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An exploration on interbank markets and the operational framework of monetary policy in Colombia

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An exploration on interbank markets and the operational framework of monetary policy in Colombia

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Abstract

We set a dynamic stochastic model for the interbank daily market for funds in Colombia. The framework features exogenous reserve requirements and requirement period, competitive trading among heterogeneous commercial banks, daily open market operations held by the Central Bank (auctions and window facilities), and idiosyncratic demand shocks and uncertainty in the daily auction. The model highlights the institutional framework and the money supply mechanisms for the interbank market. We construct a data base for the Colombian case that incorporates the principal variables of the model and give us some insights about the behavior of them in a typical requirement period. We corroborate the Martingale hypothesis for the interbank interest rate..

Key words: Interbank Market; Overnight Rates; Reserve Demand

JEL Code: E44, E52, G21.

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1 Introduction

In 1999, Colombia established a floating exchange rate regime, and started the process to converge towards an inflation targeting regime. During this process, monetary aggregates were replaced by the interest rate as the instrument used by the Central Bank.

There are some key elements within the inflation targeting framework. The starting point is the announcement of an inflation target for a future period, usually one to two years ahead. This seeks to anchor inflation expectations of agents.

In this sense, theoretically, when there are shocks to the economy, the Central Bank changes the policy interest rate to bring inflation back into line with the target and to maintain economic growth around its long-term trend. It is expected that when the Central Bank in Colombia changes its policy interest rate, this immediately affects the interbank interest rate resulting in changes in short and long term interest rates in the market.

Therefore, the alignment between the policy interest rate and the interest rate in the interbank market is a necessary condition for the success of the monetary policy. It ensures the correct operation of the monetary transmission channels and, ultimately, the fulfillment of the inflation target as well as an output gap close to zero. This requires the correct functioning of the interbank market in which financial institutions lend or borrow resources.

In Colombia, the way the monetary policy actually works is through auctions and window facilities, instead of controlling the interest rate directly. The Central Bank of Colombia supplies resources in a daily basis through auctions, with amounts announced a day before; and through deposit and lending (last resort) facilities. The aim of the monetary authority is to supply just-enough resources to keep the auction rates in line with the policy rate.

There are different strategies of implementation of monetary policy in countries with implicit or explicit inflation targeting regimes. The Colombian case shares the most with other countries operating with inflation targets. In fact, we reviewed 16 central banks (Australia, Brazil, Canada, Chile, Colombia, United States, Europe, Japan, Mexico, Norway, New Zealand, Peru, England, South Africa, Sweden and Turkey) and found the following:

- All of them have some overnight interest rate as the operative instrument (either, the interest rate for non-collateralized credit or the interest rate for collateralized credit).
- 15 out of the 16 banks use REPO at auctions, but they differ in frequency and maturity.
- Also, all these central banks have lending and deposit facilities opened.
- Furthermore, central banks regulate the liquidity in a more permanent way by buying or selling securities and international reserves. In the first case, the securities can be issued by governments or, in some cases, by central banks themselves.

Colombia is not the only country that uses REPO and lending and deposit facilities as mechanisms to implement monetary policy. In particular, as argued by Cardozo et al. (2011), there are two advantages for having a system where the central bank has auctions and window facilities compared to a system with only one explicit interest rate through which all liquidity is provided (without auctions):

1. It encourages the deepening of the interbank market, which is useful to extract signals and evaluate solvency and risks taking by its participants.
2. It reduces the possibility of excessive leverage by the financial system, which may be used to speculate on the foreign exchange or securities markets.

This paper focuses on providing an analytical tool to evaluate the first reason. We do not assess the second one.

In order to understand how the interbank market works in Colombia, we construct a framework that allows us to assess the relationship between the mechanism through which the Central Bank provides liquidity and the overall interbank market. We set a stochastic and dynamic model for the overnight funding that seeks to identify analytical determinants for supply (or demand) of resources in the interbank market, financial institutions' demand at the auctions, and equilibrium interest rates.

The empirical data for the Colombian case shows, for the period considered, that banks prefer to maintain higher levels of reserves at the beginning of the required period in order to lessen their deficiency faster in the first days. According to this deficiency reduction strategy, we also find that the spread between the interbank interest rate and the policy rate exhibit a positive relationship, and that the demand for resources at the Central Bank auctions, supply in the interbank market and the net resources brought to the Central Bank facilities are consistent with that strategy. Using the information for Colombian interbank markets, we statistically test that one of the main results of the model: the interbank interest rate follows a Martingale process.

This paper is composed by four sections, including this introduction. In section two we set a model for the interbank market in Colombia. The third section presents the data analysis with Colombian data, and the fourth section concludes with some final remarks.

2 Model

We describe a model for the interbank daily market for overnight funds in Colombia. The structure of the model follows some features of the problem setting, derivation and solution in Pérez & Rodríguez (2006).

We are not the first in building on the framework proposed by Pérez & Rodríguez (2006) [PR (2006) from now on]: Cardozo, et al. (2011) set a framework with the Colombian timing, but do not include sources of uncertainty, of which we have two. Perez & Rodríguez (2010) allow for an extra facility in which commercial banks clear their accounts between them and with the Central Bank. This facility is designed to be occasional and the banks have uncertainty over it. Kempa (2006) models common and idiosyncratic shocks, and Kempa (2007) includes expected innovations in the demand for resources faced by banks along with the demand shocks. Jurgilas (2006) introduces heterogenous banks, and the possibility for foreign funding for them. Moschitz (2004) models the supply side in detail from the perspective of the balance sheet of the Central Bank, setting an explicit objective function for it.

Trading day activity in the Colombian monetary market is not too simple. The complexity arises due to a variety of possible operations and counterparts. As explained by Cardozo, et al. (2011), there are collateralized and uncollateralized trading that take place in electronic negotiation systems or in OTC markets. Furthermore, a wide range of institutions are able to trade in these markets (banks, bank-like institutions, stockbrokers, among others) and the Central Bank realizes open market operations (OMOs from now on) at certain and known hours in a day.

Trading days start at 7 a.m., when Colombia's large-value payment system (CUD, in Spanish) and SEBRA (Electronic services provided by the Banco de la República) open. At 8 a.m. institutions start trading in electronic negotiation systems like SEN and MEC[1]. Operations in SEN go until 1 p.m., while those in MEC go until 5 p.m.

Central Bank holds two main OMOs: (i) auctions of funds by REPOS (1 p.m.) and (ii) lending and deposit facilities (4 p.m.). The amount auctioned is bounded by the Central Bank. Commercial banks and bank-like institutions compete under a Dutch auction system.

With the window facilities, the Central Bank lends or borrows funds without setting a maximum amount, but charging or paying interest rates different from the official interest policy rate.

Although banks can trade and negotiate until 5 p.m., most of the activity in the interbank market occurs before 1 p.m.[2] This fact is crucial to our model because we can assume that institutions stop trading at that time. We state the timing of events as shown in the Figure 1.

[1] SEN stands for Sistema Electrónico de Negociación (Electronic trading system) and is administrated by Banco de la República. On the other hand, MEC stands for Mercado Electrónico (Electronic market) and is administrated by the Colombian Stock Market (Bolsa de Valores de Colombia).

[2] See Cardozo et al (2011)

Start of day	Bank holds some level of liquid assets
	Bank participate in the IM and decides the net demand for funds
	The auction of REPOS takes place, bank decides its next period deficiency and supply shock is realized
	Idiosyncratic demand shock is realized
	Lending and deposit facilities are opened
End of day	Central Bank announces the amount of resources available for the next auction of REPOS

Figure 1: Typical day of the requirement period for bank j

We set a model for a generic requirement period of T days, where each bank faces an exogenous-to-the-model requirement average constraint. We modify the structure of the model in PR (2006) in four aspects: first, we allow for daily auctions instead of one in the entire requirement period; second, we alter the timing of the model to have the auction after interbank trading has taken place, at any given day, and not before; third, the banks in our model optimally decide over the amount of reserves they accumulate each day, which is a residual in PR (2006). Fourth, along with the demand shock in PR (2006), banks in our model face a second source of uncertainty: there is a probability of not getting resources at the auction.[3]

2.1 The set up

Consider a continuum of heterogenous commercial banks of size one, indexed by j, that trade in a competitive fashion over daily reserves. There is a Central Bank that provides liquidity through auctions and windows every day during the requirement period, and that has established a requirement period of T calendar days. We set the model for a single requirement period, and assume initial and tranversality conditions to make the period of T days independent of each other.

The Central Bank sets each bank's requirement exogenously from the model. Banks hold reserves at the end of each day to complete the T-days-average

[3] We recognize that there are different possible timings for the events that take place in a typical day and, actually, we are aware that the order of the REPO action, the interbank trading and the decision about the reduction of the deficiency may alter our results. Although we state that banks and banks-like institutions have an explicit strategy to fulfill their required reserve restrictions, we do not know when exactly this decision happens in a day. Initially, our primary goal allow us to focus on one of these possible timings but we are conscious that other configurations are a source for further research. We will discuss these issues in a forthcoming paper.

heterogenous amount, Q_j, required. Banks satisfy the reserve requirement when

$$Q_j \leq \frac{1}{T}\sum_{t=1}^{T} res_{j,t}, \tag{1}$$

where $res_{j,t}$ is the reserve at the end of the day t that the bank j leaves to contribute to its requirement constraint. Daily reserves have both stochastic and deterministic components that will be defined shortly.

At the beginning of every trading day $t \in [1, T]$, commercial banks meet at the interbank market and supply ($b_{j,t} > 0$) or demand ($b_{j,t} < 0$) net resources to maximize benefits, subject to their heterogenous asset holdings, $a_{j,t}$, at the beginning of the day t. Banks clear the interbank market at the interest rate i_t.

By the end of the interbank trading activity, the Central Bank holds an auction to provide the financial system with liquidity. Banks demand resources ($d_{j,t}$) at the auction interest rate $i_{omo,t}$.

Deterministic daily reserves for bank j at time t ($m_{j,t}$), are defined as

$$m_{j,t} = a_{j,t} - b_{j,t} + d_{j,t}. \tag{2}$$

Note that equation 2 does not make explicit reference to payments of previous day's auction demand or interbank activity. This is because $m_{j,t}$ is defined as a net flow: re-payments in the interbank market and to the Central Bank have taken place.

We introduce uncertainty at the auction assuming each bank gets the resources it demands with probability $p \in [0, 1]$, but with probability $1 - p$ it leaves empty handed. Furthermore, we assume p banks receive the liquidity they demanded.

At the time of the auction, banks also decide how much they want to contribute to its required reserve. We follow PR (2006) in defining the deficiency, $r_{j,t}$, as the amount of reserves the bank j is short from the total requirement of $T{\cdot}Q_j$, at time t. The deficiency is non-increasing by definition (i.e. $r_{j,t} - r_{j,t+1} \geq 0$). Banks start day t with deficiency $r_{j,t}$ and decide on its next day's deficiency $r_{j,t+1}$. Note that on the last day of the requirement period T, the requirement constraint is binding, therefore banks set its deficiency to be zero (i.e. $r_{j,T+1} = 0$).

After the daily auction, banks realize they have been hit by a demand shock for resources $\varepsilon_{j,t} \underset{iid}{\sim} F\left(\mu_\varepsilon, \sigma_\varepsilon^2\right)$, typically coming from their clients. This shock is assumed to be identically, and independently distributed across time and banks.

Daily reserves are given by

$$r_{j,t} - r_{j,t+1} + e_{j,t} = m_{j,t} + \varepsilon_{j,t}. \tag{3}$$

We have departed from PR (2006) by having a supply shock and by allowing banks to optimally decide how much of the available reserves the bank wants to use to reduce its next day deficiency, $r_{j,t+1}$. Then, we define $e_{j,t}$ as residual reserves after the demand shock, which do not contribute to reduce the deficiency. If $e_{j,t} > 0$, the bank takes those resources to the deposit facility at the

Central Bank, that yields i^d. If $e_{j,t} < 0$, the bank asks for those resources from the lending facility at the Central Bank, that costs i^l, since banks are not allowed to go overdraft through night and/or they have to honour its requirement constraint at time T.

At the beginning of the next day, assets by bank are given by[4]

$$a_{j,t+1} = a_{j,t} + \varepsilon_{j,t}. \tag{4}$$

A solution for the model is the set of equilibrium interbank and auction interest rates, $\{i_t^*, i_{omo,t}^*\}$, for each day of the requirement period, $t \in [1, T]$. The equilibrium interest rates are determined by the clearing market conditions:

$$\int_0^1 b_{j,t} \partial j \overset{i_t^*}{=} 0, \text{ and } \int_0^1 d_{j,t} I(p) \partial j \overset{i_{omo,t}^*}{=} M_t^s. \tag{5}$$

The Central Bank has the same information as commercial banks: the distribution of the supply and demand shocks.

We follow PR (2006) in solving the model by backward induction. We start describing the decisions at the auction time for the generic j-bank at any given day t. Then, we describe the decision making in the interbank market at the beginning of the day.

2.2 Bank's problem at the auction

At period t, the bank faces the auction having traded in the interbank market. It decides its demand for liquidity, $d_{j,t}$ and the next period deficiency $r_{j,t+1}$, to maximize the expected value of its daily profit function with uncertainty about the demand shock, $\varepsilon_{j,t}$, and whether it will get the demanded liquidity at the auction or not.

After the auction, the bank finds itself with or without the demanded resources, and in one of the following three situations, depending on the realization of the shocks and previous decisions: (i) the bank might need resources to honour its obligations with other commercial banks or the Central Bank, because it is not allowed to go in overdraft through the night or because its requirement constraint is binding, respectively. In either case it has to borrow the amount needed from the Central Bank facility, at the interest rate i^l. (ii) The bank is in perfect balance or (iii) it has excess liquidity and uses the Central Bank's deposit facility that yields i^d overnight.

Knowing these possible scenarios, the bank decides over its demand in the auction and its deficiency for next period by solving

$$\max_{\{d_{j,t}, r_{j,t+1}\}} E_t \left[\pi_{j,t}^{omo} + V_{t+1}\left(s_{j,t+1}; S_{t+1}\right) \right], \tag{6}$$

[4] It is useful to see it this way: bank j started day t with $a_{j,t}$. It gave away resources at the interbank market ($b_{j,t}$), received $d_{j,t}$ at the auction with probability p, and the demand shock $\varepsilon_{j,t}$. Next day, the bank recovers what it lent and pays back the money demanded at the auction. Therefore, it starts the next day with assets:

$$a_{j,t+1} = m_{j,t} + \varepsilon_{j,t} + b_{j,t} - d_{j,t} = a_{j,t} + \varepsilon_{j,t}.$$

where the objective function has two arguments. The first takes account of the expected profits at the auction, while the second takes account of the next day's expected value function. We describe these arguments in detail. Profits at the auction are given by

$$\pi_{j,t}^{omo} = $$

$$\left\{ \begin{array}{c} -i_{omo,t}d_{j,t} \\ +i^{d}\left[\begin{array}{c} m_{j,t}+\varepsilon_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right) \end{array} \right] I\left(m_{j,t}+\varepsilon_{j,t}>r_{j,t}-r_{j,t+1}\right) \\ +i^{l}\left[\begin{array}{c} m_{j,t}+\varepsilon_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right) \end{array} \right] I\left(m_{j,t}+\varepsilon_{j,t}<r_{j,t}-r_{j,t+1}\right) \end{array} \right\} I\left(d_{j,t}>0\right) \quad (7)$$

$$+\left\{ \begin{array}{c} i^{d}\left[\begin{array}{c} a_{j,t}-b_{j,t}+\varepsilon_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right) \end{array} \right] I\left(a_{j,t}-b_{j,t}+\varepsilon_{j,t}>r_{j,t}-r_{j,t+1}\right) \\ +i^{l}\left[\begin{array}{c} a_{j,t}-b_{j,t}+\varepsilon_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right) \end{array} \right] I\left(a_{j,t}-b_{j,t}+\varepsilon_{j,t}<r_{j,t}-r_{j,t+1}\right) \end{array} \right\} I\left(d_{j,T}=0\right).$$

subject to 2.

We assume the indicator function $I\left(d_{j,t}>0\right)$ takes the value of one, with probability p, when the bank j gets the demanded liquidity out of the auction, and zero if it does not. When it gets the money (in reference to the first curly bracket in 7), the bank pays the auction rate, $i_{omo,t}$, for its demand, $d_{j,t}$, in the first line. Subject to the realization of the demand shock $\varepsilon_{j,t}$, the bank either: (i) saves money at the deposit facility when resources available $(m_{j,t}+\varepsilon_{j,t})$ are greater than the optimal reduction in the deficiency for next period $(r_{j,t}-r_{j,t+1})$. Otherwise, (ii) the bank asks for reserves at the Central Bank's facility when resources available are shorter than the optimal reduction of the next period's deficiency (i.e. $m_{j,t}+\varepsilon_{j,t}<r_{j,t}-r_{j,t+1}$).

The second curly bracket in 7 is multiplied by the indicator function $I\left(d_{j,T}=0\right)$, which by assumption takes the value of one with probability $1-p$. In this case, bank j does not get any resources from the auction and deterministic reserves $(m_{j,t})$ are reduced to $a_{j,t}-b_{j,t}$. Analogously, the bank either takes any excess reserves after the optimal reduction of the deficiency to the deposit facility, or asks for any needed reserves from the lending facility, both at the Central Bank. Then, expected profits at the auction are

$$E_t\left(\pi_{j,t}^{omo}\right) =$$

$$p\left\{\begin{array}{c} -i_{omo,t}d_{j,t} \\ +i^d\left[\begin{array}{c} m_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right)\end{array}\right]\left[1-F\left(\begin{array}{c} r_{j,t}-r_{j,t+1} \\ -m_{j,t}\end{array}\right)\right] \\ +i^l\left[\begin{array}{c} m_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right)\end{array}\right]F\left(\begin{array}{c} r_{j,t}-r_{j,t+1} \\ -m_{j,t}\end{array}\right)\end{array}\right\} \tag{8}$$

$$+\left(1-p\right)\left\langle\begin{array}{c} i^d\left[\begin{array}{c} a_{j,t}-b_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right)\end{array}\right]\left\{1-F\left[\begin{array}{c} r_{j,t}-r_{j,t+1} \\ -\left(a_{j,t}-b_{j,t}\right)\end{array}\right]\right\} \\ +i^l\left[\begin{array}{c} a_{j,t}-b_{j,t} \\ -\left(r_{j,t}-r_{j,t+1}\right)\end{array}\right]F\left[\begin{array}{c} r_{j,t}-r_{j,t+1} \\ -\left(a_{j,t}-b_{j,t}\right)\end{array}\right]\end{array}\right\rangle$$

$$+p\left[\begin{array}{c} i^dE_t\left(\varepsilon_{j,t}/\varepsilon_{j,t}>r_{j,t}-r_{j,t+1}-m_{j,t}\right) \\ +i^lE_t\left(\varepsilon_{j,t}/\varepsilon_{j,t}<r_{j,t}-r_{j,t+1}-m_{j,t}\right)\end{array}\right]$$

$$+\left(1-p\right)\left\{\begin{array}{c} i^dE_t\left[\varepsilon_{j,t}/\varepsilon_{j,t}>r_{j,t}-r_{j,t+1}-\left(a_{j,t}-b_{j,t}\right)\right] \\ +i^lE_t\left[\varepsilon_{j,t}/\varepsilon_{j,t}<r_{j,t}-r_{j,t+1}-\left(a_{j,t}-b_{j,t}\right)\right]\end{array}\right\}.$$

The second argument in the objective function 6 is the expected value function of the next period, $E_t\left[V_{t+1}\left(s_{j,t+1};S_{t+1}\right)\right]$, where the state of bank j at time t is defined by its reserve position $s_{j,t}=\left(a_{j,t},r_{j,t}\right)$, and the aggregate state variable at time t is given by interbank market and auction interest rates up to $t-1$, $S_t=\left(i_1,i_2...i_{t-1},i_{omo,1},i_{omo,2},...i_{omo,t-1}\right)$.

We show later that, given the recursiveness of the bank's problem, it holds that

$$\frac{\partial}{\partial d_{j,t}}V_{t+1}\left(s_{j,t+1};S_{t+1}\right)=0,\text{ and }\frac{\partial}{\partial r_{j,t+1}}V_{t+1}\left(s_{j,t+1};S_{t+1}\right)=-i_{t+1}, \tag{9}$$

where the value function in $t+1$ does not respond to the reserves demanded at the auction in t. As it is explained in footnote (2): resources demanded at the auction are paid before the start of the trading day. Derivative of the next day value function with respect to the deficiency implies an opportunity cost: an extra reduction of the deficiency $r_{j,t+1}$, decided at time t, spares the bank of paying the interbank rate in $t+1$, if the bank were to reduce the deficiency that day.

First order conditions (FOC) of 6 give

$$\left\{d_{j,t}\right\}:p\left\{\begin{array}{c} -i_{omo,t}+i^d\left[1-F\left(r_{j,t}-r_{j,t+1}-m_{j,t}^{+}\right)\right] \\ +i^lF\left(r_{j,t}-r_{j,t+1}-m_{j,t}^{+}\right)\end{array}\right\}=0, \tag{10}$$

where $m_{j,t}^{+}=a_{j,t}-b_{j,t}+d_{j,t}^{*}$, and

$$\left\{r_{j,t+1}\right\} \quad : \quad p\left\{\begin{array}{c} i^d\left[1-F\left(r_{j,t}-r_{j,t+1}-m_{j,t}^{+}\right)\right] \\ +i^lF\left(r_{j,t}-r_{j,t+1}-m_{j,t}^{+}\right)\end{array}\right\} \tag{11}$$

$$+\left(1-p\right)\left\langle\begin{array}{c} i^d\left\{1-F\left[r_{j,t}-r_{j,t+1}-\left(a_{j,t}-b_{j,t}\right)\right]\right\} \\ +i^lF\left[r_{j,t}-r_{j,t+1}-\left(a_{j,t}-b_{j,t}\right)\right]\end{array}\right\rangle-E_t\left(i_{t+1}\right)=0.$$

Note that in both conditions, 10 and 11, we follow PR (2006) and assume that, for any $k_{j,t}$ conditioning on the expected value of the demand shock, it holds that

$$\frac{\partial}{\partial k} E_t \left(\varepsilon_{j,t} \diagup \varepsilon_{j,t} \gtrless k_{j,t} \right) \to 0. \qquad \text{(Assumption 1)}$$

Combining 10 and 11, and solving for $d^*_{j,t}$, we obtain the reaction function at the auction in time t, by bank j:

$$d^*_{j,t} = r_{j,t} - r^*_{j,t+1} - (a_{j,t} - b_{j,t}) - F^{-1} \left(\frac{i_{omo,t} - i^d}{i^l - i^d} \right), \qquad (12)$$

where

$$r^*_{j,t+1} = r_{j,t} - (a_{j,t} - b_{j,t}) - F^{-1} \left(\frac{\frac{E_t(i_{t+1}) - p i_{omo,t}}{1-p} - i^d}{i^l - i^d} \right). \qquad (13)$$

The bank demands more liquidity at the auction when the difference between the optimal reduction in the deficiency and the resources not lent to other banks in the interbank market is big, or when its expected idiosyncratic demand shock is negative. Optimal reduction of the deficiency depends positively on assets not lent in the interbank market and the effect of the probability of getting resources at the auction over the relative size of the expected idiosyncratic demand shock. Ultimately, banks demand at the auction to satisfy conditions in 12 and 13, given the market auction and next's period expected interbank rates, and the expectation of the idiosyncratic demand shock and the supply shock.

To continue with the backward induction solution of the model, we find out how previous decisions in day t affect the objective function at the auction, when evaluated at its optimum. Using 12 to replace $d^*_{j,t}$ and $r^*_{j,t+1}$ in the objective function 6, it can be shown that

$$\frac{\partial}{\partial b_{j,t}} \max_{\{d_{j,t}, r_{j,t+1}\}} E_t \left[\pi^{omo}_{j,t} + V_{t+1} \left(s_{j,t+1}; S_{t+1} \right) \right] = -E_t \left(i_{t+1} \right), \qquad (14)$$

again with the use of Assumption 1. The maximum profit of bank j at the auction, in time t, decreases at the expected interbank rate in the next period with marginal interbank borrowing in time t. In case of need, due to a negative demand shock, bank j has to pay the expected interbank rate at time $t+1$ to obtain liquidity.

Two special cases are worth to be shown in detail: the first for the last day of the requirement period (i.e. $t = T$), and the second when there is no supply shock and all banks receive the resources they demanded at the auction (i.e. $p = 1$).

2.2.1 Reserve demand and auction equilibrium in the last day of the requirement period

When $t = T$ the requirement constraint is binding and reserves at the end of the day must be enough to satisfy constraint 1. Therefore, next period's deficiency must be reduced to zero (i.e. $r_{j,T+1} = 0$ for all j). Taking this into account, from FOC in 10 and 11, the reaction function for demand of resources of bank j at the auction in time T is:

$$d^*_{j,T} = r_{j,T} - (a_{j,T} - b_{j,T}) - F^{-1}\left(\frac{i_{omo,T} - i^d}{i^l - i^d}\right). \tag{15}$$

2.2.2 Reserve demand and auction equilibrium with certainty at the auction

When $p = 1$, banks know that they get all the resources demanded at the auction, and therefore they face only the uncertainty of the demand shock. From the FOC in 10 and 11, the reaction function for demand of resources of bank j at the auction at any time t is given by

$$d^*_{j,t} = r_{j,t} - r^*_{j,t+1} - (a_{j,t} - b_{j,t}) - F^{-1}\left(\frac{i_{omo,t} - i^d}{i^l - i^d}\right). \tag{16}$$

Note that 16 is different form 12 because $r^*_{j,t+1}$ is not yet determined. At a micro level, combinations of $d^*_{j,t}$ and $r^*_{j,t+1}$ that satisfy the equilibrium auction rate for any bank j are infinite: equation 16 is a relation of optimal combinations for $d^*_{j,t}$ and $r^*_{j,t+1}$, at a given auction interest rate.

2.3 Bank's problem at the beginning of the day

We continue solving the bank's problem at time t by backward induction. Now we focus in the beginning of the day maximization problem. Bank j maximizes the day profits by deciding on his activity in the interbank market, taking as given the auction and the requirement restrictions. We define the value function at the beginning of date t, as:

$$V_t\left(s_{j,t}; S_t\right) = \max_{\left\{b^j_t\right\}}\left\{i_t b_{j,t} + \max_{\left\{d_{j,t}, r_{j,t+1}\right\}} E_T\left[\pi^{omo}_{j,t} + V_{t+1}\left(s_{j,t+1}; S_{t+1}\right)\right]\right\}. \tag{17}$$

The value function has two arguments. First, the bank decides how much to lend or borrow in the interbank market, and receives or pays the interbank interest rate respectively. Second, the activity of the bank in the interbank market affects the value function of the next day at its optimum.

FOC to 17 gives[5]

$$i_t = E_t\left(i_{t+1}\right), \tag{18}$$

which is the Martingale hypothesis proposed by PR (2006), and found frequently in the literature[6]. We claim that our model presents an analytical derivation of a Martingale process for the interbank interest rate.

[5] Given that we use a linear benefits function, it can be easily shown that the equilibrium in the interbank market is reached only when this equality holds.

[6] See a literature review on this subjetc in Domínguez & Lobato (2003).

We combine 18 with 13, and solve for the optimal supply of resources of bank j at the interbank market:

$$b_{j,t}^{*} = a_{j,t} - \left(r_{j,t} - r_{j,t+1}^{*}\right) + F^{-1}\left(\frac{\frac{i_t - p i_{omo,t}}{1-p} - i^d}{i^l - i^d}\right). \quad (19)$$

Note that, despite $d_{j,t}^{*}$ is fully determined by equations 12 and 19, there is no bank-level determination for $b_{j,t}^{*}$ and $r_{j,t+1}^{*}$. At a micro level, combinations of $b_{j,t}^{*}$ and $r_{j,t+1}^{*}$ that satisfy the equilibrium interbank rate for any bank j are infinite: equation 19 is a relation of optimal combinations for $b_{j,t}^{*}$ and $r_{j,t+1}^{*}$, at a given interbank interest rate.

Despite this indetermination, optimal supply (or demand) of resources of bank j at the interbank market grows (falls) with assets net of the optimal reduction of the deficiency, and with positive (negative) demand shocks. Regarding the indetermination, it constitutes a challenging issue to be resolved. One appealing approach is to think there is a trade-off between the supply of interbank funds and the reduction of the deficiency: if a bank decides to reduce its deficiency, it sacrifices the benefits of lending funds; in the other hand, if the bank chooses to make benefits by lending at the interbank market, there is an opportunity cost of not reducing its deficiency. In other words, to solve this puzzle it is necessary to have a function that represents the preferences over the deficiency reduction and the interbank supply. This kind of modeling will be developed in a forthcoming paper.

Evaluating the reaction function of supply of resources in the interbank market (equation 19) in the value function for bank j (equation 17), it can be shown that

$$\frac{\partial V_t\left(s_{j,t}; S_t\right)}{\partial r_{j,t}} = -i_t, \quad (20)$$

again using Assumption 1. The value function is decreasing at the rate of i_t with respect to the deficiency in t.

We now complete the analysis of the two special cases mentioned above: the first for the last day of the requirement period (i.e. $t = T$), and the second when there is no supply shock and all banks receive the resources they demanded at the auction (i.e. $p = 1$).

2.3.1 Interbank market supply or demand and equilibrium interbank rate in the last day of the requirement period

When $t = T$, next period's deficiency must be reduced to zero. Then, optimal supply of resources by bank j at the interbank market is given by:

$$b_{j,T}^{*} = a_{j,T} - r_{j,T} + F^{-1}\left(\frac{\frac{i_T - p i_{omo,T}}{1-p} - i^d}{i^l - i^d}\right). \quad (21)$$

Aggregating 21 over j, we have the equilibrium interbank rate for $t = T$:

$$i_T^{*} = p i_{omo,T} + (1-p)\left[i^d + \left(i^l - i^d\right) F\left(R_T - A_T\right)\right]. \quad (22)$$

The equilibrium interbank market interest rate is a convex combination of the auction interest rate and cost defined as another convex combination of the deposit and lending interest rates, weighted by the probability of having excess or lack of resources respectively, for the case when the Central Bank does not supply liquidity at the auction. The equilibrium interbank interest rate is also found to be between the deposit and the lending interest rates of the Central Bank's facilities.

2.3.2 Interbank market supply or demand and equilibrium interbank rate with certainty in the auction

When $p = 1$, banks face only the uncertainty of the demand shock. From the FOC in 18 and 12, the optimal supply of resources of bank j in the interbank market at any time t is given by

$$b^*_{j,t} = -\left(r_{j,t} - r^*_{j,t+1}\right) + a_{j,t} + d^*_{j,t} + F^{-1}\left(\frac{i_T - i^d}{i^l - i^d}\right), \tag{23}$$

where $d^*_{j,t}$ and $r^*_{j,t+1}$ are not yet individually determined, and are related by equation 16.

Aggregating over j, we obtain an equilibrium condition for interbank interest rate:

$$i^*_t = i^d + \left(i^l - i^d\right) F\left[\left(R_t - R^*_{t+1}\right) - \left(A_t + \bar{M}^s_t\right)\right]. \tag{24}$$

3 Data analysis

3.1 Monetary market in Colombia

Having developed an analytical framework to understand how the interest rates and the net demand for funds are set in the interbank market, and given institutional the arrangements for implementing the monetary policy, we now show some evidence form the Colombian interbank market data.

Even though we focus our attention to the Colombian case, the analytical framework is general and could be used to explore the functioning of that market in other countries. As shown in the introduction, the tools to affect and stabilize the short-term interest rates used by Central Banks are similar (i.e. reserve maintenance period, auctions, lending and deposit facilities, among others).

The key element in our model is the uncertainty about the supply and demand shocks. As it is implied in the model description, we follow a Bernoulli distribution function to model the first of these shocks.

We collect information that institutions report to the Colombian Regulator (Superintendencia Financiera): the reserve requirements and funds effectively held by banks from the Form 443. For uncollateralized markets: interest rates and the amounts lent and borrowed by each bank from the Form 441.

It is worth to mention that in Colombia the requirement constraint is set to be a 14 days average of reserves. Regarding the interest rates and amounts traded in the collateralized interbank market, we use the information from DCV (Depósito Central de Valores, Central Bank's depositary for clearing and delivering of Government bonds -TES-). This data includes records from SEN, MEC and those made OTC.

We focus only in analyzing and processing data for overnight operations and we find a *global* position in the interbank market for each bank (i.e. the amount lent in collateralized and uncollateralized markets minus the amount borrowed in both of them) and construct a weighted interest rate.

The information about OMOs is taken from the Central Bank. We use the amounts demanded and actually obtained in the daily REPO auctions by each bank, and its corresponding interest rates. This information can be used to calculate the supply shock (and the frequency of banks getting nothing out of the auction).

Central Banks usually have a big amount of information which contains key variables for the analysis of the monetary market in the short run. However, this information is often analyzed in the light of the flows or stocks available at each point in time. The model described here provides a consistent theoretical framework, which can be used for a more structured analysis of what happens through short-time transactions between the agents involved.

In order to link the model described before with the data available for the Colombian case, we build a database with variables derived according to the definitions in the model, which accurately reflects the operation of the monetary policy in Colombia with respect to the supply of liquidity by the Central Bank and reserve requirements. For this purpose, we took into account all the operations per entity each day for the period January 2012 - April 2013. It included 34 full reserve requirement periods and 57,596 observations.[7]

Because our purpose is to analyze all types of overnight operations among entities, which we called the total interbank market, we include the collateralized and non-collateralized transactions.[8] From these operations, we exclude all transactions at rates lower than the deposit rate of the Central Bank because we recognize that those are for different purposes than the borrowing-lending type.[9] With the remaining transactions of all entities, we then defined the following variables:

[7] In Colombia, a reserve requirement period includes a total of 14 days, starting a Wednesday and finishing a Tuesday two weeks later. During this period, the amount held as reserves every day (including weekends) is considered in the average for the reserve requirement.

[8] In this sense, our goal is not to explain the interbank interest rate market as it is usually understood. What is usually meant as interbank or overnight rate is that rate resulting from non-collateralized transactions among entities. However, as we mentioned, our interest is to analyze these operations along with those that are collateralized. Therefore, the resulting average rate of both transactions is what we called the total interbank interest rate.

[9] In some cases, entities conduct market transactions with the aim of getting some kind of securities that seek to fulfill other operations. To the extent that their interest is the security itself, these institutions are willing to lend their liquid resources regardless the interest rate they can get. For this reason, some operations can be celebrated with rates close to zero. To the extent we considered that these transactions are not carried out for the purposes considered in the model, we did not considered them as part of the total interbank market we are analyzing.

- The reduction of the deficiency $(r_{j,t} - r_{j,t+1})$: it corresponds to the daily value of the reserves for each institution. It does not include resources different from those used to meet reserve requirements.[10] It is important to mention that not all entities considered here have to satisfy the reserve requirement. This is only for credit institutions, for the other institutions $r_{j,t} = 0$ for al t.
- Spread between the total interbank interest rate and the policy rate. The first one was calculated as the weighted average of all collateralized and non-collateralized operations considered in the model.
- Daily operations with the Central Bank: $d_{j,t}$ for REPO and $e_{j,t}$ for window facilities.
- Daily total interbank transactions, $b_{j,t}$: the information was obtained for all overnight collateralized and non-collateralized transactions of which 57,596 observations resulted.
- Daily demand shock, $\varepsilon_{j,t}$: this was obtained taking into account daily reserves and transactions in the total interbank market and REPO operations with the Central Bank.
- Deterministic daily reserves, $m_{j,t}$: it was calculated based on the information for daily reserves and the demand shock.
- Initial daily liquid asset holding at the beginning of the day, $a_{j,t}$: it was obtained from the value for and the REPO operations with the Central Bank and all the transactions in the total interbank market.

Figure 1 exhibits aggregated average levels of the variables considered in the model. This includes the operations performed by all agents, regardless of whether they have REPO operations with the Central Bank or whether they are active or not in the total interbank market. According to the data, the plan to lessen the deficiency seems to have a strong influence in the interbank trading, the demand at the REPO auction, and the net funds brought to the Central Bank facilities. As mentioned, a forthcoming paper will deepen into the preferences over the deficiency reduction strategy of banks, and that model must be good enough to adjust its results to the empirical evidence we have until here. Nevertheless, we take that fact as granted due to our primary objective of understanding interbank trading and its link to operational framework of monetary policy can be reached. Thus, assuming that there is such strategy, our analytical outcomes are able to describe the patterns we observe.

In each panel of Figure 1 the number included in label 1 corresponds to the average of the variable for all entities for the first day of the reserve requirement period in the whole period considered. With this in mind, we can identify some patterns across the requirement period:

[10] In this sense, it does not include resources kept in deposit or lending facilities at the Central Bank.

- From Figure 1A, we can observe that there is a clear decision by the agents to reduce rapidly the deficiency since the beginning of the reserve period. This means that entities perform operations to quickly meet the required average reserve. As it can be seen, the reduction of the deficiency starts in values close to \$19,000 billion and decreases to levels close to \$17,000 billion. It corresponds to the strategy followed by the entities in the sample period. Even though we only show what has happened in sixteen months from January 2012, we have evidence that in the past the strategy for reserve requirements behaved in the opposite direction.

- In the same graph we can see that this decision about the deficiency occurs while the spread between the total interbank interest rate and the policy rate is also decreasing over the reserve requirement period. In particular, in the early days of the period the spread is on average 20 bp, but throughout the period it reduces to a level close to zero in the last day. This means that entities, following the decision to rapidly reduce the deficiency, are willing to pay a higher interest rate at the beginning of the period to ensure sufficient resources to carry out their decisions on the requirement. To the extent that entities want to quickly reduce the deficiency, the model outcomes are consistent with upward pressures on the total interbank interest rate at the beginning of the period (higher spread).

- This is also consistent with the behavior of Central Bank REPO operations. In Figure 1B we observe that at the beginning of the requirement period there is a high demand for this type of resources. In particular the entities demand on average amounts around \$5,000 - \$5,500 billion. Subsequently, to the extent that by the end of the period the entities maintain sufficient resources to meet the reserve requirement, the demand for those resources falls. This graph also shows that the level for the total interbank market behaved in the opposite direction of REPOS: as the closing day of requirement period is closer, the trading level increases.[11] Thus, the data shows that once the institutions have reduced their deficiency, they start trading more actively among them.

- It is worth to notice that demand for funds at 13th day, in REPO auctions or in the total interbank market, does not follow the trend. The reason we find to explain this pattern is that entities behave in a preventive way: since there is a chance to suffer a negative demand shock in the last day of the maintenance period (when this constraint binds), institutions then reduce lending process and ask for more funds at REPO auction and in the total interbank market.

- Figure 1C includes total the average levels for $a_{j,t}$ (from now on, we call average flows with upper bar i.e. average $a_{j,t} = \bar{a}_t$). and $\bar{m}_t$ in each day of

[11] Due to the fact that, in the aggregate, the total interbank market is equal to zero (i.e. $B_t = 0$) , for this figure we took only the observations where $b_{j,t} > 0$.

the reserve requirement period. One can notice that $\bar{m}_t$ has a decreasing behavior while $\bar{a}_t$ increases along the requirement period. The last result can be related to the agent´s decision to rapidly reduce the deficiency and is consistent with equations 12 and 24 in the model and that included in footnote 2.

- Figure 1D plots the average of net funds deposited at window facility, $\bar{e}_t$, and demand shock, $\bar{\varepsilon}_t$. The first of these series increases as the end of requirement period gets closer and the second does not exhibit any trend (actually the mean of these observations is close to zero for the whole period).

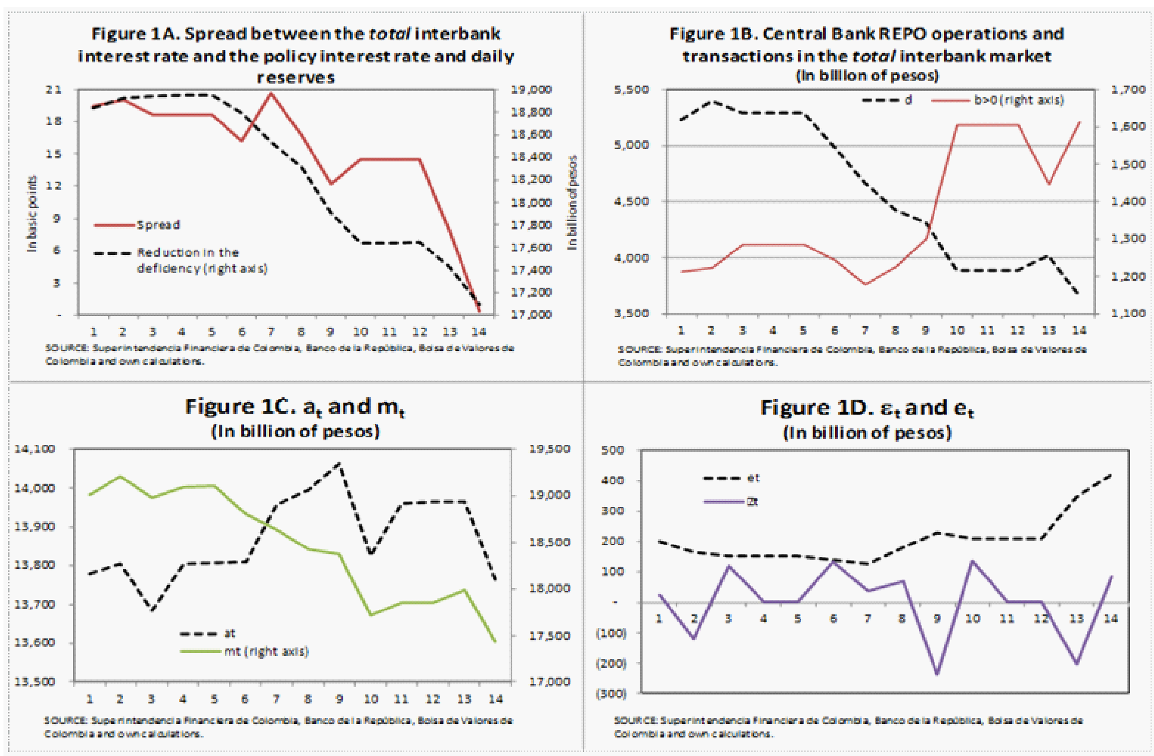

Figure 3.1: Monetary market in Colombia

3.2 The Martingale (difference) hypothesis

Equation 18 is an analytical derivation of the Martingale hypothesis proposed by PR (2006). In this section we aim to test whether the interbank interest rate follows this kind of process or not. We follow the method of Domínguez & Lobato (2003) to test Martingale difference hypothesis. From equation 18, we have

$$E_t\left(i_{t+1}\diagup i_t,...i_1\right)=i_t. \tag{25}$$

We use the definition of the conditional expectation to write

$$E_t\left(i_{t+1}-i_t\diagup i_t,...i_1\right)=0, \tag{26}$$

which implies that the conditional expected marginal return of the interbank reserves is zero. The Martingale Difference Hypothesis allows us to claim that if equation 26 holds, the interbank interest rate follows a Martingale process.

We use the Dominguez-Lobato Test for Martingale Difference Hypothesis of the package: Variance Ratio tests and other tests for Martingale Difference Hypothesis, "vrtest", that runs in R, and was programmed by Kim (2011).[12]

The test sets

$$H_0: E_t\left(i_{t+1}-i_t\diagup i_t,...i_{t-q}\right)=0,$$

calculates de Cramer von Mises test statistic (Cp) and the Kolmogorov-Smirnov test statistic (Kp), with wild bootstrap p-value of the Cp test ($Cp-pval$) and wild bootstrap p-value of the Kp test ($Kp-pval$); with 300 bootstrap iterations, and $q=5$ lags for the conditional expectation. This last value was included because five is the average lag of a two-weeks maintenance period with 10 working days (i.e. $5=\frac{1}{9}\sum\limits_{q=1}^{9}q$).

Results allow not to reject the null hypothesis with $Cp=0,0176$; $Kp=0,5050$; $Cp-pval=0,9367$; $Kp-pval=0,8133$.

Given that $Cp-pval$ and $Kp-pval$ are greater than $0,05$, we do not reject the Martingale hypothesis process for the Colombian interbank rate.[13]

4 Final Remarks

We have set a dynamic stochastic model for the interbank daily market for funds in Colombia. The framework features exogenous requirement and requirement period, competitive trading among heterogeneous commercial banks, daily open market operations held by the Central Bank (auctions and window facilities), and idiosyncratic demand shocks and uncertainty in the daily auction.

Even though a micro-level indetermination between interbank lending and the strategy to fulfill the reserve requirement still persists (and will be solved in a forthcoming paper), the model suggests the generic bank j demands more liquidity at the auction when the difference between the optimal reduction in the deficiency and the resources not lent to other banks in the interbank market is big, or when its expected idiosyncratic demand shock is negative. Also, optimal reduction of the deficiency depends positively on assets not lent in the interbank market and the effect of the probability of getting resources at the auction over the relative size of the idiosyncratic demand shock. Ultimately, banks demand

[12] See reference in Charles, Darne and Kim (2011).

[13] Code in R is available at the request of the reader.

at the auction given the market auction rate and expected interbank rate for next period, and the expectation of the idiosyncratic demand shock and the supply shock.

Optimal supply (or demand) of resources of bank j at the interbank market grows (falls) with assets net of the optimal reduction of the deficiency, and with expected positive (negative) demand shocks.

In the last day of the requirement period, the bank is constrained to reduce its deficiency to zero. Indeed, the equilibrium interbank interest rate is a convex combination of the auction interest rate and a cost defined as another convex combination of the deposit and lending interest rates, weighted by the probability of having excess or lack of resources respectively, for the case when the bank does not get funds at the Central Bank's auction. Also, the equilibrium interbank interest rate is also found to be between the deposit and the lending interest rates of the Central Bank's facilities.

Equilibrium interbank interest rate follows the Martingale hypothesis proposed by PR (2006), and found frequently in the literature. We claim that our model presents an analytical derivation of a Martingale process for the interbank interest rate.

The model results are coherent with our findings in Colombian data. We highlight: (i) the common strategy of entities with reserve requirement constraint to quickly reduce their deficiency, instead of other possible patterns (e.g. uniform reduction). (ii) A positive relation between the interbank-auction rates spread and the reduction in the deficiency. (iii) Both, the spread and the reduction of the deficiency decrease along the maintenance period. (iv) Aggregate demand at the auction, supply in the interbank market and net resources at the Central Bank facilities are consistent with the reduction of the deficiency strategy in (i).

Finally, we test the Martingale hypothesis following Domínguez and Lobato (2003) test, and Charles, et. al (2011) procedure. Our results suggest that the Martingale hypothesis should not be rejected for the Colombian interbank interest rate.

Bibliography

1. Casella, G. and R. L. Berger, 1990. Statistical Inference. Duxbury. Belmont, CA. pp. 69.

2. Cardozo, P., Huertas, C., Parra, J., & Patiño, L. (2011). Mercado Interbancario Colombiano y Manejo de la Liquidez del Banco de la República. Borradores de Economía (673).

3. Charles, A., Darne, O and J.H. Kim, 2011. Small Sample Properties of Alternative Tests for Martingale Difference Hypothesis, Economics Letters, vol. 110(2), pages 151-154.

4. Domínguez, M. A. and I. N. Lobato, 2003. "Testing the Martingale Difference Hypothesis," Econometric Reviews, vol. 22(4), pages 351-377.

5. Freedman, C. and D. Laxton, 2009. "Why Inflation Targeting?" IMF Working Paper/09/86.

6. González, C, 2013. "Mercados interbancarios no colateralizados e información asimétrica" Borradores de Economía No. 758, Banco de la República de Colombia.

7. Jurgilas, M. 2006. "Interbank Markets under Currency Boards," Working papers 2006-19, University of Connecticut, Department of Economics.

8. Kempa , M., 2006. "Money market volatility, A simulation study," Research Discussion Papers 13/2006, Bank of Finland.

9. __________, 2007. "What determines commercial banks' demand for reserves in the interbank market," Research Discussion Papers 30/2007, Bank of Finland.

10. Moschitz, J. 2004. "The determinants of the overnight interest rate in the euro area," Working Paper Series 393, European Central Bank.

11. Pérez Q., G. and H. Rodríguez M., 2006. "The Daily Market for Funds in Europe: What Has Changed with the EMU?" Journal of Money, Credit and Banking, Blackwell Publishing, vol. 38(1), pages 91-118, February.

12. __________, 2010. "Asymmetric standing facilities: an unexploited monetary policy tool," Banco de España. Working Papers 1004, Banco de España.