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BAYESIAN FORECAST COMBINATION FOR INFLATION USING ROLLING WINDOWS: AN EMERGING COUNTRY CASE

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ABSTRACT. Typically, when forecasting inflation rates, there are a variety of individual models and a combination of several of these models. We implement a Bayesian shrinkage combination methodology to include information that is not captured by the individual models using expert forecasts as prior information. To take into account two common characteristics in emerging countries' economies, possible parameter instabilities and non-stationary dynamics, we use a rolling estimation windows technique for series integrated of order one. The empirical results of Colombian inflation show that the Bayesian forecast combination model outperforms the individual models and the random walk predictions for every evaluated forecast horizon. Moreover, these results outperform shrinkage forecasts that consider other priors as equal or zero weights.

Keywords: *Forecast combination, Shrinkage, Expert forecasts, Rolling window estimation, Inflation forecasts.*

JEL Classification: *C22, C53, C11, E31.*

1. INTRODUCTION

Since the adoption in New Zealand of inflation targeting to steer monetary policy, many other countries have implemented inflation targeting. These countries require accurate inflation forecasts to accomplish successful monetary policy interventions. Therefore, it is important to establish a set of forecast methodologies that model different features of inflation. Central banks commonly use both individual forecast models and a combination of these models. The individual models contain information about characteristics of the data-generating process, such as persistence, non-linearities, and asymmetries, among others. However, one single model cannot all of the relevant information. Hence, forecast combination usually outperform individual models.

Forecast combination methodology has been widely applied since the pioneering works of Reid [1968] and Bates and Granger [1969]. There is consensus that pooling forecasts outperform individual forecasts in terms of predictive ability. Reviews of the most relevant contributions of forecast combination can be found in Clemen [1989] and Timmermann [2006].

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Information may exist about further events that would improve the predictive performance of forecast combination. To include prior information in this framework, Diebold and Pauly [1990] propose a Bayesian shrinkage methodology. This procedure results in a linear combination of Granger and Ramanathan [1984], classical approach weights and prior weights. An important issue in this methodology is the prior specification. Some applications, such as Wright [2008], Zellner and Hong [1989] and Koop and Potter [2003], use equal weights or a zero vector as the prior mean. We propose to compute the prior mean in terms of an expert forecast inflation series.

Some empirical works have examined forecast combination for the Colombian inflation case (Castaño and Melo [2000], Melo and Núñez [2004]). However, a Bayesian approach has not been used in this context. In light of the changes and non-stationary dynamics of Colombian inflation (Melo and Misas [2004], Gómez, González, and Melo [2012]), we use a rolling window estimation method for series integrated of order one to compute the Bayesian shrinkage weights. The aim of this paper is to implement a rolling Bayesian forecast combination technique for inflation using prior information from expert forecasts.

The rest of the paper is organized as follows. Section 2 explains the Bayesian shrinkage methodology in terms of forecast combination and the specification of the prior distribution. An application of this methodology to Colombian inflation is presented in section 3. Finally, Section 4 concludes.

2. Methodology

Let $f^1_{t|t-h},\dots,f^m_{t|t-h}$ be the set of m $h-$step ahead forecasts of y_t. Following Granger and Ramanathan [1984], a typical way to combine these forecasts is as follows:

$$y_t = \boldsymbol{\beta}'\boldsymbol{f}_{t|t-h} + \varepsilon_t, \tag{1}$$

where $\boldsymbol{\beta} = (\beta_0,\beta_1,\dots,\beta_m)'$ is the regression coefficients vector, and $\boldsymbol{f}_{t|t-h} = (1, f^1_{t|t-h},\dots,f^m_{t|t-h})'$ is a $m+1$ vector related to the intercept and the m forecasts. The intercept plays an important role in this model because it ensures that the bias correction of the combined forecast is optimally determined.

Diebold and Pauly [1990] consider a methodology that allows prior information to be incorporated into a regression-based forecast combination framework. They use the g-prior model of Zellner [1986] for a Bayesian estimation of the parameters of model (1). They assume that $\varepsilon_t \overset{iid}{\sim} N(0,\sigma^2)$ and the natural conjugate normal-gamma prior

$$P_0(\boldsymbol{\beta},\sigma) \propto \sigma^{-K-\nu_0-1}\exp\left\{-\tfrac{1}{2}\sigma^2\left[\nu_0 s_0^2 + \left(\boldsymbol{\beta}-\underline{\boldsymbol{\beta}}\right)' M\left(\boldsymbol{\beta}-\underline{\boldsymbol{\beta}}\right)\right]\right\}$$

with $K = m+1$. The likelihood is

$$L(\boldsymbol{\beta},\sigma \mid \boldsymbol{Y},F) \propto \sigma^{-T}\exp\left\{-\tfrac{1}{2}\sigma^2(\boldsymbol{Y}-F\boldsymbol{\beta})'(\boldsymbol{Y}-F\boldsymbol{\beta})\right\}$$

where $\boldsymbol{Y} = (y_1,\dots,y_{t-h})'$ and $F = (\boldsymbol{f}_{1|1-h},\dots,\boldsymbol{f}_{t-h|t-2h})'$.

Then, the marginal posterior of $\boldsymbol{\beta}$ is given by

$$P_1(\boldsymbol{\beta} \mid \boldsymbol{Y}, F) \propto \left[1 + \frac{1}{v_1}\left(\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}\right)' s_1^{-2}\left(M + F'F\right)\left(\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}\right)\right]^{-\frac{K+v_1}{2}}.$$

The marginal posterior mean is as follows:

$$\overline{\boldsymbol{\beta}} = \left(M + F'F\right)^{-1}\left(M\underline{\boldsymbol{\beta}} + F'F\widehat{\boldsymbol{\beta}}\right),$$

where $v_1 = T + v_0$, $s_1^2 = \frac{1}{v_1}\left[v_0\, s_0^2 + \boldsymbol{Y}'\boldsymbol{Y} + \underline{\boldsymbol{\beta}}'M\underline{\boldsymbol{\beta}} - \overline{\boldsymbol{\beta}}'\left(M + F'F\right)\overline{\boldsymbol{\beta}}\right]$ and $\widehat{\boldsymbol{\beta}} = \left(F'F\right)^{-1}F'\boldsymbol{Y}$.

Under $g-$prior analysis with $M = gF'F$, Diebold and Pauly [1990] show that

$$\overline{\boldsymbol{\beta}} = \frac{g}{1+g}\underline{\boldsymbol{\beta}} + \frac{1}{1+g}\widehat{\boldsymbol{\beta}}, \tag{2}$$

where $g \in [0, \infty)$ is the shrinkage parameter that controls the relative weight between the prior mean and the maximum likelihood estimator in the posterior mean.

However, Diebold and Pauly [1990] do not control for the possible presence of structural breaks. Nevertheless, expression (1) can be extended to consider these instabilities by using time-varying forecast combination weights, as follows:

$$y_t = \boldsymbol{\beta}_t'\boldsymbol{f}_{t|t-h} + \varepsilon_t. \tag{3}$$

The Bayesian shrinkage forecast combination methodology of Diebold and Pauly [1990] can be easily generalized to consider (3) by using rolling estimates with a w window size. This procedure yields the following posterior mean:

$$\overline{\boldsymbol{\beta}}_t = \frac{g}{1+g}\underline{\boldsymbol{\beta}}_t + \frac{1}{1+g}\widehat{\boldsymbol{\beta}}_t, \tag{4}$$

where $\widehat{\boldsymbol{\beta}}_t = \left(F_{t-h-w+1,t-h}'F_{t-h-w+1,t-h}\right)^{-1}F_{t-h-w+1,t-h}'\boldsymbol{Y}_{t-h-w+1,t-h}$ and $F_{t-h-w+1,t-h} = (\boldsymbol{f}_{t-h-w+1|t-2h-w+1}, \ldots, \boldsymbol{f}_{t-h|t-2h})'$, $\boldsymbol{Y}_{t-h-w+1,t-h} = (y_{t-h-w+1}, \ldots, y_{t-h})$.

Diebold and Pauly [1990] use equal weights as the prior mean $\underline{\boldsymbol{\beta}}_t$. Wright [2008] uses zero weight as the prior mean in a Bayesian shrinkage exercise. Following Geweke and Whiteman [2006], one way to specify the prior distribution in Bayesian economic forecasting is to incorporate expert forecast information. Thus, we propose to use as prior weights the OLS estimated parameters of the regression between expert $h-$step forecast series $f_{t|t-h}^{ex}$ and the set of individual $h-$step forecasts. The prior mean is calculated as follows:

$$f_{t|t-h}^{ex} = \boldsymbol{\beta}_t'\boldsymbol{f}_{t|t-h} + \varepsilon_t. \tag{5}$$

Then, $\underline{\boldsymbol{\beta}}_t = \left(F_{t-w+1,t}'F_{t-w+1,t}\right)^{-1}F_{t-w+1,t}'\boldsymbol{F}_{t-w+1,t}^{ex}$ with $F_{t-w+1,t} = (\boldsymbol{f}_{t-w+1|t-h-w+1}, \ldots, \boldsymbol{f}_{t|t-h})'$, $\boldsymbol{F}_{t-w+1,t}^{ex} = (f_{t-w+1|t-h-w+1}^{ex}, \ldots, f_{t|t-h}^{ex})$.

When the forecasting series are non-stationary, Coulson and Robins [1993] propose a combination method based on the linear model:

$$y_t - y_{t-h} = \boldsymbol{\beta}' \widetilde{\boldsymbol{f}}_{t|t-h} + \varepsilon_t, \tag{6}$$

where $\widetilde{\boldsymbol{f}}_{t|t-h} = (1, f^1_{t|t-h} - y_{t-h}, \ldots, f^m_{t|t-h} - y_{t-h})'$. Following (3), (4) and (5), equation (6) can be easily modified to consider a rolling Bayesian shrinkage methodology. In this case, $\underline{\boldsymbol{\beta}}_t$ is obtained as the rolling OLS estimation of $\boldsymbol{\beta}_t$:

$$f^{ex}_{t|t-h} - f^{ex}_{t-h|t-2h} = \boldsymbol{\beta}'_t \,\widetilde{\widetilde{\boldsymbol{f}}}_{t|t-h} + \varepsilon_t, \tag{7}$$

where $\widetilde{\widetilde{\boldsymbol{f}}}_{t|t-h} = \left(1, f^1_{t|t-h} - f^{ex}_{t-h|t-2h}, \ldots, f^m_{t|t-h} - f^{ex}_{t-h|t-2h}\right)'$.

The polar or extreme cases of the posterior mean in terms of the shrinkage parameter are obtained under the Coulson and Robins modified methodology in Table 1. These cases are shown for different priors.

Prior	Shrinkage Parameter $g = 0$	Shrinkage Parameter $g \to \infty$
Zero weights	GR-CR	Random walk weights
Equal weights	GR-CR	Equal weights
Expert forecast	GR-CR$^{(-1)}$	Expert forecast weights

TABLE 1. Posterior mean polar cases for the Coulson and Robins modified methodology. GR-CR indicates the MLE weights obtained by rolling estimation of the parameters in (6) including expert forecast as a covariate. GR-CR$^{(-1)}$ indicates the MLE weights obtained by rolling estimation of the parameters in (6), excluding expert forecast as a covariate.

Two results of Table 1 should noted. First, when $g \to \infty$ with zero weights prior, the posterior mean is equal to a zero weight vector. In this case, equation (6) implies a random walk forecast. Second, when $g = 0$, the posterior mean corresponds to the MLE weights. However, the posterior mean of the three priors are slightly different because they do not contain the same information. The Bayesian combination with zero and equal weight priors is calculated using the expert forecast as a covariate, whereas the expert forecast prior does not include this covariate.

3. EMPIRICAL RESULTS

The data consist of monthly Colombian inflation, measured as the first difference of the logarithm of the CPI and nine competing forecasts employed by the Colombian central bank. Forecasts from one step to nine steps ahead from September 2002 to December 2011 are considered. A brief description of the set of competing forecasts is presented in Table 11 of Appendix B. In addition, expert forecasts from the staff consensus predictions provided by the programming and inflation department of the Colombian central bank are used to specify the prior in the shrinkage methodology.

The available data from September 2002 to December 2011 are divided into two samples. The first sample is used to estimate the proposed rolling Bayesian forecast combination model, and the second is used to evaluate the predictive ability of the individual models and their combination. The first rolling window estimation of size w goes to September 2007. With this information, an h-step forecast is estimated. Next, the parameters of the combination are re-estimated after including the next month's data point and ruling out the oldest one. A new set of forecasts is obtained until the last available observation is included.

The root mean square error (RMSE) criterion is used to compare the forecast accuracy of the models. The U-Theil statistic is also calculated to assess the performance of each model relative to a random walk. Table 2 and Tables 3 to 10, in Appendix A, show the forecast performance statistics for windows size $w = 20, 30, 40$ and 50 months[1], shrinkage parameters $g = 0, 1, 3, 5, 20$ and $g \rightarrow \infty$ and forecast horizons from 1 to 9 months ahead. In addition to the proposed prior, which is based on the expert forecasts, equal- and zero-weight priors were used.

The upper parts of Tables 2 to 10 contain the results of the individual models, and the combined forecast statistics are shown in the lower parts. These results show that forecast accuracy is improved by using a rolling Bayesian shrinkage forecast combination methodology with expert forecasts as prior information (hereafter, the RSFC methodology) because they correspond to the minimum RMSE and U-Theil. For example, for a 1-month forecast horizon, Table 2 shows that the minimum RMSE is 0.177, which corresponds to the RSFC methodology with a shrinkage parameter $g = 20$ and a rolling window size $w = 20$. However, the forecast performance of the RSFC methodology depends on the magnitude of the shrinkage parameter and the window size. For the longest forecast horizons, $h = 6, 7, 8$ and 9, the best performance is obtained when $g \rightarrow \infty$, as shown in Tables 7 to 10. This result suggests that for further horizons, the expert forecasts are more informative.

These results also indicate that the RSFC methodology tends to produce the most accurate inflation forecasts when compared with the shrinkage methodology with other priors as equal and zero weights. This finding indicates that the prior computed with the expert forecasts is more informative than the other priors. In the few cases where the other priors have better performance, almost all of them are associated with a zero shrinkage parameter because the equal and zero priors contain more information when $g = 0$, as noted in section 2.

As expected, when the shrinkage parameter g is zero, all three Bayesian shrinkage forecast have similar forecast performance because the prior mean has zero weight in the posterior mean. As explained previously, in this case, the forecast statistics of the expert forecast prior differ slightly

[1]The maximum possible rolling window size for forecast horizon $h = 7, 8$ and 9 months ahead is 40.

because they are computed with less information. Furthermore, when $g \to \infty$, the U-Theil statistic is equal to one for the Bayesian shrinkage forecast combination methodology with zero weights prior. In this case, equation (6) implies a random walk forecast; thus, the U-Theil statistic is one.

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		0.280	0.751	0.280	0.751	0.280	0.751	0.280	0.751
ARIMA.C4		0.276	0.740	0.276	0.740	0.276	0.740	0.276	0.740
ARIMA.C6		0.216	0.579	0.216	0.579	0.216	0.579	0.216	0.579
ARIMA.C10		0.258	0.690	0.258	0.690	0.258	0.690	0.258	0.690
FLS		0.267	0.715	0.267	0.715	0.267	0.715	0.267	0.715
LSTR		0.353	0.946	0.353	0.946	0.353	0.946	0.353	0.946
Neural.Network		0.248	0.665	0.248	0.665	0.248	0.665	0.248	0.665
Neural.Network.C		0.249	0.668	0.249	0.668	0.249	0.668	0.249	0.668
Non.Parametric		0.351	0.941	0.351	0.941	0.351	0.941	0.351	0.941
Exp.Forecasts		0.185	0.495	0.185	0.495	0.185	0.495	0.185	0.495
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	0.296	0.793	0.246	0.660	0.240	0.643	0.244	0.653
g=0	Equal Weights	0.305	0.818	0.254	0.680	0.230	0.617	0.223	0.598
	Zero Weights	0.305	0.818	0.254	0.680	0.230	0.617	0.223	0.598
	Exp.Forecasts	0.207	0.555	0.208	0.557	0.208	0.557	0.215	0.575
g=1	Equal Weights	0.220	0.589	0.217	0.581	0.208	0.556	0.207	0.555
	Zero Weights	0.258	0.691	0.254	0.679	0.247	0.661	0.248	0.665
	Exp.Forecasts	0.182	0.488	0.197	0.527	0.198	0.530	0.205	0.551
g=3	Equal Weights	0.210	0.564	0.217	0.580	0.213	0.570	0.214	0.572
	Zero Weights	0.301	0.806	0.304	0.814	0.302	0.809	0.304	0.814
	Exp.Forecasts	0.178	0.478	0.194	0.520	0.196	0.524	0.203	0.545
g=5	Equal Weights	0.214	0.572	0.219	0.588	0.217	0.582	0.218	0.583
	Zero Weights	0.323	0.864	0.325	0.872	0.324	0.869	0.326	0.873
	Exp.Forecasts	0.177*	0.476	0.192	0.514	0.193	0.518	0.201	0.539
g=20	Equal Weights	0.223	0.599	0.226	0.605	0.225	0.603	0.225	0.604
	Zero Weights	0.358	0.959	0.359	0.962	0.359	0.961	0.359	0.963
	Exp.Forecasts	0.179	0.479	0.191	0.513	0.193	0.517	0.200	0.537
$g \to \infty$	Equal Weights	0.229	0.614	0.229	0.614	0.229	0.614	0.229	0.614
	Zero Weights	0.373	1.000	0.373	1.000	0.373	1.000	0.373	1.000

TABLE 2. Performance of Colombian inflation for 1-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

4. Final Remarks

This study implements a Bayesian shrinkage forecast combination methodology for an emerging country case using Colombian inflation data from September 2002 to December 2011. Our estimation method takes into account two important characteristics of these economies: instability by using rolling estimation, and non-stationarity by implementing methods for series integrated of order one.

We find that forecast accuracy can be improved by using a Bayesian shrinkage forecast combination methodology that considers expert forecasts as prior information. Our results indicate that the accuracy of the combined forecast tends to outperform the forecast of individual models. Moreover, these results outperform shrinkage forecasts that consider other priors as equal or zero weights. This finding indicates that the prior computed with the expert forecasts is more informative than the other priors.

However, the forecast performance of the RSFC methodology depends on the magnitude of the shrinkage parameter and window size. For the longest forecast horizons, $h = 6, 7, 8$ and 9, the best performance is obtained when the shrinkage parameter tends to infinity.

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APPENDIX A. PERFORMANCE OF COLOMBIAN INFLATION FOR 2-MONTH TO 9-MONTH AHEAD FORECASTS

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		0.540	0.816	0.540	0.816	0.540	0.816	0.540	0.816
ARIMA.C4		0.554	0.837	0.554	0.837	0.554	0.837	0.554	0.837
ARIMA.C6		0.486	0.735	0.486	0.735	0.486	0.735	0.486	0.735
ARIMA.C10		0.485	0.734	0.485	0.734	0.485	0.734	0.485	0.734
FLS		0.536	0.810	0.536	0.810	0.536	0.810	0.536	0.810
LSTR		0.643	0.972	0.643	0.972	0.643	0.972	0.643	0.972
Neural.Network		0.444	0.671	0.444	0.671	0.444	0.671	0.444	0.671
Neural.Network.C		0.497	0.751	0.497	0.751	0.497	0.751	0.497	0.751
Non.Parametric		0.644	0.974	0.644	0.974	0.644	0.974	0.644	0.974
Exp.Forecasts		0.430	0.650	0.430	0.650	0.430	0.650	0.430	0.650
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	0.595	0.899	0.492	0.744	0.440	0.666	0.427	0.646
g=0	Equal Weights	0.605	0.914	0.506	0.765	0.422	0.639	0.414	0.626
	Zero Weights	0.605	0.914	0.506	0.765	0.422	0.639	0.414	0.626
	Exp.Forecasts	0.428	0.647	0.411	0.621	0.401	0.606	0.400	0.604
g=1	Equal Weights	0.453	0.685	0.432	0.654	0.401	0.607	0.403	0.609
	Zero Weights	0.504	0.762	0.485	0.733	0.450	0.681	0.457	0.692
	Exp.Forecasts	0.389	0.588	0.395	0.597	0.399	0.603	0.400	0.605
g=3	Equal Weights	0.433	0.654	0.432	0.654	0.421	0.637	0.423	0.640
	Zero Weights	0.556	0.841	0.555	0.839	0.542	0.819	0.547	0.828
	Exp.Forecasts	0.385*	0.583	0.394	0.595	0.401	0.606	0.402	0.609
g=5	Equal Weights	0.436	0.660	0.438	0.663	0.432	0.653	0.434	0.656
	Zero Weights	0.586	0.886	0.587	0.888	0.579	0.876	0.583	0.882
	Exp.Forecasts	0.389	0.588	0.396	0.599	0.406	0.614	0.408	0.616
g=20	Equal Weights	0.450	0.681	0.452	0.683	0.450	0.681	0.451	0.682
	Zero Weights	0.638	0.965	0.639	0.966	0.637	0.963	0.638	0.965
	Exp.Forecasts	0.393	0.595	0.399	0.603	0.409	0.619	0.410	0.620
$g\rightarrow \infty$	Equal Weights	0.459	0.693	0.459	0.693	0.459	0.693	0.459	0.693
	Zero Weights	0.661	1.000	0.661	1.000	0.661	1.000	0.661	1.000

TABLE 3. Performance of Colombian inflation for 2-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		0.799	0.875	0.799	0.875	0.799	0.875	0.799	0.875
ARIMA.C4		0.832	0.911	0.832	0.911	0.832	0.911	0.832	0.911
ARIMA.C6		0.777	0.852	0.777	0.852	0.777	0.852	0.777	0.852
ARIMA.C10		0.751	0.823	0.751	0.823	0.751	0.823	0.751	0.823
FLS		0.778	0.852	0.778	0.852	0.778	0.852	0.778	0.852
LSTR		0.938	1.028	0.938	1.028	0.938	1.028	0.938	1.028
Neural.Network		0.695	0.762	0.695	0.762	0.695	0.762	0.695	0.762
Neural.Network.C		0.749	0.820	0.749	0.820	0.749	0.820	0.749	0.820
Non.Parametric		0.900	0.986	0.900	0.986	0.900	0.986	0.900	0.986
Exp.Forecasts		0.695	0.761	0.695	0.761	0.695	0.761	0.695	0.761
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	1.167	1.279	0.840	0.920	0.690	0.756	0.613	0.672
g=0	Equal Weights	1.333	1.461	0.924	1.013	0.724	0.793	0.625	0.685
	Zero Weights	1.333	1.461	0.924	1.013	0.724	0.793	0.625	0.685
	Exp.Forecasts	0.798	0.874	0.694	0.761	0.633	0.694	0.601*	0.658
g=1	Equal Weights	0.855	0.937	0.727	0.797	0.648	0.711	0.612	0.671
	Zero Weights	0.926	1.015	0.799	0.875	0.696	0.763	0.657	0.720
	Exp.Forecasts	0.695	0.762	0.654	0.717	0.631	0.691	0.614	0.673
g=3	Equal Weights	0.711	0.779	0.685	0.751	0.655	0.718	0.642	0.703
	Zero Weights	0.856	0.938	0.827	0.906	0.781	0.856	0.766	0.840
	Exp.Forecasts	0.681	0.746	0.647	0.709	0.634	0.695	0.621	0.681
g=5	Equal Weights	0.689	0.755	0.682	0.747	0.665	0.728	0.657	0.720
	Zero Weights	0.861	0.944	0.850	0.931	0.821	0.899	0.812	0.890
	Exp.Forecasts	0.679	0.745	0.642	0.704	0.642	0.703	0.634	0.695
g=20	Equal Weights	0.685	0.751	0.687	0.753	0.683	0.749	0.681	0.747
	Zero Weights	0.892	0.978	0.892	0.978	0.885	0.970	0.883	0.967
	Exp.Forecasts	0.685	0.751	0.642	0.704	0.646	0.708	0.640	0.701
$g \to \infty$	Equal Weights	0.692	0.759	0.692	0.759	0.692	0.759	0.692	0.759
	Zero Weights	0.912	1.000	0.912	1.000	0.912	1.000	0.912	1.000

TABLE 4. Performance of Colombian inflation for 3-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		1.025	0.899	1.025	0.899	1.025	0.899	1.025	0.899
ARIMA.C4		1.066	0.935	1.066	0.935	1.066	0.935	1.066	0.935
ARIMA.C6		1.039	0.912	1.039	0.912	1.039	0.912	1.039	0.912
ARIMA.C10		0.978	0.858	0.978	0.858	0.978	0.858	0.978	0.858
FLS		1.000	0.877	1.000	0.877	1.000	0.877	1.000	0.877
LSTR		1.231	1.080	1.231	1.080	1.231	1.080	1.231	1.080
Neural.Network		0.909	0.797	0.909	0.797	0.909	0.797	0.909	0.797
Neural.Network.C		0.951	0.834	0.951	0.834	0.951	0.834	0.951	0.834
Non.Parametric		1.126	0.988	1.126	0.988	1.126	0.988	1.126	0.988
Exp.Forecasts		0.896	0.786	0.896	0.786	0.896	0.786	0.896	0.786
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	1.852	1.624	1.236	1.084	0.943	0.827	0.807	0.708
g=0	Equal Weights	2.019	1.771	1.314	1.152	1.030	0.904	0.890	0.781
	Zero Weights	2.019	1.771	1.314	1.152	1.030	0.904	0.890	0.781
	Exp.Forecasts	1.168	1.025	0.961	0.843	0.831	0.729	0.777*	0.681
g=1	Equal Weights	1.297	1.138	1.048	0.919	0.908	0.797	0.851	0.746
	Zero Weights	1.433	1.257	1.153	1.012	0.970	0.851	0.915	0.803
	Exp.Forecasts	0.925	0.812	0.855	0.750	0.801	0.703	0.777	0.682
g=3	Equal Weights	1.026	0.900	0.954	0.837	0.887	0.779	0.863	0.757
	Zero Weights	1.232	1.081	1.127	0.988	1.029	0.903	1.008	0.884
	Exp.Forecasts	0.874	0.766	0.826	0.725	0.796	0.698	0.780	0.684
g=5	Equal Weights	0.963	0.845	0.930	0.816	0.887	0.778	0.872	0.765
	Zero Weights	1.187	1.041	1.127	0.988	1.061	0.931	1.048	0.920
	Exp.Forecasts	0.836	0.733	0.793	0.695	0.792	0.695	0.785	0.689
g=20	Equal Weights	0.907	0.796	0.905	0.794	0.893	0.783	0.889	0.780
	Zero Weights	1.147	1.007	1.134	0.995	1.116	0.979	1.112	0.976
	Exp.Forecasts	0.833	0.731	0.782	0.686	0.792	0.695	0.788	0.691
$g \to \infty$	Equal Weights	0.897	0.787	0.897	0.787	0.897	0.787	0.897	0.787
	Zero Weights	1.140	1.000	1.140	1.000	1.140	1.000	1.140	1.000

TABLE 5. Performance of Colombian inflation for 4-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		1.224	0.895	1.224	0.895	1.224	0.895	1.224	0.895
ARIMA.C4		1.272	0.931	1.272	0.931	1.272	0.931	1.272	0.931
ARIMA.C6		1.266	0.926	1.266	0.926	1.266	0.926	1.266	0.926
ARIMA.C10		1.170	0.856	1.170	0.856	1.170	0.856	1.170	0.856
FLS		1.205	0.882	1.205	0.882	1.205	0.882	1.205	0.882
LSTR		1.477	1.081	1.477	1.081	1.477	1.081	1.477	1.081
Neural.Network		1.123	0.822	1.123	0.822	1.123	0.822	1.123	0.822
Neural.Network.C		1.134	0.830	1.134	0.830	1.134	0.830	1.134	0.830
Non.Parametric		1.348	0.986	1.348	0.986	1.348	0.986	1.348	0.986
Exp.Forecasts		1.056	0.773	1.056	0.773	1.056	0.773	1.056	0.773
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	2.112	1.545	1.760	1.288	1.365	0.999	1.053	0.770
g=0	Equal Weights	2.251	1.647	1.821	1.333	1.485	1.086	1.198	0.877
	Zero Weights	2.251	1.647	1.821	1.333	1.485	1.086	1.198	0.877
	Exp.Forecasts	1.361	0.996	1.263	0.924	1.067	0.781	0.958	0.701
g=1	Equal Weights	1.505	1.102	1.359	0.994	1.193	0.873	1.077	0.788
	Zero Weights	1.673	1.224	1.480	1.083	1.292	0.945	1.164	0.851
	Exp.Forecasts	1.108	0.811	1.077	0.788	0.980	0.717	0.937	0.686
g=3	Equal Weights	1.225	0.896	1.185	0.867	1.108	0.810	1.060	0.776
	Zero Weights	1.472	1.077	1.389	1.016	1.295	0.948	1.240	0.907
	Exp.Forecasts	1.058	0.774	1.031	0.754	0.963	0.705	0.935*	0.684
g=5	Equal Weights	1.157	0.847	1.140	0.834	1.091	0.798	1.062	0.777
	Zero Weights	1.425	1.042	1.373	1.005	1.312	0.960	1.277	0.934
	Exp.Forecasts	1.026	0.751	0.981	0.718	0.951	0.696	0.935	0.684
g=20	Equal Weights	1.091	0.798	1.091	0.798	1.078	0.788	1.071	0.783
	Zero Weights	1.378	1.008	1.365	0.999	1.348	0.986	1.339	0.980
	Exp.Forecasts	1.027	0.752	0.968	0.708	0.950	0.695	0.936	0.685
$g \to \infty$	Equal Weights	1.076	0.787	1.076	0.787	1.076	0.787	1.076	0.787
	Zero Weights	1.367	1.000	1.367	1.000	1.367	1.000	1.367	1.000

TABLE 6. Performance of Colombian inflation for 5-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40		Window Size=50	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS									
ARIMA		1.416	0.888	1.416	0.888	1.416	0.888	1.416	0.888
ARIMA.C4		1.455	0.912	1.455	0.912	1.455	0.912	1.455	0.912
ARIMA.C6		1.460	0.916	1.460	0.916	1.460	0.916	1.460	0.916
ARIMA.C10		1.346	0.844	1.346	0.844	1.346	0.844	1.346	0.844
FLS		1.430	0.897	1.430	0.897	1.430	0.897	1.430	0.897
LSTR		1.741	1.091	1.741	1.091	1.741	1.091	1.741	1.091
Neural.Network		1.346	0.844	1.346	0.844	1.346	0.844	1.346	0.844
Neural.Network.C		1.283	0.804	1.283	0.804	1.283	0.804	1.283	0.804
Non.Parametric		1.570	0.984	1.570	0.984	1.570	0.984	1.570	0.984
Exp.Forecasts		1.223	0.767	1.223	0.767	1.223	0.767	1.223	0.767
COMBINED MODELS									
Shrinkage	Prior								
	Exp.Forecasts	2.452	1.537	2.345	1.470	1.798	1.127	1.273	0.798
g=0	Equal Weights	2.618	1.641	2.442	1.531	1.932	1.211	1.400	0.878
	Zero Weights	2.618	1.641	2.442	1.531	1.932	1.211	1.400	0.878
	Exp.Forecasts	1.642	1.030	1.648	1.033	1.378	0.864	1.129	0.708
g=1	Equal Weights	1.834	1.150	1.741	1.092	1.515	0.950	1.258	0.789
	Zero Weights	2.038	1.278	1.905	1.195	1.661	1.041	1.357	0.850
	Exp.Forecasts	1.333	0.836	1.350	0.847	1.213	0.761	1.083	0.679
g=3	Equal Weights	1.501	0.941	1.454	0.912	1.357	0.851	1.236	0.775
	Zero Weights	1.792	1.124	1.712	1.073	1.599	1.002	1.445	0.906
	Exp.Forecasts	1.257	0.788	1.265	0.793	1.169	0.733	1.073	0.672
g=5	Equal Weights	1.405	0.881	1.375	0.862	1.315	0.824	1.237	0.775
	Zero Weights	1.720	1.078	1.664	1.043	1.591	0.997	1.489	0.934
	Exp.Forecasts	1.181	0.740	1.160	0.727	1.116	0.700	1.061	0.665
g=20	Equal Weights	1.289	0.808	1.281	0.803	1.266	0.794	1.245	0.781
	Zero Weights	1.628	1.021	1.611	1.010	1.591	0.998	1.563	0.980
	Exp.Forecasts	1.162	0.729	1.125	0.705	1.099	0.689	1.058*	0.664
$g \to \infty$	Equal Weights	1.250	0.784	1.250	0.784	1.250	0.784	1.250	0.784
	Zero Weights	1.595	1.000	1.595	1.000	1.595	1.000	1.595	1.000

TABLE 7. Performance of Colombian inflation for 6-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS							
ARIMA		1.634	0.895	1.634	0.895	1.634	0.895
ARIMA.C4		1.652	0.905	1.652	0.905	1.652	0.905
ARIMA.C6		1.656	0.907	1.656	0.907	1.656	0.907
ARIMA.C10		1.542	0.845	1.542	0.845	1.542	0.845
FLS		1.673	0.916	1.673	0.916	1.673	0.916
LSTR		1.989	1.090	1.989	1.090	1.989	1.090
Neural.Network		1.527	0.836	1.527	0.836	1.527	0.836
Neural.Network.C		1.472	0.806	1.472	0.806	1.472	0.806
Non.Parametric		1.801	0.987	1.801	0.987	1.801	0.987
Exp.Forecasts		1.388	0.760	1.388	0.760	1.388	0.760
COMBINED MODELS							
Shrinkage	Prior						
	Exp.Forecasts	4.065	2.227	3.306	1.811	2.568	1.407
g=0	Equal Weights	4.215	2.309	3.452	1.891	2.709	1.484
	Zero Weights	4.215	2.309	3.452	1.891	2.709	1.484
	Exp.Forecasts	2.472	1.354	2.184	1.197	1.835	1.005
g=1	Equal Weights	2.651	1.452	2.314	1.268	1.972	1.080
	Zero Weights	2.909	1.594	2.525	1.383	2.147	1.176
	Exp.Forecasts	1.773	0.971	1.686	0.924	1.518	0.832
g=3	Equal Weights	1.957	1.072	1.818	0.996	1.667	0.913
	Zero Weights	2.319	1.270	2.132	1.168	1.947	1.066
	Exp.Forecasts	1.577	0.864	1.540	0.844	1.426	0.781
g=5	Equal Weights	1.756	0.962	1.675	0.918	1.581	0.866
	Zero Weights	2.140	1.172	2.018	1.105	1.896	1.039
	Exp.Forecasts	1.354	0.742	1.358	0.744	1.310	0.718
g=20	Equal Weights	1.517	0.831	1.500	0.822	1.477	0.809
	Zero Weights	1.908	1.045	1.875	1.027	1.842	1.009
	Exp.Forecasts	1.292	0.708	1.297	0.711	1.270*	0.696
$g \to \infty$	Equal Weights	1.442	0.790	1.442	0.790	1.442	0.790
	Zero Weights	1.825	1.000	1.825	1.000	1.825	1.000

TABLE 8. Performance of Colombian inflation for 7-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS							
ARIMA		1.846	0.904	1.846	0.904	1.846	0.904
ARIMA.C4		1.832	0.898	1.832	0.898	1.832	0.898
ARIMA.C6		1.835	0.899	1.835	0.899	1.835	0.899
ARIMA.C10		1.731	0.848	1.731	0.848	1.731	0.848
FLS		1.920	0.941	1.920	0.941	1.920	0.941
LSTR		2.190	1.073	2.190	1.073	2.190	1.073
Neural.Network		1.729	0.847	1.729	0.847	1.729	0.847
Neural.Network.C		1.642	0.805	1.642	0.805	1.642	0.805
Non.Parametric		2.014	0.987	2.014	0.987	2.014	0.987
Exp.Forecasts		1.515	0.742	1.515	0.742	1.515	0.742
COMBINED MODELS							
Shrinkage	Prior						
	Exp.Forecasts	4.659	2.283	3.531	1.730	2.866	1.404
g=0	Equal Weights	4.679	2.292	3.514	1.722	3.133	1.535
	Zero Weights	4.679	2.292	3.514	1.722	3.133	1.535
	Exp.Forecasts	2.869	1.406	2.375	1.164	2.098	1.028
g=1	Equal Weights	2.938	1.439	2.380	1.166	2.258	1.106
	Zero Weights	3.163	1.550	2.574	1.261	2.470	1.210
	Exp.Forecasts	2.045	1.002	1.864	0.913	1.757	0.861
g=3	Equal Weights	2.172	1.064	1.921	0.941	1.892	0.927
	Zero Weights	2.516	1.233	2.233	1.094	2.216	1.086
	Exp.Forecasts	1.797	0.881	1.714	0.840	1.654	0.810
g=5	Equal Weights	1.954	0.957	1.798	0.881	1.788	0.876
	Zero Weights	2.333	1.143	2.150	1.053	2.148	1.052
	Exp.Forecasts	1.489	0.729	1.526	0.748	1.520	0.745
g=20	Equal Weights	1.697	0.832	1.660	0.813	1.661	0.814
	Zero Weights	2.112	1.035	2.064	1.011	2.067	1.013
	Exp.Forecasts	1.388*	0.680	1.462	0.716	1.471	0.721
$g \rightarrow \infty$	Equal Weights	1.619	0.793	1.619	0.793	1.619	0.793
	Zero Weights	2.041	1.000	2.041	1.000	2.041	1.000

TABLE 9. Performance of Colombian inflation for 8-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

		Window Size=20		Window Size=30		Window Size=40	
		RMSE	U-Theil	RMSE	U-Theil	RMSE	U-Theil
INDIVIDUAL MODELS							
ARIMA		2.019	0.904	2.019	0.904	2.019	0.904
ARIMA.C4		1.960	0.877	1.960	0.877	1.960	0.877
ARIMA.C6		1.971	0.882	1.971	0.882	1.971	0.882
ARIMA.C10		1.880	0.841	1.880	0.841	1.880	0.841
FLS		2.129	0.953	2.129	0.953	2.129	0.953
LSTR		2.402	1.075	2.402	1.075	2.402	1.075
Neural.Network		1.904	0.852	1.904	0.852	1.904	0.852
Neural.Network.C		1.792	0.802	1.792	0.802	1.792	0.802
Non.Parametric		2.199	0.984	2.199	0.984	2.199	0.984
Exp.Forecasts		1.709	0.764	1.709	0.764	1.709	0.764
COMBINED MODELS							
Shrinkage	Prior						
	Exp.Forecasts	4.628	2.071	3.638	1.628	3.375	1.510
g=0	Equal Weights	4.574	2.047	3.628	1.623	3.473	1.554
	Zero Weights	4.574	2.047	3.628	1.623	3.473	1.554
	Exp.Forecasts	2.941	1.316	2.497	1.117	2.451	1.097
g=1	Equal Weights	2.954	1.322	2.543	1.138	2.510	1.123
	Zero Weights	3.198	1.431	2.771	1.240	2.745	1.228
	Exp.Forecasts	2.185	0.978	2.002	0.896	2.034	0.910
g=3	Equal Weights	2.260	1.011	2.093	0.937	2.096	0.938
	Zero Weights	2.632	1.178	2.446	1.095	2.453	1.097
	Exp.Forecasts	1.963	0.878	1.858	0.832	1.907	0.853
g=5	Equal Weights	2.066	0.924	1.969	0.881	1.976	0.884
	Zero Weights	2.476	1.108	2.362	1.057	2.371	1.061
	Exp.Forecasts	1.691	0.756	1.681	0.752	1.739	0.778
g=20	Equal Weights	1.841	0.824	1.820	0.814	1.825	0.817
	Zero Weights	2.293	1.026	2.265	1.013	2.270	1.015
	Exp.Forecasts	1.602*	0.717	1.621	0.725	1.677	0.751
$g \rightarrow \infty$	Equal Weights	1.773	0.793	1.773	0.793	1.773	0.793
	Zero Weights	2.235	1.000	2.235	1.000	2.235	1.000

TABLE 10. Performance of Colombian inflation for 9-month ahead forecasts. The symbol (*) indicates the smallest RMSE.

APPENDIX B. FORECAST MODELS

Forecast model	Abbreviation	Characteristics	Reference
ARIMA by components	$ARIMA.C4$, $ARIMA.C6$, $ARIMA.C10$	Weighted average between ARIMA models with different aggregation levels of the CPI basket	Gómez, González, and Melo [2012]
ARIMA	$ARIMA$	ARIMA model	–
Non parametric	$Non.Parametric$	Non-parametric regression model	Rodríguez and Siado [2003]
Neural Networks	$Neural.Network$	Neural Networks model	Misas, López, and Querubín [2002]
Neural Networks by components	$Neural.Network.C$	Weighted average between an NN for food inflation and an NN for non-food inflation	–
LSTR	$LSTR$	Logistic smooth transition regression model	Jalil and Melo [1999]
FLS	FLS	Flexible Least Squares approach	Melo and Misas [2004]

TABLE 11. Forecast models included in the combination