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A multi-layer network of the sovereign securities market[1]

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Abstract

We study the network of Colombian sovereign securities settlements. With data from the settlement market infrastructure we study financial institutions' transactions from three different trading and registering individual networks that we combine into a multi-layer network. Examining this network of networks enables us to confirm that (i) studying isolated single-layer trading and registering networks yields a misleading perspective on the relations between and risks induced by participating financial institutions; (ii) a multi-layer approach produces a connective structure consistent with most real-world networks (e.g. sparse, inhomogeneous, and clustered); and (iii) the multi-layer network is a multiplex that preserves the main connective features of its constituent layers due to positively correlated multiplexity. The results highlight the importance of mapping and understanding how financial institutions relate to each other across multiple financial environments, and the value of financial market infrastructures as sources of data that may help to overcome the main obstacles for working on multi-layer financial networks.

JEL: D85, D53, G20, G01, L14

Keywords: multiplex networks, financial market infrastructures, correlated multiplexity, settlement.

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1. Introduction

Financial market infrastructures are considered the "plumbing" of financial systems (Bernanke, 2011). Financial market infrastructures responsible for settling transactions between financial institutions fulfill a pivotal position within that plumbing, thus they are of particular interest to disentangle and dissect this complex structure of financial systems by means of network analysis. Under this analytical framework, we study the local sovereign securities settlement infrastructure in order to attain a more detailed and actual illustration of how financial intuitions organize themselves in the corresponding legal, operational, and economic contexts.

As transactions settled originate in different trading and registering platforms, a single network of settlements should necessarily aggregate networks from other financial market infrastructures. In this vein, the settlement network is a *network of networks*, which entails additional sources of complexity, critical for understanding financial markets. This is consistent with recent efforts to examine the consequences of considering financial systems as multi-layer networks, in which financial institutions interact across several environments or layers (e.g. markets, asset classes, trading and registering platforms, jurisdictions).

The theoretical development of multi-layer networks (and its empirical validation) is still in scaffolding, and consequently most literature on multi-layer financial networks is still preliminary. Nevertheless, there is a consensus regarding the need to examine how the basic assumptions of real-world networks are altered in a context of multi-layer networks or networks of networks. Most examinations have focused on how real-world networks' coupling affects their well-documented inhomogeneous connective architecture, in which connections being distributed in an extremely skewed fashion (presumably following a power-law distribution) contribute to their robustness; as most participants are weakly connected, most real-world networks, either biological or man-made, tend to be robust to the random failure of their constituents (see Newman (2010), Barabási (2003), Strogatz (2003)).

In this sense, studying networks of financial networks allows to confirm if their individual well-documented *robust-yet-fragile* feature (Haldane, 2009) is also valid within a financial multi-layer network. Moreover, studying how single-layer financial networks' couple allows understanding the rationale behind the main connective features of financial multi-layer networks.

Consequently, in the light of the above view of the transactions in a settlement market infrastructure as a network of networks, this paper has two main purposes: First, we examine how Colombian sovereign securities' transactions that come from single-layer networks (corresponding to distinct trading and registering platforms) can be aggregated into a multiplex network of the sovereign securities settlement system. Second, we verify whether or not viewing a financial system as a multiplex network is a superior view in that it preserves the main connective properties of its constituents, while revealing some additional layers of complexity.

This paper contributes to the financial literature by examining the connective structure of financial institutions' interactions under different economic and operational environments, and by investigating how those interactions aggregate and result in the local sovereign securities' market. Moreover, in the context of the networks of networks literature, our work contributes to the study of financial market infrastructures by revealing that they critically depend on their interaction with other financial market infrastructures; in our case, the local sovereign securities settlement market infrastructure (DCV) depends on its interaction with trading and registering platforms (SEN, MEC and MEC-R). Finally, our work highlights that financial market infrastructures are a vital source of granular and timely data for examining single- and multi-layer networks as a way to better understand financial markets.

2. Background

Most efforts to characterize the topology of complex systems by means of network analysis have assumed that each system is isolated (i.e. non-coupled) from other networks. In the case of single-layer networks (i.e. monoplex networks) such efforts have converged to an inhomogeneous connective structure, typically in the form of an approximate power-law distribution of links and their weights of real-world networks. The power-law or Pareto distribution of links is commonly referred as a *scale-free network* (Barabási and Albert, 1999), and it corresponds to the most documented type of network in social, biological and man-made complex systems.[5] In the case of financial networks, most literature confirms their scale-free nature (León et al., 2014; León and Berndsen, 2014; Bargigli et al., 2013; Fricke and Lux, 2012; Bech and Atalay, 2008; Pröpper et al., 2008; May et al., 2008; Cepeda, 2008; Renault et al. 2007; Soramäki et al., 2006; Inaoka et al., 2004; Boss et al., 2004), whereas other papers confirm their inhomogeneity but report a divergence from a strict power-law distribution of links (Martínez-Jaramillo et al., 2012; van Lelyveld and in 't Veld, 2012; Craig and von Peter, 2010).[6]

Yet, as highlighted by Cardillo et al. (2013), "many biological and man-made networked systems are characterized by the simultaneous presence of different sub-networks organized in separate layers, with connections and participants of qualitatively different types" (p.1). This multi-layered nature of networks, also known as network of networks, has been the focus of network scientists rather recently, among whom Kurant and Thiran (2006) provided one of the most seminal contributions.

Research on multi-layer financial networks has recently emerged. The standard multi-layer framework in finance corresponds to the so-called *multiplex* network, which may be described as networks containing participants of one sort but with several kinds of connections between

[5] In scale-free networks the number of connections is distributed as in a power-law (i.e. extreme heterogeneity and skewness), with a few heavily connected participants and many poorly connected participants. Due to this particular type of inhomogeneity there is no typical participant in the network, thus it has no scale (i.e. it is scale-free or scale-invariant).

[6] As in van Lelyveld and in 't Veld (2012), the power-law distribution of links is an asymptotic property, thus testing the scale-free features of a network is problematic. In this sense, a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks –as those examined here- may be impractical.

them (Baxter et al., 2014).[7] Figure 1 displays a two-layer network composed by layers X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertices are the participants, whereas each vertex has a function in the corresponding layer.

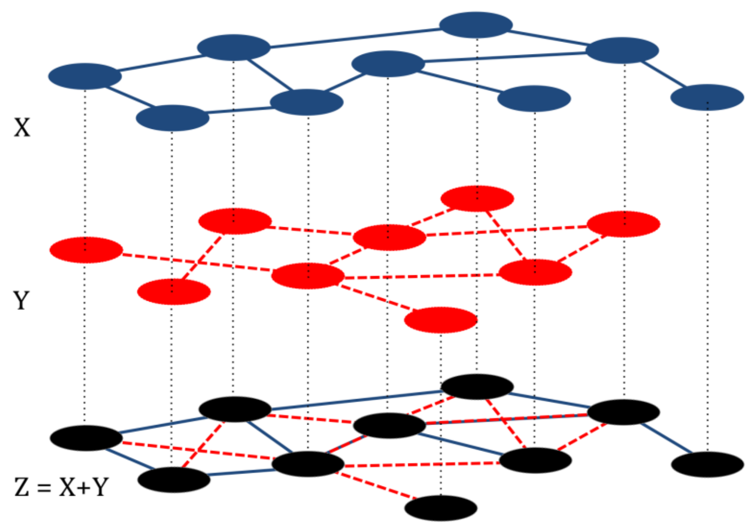

Figure 1. A multiplex network. Two-layer networks, X and Y, and the multiplex (Z) resulting from merging X and Y. Vertical lines connecting superimposed vertices are the participants, whereas each vertex is a role in the corresponding layer.

Montagna and Kok (2013) model interbank contagion in the US with a triple-layer multiplex network consisting of long-term direct bilateral exposures, short-term bilateral exposures, and common exposures to financial assets. Bargigli et al. (2013) examine the Italian interbank multiplex network by transaction type (i.e. secured and non-secured) and by maturity (i.e. overnight, short-term and long-term).

In our case, we examine the Colombian sovereign securities' market. We use a triple-layer multiplex network corresponding to the three Colombian sovereign securities' trading and registering environments (i.e. SEN, MEC, MEC-R), that altogether meet in the settlement market infrastructure for local sovereign securities: DCV.[8] As is the case in other countries, the sovereign securities settlement infrastructure is owned and operated by the central bank (Banco de la República), and works under a delivery versus payment setting, and it interacts with the large-value payment system within a real-time gross settlement framework.

As our network of networks deals with three different monoplex networks composed by a single type of participant (i.e. financial institutions) but distinct kinds of connections

[7] Several applications of multiplex networks have been documented for non-financial complex systems, such as transport systems (Kurant and Thiran, 2006; Cardillo et al., 2013), electrical networks (Pahwa et al., 2014), physiological systems (Ivanov and Bartsch, 2014), critical infrastructures (Martí, 2014; Rome et al., 2014) and cooperation networks (Gómez-Gardenes et al., 2012). Yet, other types of multi-layer networks are available (e.g. with layers containing participants of different sorts).

[8] *Sistema Electrónico de Negociación* (SEN, Electronic Trading System); *Mercado Electrónico Colombiano* (MEC, Colombian Electronic Market); *Módulo de Registro del Mercado Electrónico Colombiano* (MEC-R, Colombian Electronic Market Register Module); *Depósito Centralizado de Valores* (DCV, Centralized Securities Depository).

corresponding to different trading and registering platforms with divergent operational and economic features, it may be well defined as a multiplex network. Accordingly, the literature suggests that working on multi-layer networks may alter the basic assumptions that according to network theory lie at the basis of single networks (Kenett et al., 2014).

Contrary to what one would expect, the literature shows that the coupling of scale-free networks may yield a less robust network (Gao et al., 2012; Buldyrev et al., 2010). Exceptions to this finding would occur when the number of links (i.e. the *degree*) of interdependent participants coincides across the layers. This is, scale-free networks' robustness is likely to be preserved if *positively correlated multiplexity* exists, such that a high-degree vertex in one layer likely is high-degree in the other layers as well (Lee et al., 2014; Kenett et al., 2014).

Our results contribute to the related literature in two ways. First, our results help to understand the economics of financial market infrastructures as the collective function of several layers of interaction between financial institutions. In our case, the settlement network is envisaged as a collection of the transactions routed via distinct trading and registering platforms with different economic and operational features. Second, our empirical analysis illustrates how the main features of individual networks aggregate into a multiplex network. As obtaining empirical data on multi-layer networks is particularly difficult (D'Souza et al., 2014), and given the novelty of multi-layer network analysis, both these contributions could be valuable for financial authorities, market practitioners, and the academics.

3. The data set

The clearing and settlement of local sovereign securities market takes place in DCV. Colombia's Central Bank owns and operates DCV, a financial market infrastructure that is both the local sovereign securities' clearing and settlement system, and also their central securities depository. Working on a real-time gross settlement system and a delivery-versus-payment mechanism, DCV clears and settles spot market transactions, repos and sell-buy back transactions. DCV is the second most systemically important local financial market infrastructure according to León and Pérez (2014), only surpassed by the large-value payment system.

Most transactions in DCV result from three different Colombian sovereign securities' trading and registering platforms: SEN, MEC and MEC-R.[9] SEN is the main local sovereign securities' trading platform, owned and operated by Colombia's Central Bank, in which a group of 14 market makers trade and settle anonymously, and free of any counterparty limit.[10] MEC is an anonymous trading platform privately owned and operated by the Colombian Stock Exchange (*Bolsa de Valores de Colombia*), which also serves as trading platform for other non-sovereign fixed income securities. SEN and MEC, are both multilateral trading platforms, but differ in particular issues. SEN is a "rich-club" of participants brought together by the Ministry of

[9] Other trading platforms exist (i.e. GFI, Icap and Tradition), but their contribution has always been practically nil, which is why they are excluded from the analysis.

[10] SEN also provides an open trading platform for non-market makers and a registering platform for over-the-counter trades, but none of the market participants use it.

Finance into a market-maker scheme, in which they trade and settle anonymously without counterparty limits, whereas MEC is an open platform that allows participants to manage counterparty risk by imposing limits amid an anonymous trading environment.

Unlike MEC and SEN, MEC-R is not a multilateral trading platform; it is a facility provided by the Colombian Stock Exchange that enables local over-the-counter (OTC) sovereign securities market transactions to be registered for settlement purposes. Transactions registered in MEC-R come from trades agreed outside the electronic trading systems (e.g. by phone), in which counterparty limits play a vital role due to its bilateral (i.e. non anonymous) nature.

DCV daily transactions corresponding to the third quarter of 2013 (i.e. from July 2 to September 30, 63 days) are used to construct the monoplex and multiplex networks analyzed in this paper. The choice of our data is induced by practical reasons, such as that fact that it is a recent period that does not display seasonal effects (e.g. Easter, Christmas, end of the year, large firms' tax collection deadlines) and that can be considered typical according to the recent local sovereign securities' market dynamics.[11] Our database consists of 35,775 consolidated registers between 159 financial institutions.

4. Methodological approach

We examine the main connective features of the monoplex networks and the resulting multiplex network. Particularly, we try to determine whether or not the monoplex and multiplex networks follow the sparse, scale-free, small-world, and clustered patterns that are ubiquitous in real-world networks. Therefore, each network is examined for its main connective features, while each participant is evaluated for its importance (i.e. centrality) within every network.

4.1. Main connective features of the network

A network, or graph, represents patterns of connections between the parts of a system. The most common mathematical representation of a network is the *adjacency matrix*. For a directed network, in which the direction of the linkages between n elements or vertices is relevant, the adjacency matrix (Ω) is a square matrix of dimensions $n \times n$, potentially non-symmetrical, with elements Ω_{ij} such that

$$\Omega_{ij} = \begin{Bmatrix} 1 \text{ if there is an edge from } i \text{ to } j, \\ 0 \text{ otherwise.} \end{Bmatrix} \quad [1]$$

[11] The local sovereign securities' market has not experienced structural shifts as a consequence of recent external events such as the Global Financial Crisis. With regard to the local market, the intervention of one of the largest and most connected broker dealers in the local financial market in November 2012 is the only recent event worth mentioning. Based on our previous research on the Colombian sovereign securities' network the choice of the sample period for studying its main structural features is rather trivial as long as well-known seasonal effects are avoided.

It may be useful to assign real numbers to the edges. These numbers may represent distance, frequency or value, in what is called a weighted network and its corresponding weighted adjacency matrix.

The simplest metric for approximating the connective pattern of a network is *density* (d), a statistic that measures its cohesiveness. Let n be the number of participants or vertices in the network, the density of a graph with no self-edges is the ratio of the number of actual edges (m) to the maximum possible number of edges [2].

$$d = \frac{m}{n(n-1)} \qquad [2]$$

By construction, the density is restricted to the $0 < d \leq 1$ range. Formally, Newman (2010) states that a sufficiently large network for which the density (d) tends to a constant as n tends to infinity is considered as *dense*, whereas a network for which the density tends to zero as n tends to infinity is called *sparse*. However, as one frequently works with non-sufficiently large networks, networks are commonly characterized as sparse when the density is much smaller than the upper limit ($d \ll 1$), and as *dense* when the density approximates the upper limit ($d \cong 1$).

An informative alternative to density is to examine the degree's probability distribution ($\mathcal{P}_k$), with degree (k_i) corresponding to the number of edges or links of the i-vertex. Such distribution provides a natural summary of the connectivity in the graph (Kolaczyk, 2009). Akin to density, the first moment of the distribution of degree (μ_k) measures the cohesion of the network, and is usually restricted to the $0 < \mu_k < n - 1$ range. A sparse graph has an average degree that is much smaller than the size of the graph ($\mu_k \ll n - 1$).

Most real-world networks display right-skewed distributions, in which the majority of vertices are of very low degree, and few vertices are of very high degree, hence inhomogeneous. Such right-skew of real-world networks' degree distributions has been found to approximate a power-law distribution, in what is commonly known as scale-free networks (Barabási and Albert, 1999).

The power-law (or Pareto-law) distribution suggests that the probability of observing a vertex with k edges obeys the potential functional form in [3], where z is a (arbitrary) constant, and γ is known as the *exponent* of the power-law.

$$\mathcal{P}_k \propto z k^{-\gamma} \qquad [3]$$

According to Newman (2010), values in the range $2 \leq \gamma \leq 3$ are typical of scale-free networks, although values slightly outside this range are possible and are occasionally observed. On the other hand, values much greater than 3 are considered typical of homogeneous or random networks.[12]

Regarding the small-world feature (i.e. every vertex can be reached in a limited number of steps), the mean geodesic distance (ℓ) reflects the global structure and measures how big the network is. This average distance depends on the way the entire network is connected, and cannot be inferred from any local measurement (Strogatz, 2003). Let g_{ij} be the *geodesic distance* (i.e. the shortest path in terms of number of edges) from vertex i to j, the mean geodesic distance for vertex i (ℓ_i) corresponds to the mean of g_{ij}, averaged over all reachable vertices j in the network, as in [4]. Respectively, the mean geodesic distance or average path length of a network (i.e. for all pairs of vertices) is denoted as ℓ (without the subscript), and corresponds to the mean of ℓ_i over all vertices.

$$\ell_i = \frac{1}{(n-1)} \sum_{j(\neq i)} g_{ij} \qquad\qquad \ell = \frac{1}{n} \sum_{i} \ell_i \qquad [4]$$

The mean geodesic distance (ℓ) of random or Poisson networks is small, and increases slowly with the size of the network. Therefore, as stressed by Albert and Barabási (2002), random graphs are *small-world* because, in spite of their often large size, in most networks there is relatively a short path between any two vertices. According to Newman et al. (2006), ℓ approximates $\ln n$ for random networks ($\ell \sim \ln n$), where such slow logarithmic increase with the size of the network coincides with the small-world effect (i.e. short average path lengths). In the case of scale-free networks, the mean geodesic distance has been found to be smaller than $\ell \sim \ln n$, about $\ell \sim \ln\ln n$, which Cohen and Havlin (2003) refer to as *ultra-small-world*

The clustering coefficient (c) corresponds to the property of network transitivity. It measures the average probability that two neighbors of a vertex are themselves neighbors; or, in other words, the frequency with which loops of length three (i.e. triangles) appear in the network. Let a *triangle* be a graph of three vertices that is fully connected, and a *connected triple* be a graph of three vertices with at least two connections, the calculation of the network's clustering coefficient is as follows: [13]

[12] Random networks correspond to those originally studied by Erdös and Rényi (1960), in which connections are homogeneously distributed between participants due to the assumption of exponentially decaying tail processes for the distribution of links –such as the Poisson distribution. This type of network, also labeled as "random" or "Poisson", was –explicitly or implicitly- the main assumption of most literature on networks before the seminal work of Barabási and Albert (1999) on scale-free networks.

[13] If three vertices (a, b, c) exist in a graph, a triangle exists when edges (a,b), (b,c) and (c,a) are present (i.e. the graph is complete), whereas a connected triple exists if at least two of these edges are present. In this sense, a triangle occurs when there is transitivity (i.e. two neighbors of a vertex are themselves neighbors). The factor of three in the numerator arises because each triangle is counted three times when connected triplets are counted (Newman, 2010).

$$c = \frac{(\text{number of triangles in the network}) \times 3}{\text{number of connected triples}} \qquad [5]$$

Hence, by construction, clustering reflects the local structure. It depends only on the interconnectedness of a typical neighborhood, the inbreeding among vertices tied to a common center, and thus it measures how incestuous the network is (Strogatz, 2003). Intuitively, the probability of connection of two vertices in a random or homogeneous graph tends to be the same for all vertices regardless the existence of a common neighbor. Therefore, the clustering coefficient is expected to be low in the case of random graphs, and tends to zero in the limit for large random networks.

Real-world complex networks tend to exhibit a large degree of clustering. Albert and Barabási (2002) report that in most –if not all- real networks the clustering coefficient is typically much larger then in comparable random networks, and that this factor slowly increases with the number of vertices. Accordingly, in inhomogeneous graphs, as those resulting from real-world networks, the probability of two neighbors of a vertex being themselves neighbors is reported to be in the 10% and 60% range in most cases (Newman, 2010). In this sense, a particularly low mean geodesic distance combined with high clustering in scale-free networks implies that a few too-connected vertices play a key role in bringing the other vertices close to each other. In the context of studies on robustness of and contagion in financial systems, it is important to identify those too-connected players that are central in a network and could threaten financial stability in case they would fail.

4.2. Importance within the network

Regarding the importance of each participant within every network, *centrality* is the most prevalent and widely used concept. Centrality's most common and simple measure is degree centrality ($\mathcal{k}_i$), which assesses how connected a vertex is to the network. For weighted networks, the strength ($\mathcal{s}_i$) measures the total weight of connections for a given vertex and hence provides an assessment of the intensity of the interaction between participants.

Intuitively, the larger the degree or the strength, the more important the vertex is for the network. Nevertheless, the analytical reach of these two metrics as measures of the relative importance of a vertex is limited because they do not take into account the global properties of the network; this is, they are local measures of importance, and they do not capture neighbors' importance.

The simplest global measure of centrality is eigenvector centrality, whereby the centrality of a vertex is proportional to the sum of the centrality of its adjacent vertices; accordingly, the centrality of a vertex is the weighted sum of centrality at all possible order adjacencies. Hence, in this case centrality arises from (i) being connected to many vertices; (ii) being connected to central vertices; (iii) or both.[14] Alternatively, as put forward by Soramäki and Cook (2012),

[14] For instance, Markose et al. (2012) use eigenvector centrality to determine the most dominant financial institutions in the US credit default swap market, and to design a super-spreader tax that mitigates potential socialized losses.

eigenvector centrality may be thought of as the proportion of time spent visiting each participant in an infinite random walk through the network.

Eigenvector centrality is based on the *spectral decomposition* of a matrix. Let Ω be an adjacency matrix (weighted or non-weighted), Λ a diagonal matrix containing the eigenvalues of Ω, and Γ an orthogonal matrix satisfying $\Gamma\Gamma' = \Gamma\Gamma = \mathrm{I}_n$, whose columns are eigenvectors of Ω, such that

$$\Omega = \Gamma\Lambda\Gamma' \qquad [6]$$

If the diagonal matrix of eigenvalues (Λ) is ordered so that $\lambda_1 \geq \lambda_2 \cdots \lambda_n$, the first column in Γ corresponds to the principal eigenvector of Ω. The principal eigenvector (Γ_1) may be considered as the leading vector of the system, the one that is able to explain most of the underlying system. The positive n-scaled scores corresponding to each element in the principal eigenvector may be considered as their weights within an index. Because the largest eigenvalue and its corresponding eigenvector provide the highest accuracy (i.e. explanatory power) for reproducing the original matrix and capturing the main features of networks (Straffin, 1980), Bonacich (1972) envisaged $\Gamma_1$as a global measure of popularity or centrality within a social network.

However, eigenvector centrality has some drawbacks. As stated by Bonacich (1972), eigenvector centrality works for symmetric structures only (i.e. undirected graphs); yet, it is possible to work with the right (or left) eigenvector (as in Markose et al. (2012)), but this may entail some information loss. Yet, the most severe inconvenience from estimating eigenvector centrality on asymmetric matrices arises from vertices with only outgoing or incoming edges, which will may result in zero eigenvector centrality, and may cause some other non-strongly connected vertices to have zero eigenvector centrality as well (Newman, 2010).

Among some alternatives to surmount the drawbacks of eigenvector centrality (e.g. PageRank, Katz centrality), the HITS (Hypertext Induced Topic Search) algorithm by Kleinberg (1998) has two main advantages that matter for our empirical design. First, it provides two separate centrality measures, *authority centrality* and *hub centrality*, which correspond to the eigenvector centrality as recipient and as originator of links, respectively. Second, it avoids introducing inconvenient stochastic or arbitrary adjustments to the network –as in PageRank or Katz centrality.

The estimation of authority and hub centrality results from estimating standard eigenvector centrality [6] on two modified versions of the adjacency matrix, $\mathcal{A}$ and $\mathcal{H}$, as in [7].

$$\mathcal{A} = \Omega^T\Omega \qquad \mathcal{H} = \Omega\Omega^T \qquad [7]$$

Multiplying the adjacency matrix with a transposed version of itself enables one to identify directed (*in* or *out*) second order adjacencies. Regarding $\mathcal{A}$, multiplying Ω^T with Ω sends weights backwards –against the arrows, towards the pointing vertex-, whereas multiplying Ω with Ω^T (as in $\mathcal{H}$) sends scores forwards –with the arrows, towards the pointed-to vertex (Bjelland et al., 2008). Thus, the HITS algorithm works on a circular thesis: the authority centrality of each participant is defined to be proportional to the sum of the hub centrality of the participants that point to it, and the hub centrality of each participant is defined to be proportional to the sum of the authority centrality of the participant it points to.

Therefore, authority ($\mathcal{a}$) and hub ($\mathcal{h}$) centrality provide global measures of participants' centrality that supplements traditional local measures such as degree ($\mathcal{k}$) and strength ($\mathcal{s}$). Together, local and global measures of centrality will be useful to comprehensively test the presence of positively correlated multiplexity in a system.

5. Main results

The multiplex network here analyzed consists of aggregating three different networks: SEN, MEC, MEC-R. In our sample period (third quarter of year 2013), SEN accounted for 52.6% of the value of transactions in DCV, whereas MEC and MEC-R accounted for 18.5% and 28.7%, respectively. The main connective features of SEN, MEC, MEC-R and the resulting sovereign securities market multiplex are presented in Table 1. All statistics in Table 1 correspond to the mean of those estimated on daily networks; no serial aggregation of daily networks was implemented for estimating the statistics.

SEN, the largest contributor to the sovereign securities multiplex by market value of transactions, exhibits a rather odd connective structure. Against most documented real-world networks, SEN appears to be non-sparse, with a density about 0.60 and with participants directly connecting on average with more than half of their counterparties. Moreover, it appears to be homogeneous by degree, with the corresponding power-law exponent (6.05) well above typical values for scale-free networks (i.e. $\gamma_{\mathcal{k}} \gg 3$) and inhomogeneous by strength. SEN is also non-clustered because the clustering coefficient is below the expected value for random networks of the same size ($\mu_{\mathcal{k}}/n = 0.55$).[15] Yet, we could consider it as approximately ultra-small-world network due to the low mean geodesic distance ($\ell = 1.36$). Such a unique connective structure results from its design: it is a 14-vertex "rich-club" network created by the Ministry of Finance to foster sovereign securities market's liquidity, in which financial institutions agree to trade anonymously, free of any counterparty limit, while observing requisites that determine their permanence on the network.[16] Taken all together, these special features of SEN ultimately govern how participants relate to each other in the network.

[15] Because the clustering coefficient for the SEN monoplex network ($c = 0.43$) is lower than the probability of two vertexes being connected in this dense network ($\mu_{\mathcal{k}}/n = 0.55$), the clustering level may be considered trivial.
[16] The permanence in the SEN network is determined by means of a ranking revised each year. The score obtained by each participant depends on fulfilling requisites such as maximum bid-ask spreads; consistently quoting bid and ask prices; participating in primary auctions of sovereign securities; and maintaining a minimum level of capital.

Table 1
Main connective features of networks [a]

Statistics	SEN	MEC	MEC-R	Sovereign securities multiplex
Contribution (by value of transactions)	0.53	0.18	0.29	1,00
Participants (n)	14	90	91	159
Density (d)	0.60	0.05	0.02	0.02
Mean degree (μ_k)	7.75	4.31	1.45	3.46
Degree power-law exponent (γ_k)	6.05	2.76	2.99	3.66
Strength power-law exponent (s)	2.92	2.43	2.66	2.40
Mean geodesic distance (ℓ)	1.36 [2.63]	2.23 [4.50]	3.11 [4.51]	2.78 [5.07]
Clustering coefficient (c)	0.43 [0.55]	0.10 [0.05]	0.03 [0.02]	0.10 [0.02]

[a] All statistics correspond to sample means estimated on daily statistics for the analyzed quarter (63 days). Expected values for homogeneous networks of n size appear in brackets for the mean geodesic distance (ℓ) and the clustering coefficient (c). We use the algorithm by Clauset et al. (2009) for estimating the power-law exponents.

Figure 2 presents the graph corresponding to SEN's network, in which the diameter of the vertices and the width of the arrows are determined by their strength and the market value of the sovereign securities' deliveries, respectively. The number of edges appears to be evenly distributed, but their width and the diameter of the vertices is inhomogeneous. Likewise, it can be easily observed that participants are well-connected and –therefore- the network is rather dense.

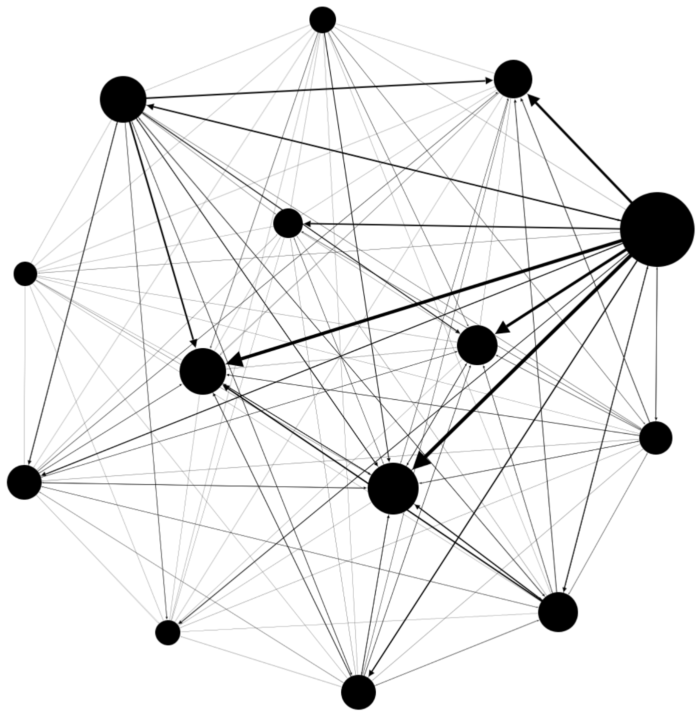

Figure 2. SEN graph. Vertices correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertices' diameter correspond to their strength.

When we turn to the connective structure of our second network, MEC, we find that its characteristics agree with what the literature considers a typical real-world network: it is sparse (the average density is about 0.05), scale-free (the average power law exponent estimated on degree and strength is around 2.76 and 2.43, respectively), and it is approximately ultra-small-world (the mean geodesic distance averages to 2.23 edges).[17] Although the clustering level is not particularly high ($c = 0.10$), it is still higher than the probability of any two vertices in MEC being connected ($\mu_{k}/n = 0.05$), such that we could consider it as a clustered network. Figure 3 presents the graph corresponding to MEC's network, in which there is a high degree of correspondence between the participants pertaining to SEN network (in black) and those vertices at which most edges are concentrated (i.e. with the highest degree) and which display the largest diameters (i.e. with the highest strength).

[17] Some aggregated statistics for MEC and MEC-R reported by Saade (2010) concur with ours regarding their sparseness, inhomogeneity and clustering.

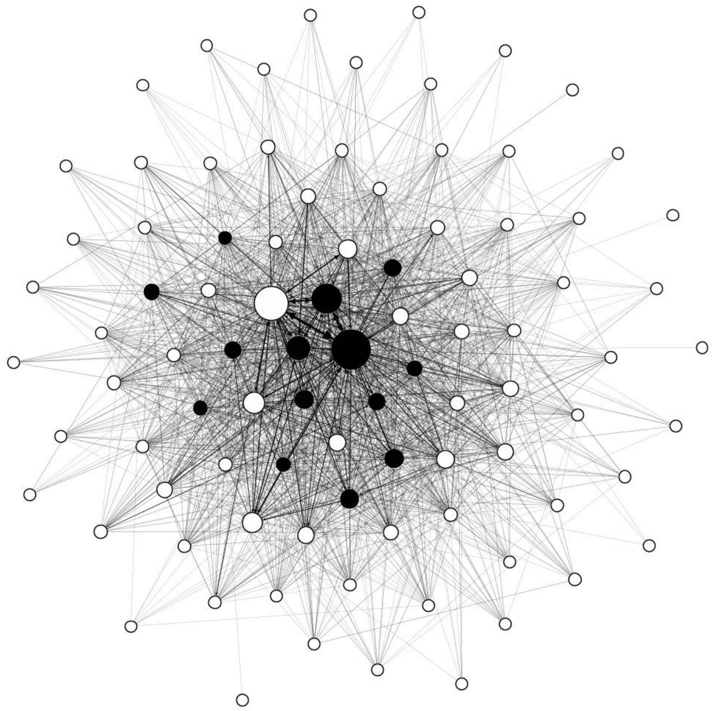

Figure 3. MEC graph. Vertices correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertices' diameter correspond to their strength; SEN participants are in black.

Akin to MEC, the connective structure of MEC-R, is very much like what we would expect of real-world networks. The MEC-R network is sparse, with average density about 0.02; scale-free, with the average power-law exponent estimated on degree and strength around 2.99 and 2.66, respectively; and approximately ultra-small-world, with the mean geodesic distance averaging 3.11 edges. The clustering level is not particularly high ($c = 0.03$), but it is slightly higher than the probability of any two vertices of MEC-R being connected ($\mu_{k}/n = 0.02$), and could thus be considered as somewhat clustered. Figure 4 presents the graph corresponding to MEC-R's network. As with MEC, there is a high degree of correspondence between the participants pertaining to SEN network (in black) and those vertices at which most edges are concentrated (i.e. vertices with the highest degree) and which display the largest diameters (i.e. vertices with the highest strength).

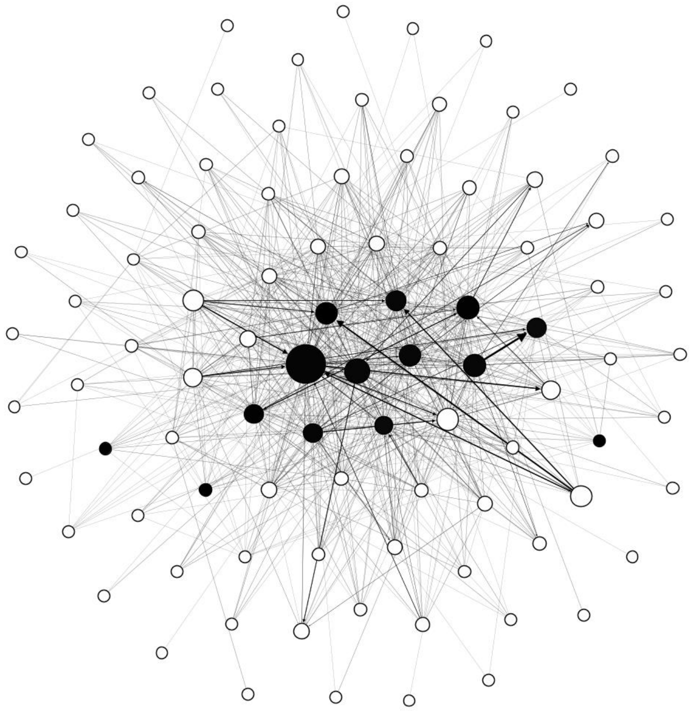

Figure 4. MEC-R graph. Vertices correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value, respectively; vertices' diameter correspond to their strength; SEN participants are filled in black.

According to Table 1, the sovereign securities market multiplex network displays the sparse, inhomogeneous, approximately scale-free[18], approximately ultra-small-world features documented in real-world networks. The clustering coefficient ($c = 0.10$) of the multiplex network is higher than the probability of any two vertices being connected within ($\mu_{k}/n = 0.02$), which indicates that significant clustering is present. Consequently, the multiplex reproduces the overall connective features of MEC and MEC-R, and ignores those of SEN, which is –paradoxically- its main contributor in terms of the market value of transactions. In this sense, the multiplex network of the Colombian sovereign securities market appears to largely preserve the robust-yet-fragile (Haldane, 2009) features of MEC and MEC-R.

Figure 5 presents the graph corresponding to the sovereign securities market multiplex, with the vertices filled out in black pertaining to the SEN network. As in MEC and MEC-R, there is a high degree of correspondence between the participants pertaining to SEN network and the participants with the highest concentration of edges (i.e. the highest degree) and with the largest diameters (i.e. the highest strength).

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[18] As in Newman (2010), exponent values slightly outside the $2 \leq \gamma \leq 3$ range are possible and are observed occasionally. Moreover, the level of the exponent is consistent with the inhomogeneous and skewed distribution of degree, a feature that may be easily verified by visual inspection of Figure 5. As before, the power-law distribution of links is an asymptotic property, thus a strict match between observed and expected theoretical properties for determining the scale-free properties of non-large networks may be impractical.

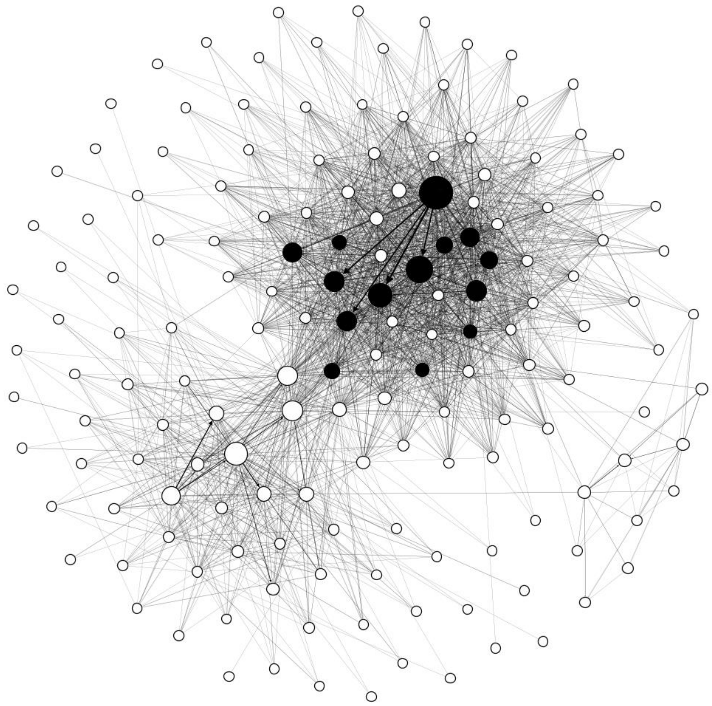

Figure 5. Sovereign securities market graph. Vertices correspond to participating financial institutions; the direction and width of each arrow represents the delivery of local sovereign securities and its market value in the three monoplex networks, respectively; vertices' diameter correspond to their strength; SEN participants are filled in black.

The main features of the multiplex of the sovereign securities market point out that it is a network of networks that preserves the main characteristics of the two networks that contribute the least (47%) to the total market value of sovereign securities transactions, whereas the most salient connective features of SEN appear to contribute less. Therefore, using SEN as a benchmark for the local sovereign securities market may be particularly misleading.[19]

Interestingly, as the three monoplex networks result from different platforms with distinct economic settings and dissimilar operational frameworks, our multiplex network approach reveals how each of the monoplex networks shapes the multiplex one. First, the statistical and graphical resemblance between the multiplex and the MEC and MEC-R monoplex networks is evident from Table 1 and the corresponding graphs, respectively. Second, when coupling SEN, MEC, and MEC-R, it is evident that SEN is no longer a "rich-club" network of 14-financial institutions densely interconnected, but a network in which there is a core of densely connected vertices surrounded by a vast periphery of non-linked participants, and with a connective structure approximating that of MEC and MEC-R. Under this comprehensive view revealed by the multi-layer approach, the SEN network is turned into a sparse,

[19] This is the case of Laverde and Gutiérrez (2012), and Estrada and Morales (2008).

inhomogeneous and potentially scale-free network that preserves its ultra-small-world and non-clustered nature.[20]

The connective coincidence across the layers in the sovereign securities multiplex network may suggest the presence of positively correlated multiplexity in the sense of Lee et al. (2014) and Kenett et al. (2014). This means that the main connective features of the constituent monoplex networks may be preserved as a consequence of consistency in the importance of participants across networks (i.e. vertices' centrality overlapping across layers). The correlation matrix estimated on financial institutions' degree (k), strength (s), hub centrality (h) and authority centrality (a) in Table 2 confirms that there is indeed a positive and non-negligible linear dependence in financial institutions' role and importance across the three different trading and registering platforms, either by local (i.e. degree and strength) or by global (i.e. hub and authority) measures of centrality. Thus, our evidence confirms that the presence of positively correlated multiplexity prompts a multiplex network that tends to preserve the main features of its constituent monoplex networks.

Table 2

Monoplex networks' degree, strength, authority and hub centrality correlation matrix [a]

	k_{MEC}	k_{SEN}	k_{MEC-R}	s_{MEC}	s_{SEN}	s_{MEC-R}	h_{MEC}	h_{SEN}	h_{MEC-R}	a_{MEC}	a_{SEN}	a_{MEC-R}
k_{MEC}	1											
k_{SEN}	0.55	1										
k_{MEC-R}	0.81	0.71	1									
s_{MEC}				1								
s_{SEN}				0.48	1							
s_{MEC-R}				0.67	0.70	1						
h_{MEC}							1					
h_{SEN}							0.29	1				
h_{MEC-R}							0.41	0.49	1			
a_{MEC}										1		
a_{SEN}										0.53	1	
a_{MEC-R}										0.63	0.70	1

[a] The correlation matrix is estimated based on the contribution of each financial institution to the total degree, strength, hub and authority centrality in the whole sample; non-participating financial institutions are assigned a contribution equal to zero. Correlations across centrality measures are omitted to enhance readability.

6. Final remarks

Financial markets are complex systems that we can understand better by examining how its constituents, the financial institutions, relate to one another. This type of research is particularly important to detect the pivotal players in a system whose robustness is vital for

[20] As can be derived from equations 1, 4 and 3, respectively, a large number of non-participating financial institutions will turn the network sparse and less clustered, whereas the small-world characteristic will in this case remain the same because the mean geodesic distance considers reachable vertexes only.

financial stability. Analyses on financial contagion enable us to chart systemic risks. The right approach to gain insight on the interrelated functioning of financial institutions is precisely by means of applying network science to financial data. The main objective is to get a comprehensive view of how financial institutions interact, and to avoid the pitfalls of traditional institution-centric analyses by means of a macro-prudential approach to financial systems.

As has been acknowledged rather recently, it is critical to go beyond single-layer networks to better understand how financial institutions interrelate without assuming that each financial network is isolated from the others. Data recorded in financial market infrastructures is a convenient source of information to take a step towards examining how financial institutions are linked across different financial environments.

We have examined how financial institutions relate to each other in different trading and registering environments of sovereign securities (i.e. the platforms). We have also studied how those environments aggregate into a multiplex network that portrays the local sovereign securities as a network of networks. When we build the multiplex network, we observe strong non-linear effects: the network characteristics of the most important monoplex network (i.e. SEN) in terms of the market value of its transactions do not hand over its characteristics to the multiplex network. SEN appears to be a non-sparse network with its participants directly connecting on average with more than half of their counterparties. It is also homogeneous by degree, inhomogeneous by strength, and non-clustered. In contrast, the multiplex resulting from the three constituting networks conforms to most real-world networks' features, i.e. sparse, inhomogeneous, scale-free, ultra-small-world, and clustered, which are the traits of the lesser important networks MEC and MEC-R.

The unusual connective features of SEN actually reinforce the real-world networks' characteristics displayed by MEC and MEC-R by means of a strong linear dependence between most central participants across all networks. This important finding would have been omitted if one studies monoplex networks in isolation, and could have misled the perspective on the relations between and risks induced by participating financial institutions. Likewise, this finding would have been elusive without the data from the corresponding settlement market infrastructure (i.e. DCV).

Furthermore, the evidence of positively correlated multiplexity reveals some interesting characteristics of the local sovereign securities market. Three characteristics are worth reporting: First, the inhomogeneous and approximate scale-free connective structure, consistent with the robust-yet-fragile nature of financial networks (Haldane, 1999), is the result of structural similarity (i.e. positive correlated multiplexity) across networks within a multi-layer analytical framework. Second, due to the ultra-small-world nature of all networks analyzed in this framework, it is evident that the role of too-connected financial institutions is critical for the efficiency of the market, not only at the single-layer level, but also for the whole market when the coupling of single-layer networks is considered. Third, notwithstanding the economic and operational differences of each monoplex network, financial institutions that are considered "too-connected to fail" tend to overlap across networks, and their role in financial stability is critical due to their contribution to *cross-system risk*.[21] These three characteristics are also an essential focus in order to better understand the local sovereign securities market and its contribution to financial stability.

[21] Cross-system risk corresponds to the potential effects caused by a financial institution experiencing problems across different markets or layers (CPSS, 2008).

This research has contributed to the literature by highlighting the relevance of network analysis on data from financial market infrastructures. First, data gathered by financial market infrastructures have the potential to overcome the main obstacles for working on multi-layer networks highlighted by D'Souza et al. (2014), namely the independent ownership and operation of layers, and the lack of data standardization and sharing. Second, unlike traditional balance sheet data, financial market infrastructures gather granular information from distinct financial environments, which may allow breaking down aggregated financial data into very different layers of complexity in order to reach a deeper understanding of financial markets.

To conclude, we would like to highlight some interesting research extensions. A multi-layer analysis can be done at the level of transactions' or participants' intrinsic characteristics which include – non-exhaustively - the maturity of the traded sovereign securities, the volatility of the securities; the ownership structures of the participating financial institutions; institutions' credit risk ratings; the types of institutions (e.g. banking, non-banking), and the final beneficiary of the transactions (e.g. proprietary or non-proprietary). Likewise, other financial markets (such as foreign exchange, non-sovereign fixed income, derivatives, etc.) may profit from a multi-layer approach based on data provided by the corresponding financial market infrastructures. All these potential multi-layer networks may help understand how financial institutions relate into each other in distinct financial environments, and in the entire financial system.

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