaves external savings (current account deficit) and the composite good price as endogenous variables; and ii) a closure in which external savings and the composite good price are exogenous, therefore leaving private investment and the exchange rate as endogenous variables. In this paper, the main interest of this Macro-CGEM users is to build counterfactual scenarios related to commodities prices shocks, external demand growth, public and private investment shocks, and current account deficit levels shifts.

The sensitivity analyses exercises suggested that the elasticities used in the model are coherent with economic theory and similar to those used in previous models. However, these elasticities shall be estimated through econometric methods in posterior updates of the model. A comparative analysis is carried using 2012 national accounts information. Results show that this model captures the main characteristics of the Colombian economy, and although greater accuracy could be achieved for the international trade accounts, once the structure of the model is expanded we expect to obtain more detailed results.

Lastly, we recall that this model admits further extensions. Firstly, an extended version of the production block of the model which includes different sectors is currently being developed. Secondly, a more detailed taxation structure shall be explored, as well as a broader modeling of the Colombian trading partners and conditions. Also, econometric estimation of the model elasticities shall be conducted, as it has been already stated. *GAMS codes and SAMs are available upon request from the corresponding author.*

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Appendix

List of Equations

Table 2: Macro-CGEM Summary. List of Equations

	Equation	Eq. Number	Variable
1.	$FAC = \bar{\theta}_F \left(\sum_{f=\{\bar{L},\bar{K},\bar{Z}\}} \pi_f f^{\frac{\sigma_F-1}{\sigma_F}} \right)^{\frac{\sigma_F}{\sigma_F-1}}$	(2)	FAC
2.	$p_{AV} AV = p_F FAC + TX_{va}$	(7)	p_F
3.	$AV = FAC \ (1 + tx_{va})$	(9)	AV
4.	$TX_{va} = tx_{va} p_F FAC$	(8)	TX_{va}
5.	$p_Y Y = p_{AV} AV + TX_{yy} + TR_{ff}$	(10)	p_{AV}
6.	$TX_{yy} = tx_{yy} p_{AV} AV$	(11)	TX_{yy}
7.	$TR_{ff} = tr_{ff} p_M M$	(11)	TR_{ff}
8.	$Y = \left(\theta_O \pi_Y \frac{p_O}{p_Y} \right)^{\sigma_o} \frac{OUT}{\theta_O}$	(14)	Y
9.	$ICD = \left(\theta_O \pi_{ICD} \frac{p_O}{p_{IC}} \right)^{\sigma_o} \frac{OUT}{\theta_O}$	(15)	IDC
10.	$p_Y = \left[\frac{p_{AV} AV + TX_{yy} + TR_{ff}}{(\theta_O \pi_Y p_O)^{\sigma_O} \frac{OUT}{\theta_O}} \right]^{\frac{1}{1-\sigma_o}}$	(17)	p_Y
11.	$OUT = \left(\theta_A \pi_O \frac{p_A}{p_O} \right)^{\sigma_A} \frac{ACT}{\theta_A}$	(20)	OUT
12.	$M = \left(\theta_A \pi_M \frac{p_A}{p_M} \right)^{\sigma_A} \frac{ACT}{\theta_A}$	(20)	M
13.	$p_O = \left[\frac{(\theta_A \pi_O p_A)^{\sigma_A} \frac{ACT}{\theta_A}}{\theta_O \left(\pi_Y Y^{\frac{\sigma_O-1}{\sigma_O}} + \pi_{ICD} ICD^{\frac{\sigma_O-1}{\sigma_O}} \right)^{\frac{\sigma_O}{\sigma_O-1}}} \right]^{\frac{1}{\sigma_A}}$	(23)	p_O
14.	$ACT = \frac{p_O OUT + p_M M}{p_A}$	(24)	ACT
15.	$p_M = e \cdot p_M^*$	(25)	p_M
16.	$ICS = \left(\theta_{AD} \pi_{ICS} \frac{p_A}{p_{IC}} \right)^{\tau_A} \frac{ACT}{\theta_{AD}}$	(28)	ICS
17.	$FC = \left(\theta_{AD} \pi_{FC} \frac{p_A}{p_{FC}} \right)^{\tau_A} \frac{ACT}{\theta_{AD}}$	(29)	FC
18.	$p_A = \frac{p_{IC} ICS + p_{FC} FC}{ACT}$	(31)	p_A
19.	$p_{IC} = \left[\frac{(\theta_O \pi_{ICD} p_O)^{\sigma_O} \frac{OUT}{\theta_O}}{(\theta_{AD} \pi_{ICS} p_A)^{\tau_A} \frac{ACT}{\theta_{AD}}} \right]^{\frac{1}{\sigma_O - \tau_A}}$	(32)	p_{IC}

Table 2 - continued from previous page

	Equation	Eq. Number	Variable
20.	$I = I_{PR} + I_{GV}$	(79)	I
21.	$X_N = \left[\theta_{FC}\pi_{XN}\frac{P_{FC}FC - P_{XT}\bar{X}_T}{p_{XN}(FC - \bar{X}_T)}\right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}}$	(38)	X_{NT}
22.	$p_{FC} = \left[\dfrac{(\theta_{AD}\pi_{FC}p_A)^{\tau_A}\frac{ACT}{\theta_{AD}}}{\bar{X}_T + \theta_{FC}\left(\sum \pi_i i^{\frac{\tau_{FC}-1}{\tau_{FC}}}\right)^{\frac{\tau_{FC}}{\tau_{FC}-1}}}\right]^{\frac{1}{\tau_A}}$	(40)	$p_F C$
23.	$p_L \bar{L} = REM$	(44)	REM
24.	$p_K \bar{K} = EBE$	(45)	EBE
25.	$p_Z \bar{Z} = MIX$	(46)	MIX
26.	$p_L = \theta_F \pi_L \left(\frac{FAC}{\theta_F \bar{L}}\right)^{\frac{1}{\sigma_F}} p_F$	(41)	p_L
27.	$p_K = \theta_F \pi_K \left(\frac{FAC}{\theta_F \bar{K}}\right)^{\frac{1}{\sigma_F}} p_F$	(42)	p_K
28.	$p_Z = \theta_F \pi_Z \left(\frac{FAC}{\theta_F \bar{Z}}\right)^{\frac{1}{\sigma_F}} p_F$	(43)	p_Z
29.	$REM_{HH} = \pi_{HH}^{REM} REM$	(47)	REM_{HH}
30.	$F_L = \pi_{RW}^{REM} REM$	(47)	F_L
31.	$EBE_{HH} = \pi_{HH}^{EBE} EBE$	(48)	EBE_{HH}
32.	$EBE_{FR} = \pi_{FR}^{EBE} EBE$	(48)	EBE_{FR}
33.	$EBE_{GV} = \pi_{GV}^{EBE} EBE$	(48)	EBE_{GV}
34.	$R^{HH} = \pi_R^{HH} EBE_{HH}$	(49)	R^{HH}
35.	$R^{FR} = \pi_R^{FR} EBE_{FR}$	(49)	R^{FR}
36.	$R^{GV} = \pi_R^{GV} EBE_{GV}$	(49)	R^{GV}
37.	$R_{HH} = \pi_{HH}^R R$	(50)	R_{HH}
38.	$R_{FR} = \pi_{FR}^R R$	(50)	R_{FR}
39.	$R_{GV} = \pi_{GV}^R R$	(50)	R_{GV}
40.	$F_K = \pi_{RW}^R R$	(50)	F_K
41.	$R = R^{HH} + R^{FR} + R^{GV}$	(51)	R
42.	$Y_{HH} = REM_{HH} + EBE_{HH} + MIX_{HH} + \left(R_{HH} - R^{HH}\right)$	(52)	Y_{HH}
43.	$Y_{FR} = EBE_{FR} + \left(R_{FR} - R^{FR}\right)$	(53)	Y_{FR}
44.	$Y_{GV} = EBE_{GV} + \left(R_{GV} - R^{GV}\right)$	(54)	Y_{GV}
45.	$TXhh = txhh Y_{HH}$	(55)	TXhh
46.	$TXac_{FR} = txac_{FR} Y_{FR}$	(56)	$TXac_{FR}$
47.	$TXac_{GV} = txac_{GV} Y_{GV}$	(57)	$TXac_{GV}$
48.	$T = TXhh + TXac_{HH} + TXac_{GV}$	(58)	T
49.	$SC_{FR}^{HH} = \pi_{FR}^{SC} \bar{SC}^{HH}$	(59)	SC_{FR}^{HH}
50.	$SC_{GV}^{HH} = \pi_{GV}^{SC} \bar{SC}^{HH}$	(60)	SC_{GV}^{HH}
51.	$SB_{HH}^{FR} = \pi_{FR}^{SB} \bar{SB}_{HH}$	(61)	SB_{HH}^{FR}
52.	$SB_{HH}^{GV} = \pi_{GV}^{SB} \bar{SB}_{HH}$	(62)	SB_{HH}^{GV}

Table 2 - continued from previous page

	Equation	Eq. Number	Variable
53.	$\bar{CT}^{RW} + \bar{CT}^{FR} = CT$	(63)	CT
54.	$CT_{HH} = \pi_{HH}^{CT} CT$	(64)	CT_{HH}
55.	$CT_{GV} = \pi_{GV}^{CT} CT$	(65)	CT_{GV}
56.	$NT_{HH} = -\bar{SC}^{HH} + \bar{SB}_{HH} + CT_{HH} + \bar{PT}_{HH}^{GV}$	(67)	NT_{HH}
57.	$NT_{FR} = SC_{FR}^{HH} - SB_{HH}^{FR} - \bar{CT}^{FR}$	(68)	NT_{FR}
58.	$NT_{GV} = SC_{GV}^{HH} - SB_{HH}^{GV} + CT_{GV} - \bar{PT}_{HH}^{GV}$	(69)	NT_{GV}
59.	$C = \alpha \frac{DY_{HH}}{p_C}$	(71)	C
60.	$S_{HH} = DY_{HH} - p_C C$	(72)	S_{HH}
61.	$DY_{HH} = Y_{HH} + NT_{HH} - TXhh$	(73)	DY_{HH}
62.	$I_{HH} = \beta \cdot I_{PR}$	(74)	I_{HH}
63.	$I_{FR} = (1 - \beta) \cdot I_{PR}$	(75)	I_{PR}
64.	$S_{FR} = Y_{FR} + NT_{FR} - TXac_{FR}$	(76)	S_{FR}
65.	$p_I = \theta_{FC} \pi_I \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{\left(FC - \bar{X}_T\right)} \left[\frac{FC - \bar{X}_T}{\theta_{FC}\left(I_{PR} + \bar{I}_{GV}\right)}\right]^{\frac{1}{\tau_{FC}}}$	(80)	p_I
66.	$p_G = \theta_{FC} \pi_G \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{\left(FC - \bar{X}_T\right)} \left(\frac{FC - \bar{X}_T}{\theta_{FC} \bar{G}}\right)^{\frac{1}{\tau_{FC}}}$	(81)	p_G
67.	$S_{GV} = Y_{GV} + NT_{GV} + Tx + T - TXac_{GV} - p_G \bar{G}$	(82)	S_{GV}
68.	$Tx = TX_{va} + TX_{yy} + TR_{ff}$	(83)	Tx
69.	$X = X_N + \bar{X}_T$	(85)	X
70.	$p_X = \frac{p_{X_T} X_T + p_{X_{NT}} X_{NT}}{X}$	(86)	p_X
71.	$p_{X_{NT}} = \left[\frac{\left(\theta_{\bar{M}*} \cdot \pi_{COL} \cdot e \cdot \bar{p}_{\bar{M}*}^*\right)^{\sigma_{\bar{M}}^*} \frac{\bar{M}^*}{\theta_{\bar{M}*}}}{\left(\theta_{FC} \cdot \pi_{X_{NT}} \cdot \frac{p_{FC} FC - p_{X_T} \bar{X}_T}{\left(FC - \bar{X}_T\right)}\right)^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}}}\right]^{\frac{1}{\sigma_{\bar{M}}^* - \tau_{FC}}}$	(87)	$p_{X_{NT}}$
72.	$p_{XT} = e \cdot p_{X_T}^*$	(88)	p_{X_T}
73.	$Y_{RW} = F_L + F_K - \bar{CT}^{RW}$	(89)	Y_{RW}

Closure equations

Table 3: Macro-CGEM Summary. List of Closure Equations

	Equation	Eq. Number	Variable
Investment	$e = \bar{e}$	(90)	e
	$I_{PR} = \bar{I}_{PR}$	(91)	I_{PR}
	$S_{RW} = Y_{RW} + p_M M - p_X X$	(92)	S_{RW}
	$p_C = \frac{DY_{HH} + S_{FR} + S_{GV} + S_{RW} - p_I \bar{I}}{C}$	(93)	p_C

Table 3 - continued from previous page

	Equation	**Eq. Number**	**Variable**
Savings	$p_C = \bar{p}_C$	(94)	p_C
	$S_{RW} = \bar{S}_{RW}$	(95)	S_{RW}
	$\bar{S}_{RW} = Y_{RW} + p_M(e)M(e) - p_X(e)X(e)$	(96)	e
	$I_{PR} = \frac{S_{HH}+S_{FR}+S_{GV}+\bar{S}_{RW}}{p_I} - \bar{I}_{GV}$	(97)	I_{PR}