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#### Abstract

We study financial crises in a small open production economy subject to credit constraint and uncertainty on the value of debt repayments. We find that the possibility of reducing the severity of future crises encourages the central planner (CP) to increase both the crisis frequency and current debt. The CP equilibrium can be implemented by a macro-prudential tax on debt and, only during crises, subsidies on consumption and a tax on non-tradable labor. The welfare gain of implementing such equilibrium is small for the baseline scenario but very sensitive to changes in debt volatility and the economy's degree of openness.


Keywords: financial crisis, capital controls, debt shocks, optimal tax
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## 1 Introduction

Recent economic literature has suggested the adoption of macro-prudential policies to reduce financial vulnerability and prevent the occurrence of crisis events. ${ }^{1}$ In this paper, we present a model where the Central Planner (CP) intervention leads, instead, to an increase in the probability of crisis, although it improves social welfare by reducing the crisis severity.

In models of endowment economies where agents face a financial constraint, the decentralized (DC) decisions imply overborrowing in the sense that a benevolent Central Planner (CP), subject to the same constraint but able to internalize the social costs of his decisions, would choose a lower level of debt during normal times. The CP and the DC agents face equally-severe crises, given a level of previous debt; however, since the CP chooses a lower level of debt, it faces a lower probability of crisis. ${ }^{2}$ In contrast, Benigno et al. (2013) analyze the same problem in a model where production is endogenous thereby allowing the CP to reallocate resources across sectors. They find that by this reallocation of resources the CP can reduce both the probability and the intensity of crises which, in turn, reduces the social value of savings and the DC economy ends up displaying underborrowing, i.e. a better crisis management allows the CP to borrow more in normal times.

In this paper, we present a model where production is endogenous and the CP decisions imply that the economy exhibits underborrowing as in Benigno et al. (2013). However, a crucial implication of the CP decision in our model is that the economy faces a higher likelihood of crises although of lower intensity. Following the standard practice, we define a state of crisis as one in which the economy is financially constrained.

Ours is a three-period model of financial amplification for a small open production economy in which the access to credit in period two is limited by a fraction of income net of previous debt. The repayment value of first-period debt is subject to uncertainty. Using this model we find that the economy exhibits underborrowing. DC agents face more intense crises and therefore end up being more cautious when borrowing in the initial period. However, the probability of crisis is increased rather than reduced by the intervention of the CP. The possibility of reallocating resources during crises increases the expected value of future production and gives incentives to the CP to increase the level of current debt. This in turn, on the one hand, increases the expected level of future debt that the CP can take but, on the other hand, also increases the level he would like to take. Since the latter effect is stronger, there is a higher probability of being constrained. The CP is willing to assume additional costs in terms of a higher crisis probability because such costs are compensated by the reduced intensity of the worst crises while additional social benefits are obtained as a greater initial debt supports higher consumption in period one.

There is some empirical evidence showing a positive association between capital controls and the frequency of crises. ${ }^{3}$ Our theoretical results suggest that when the controls are part of an optimal intervention, this increase in the probability of crises would be linked to a reduction in their severity and, therefore, would result in an increase in consumer's welfare.

In the present paper, we incorporate into a single model three important elements from

[^0]previous literature. First, in our model production is endogenous and, in that sense, we follow Benigno et al. (2013). Second, we follow Parra-Polania and Vargas (2014) in modifying the financial constraint to incorporate a fact neglected by the traditional constraint, that is, the effect of previous liabilities on the borrowing capacity. A lender will not regard two people with the same income but different levels of previous debt as equals. Consequently, when evaluating the debt capacity of potential borrowers, lenders take into account not only the borrowers' income but also their previously acquired debt. This fact is also relevant because borrowers (either DC agents or the CP) are aware of this effect and they have additional incentives to limit their debt during normal times. Third, we incorporate debt volatility into the analysis, as in Parra-Polania and Vargas (2014), although we use a more general stochastic payoff profile of financial assets. By including debt volatility we intend to capture the fact that different asset types, associated with debt, are subject to uncertainty, and hence changes in economic conditions may produce significant variations in the real value of debt repayments (i.e. there is a debt shock).

Following Benigno et al. (2013), we calibrate the model at quarterly frequency for Mexican data and then allow for variations to analyze the impact on results of changing some parameter values. For the baseline scenario, we find that the welfare gain from the intervention of the CP is small and equivalent to $0.007 \%$ of total consumption (over the three periods). The decomposition of this overall gain shows that the CP intervention is especially significant in the states where the crisis is the most severe. The particular gain in such states can be equivalent to an increase of $0.58 \%$ of consumption. Furthermore, the welfare gain is especially sensitive to changes in debt volatility and changes in the degree of openness of the economy (measured by the ratio of tradable to non-tradable goods). Small increases in the parameters related to these features produce significant increases in the gain derived from the CP intervention.

An important contribution of this paper is that we propose a tax/subsidy scheme to implement the CP equilibrium in the DC economy. This equilibrium can be implemented by means of a tax on initial debt (a macro-prudential policy) and, only in periods of crisis, subsidies on both tradable and non-tradable consumption and a tax on non-tradable labor. The subsidies to consumption and the tax on labor reallocate resources during periods of crisis such that the price of non-tradable goods and the total production value are higher thereby making it possible for DC agents to borrow more during these events. This reduces the severity of crises and therefore the value of saving in period one, changing the incentives of DC agents from underborrowing to overborrowing. Therefore, it is necessary to impose a tax, rather than a subsidy, on period-one debt.

Specifically, for the baseline scenario we find that we can implement the CP equilibrium by means of a small tax on initial debt equal to $0.09 \%$, an average value of the tax on nontradable labor equal to $6.39 \%$ and average values of the subsidies on tradable and non-tradable consumption equal to $4.53 \%$ and $4.57 \%$, respectively. The subsidies on consumption and the tax on labor during crisis vary with each particular state of the economy (the particular value of the debt shock). The more severe the crisis, the higher the values of the tax and the subsidies.

In the following section we present the model and in Section 3 we describe the equilibrium for both the DC and the CP cases. In Section 4, we present the results, welfare implications and our proposal for the implementation of the CP equilibrium. In the same section, we analyze the results under some parameter changes. In Section 5, we conclude.

## 2 The Model

The model is based on that used by Parra-Polania and Vargas (2014) to differentiate the optimal tax on capital inflows by debt-risk profile. They intend to capture the fact that debt is subject to uncertainty and its real value may change from the moment it is acquired until it is repaid. This risk associated to debt has an impact on the size of the externality generated by private decisions on debt, and therefore affects the size of the optimal tax. A state of crisis is defined as one in which the economy is financially constrained.

We add two important elements to that model, both of which endogenize the probability of crisis. First, we incorporate a more general distribution for the stochastic payoff profile of financial assets. Second, we endogenize production.

This is a three-period model of financial amplification for a small open production economy. In periods one and three, for the sake of simplicity, there is only one type of good, tradable $(T)$, which is the numeraire. In the second period, there is also a non-tradable $(N)$ good with a relative price $p$ that can be interpreted as the inverse of the real exchange rate. Secondperiod production of tradable and non-tradable goods is endogenously determined. In periods one and three, income (production) is exogenous and equal to $\bar{y}_{1}$ and $\bar{y}_{3}$, respectively. First and second-period consumption must be partially financed by debt.

## Households

The utility of the representative consumer is given by

$$
\begin{equation*}
u\left(c_{T, 1}\right)+\beta u\left(C_{2}, l\right)+\beta^{2} u\left(c_{T, 3}\right) \tag{1}
\end{equation*}
$$

where

$$
C_{2} \equiv\left[\sigma^{\frac{1}{\gamma}}\left(c_{T, 2}\right)^{\frac{\gamma-1}{\gamma}}+(1-\sigma)^{\frac{1}{\gamma}}\left(c_{N, 2}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}
$$

is a consumption index that aggregates tradable $\left(c_{T}\right)$ and non-tradable consumption ( $c_{N}$ ) with shares $\sigma$ and $1-\sigma$ respectively, and elasticity of substitution $\gamma$. The parameter $\beta$ is the discount factor, and $l$ is the individual's labor supply. We assume perfect substitutability between labor in the two sectors, and hence $l=l_{T}+l_{N}$.

For the period-two utility function, we consider the Greenwood et al. (1988) form (commonly known as GHH)

$$
u\left(C_{2}, l\right)=\frac{1}{1-\rho}\left(C_{2}-\frac{l^{\delta}}{\delta}\right)^{1-\rho}
$$

and the utility function in periods one and three is given by

$$
u\left(c_{T, j}\right)=\frac{c_{T, j}^{1-\rho}}{1-\rho}, \quad j \in\{1,3\}
$$

The consumer may acquire one-period (tradable) debt in periods one ( $d_{1}$ ) and two ( $d_{2}$ ). The repayment value of first-period debt is subject to uncertainty because it depends on the state of nature $(i)$ in period two such that the consumer repays $d_{1}\left(1+\theta^{i}\right)$ where $\theta^{i}$ is a zero mean shock. We assume there is a continuum of possible states whose probabilities are described by a density function $f\left(\theta^{i}\right)$. The repayment value of second-period debt is not subject to uncertainty.

Budget constraints for periods one, two and three can be respectively expressed as follows:

$$
\begin{gather*}
c_{T, 1}=\bar{y}_{1}+d_{1} / R  \tag{2}\\
c_{T, 2}^{i}+p^{i} c_{N, 2}^{i}+d_{1}\left(1+\theta^{i}\right)=w^{i} l^{i}+\pi^{i}+d_{2}^{i} / R  \tag{3}\\
c_{T, 3}^{i}+d_{2}^{i}=\bar{y}_{3} \tag{4}
\end{gather*}
$$

where $R$ is the gross interest rate (assumed to be constant and equal to $1 / \beta$ ), and the superscript $i$ indicates the state of nature realized at the beginning of period two, which conditions decisions in periods two and three. In the first period, the consumer partially finances consumption by borrowing. In the second period, consumption and debt repayment are financed by the labor income ( $w$ is the wage), the profits received from firms $(\pi)$ and new debt $\left(d_{2}\right)$. In the third period, income is used to finance consumption and to pay off all remaining debt.

We assume that access to international financial markets is imperfect and the consumer has limited access to credit. In evaluating the debt capacity of potential borrowers, lenders take into account not only a fraction $k$ of their current income (the fraction that agents can use as collateral), but also their previously acquired debt. The financial constraint is

$$
\begin{equation*}
d_{2}^{i} / R \leq k\left[w^{i} l^{i}+\pi^{i}-d_{1}\left(1+\theta^{i}\right)\right] \tag{5}
\end{equation*}
$$

As explained by Parra-Polania and Vargas (2014), including previously acquired liabilities incorporates the fact that the lender does not regard two individuals with the same income but with different levels of initial debt as equals. The maximum amount of credit that the borrower with higher initial debt can obtain should be lower than that of the other because the former already owes a higher proportion of his income. This element, although neglected by traditional constraints, is important for the calculation of the externality size because it provides additional incentives for consumers to limit their debt, and thus the externality that arises during crises is smaller than the one calculated without such effect.

## Firms

The problem for a representative firm is static and simple. In the second period, it uses labor to produce tradable and non-tradable goods according to the following constant returns to scale technologies:

$$
\begin{align*}
y_{T}^{i} & =A_{T}\left(l_{T}^{i}\right)^{\alpha_{T}}  \tag{6}\\
y_{N}^{i} & =A_{N}\left(l_{N}^{i}\right)^{\alpha_{N}} \tag{7}
\end{align*}
$$

where $A_{T}$ and $A_{N}$ are productivity levels. The representative firm maximizes benefits:

$$
\begin{equation*}
\pi^{i}=y_{T}^{i}+p^{i} y_{N}^{i}-w^{i} l^{i} \tag{8}
\end{equation*}
$$

## 3 Equilibrium

We obtain solutions for both the DC economy and that with a CP. The solution is obtained by backward induction so we first solve for periods two and three, taking the initial debt level as given. Since the state of nature is observed at the beginning of period two, this part of the solution does not imply uncertainty. Then we proceed to solve for period one, where there is uncertainty about the state of period two.

### 3.1 Periods two and three

### 3.1.1 Decentralized economy

To solve the households' problem, we substitute (4) into (1) and denote the Lagrange multipliers associated with budget constraint (3) and financial constraint (5) by $\mu$ and $\lambda$, respectively. Maximizing the Lagrangian and considering the market-clearing conditions ( $c_{N, 2}^{i}=y_{N}^{i}$, for non-tradables and $c_{T, 2}^{i}=y_{T}^{i}+d_{2}^{i} / R-d_{1}\left(1+\theta^{i}\right)$, for tradables) we obtain the following first-order conditions with respect to $c_{T, 2}, c_{N, 2}, d_{2}$ and $l$ (for state $i$ ):

$$
\begin{gather*}
\left(C_{2}^{i}-\frac{\left(l^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left(\frac{\sigma C_{2}^{i}}{c_{T, 2}^{i}}\right)^{\frac{1}{\gamma}}=\mu^{i}  \tag{9}\\
\left(C_{2}^{i}-\frac{\left(l^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left(\frac{(1-\sigma) C_{2}^{i}}{c_{N, 2}^{i}}\right)^{\frac{1}{\gamma}}=p^{i} \mu^{i}  \tag{10}\\
\left(c_{T, 3}^{i}\right)^{-\rho}+\lambda^{i}=\mu^{i}  \tag{11}\\
\left(C_{2}^{i}-\frac{\left(l^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left(l^{i}\right)^{\delta-1}=w^{i}\left(\mu^{i}+k \lambda^{i}\right) \tag{12}
\end{gather*}
$$

Condition (9) equalizes the marginal utility of consumption to the shadow value of current wealth. Condition (10) equalizes the marginal rate of substitution of goods (tradable and nontradable) to their relative price. Equation (11) is the Euler equation for assets. If the financial constraint is binding, there is a gap between the shadow value of current wealth and the value of transferring income between periods, due to the shadow price of relaxing the financial constraint $\left(\lambda^{i}\right)$. Equation (12) indicates that when the household is financially constrained, it is more willing to supply one extra unit of labor as a way of relaxing the constraint.

Using (9) and (10), we find the following expression for the price of non-tradable goods:

$$
\begin{equation*}
p^{i}=\left(\frac{1-\sigma}{\sigma} \frac{c_{T, 2}^{i}}{c_{N, 2}^{i}}\right)^{\frac{1}{\gamma}} \tag{13}
\end{equation*}
$$

With regard to the firms' problem, the first order conditions with respect to $l_{T}$ and $l_{N}$ (for state $i$ ) are:

$$
\begin{gather*}
w^{i}=\alpha_{T} A_{T}\left(l_{T}^{i}\right)^{\alpha_{T}-1}  \tag{14}\\
w^{i}=\alpha_{N} p^{i} A_{N}\left(l_{N}^{i}\right)^{\alpha_{N}-1} \tag{15}
\end{gather*}
$$

When the economy is unconstrained we have that $\lambda^{i}=0, \mu^{i}=\left(c_{T, 3}^{i}\right)^{-\rho}$ and the DC equilibrium (of periods two and three given a level of initial debt) is described by equations (3), (4), (9), (10) and (12), from the households' problem; equations (6), (7), (14) and (15), from the firms' problem and aggregate conditions, $c_{N, 2}^{i}=y_{N}^{i}$ and $l^{i}=l_{T}^{i}+l_{N}^{i}$.

When private agents are financially constrained we need to take into account that $\lambda^{i} \geq 0$, and therefore we need to add, to the foregoing equation system, condition (11) and the financial constraint (equation (5) with equality).

### 3.1.2 Central Planner

We proceed to solve the problem (for periods two and three) faced by a benevolent CP who is subject to the same financial constraint and uncertainty conditions as private agents, but is capable of internalizing the effect of consumption and labor decisions on prices and wages. ${ }^{4}$ In this case, the first-order conditions with respect to $c_{T, 2}, d_{2}, l_{T}$ and $l_{N}$ (for state $i$ ) are:

$$
\begin{align*}
& \left(C_{c p, 2}^{i}-\frac{\left(l_{c p}^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left(\frac{\sigma C_{c p, 2}^{i}}{c_{c p, T, 2}^{i}}\right)^{\frac{1}{\gamma}}+\left(\frac{1-\sigma}{\sigma}\right)^{\frac{1}{\gamma}}\left(\frac{c_{c p, T, 2}^{i}}{c_{c p, N, 2}^{i}}\right)^{\frac{1-\gamma}{\gamma}} \frac{k}{\gamma} \lambda_{c p}^{i}=\mu_{c p}^{i}  \tag{16}\\
& \left(c_{c p, T, 3}^{i}\right)^{-\rho}+\lambda_{c p}^{i}=\mu_{c p}^{i}  \tag{17}\\
& \left(C_{c p, 2}^{i}-\frac{\left(l_{c p}^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left(l_{c p}^{i}\right)^{\delta-1}=\left(\mu_{c p}^{i}+k \lambda_{c p}^{i}\right) \alpha_{T} A_{T}\left(l_{c p, T}^{i}\right)^{\alpha_{T}-1}  \tag{18}\\
& \left(C_{c p, 2}^{i}-\frac{\left(l_{c p}^{i}\right)^{\delta}}{\delta}\right)^{-\rho}\left[\left(l_{c p}^{i}\right)^{\delta-1}-\left(\frac{(1-\sigma) C_{c p, 2}^{i}}{c_{c p, N, 2}^{i}}\right)^{\frac{1}{\gamma}} \alpha_{N} A_{N}\left(l_{c p, N}^{i}\right)^{\alpha_{N}-1}\right] \\
& =\lambda_{c p}^{i} k\left(\frac{1-\sigma}{\sigma} \frac{c_{c p, T, 2}^{i}}{c_{c p, N, 2}^{i}}\right)^{\frac{1}{\gamma}} \frac{\gamma-1}{\gamma} \alpha_{N} A_{N}\left(l_{c p, N}^{i}\right)^{\alpha_{N}-1} \tag{19}
\end{align*}
$$

Since $c_{N, 2}^{i}=y_{N}^{i}$ is always satisfied in the aggregate, $c_{N, 2}^{i}$ is not relevant to the CP problem. We have used the subscript $c p$ to distinguish the endogenous variables associated with this problem.

When the economy is unconstrained ( $\lambda_{c p}^{i}=0, \mu_{c p}^{i}=\left(c_{c p, T, 3}^{i}\right)^{-\rho}$ ) the CP equilibrium (of periods two and three given a level of initial debt) is described by equations (3), (16), (18), (19), aggregate conditions $c_{N, 2}^{i}=y_{N}^{i}$ and $l^{i}=l_{T}^{i}+l_{N}^{i}$ and the pricing rule from the DC equilibrium (equation (13)). It can be shown that this equilibrium is equal to that of the (unconstrained) DC economy. ${ }^{5}$

When the economy is financially constrained $\left(\lambda^{i} \geq 0\right)$ we need to add, to the foregoing equation system, condition (17) and the financial constraint (equation (5) with equality). In this case the equilibrium value of endogenous variables could be different from those of the DC case. By comparing condition (16) for the CP problem with condition (9) for the DC economy, we can see that the marginal valuation of liquidity for the CP differs from that for the DC agent when the economy is in crisis $\left(\lambda^{i} \geq 0\right)$. Unlike previous literature, it does not follow that the marginal valuation for the CP will be higher than that for the DC agent. The reason behind it is that for endowment economies the decisions on consumption for the

[^1]CP and the DC agent, given a level of initial debt $d_{1}$, are the same even when the economy is constrained. In that case, it is possible to show that the CP would choose a lower level of initial debt and therefore the economy would experience overborrowing. In models with endogenous production, as in the present model, the optimal decisions on consumption and labor for a given level of $d_{1}$ for the CP and the DC agent may differ, because the CP can reallocate resources among sectors, and therefore the valuation of liquidity when constrained may be higher or lower for the CP than for the DC agent.

### 3.2 First-Period Debt

In period one there is uncertainty about the state of nature of period two, and therefore neither private agents nor the CP can be certain about whether or not the economy will be constrained.

Using equations (3), (5), (8) and $c_{N, 2}^{i}=y_{N}^{i}$, for the unconstrained economy we can write:

$$
\begin{equation*}
d_{2}^{*} / R=c_{T, 2}^{*}-y_{T}^{*}+d_{1}\left(1+\theta^{i}\right) \tag{20}
\end{equation*}
$$

and

$$
\begin{equation*}
d_{2}^{*} / R \leq k\left[y_{T}^{*}+p^{*} y_{N}^{*}-d_{1}\left(1+\theta^{i}\right)\right] \tag{21}
\end{equation*}
$$

where we denote by $X^{*}$ the value of any variable $X$ in the unconstrained equilibrium. Then, from the foregoing equations and taking into account (13), we can obtain the following expression:

$$
\begin{equation*}
\theta^{i}=\frac{1}{d_{1}}\left(y_{T}^{*}\left(d_{1}, \theta^{i}\right)-\frac{c_{T, 2}^{*}\left(d_{1}, \theta^{i}\right)-k p^{*}\left(d_{1}, \theta^{i}\right) y_{N}^{*}\left(d_{1}, \theta^{i}\right)}{1+k}\right)-1 \tag{22}
\end{equation*}
$$

The value for $\theta^{i}$ that solves this equation corresponds to the critical value of $\theta$ above which the economy will be constrained in period two (we will denote it by $\widetilde{\theta}\left(d_{1}\right)$ ). Notice that the value of $\widetilde{\theta}\left(d_{1}\right)$ depends on the optimal decisions of consumption and production as well as on the level of initial debt, and hence the probability of crisis for this economy is endogenously determined.

If we assume that the support of $\theta^{i}$ is the interval $[\underline{\theta}, \bar{\theta}]$, then the function to maximize in period one with respect to $d_{1}$ takes the following form:

$$
u\left(\bar{y}_{1}+\frac{d_{1}}{R}\right)+\beta\left(\int_{\underline{\theta}}^{\tilde{\theta}\left(d_{1}\right)} V^{U E}\left(\theta^{i}, d_{1}\right) f\left(\theta^{i}\right) d \theta^{i}+\int_{\tilde{\theta}\left(d_{1}\right)}^{\bar{\theta}} V^{C E}\left(\theta^{i}, d_{1}\right) f\left(\theta^{i}\right) d \theta^{i}\right)
$$

where $V^{U E}=u\left(C_{2}^{*}\left(d_{1}, \theta^{i}\right), l^{*}\left(d_{1}, \theta^{i}\right)\right)+\beta u\left(\bar{y}_{3}-d_{2}^{*}\left(d_{1}, \theta^{i}\right)\right)$ for the Unconstrained Equilibrium $(U E)$ and $V^{C E}=u\left(C_{2}\left(d_{1}, \theta^{i}\right), l\left(d_{1}, \theta^{i}\right)\right)+\beta u\left(\bar{y}_{3}-d_{2}\left(d_{1}, \theta^{i}\right)\right)$ for the Constrained Equilibrium $(C E)$ are the value functions resulting from utility maximization of periods two and three.

For the DC economy, the first order condition of this problem implies:

$$
u^{\prime}\left(\bar{y}_{1}+\frac{d_{1}}{R}\right)=-\binom{\int_{\underline{\theta}}^{\tilde{\theta}\left(d_{1}\right)} \frac{\partial V^{U E}\left(\theta^{i}, d_{1}\right)}{\partial d_{1}} f\left(\theta^{i}\right) d \theta^{i}}{+\int_{\tilde{\theta}\left(d_{1}\right)}^{\bar{\theta}} \frac{\partial V^{C E}\left(\theta^{i}, d_{1}\right)}{\partial d_{1}} f\left(\theta^{i}\right) d \theta^{i}}
$$

Similarly, for the CP problem, the condition is

$$
u^{\prime}\left(\bar{y}_{1}+\frac{d_{c p, 1}}{R}\right)=-\binom{\int_{\underline{\theta}}^{\tilde{\theta}\left(d_{c p, 1}\right)} \frac{\partial V^{U E}\left(\theta^{i}, d_{c p, 1}\right)}{\partial d_{c p}} f\left(\theta^{i}\right) d \theta^{i}}{+\int_{\tilde{\theta}\left(d_{c p, 1}\right)}^{\bar{\theta}} \frac{\partial V_{c p}^{C E}\left(\theta^{2}, d_{c p, 1}\right)}{\partial d_{c p, 1}} f\left(\theta^{i}\right) d \theta^{i}}
$$

where we have taken into account that the value function for the constrained equilibrium is different for the CP.

## 4 Results and Implementation of the CP Equilibrium

In this section, we describe the calibration of the model, solve for the competitive equilibrium and the CP allocations numerically, and propose a mechanism to implement the CP allocations through a set of taxes and subsidies.

The specific parameter values of our baseline scenario are reported in Table 1.

| Table 1 <br> Baseline parameter values <br> Parameter |  |
| :--- | :--- |
| Elasticity of substitution between tradable | Value |
| $\quad$ and non-tradable goods | $\gamma=0.76$ |
| Discount factor | $\beta=0.9717$ |
| Steady-state productivity levels | $A_{T}=A_{N}=1$ |
| Weight of tradable consumption | $\sigma=0.3526$ |
| Intertemporal elasticity of substitution | $\rho=2$ |
| Labor supply elasticity | $\delta=1.75$ |
| Labor share in production | $\alpha_{T}=\alpha_{N}=0.66$ |
| Financial constraint parameter | $k=1.899$ |
| Exogenous income | $\bar{y}_{1}=0.6486, \bar{y}_{3}=3.2$ |
| Standard deviation of the debt shock | $s t d e v(\theta)=0.1$ |

All of these parameter values but three $\left(k, \bar{y}_{3}\right.$ and $\left.\operatorname{stdev}(\theta)\right)$ are taken from Benigno et al. (2013). They study financial crises by setting a two-sector production model which includes an occasionally binding collateral constraint (as in our model) and intend to fit the data for the Mexican economy over the period 1993-2007. We calibrate the financial constraint coefficient $k$ and the value of $\bar{y}_{3}$ to match a crisis probability of $2 \%$ and a debt-to-GDP ratio of $35 \%$ as targeted in Benigno et al. (2013). Since these authors do not incorporate the debt-risk profile, for modelling the shock $\theta$ we assume that it is normally distributed with zero mean and follow Parra-Polania and Vargas (2014) to approximate debt volatility (i.e. the standard deviation) using the real variation of credit contracted in dollars by means of the average absolute value of $(1+\operatorname{dev}) /(1+\pi)-1$ (where dev is devaluation and $\pi$ inflation) of quarterly Mexican data over the period 1993Q1-2007Q4.

Using these parameter values, we compare the DC and CP equilibria and find that, unlike
related literature, the economy in our model displays underborrowing ${ }^{6}$ (in this sense our paper is similar to Benigno et al., 2013) and the optimal intervention of the CP reduces the severity of crises but increases their probability ( $2.0 \%$ for the DC agents vs. $2.5 \%$ for the CP , in the baseline scenario).

If, for each level of initial debt and each particular state of nature, both DC agents and the CP were expected to face crises of equal severity, the former would underestimate the social value of saving in period one, that is, there would be overborrowing because DC agents do not fully take into account the impact of their debt decisions in period one on the economy's borrowing capacity in period two. As remarked in Section 1, this is precisely the result found by previous literature in models of endowment economies ${ }^{7}$.

In a production economy, like the one in this paper, an additional, opposing and dominant force affects the marginal value of saving. During the crisis, the CP can reallocate resources across sectors such that the price of non-tradable goods and the total production value are higher and, as a result, he can borrow more and reduce the crisis severity. This reduces the social value of period-one saving so much so that the DC economy displays underborrowing, i.e. because DC agents face more intense crises, they end up being more cautious when borrowing in the first period.

However, the probability of crisis is increased rather than reduced by the intervention of the CP. The possibility of reallocating resources during crises and the higher level of current debt, on the one hand, increase the expected level of future debt that the CP can take but, on the other hand, also increase the level he would like to take. Since the latter effect is stronger, there is a higher probability of being constrained. The CP is willing to assume additional costs in terms of a higher crisis probability because such costs are compensated by the reduced intensity of the worst crises while additional social benefits are obtained because a greater initial debt supports higher consumption in period one.

Figure 1 compares the expected utility (lower panel) and the critical theta $\widetilde{\theta}$ (upper panel) at different levels of initial debt $\left(d_{1}\right)$ for both the CP and the DC case in the baseline scenario. At point A, with $d_{1}^{A}=0.9080$, the DC agents maximize their utility. With the same level of initial debt but reallocating resources during crises, the CP can attain a higher level of expected utility (point B). However, as explained above, the planner can gain even more by increasing the level of initial debt up to point C , where $d_{1}^{C}=0.9154$. Instead, if such level of debt were chosen by DC agents without reallocating resources, the utility level would be lower (point D) than that originally obtained due to the increase in the probability of crisis as well as of its severity.

Since the value of $\theta$ above which the economy will be constrained in period two (i.e. $\widetilde{\theta}$ ) is decreasing in the level of initial debt, the fact that DC agents borrow less than the CP in period one implies that the latter faces a higher probability of being constrained in period two.

For the baseline scenario the intervention of the CP represents a small overall gain equivalent to $0.007 \%$ of total consumption (over the three periods). ${ }^{8}$ This welfare improvement can

[^2]be decomposed into specific gains and losses. Increasing the level of initial debt represents an increase of $0.47 \%$ of consumption in period one. A higher initial debt reduces the borrowing capacity in period two and increases the probability of crisis; however, resource reallocation reduces the crisis intensity. These two effects produce a welfare gain in the states of nature where crises are the most severe (specifically in those that occur with $0.76 \%$ of probability, and correspond to $38 \%$ of all crisis events). Such gain is equivalent to an increase of $0.58 \%$ in consumption of periods two and three in those worst states. In the other states, there is a welfare loss equivalent to a reduction of $0.17 \%$ in consumption in such states.


Fig. 1. Expected Utility and Critical Theta levels

### 4.1 Implementation of the CP equilibrium

In our model, the CP equilibrium can be implemented in the DC economy by means of different taxes and subsidies that, on the one hand, reallocate resources during the crisis, just
imum expected utility in the DC equilibrium equal to that in the CP case, i.e. $u\left(c_{T, 1}(1+x)\right)+$ $\beta\left(\int_{\underline{\theta}}^{\bar{\theta}}\left(u\left(C_{2}(1+x), l^{*}\right)+\beta u\left(c_{T, 3}(1+x)\right)\right) d \theta^{i}\right)=u\left(c_{T, 1}^{c p}\right)+\beta\left(\int_{\underline{\theta}}^{\bar{\theta}}\left(u\left(C_{2}^{c p}, l^{c p}\right)+\beta u\left(c_{3}^{c p}\right)\right) d \theta^{i}\right)$
as the CP would and, on the other hand, give the incentives to DC agents to choose the same level of initial debt that would be set by the CP. Specifically, we propose a tax/subsidy scheme to equalize DC decisions to those of the CP. In our proposal, first-period debt is taxed, and second-period consumption and labor are taxed only in states of crisis.

We find that, given an initial debt level and a particular state of nature in which the economy is in crisis (i.e. constrained), we can implement the CP equilibrium for periods two and three by means of subsidies on both tradable and non-tradable consumption ( $\tau_{C T}$ and $\tau_{C N}$, respectively) and a tax on non-tradable labor ( $\tau_{L N}$ ). These taxes (subsidies) are returned to (paid back by) private agents as a lump sum transfer (tax). ${ }^{9}$ The subsidies to consumption and the tax on labor reallocate resources such that the price of non-tradable goods and the total production value are higher thereby making it possible for DC agents to borrow more during the crisis. As a result, the severity of crises is reduced and there is a lower value of saving in period one, changing the incentives of DC agents from underborrowing to overborrowing. Therefore, we also need to impose a tax on period-one debt $\left(\tau_{d 1}\right)$ so that first-period DC decisions are equal to those of the CP.

Since the debt shock $\theta^{i}$ is observed at the beginning of period two, subsidies and taxes vary with each particular state of the economy. For the baseline scenario, the average values of these subsidies and taxes are $\tau_{C T}=-4.53 \%, \tau_{C N}=-4.57 \%, \tau_{L N}=6.39 \%$ and $\tau_{d 1}=0.09 \%$. These values are calculated as averages of all the taxes/subsidies imposed across states in which the economy is constrained (i.e. for all $\theta^{i} \geq \widetilde{\theta}$ ), weighted by the probability of each state. The more severe the crisis (i.e. the greater the $\theta^{i}$ ), the higher the tax and the subsidies. In other words, the (absolute) values of the taxes are decreasing in the probability of $\theta \geq \theta^{i} .{ }^{10}$ With the appropriate taxes and subsidies for each state, DC agents choose exactly the same levels of initial debt and consumption (in each state of periods two and three) and labor (in each state of period two) that the CP would choose.

### 4.2 Analysis of sensitivity

Now we proceed to analyze the impact on results of changing, one at a time, some parameter values. We change all parameters but report the results only for those which produce the most significant impacts (see Table 2), in particular those which imply a substantial increase in the overall welfare gain.

[^3]Table 2
Changes in Parameter Values

|  | Overall Gain (\%) | Crisis Prob.(\%) |  | Avrg. Taxes (\%) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | DC | CP | $\tau_{L N}$ | $\tau_{C N}$ | $\tau_{C T}$ | $\tau_{d 1}$ |
| Benchmark | 0.007 | 2.0 | 2.5 | 6.4 | -4.6 | -4.5 | 0.09 |
| $\delta=1.84$ | 0.012 | 2.8 | 3.8 | 5.5 | -5.3 | -4.3 | 0.11 |
| $\alpha_{T}=0.7$ | 0.012 | 3.3 | 4.4 | 7.9 | -4.3 | -5.2 | 0.20 |
| $\gamma=0.85$ | 0.013 | 3.8 | 5.1 | 7.9 | -4.7 | -5.3 | 0.25 |
| $\rho=1.8$ | 0.014 | 3.2 | 4.6 | 5.3 | -5.4 | -4.1 | 0.13 |
| $A_{T}=0.95$ | 0.016 | 2.9 | 4.3 | 8.2 | -5.0 | -5.6 | 0.19 |
| $\operatorname{stdev}(\theta)=0.12$ | 0.020 | 3.3 | 4.8 | 8.6 | -5.7 | -6.0 | 0.24 |
| $\sigma=0.38$ | 0.027 | 6.0 | 9.5 | 8.6 | -4.9 | -5.6 | 0.47 |

Specifically, we bring attention to the two most significant changes reported in Table 2. A small increase of the standard deviation of the debt shock $(\operatorname{stdev}(\theta)$, from 0.1 to 0.12$)$ multiplies the overall gain almost by three. Similarly, a small increase in the weight of tradable goods ( $\sigma$, from 0.3526 to 0.38 ) multiplies the overall gain almost by four.

Although the welfare gain from the intervention (i.e. the macro-prudential tax on debt and the tax on labor and subsidies on consumption during crises) is small for the baseline scenario, such gain is very sensitive to changes in debt volatility and the weight of tradable goods in the economy. Therefore, countries with debt represented by highly volatile assets or those with a high degree of openness (measured by the ratio of tradable to non-tradable goods) should be more willing to intervene. Furthermore, notice from Table 2 that in these cases the CP is willing to allow for significant increases in the probability of crisis. As explained above, this is a result of the important benefits that can be obtained from increasing borrowing, and hence consumption, in periods where income is low (i.e. period one in the model) and from reducing the intensity of the worst crises.

## 5 Conclusions

In this paper we study sudden stops in capital flows in a small open economy model. It is a three-period model in which the access to credit in period two is limited by a fraction of income net of previous debt (i.e. there is a financial constraint), production is endogenous and there is uncertainty on the repayment value of first-period debt.

We find that, due to the possibility of reallocating resources between sectors, the CP faces less severe crises than DC agents which gives the incentives to the CP to increase current debt (i.e. the model displays underborrowing). This in turn increases both the expected level of future debt that the CP can take and the level he would like to take. Since the latter effect is stronger, the CP faces a higher frequency of being constrained (i.e. facing a crisis). However, the cost of this increase in the crisis probability is compensated by the benefits derived from the reduced intensity of the worst crises and the fact that a greater initial debt supports higher consumption in period one.

We also propose a tax/subsidy scheme to implement the CP equilibrium in the DC economy. Specifically we find that the CP equilibrium can be implemented by means of a tax on debt (a macro-prudential policy) and, only during crises, subsidies on both tradable and non-tradable consumption and a tax on non-tradable labor. The welfare gain of moving to
the CP equilibrium is small for the baseline scenario but is especially sensitive to changes in debt volatility and the degree of openness of the economy (measured by the ratio of tradable to non-tradable goods).

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[^0]:    ${ }^{1}$ e.g. Bianchi (2011), Korinek (2011), Parra-Polania and Vargas (2014).
    ${ }^{2}$ e.g. (in addition to those papers mentioned in footnote 1) Korinek (2010), Jeanne and Korinek (2010a,b), Bianchi and Mendoza (2011).
    ${ }^{3}$ See Glick, Guo and Hutchison (2006), Glick and Hutchinson (2005), Leblang (2003), and Bordo et al. (2001).

[^1]:    ${ }^{4}$ Since there are neither prices nor wages in the centralized economy, the financial constraint for the CP is given by $d_{2}^{i} / R \leq k\left[y_{T}^{i}+\left(\frac{1-\sigma}{\sigma} \frac{c_{T, 2}^{i}}{c_{N, 2}^{i}}\right)^{\frac{1}{\gamma}} y_{N}^{i}-d_{1}\left(1+\theta^{i}\right)\right]$, where we take into account equation (8) and the pricing rule (13).
    ${ }^{5}$ Comparing both equation systems, when $\lambda=0$, for a given $d_{1}$ and using equations (13), (14) and (15) we can verify that: equation (16) is equal to equation (9), equation (17) is equal to equation (11), equation (18) is equal to equation (12) and using equations (16), (18) and (19) we obtain equation (10).

[^2]:    ${ }^{6}$ The difference between the optimal initial debt level for DC agents and that for CP is small both in levels ( 0.9080 vs. 0.9154 , respectively) and as shares of the annual GDP ( $35.0 \%$ vs. $35.3 \%$, respectively).
    ${ }^{7}$ This can be seen in our model by comparing equations (9) and (16) given a value of $d_{1}$ and assuming the same values for the consumer's endogenous values. Then, the valuation of liquidity for the CP is higher than the one for the DC agent, resulting in overborrowing.
    ${ }^{8}$ This gain is calculated as the increment $x$, in consumption, that is required to make the max-

[^3]:    ${ }^{9}$ We assume that lenders evaluate the borrowing capacity of agents based on their income before taxes, and therefore the financial constraint for an economy with taxes is the same as in Equation (5).
    ${ }^{10}$ For instance, if $\theta^{i}=0.3, \operatorname{Pr}\left[\theta \geq \theta^{i}\right]=0.13 \%, \tau_{C T}=-12.1 \%, \tau_{C N}=-12.2 \%$ and $\tau_{L N}=16.9 \%$ while if $\theta^{i}=0.2$ (a less severe crisis), $\operatorname{Pr}\left[\theta \geq \theta^{i}\right]=2.28 \%, \tau_{C T}=-0.55 \%, \tau_{C N}=-0.59 \%$ and $\tau_{L N}=0.77 \%$. Note that $\tau_{d 1}$ does not change with each state of nature since it is imposed in period one, before the state of nature of period 2 is revealed.

