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Por:
Juan David Prada-Sarmiento

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Multiple policymakers and the social value of public information

Juan David Prada-Sarmiento*
jdprada@u.northwestern.edu

Northwestern University and Banco de la República de Colombia

Abstract

In most economies, macroeconomic policy is conducted by two or more independent authorities. In general, each policymaker has a different piece of information about the state of the economy, and this information is different from the one held by the private sector. We extend the model of James and Lawler [2011], of asymmetric and imperfect information, to account for the existence of two independent policymakers. The active policymakers can choose optimally what policy rule to follow, what information to share with the other policymakers, and what information to share with the public. This paper studies the social value of public information when the policymakers are active. We find that improving the quality of the signals transmitted to the private sector can increase the expected value of welfare. If both policymakers seek to maximize expected welfare, full information sharing between authorities achieves higher expected welfare than no information sharing at all.

Keywords: Macroeconomic Policy, Asymmetric Information, Policy Coordination.

JEL Classification Numbers: D82, D83, E52, E61, E62

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1 Introduction

Morris and Shin [2002] study the social value of information in an economy with a continuum of private agents that differ in the information they possess. Each agent receives a noisy private signal about the state of the economy, which is otherwise unobservable. A policymaker produces a public signal about the underlying state, possibly adding noise to its own information. The question is what is the precision of the public signal that maximizes expected welfare. They show that disclosure of more precise information to the private sector by a single policymaker is not always welfare improving. James and Lawler [2011] extend Morris and Shin [2002]'s framework to allow for an active policymaker that is choosing its policy optimally. Under active policy, revealing more precise information to the private sector is never optimal, in a global sense.

However, policy in most economies in the world is dictated by two or more policymakers. In general, each policymaker has a different piece of information about the state of the economy, and this information is different from the one held by the private sector. Moreover, in recent years, there has been a trend to grant independence to monetary authorities. Thus, policymakers tend to be independent from each other. In this paper we study what is the social value of information when there are two policymakers, instead of only one.

We find that improving the quality of the signals transmitted to the private sector can increase the expected value of welfare. If both policymakers seek to maximize expected welfare, full information sharing between authorities achieves higher expected welfare than no information sharing at all. Then full information sharing and partial transparency can be a local maximizer of expected welfare.

2 The James and Lawler [2011] framework with two policymakers

This section is heavily based on James and Lawler [2011].

The economy is inhabited by a continuum of private sector agents, indexed by $i \in [0,1]$, and two policymakers, indexed by $l \in \{1,2\}$. Agent i chooses the action $a_i \in \mathbb{R}$ to maximize the payoff function

$$\begin{aligned} u_i &= -r\left(a_i - \theta - g_1 - g_2\right)^2 - (1-r)\left(L_i - \overline{L}\right) \\ & L_i = \int_0^1 \left(a_j - a_i\right)^2 dj \\ & \overline{L} = \int_0^1 L_j dj \end{aligned}$$

where $0 < r < 1$ is a known constant, θ is a random variable representing the underlying state of the economy and g_l is policymaker l's instrument. As in Morris and Shin [2002], θ follows a uniform distribution over the real line.

The payoff function corresponds exactly to James and Lawler [2011]'s formulation, with the exception of the presence of two policy instruments. In particular, it implies that the effect of the policy instruments are perfectly aligned: whatever can be achieved with g_1, can be achieved with g_2. The policy instruments are perfect substitutes in the payoff function. Note also the presence of a "beauty contest" term: deviations of the chosen action from the average action are costly for the private agents. This generates the strategic complementarity in the actions of the private sector.

Before making its choice of g_l, the policymaker l observes a noisy signal, z_l, of θ, where

$$z_l = \theta + \phi_l$$

with the noise term, $\phi_l \sim N\left(0, \sigma_\phi^2\right)$, assumed to be independent of θ and other noise terms to be defined next, for all l, with $E\left(\phi_l \phi_{3-l}\right) = 0$ for $l = 1, 2$.

We assume that the policymakers are able to share information without revealing it to the private sector. In particular, each policymaker can transmit the signal

$$w_l = z_l + \omega_l$$

and this signal is received only by the other policymaker. The noise term, $\omega_l \sim N\left(0, \sigma_{l,\omega}^2\right)$, is assumed independent of θ, ϕ_l and all other noise terms that remain to be defined.

With both signals at hand, the l-policymaker updates its expectation about θ,

$$E_l\left(\theta\right) = \beta_\theta^l z_l + \left(1 - \beta_\theta^l\right) w_{3-l}$$

and produces the public signal

$$y_l = E_l\left(\theta\right) + \xi_l$$

where $\beta_\theta^l \equiv \frac{\sigma_\phi^2 + \sigma_{3-l,\omega}^2}{2\sigma_\phi^2 + \sigma_{3-l,\omega}^2}$ and $\xi_l \sim N\left(0, \sigma_{l,\xi}^2\right)$. The noise term is assumed independent of θ, ϕ, ω and all other noise terms that remain to be defined.

The policymakers commit to follow a disclosure rule: they add noise to their information, sampling from normal distributions with given variances. These variances controls the degree of disclosure of the signals. The case of full disclosure to the private sector is captured by $\sigma_{l,\xi}^2 = 0$. Zero disclosure to the private sector arises as $\sigma_{l,\xi}^2 \to \infty$. In the same way, full disclosure to the other policymaker is captured by $\sigma_{l,\omega}^2 = 0$, and zero disclosure (no sharing of information to the other policymaker) arises as $\sigma_{l,\omega}^2 \to \infty$.

Each private sector agent observes $\mathbf{y} \equiv [y_1, y_2]'$ and its own idiosyncratic noisy private signal

$$x_i = \theta + \varepsilon_i$$

prior to deciding its own action a_i. The noise term, $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$, is assumed independent of θ, ϕ, ω and ξ, with $E\left(\varepsilon_i \varepsilon_j\right) = 0$ for $j \neq i$ and $\int_0^1 \varepsilon_j dj = 0$.

We assume that private signals are observed only by the agent that receives them. Moreover, no agent is able to observe the chosen action of any other agent before making its own decision. Therefore, agent i's expectation of any variable is conditioned only on the observed values of $\mathbf{y}$ and x_i, while that of the policymaker l is conditioned on z_l, ω_k and y_k.

Agent i's conditional expectation of θ, denoted by $E_i(\theta)$, is given by

$$\begin{aligned} E_i(\theta) &= \frac{\frac{1}{\sigma_\varepsilon^2}x_i + \sum_{l=1}^2 \frac{\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2}{\left(\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2\sigma_\phi^2+\left(\sigma_\phi^2\right)^3+\left(\sigma_\phi^2\right)^2\sigma_{3-l,\omega}^2+\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2\sigma_{l,\xi}^2}y_l}{\frac{1}{\sigma_\varepsilon^2} + \sum_{l=1}^2 \frac{\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2}{\left(\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2\sigma_\phi^2+\left(\sigma_\phi^2\right)^3+\left(\sigma_\phi^2\right)^2\sigma_{3-l,\omega}^2+\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)^2\sigma_{l,\xi}^2}} \\ &= \alpha_\theta^x x_i + \sum_{k=1}^2 \alpha_\theta^k y_k \end{aligned}$$

Agent i's conditional expectation of the signal, z_l, observed by the l-policymaker, is

$$\begin{aligned} E_i(z_l) &= \frac{\frac{1}{\sigma_\varepsilon^2+\sigma_\phi^2}x_i + \frac{2\sigma_\phi^2+\sigma_{3-l,\omega}^2}{\left(\sigma_\phi^2\right)^2+\sigma_{l,\xi}^2\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)}y_l + \frac{\left(2\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2}{\left(\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2 2\sigma_\phi^2+\left(\sigma_\phi^2\right)^2\sigma_{l,\omega}^2+\left(2\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2\sigma_{3-l,\xi}^2}y_{3-l}}{\frac{1}{\sigma_\varepsilon^2+\sigma_\phi^2} + \frac{2\sigma_\phi^2+\sigma_{3-l,\omega}^2}{\left(\sigma_\phi^2\right)^2+\sigma_{l,\xi}^2\left(2\sigma_\phi^2+\sigma_{3-l,\omega}^2\right)} + \frac{\left(2\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2}{\left(\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2 2\sigma_\phi^2+\left(\sigma_\phi^2\right)^2\sigma_{l,\omega}^2+\left(2\sigma_\phi^2+\sigma_{l,\omega}^2\right)^2\sigma_{3-l,\xi}^2}} \\ &= \alpha_{z,l}^x x_i + \sum_{k=1}^2 \alpha_{z,l}^k y_k \end{aligned}$$

and the conditional expectation of the signal w_k, is

$$\begin{aligned} E_i(w_k) &= \frac{\frac{1}{\sigma_\varepsilon^2+\sigma_\phi^2+\sigma_{k,\omega}^2}x_i + \frac{2\sigma_\phi^2+\sigma_{k,\omega}^2}{\left(\sigma_\phi^2+\sigma_{k,\omega}^2\right)^2+\sigma_{3-k,\xi}^2\left(2\sigma_\phi^2+\sigma_{k,\omega}^2\right)}y_{3-k} + \frac{2\sigma_\phi^2+\sigma_{3-k,\omega}^2}{\left(\sigma_\phi^2\right)^2+\left(2\sigma_\phi^2+\sigma_{3-k,\omega}^2\right)\left(\sigma_{k,\omega}^2+\sigma_{k,\xi}^2\right)}y_k}{\frac{1}{\sigma_\varepsilon^2+\sigma_\phi^2+\sigma_{k,\omega}^2} + \frac{2\sigma_\phi^2+\sigma_{k,\omega}^2}{\left(\sigma_\phi^2+\sigma_{k,\omega}^2\right)^2+\sigma_{3-k,\xi}^2\left(2\sigma_\phi^2+\sigma_{k,\omega}^2\right)} + \frac{2\sigma_\phi^2+\sigma_{3-k,\omega}^2}{\left(\sigma_\phi^2\right)^2+\left(2\sigma_\phi^2+\sigma_{3-k,\omega}^2\right)\left(\sigma_{k,\omega}^2+\sigma_{k,\xi}^2\right)}} \\ &= \alpha_{w,k}^x x_i + \sum_{l=1}^2 \alpha_{w,k}^l y_l \end{aligned}$$

Agent i solves the problem

$$\max_{a_i} \quad E_i\left(-r\left(a_i-\theta-\sum_{l=1}^2 g_l\right)^2-(1-r)\left(L_i-\overline{L}\right)\right)$$

and the optimal action of agent i is determined according to

$$a_i = r\left(E_i(\theta)+\sum_{l=1}^2 E_i(g_l)\right)+(1-r)E_i(\overline{a})$$

where $\overline{a} = \int_0^1 a_i di$ denotes the average action. Thus, the optimal action a_i depends on agent i's expectations on θ, z_l, w_l and $\overline{a}$.

We characterize policy in terms of commitment to a rule. Policy g_l is assumed to be set according to

$$g_l = \rho_l^l z_l + \rho_l^{3-l} w_{3-l}$$

where the value of the rule parameters, ρ_l^l, ρ_l^{3-l}, is public knowledge, for $l = 1, 2$. Here we have an additional degree of disclosure of the policymakers.

3 Linear equilibrium

3.1 Private sector

We first determine each agent's action, taking the value of the rule parameters and the quality of the public signals as given. We then identify the values of ρ_l^k which maximize social welfare $(E(W|\theta))$ as a function of $\sigma_{l,\omega}^2$, $\sigma_{l,\xi}^2$. This is the optimal policy.

Following Morris and Shin [2002], we guess that agent i's action is a linear function of the observed signals,

$$a_i = \kappa_0 x_i + \kappa_1 y_1 + \kappa_2 y_2$$

Given $\int_0^1 \varepsilon_i di = 0$, it follows that

$$E_i(\overline{a}) = \kappa_0 E_i(\theta) + \sum_{l=1}^{2} \kappa_l y_l$$

After substituting policy rules and agent i's conditional expectations, we can solve for the unknown coefficients in the equation

$$\kappa_0 x_i + \kappa_1 y_1 + \kappa_2 y_2 = r\left(E_i(\theta) + \sum_{l=1}^{2} E_i(g_l)\right) + (1-r) E_i(\overline{a})$$

After some algebra, it can be shown that $\kappa_0, \kappa_1, \kappa_2$ satisfy

$$\kappa_0 = r\frac{\alpha_\theta^x + \sum_{l=1}^2 \left(\rho_l^l \alpha_{z,l}^x + \rho_l^{3-l}\alpha_{w,3-l}^x\right)}{1-(1-r)\alpha_\theta^x}$$

and

$$\kappa_k = \frac{(r+(1-r)\kappa_0)\alpha_\theta^k}{r} + \sum_{l=1}^{2}\left(\rho_l^l \alpha_{z,l}^k + \rho_l^{3-l}\alpha_{w,3-l}^k\right)$$

for $k = 1, 2$. Note that $\kappa_0 + \sum_{k=1}^2 \kappa_k = 1 + \sum_{l=1}^2 \sum_{k=1}^2 \rho_l^k$.

3.2 Optimal policy

We assume that both policymakers want to maximize the expected value of the normalized welfare, defined as

$$W = \frac{1}{r}\int_0^1 u_i di$$

Thus we have perfect substitution of policy instruments and perfect alignment of objectives. This of course will make cooperation between the policymakers more attractive to them.

Substituting the equilibrium actions $a_i = \kappa_0 x_i + \sum_{l=1}^{2}\kappa_l y_l$ and the policy rules $g_l = \rho_l^l z_l + \rho_l^{3-l} w_{3-l}$ in the utility function u_i we get

$$u_i = -r\left(\kappa_0 x_i + \sum_{l=1}^{2}\kappa_l y_l - \theta - \sum_{l=1}^{2}\rho_l^l z_l - \sum_{l=1}^{2}\rho_l^{3-l} w_{3-l}\right)^2 - (1-r)\left(L_i - \overline{L}\right)$$

and integrating over i and taking expected values we have

$$\begin{aligned} E\left(W|\theta\right) &= -\left(\kappa_0^2\sigma_\varepsilon^2 + \sum_{l=1}^{2}\left(\kappa_l\beta_\theta^l + \kappa_{3-l}\left(1-\beta_\theta^{3-l}\right) - \rho_l^l - \rho_{3-l}^l\right)^2\sigma_\phi^2\right. \\ &\quad \left. + \sum_{l=1}^{2}\left(\kappa_{3-l}\left(1-\beta_\theta^{3-l}\right) - \rho_{3-l}^l\right)^2\sigma_{l,\omega}^2 + \sum_{l=1}^{2}\kappa_l^2\sigma_{l,\xi}^2\right) \end{aligned}$$

The FOC w.r.t. ρ_s^t, for any $s = 1,2$ and $t = 1,2$, is

$$\begin{aligned} &\frac{\partial E\left(W|\theta\right)}{\partial\rho_s^t} \\ &= -2\left(\kappa_0\frac{\partial\kappa_0}{\partial\rho_s^t}\sigma_\varepsilon^2\right. \\ &+ \sum_{l=1}^{2}\left(\kappa_l\beta_\theta^l + \kappa_{3-l}\left(1-\beta_\theta^{3-l}\right) - \rho_l^l - \rho_{3-l}^l\right)\left(\frac{\partial\kappa_l}{\partial\rho_s^t}\beta_\theta^l + \frac{\partial\kappa_{3-l}}{\partial\rho_s^t}\left(1-\beta_\theta^{3-l}\right) - \delta_{(s,t)}^{(l,l)} - \delta_{(s,t)}^{(3-l,l)}\right)\sigma_\phi^2 \\ &\left. + \sum_{l=1}^{2}\left(\kappa_{3-l}\left(1-\beta_\theta^{3-l}\right) - \rho_{3-l}^l\right)\left(\frac{\partial\kappa_{3-l}}{\partial\rho_s^t}\left(1-\beta_\theta^{3-l}\right) - \delta_{(s,t)}^{(3-l,l)}\right)\sigma_{l,\omega}^2 + \sum_{l=1}^{2}\kappa_l\frac{\partial\kappa_l}{\partial\rho_s^t}\sigma_{l,\xi}^2\right) = 0 \end{aligned}$$

where $\delta_{(s,t)}^{(p,q)} = \begin{cases} 1 & \text{if } (s,t) = (p,q) \\ 0 & \text{otherwise} \end{cases}$.

Note that

$$\frac{\partial\kappa_0}{\partial\rho_l^l} = r\frac{\alpha_{z,l}^x}{1-(1-r)\alpha_\theta^x} \qquad \frac{\partial\kappa_0}{\partial\rho_l^{3-l}} = r\frac{\alpha_{w,3-l}^x}{1-(1-r)\alpha_\theta^x}$$

$$\frac{\partial\kappa_k}{\partial\rho_l^l} = \frac{(1-r)\alpha_\theta^k\alpha_{z,l}^x}{1-(1-r)\alpha_\theta^x} + \alpha_{z,l}^k \qquad \frac{\partial\kappa_k}{\partial\rho_l^{3-l}} = \frac{(1-r)\alpha_\theta^k\alpha_{w,3-l}^x}{1-(1-r)\alpha_\theta^x} + \alpha_{w,3-l}^k$$

and this derivatives are independent of ρ.

Thus the FOCs give us a system of four linear equations in four unknowns, and generically we have a unique solution. We can solve for the optimal ρ_l^{l*}, ρ_l^{3-l*} as functions of σ_ε^2, σ_ϕ^2, $\sigma_{l,\omega}^2$ and $\sigma_{l,\xi}^2$ for $l=1,2$.

Let $W^*\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_{1,\xi}^2, \sigma_{2,\xi}^2, \sigma_{1,\omega}^2 \sigma_{2,\omega}^2\right)$ be the value function $E\left(W|\theta\right)|_{\rho=\rho^*}$. We can ask the comparative statics question: does more precision of the signals produced by the policymakers improve expected welfare?

3.3 Passive policymakers

In Morris and Shin [2002]'s model, the policymakers commit themselves to follow the policy rules, announcing the coefficients ρ_l^l, ρ_l^{3-l}, $l=1,2$. Other agents believe the announcement and choose their actions. But the policymakers are passive, in the sense that their policy is not optimally chosen. We start our analysis by asking how the expected welfare depends on the precision of the signals produced by the policymakers, when they do not choose their policy optimally.

For simplicity, we focus on the case in which the policymakers fully share information between them. We want to know under what condition is better to fully disclose information to the private sector than to reveal nothing. Let $\sigma_{l,\omega}^2 = 0$ for $l=1,2$.

If in addition $\sigma_{l,\xi}^2 \to \infty$ for $l=1,2$ (zero disclosure to the private sector), then

$$\begin{aligned}\kappa_0^\infty &= 1 + \sum_{l=1}^2 \sum_{k=1}^2 \rho_l^k \\ \kappa_k^\infty &= 0\end{aligned}$$

and

$$E\left(W|\theta, \sigma_{l,\omega}^2 = 0, \sigma_{l,\xi}^2 \to \infty\right) = -\left(\left(1 + \sum_{l=1}^2 \sum_{k=1}^2 \rho_l^k\right)^2 \sigma_\varepsilon^2 + \sum_{l=1}^2 \left(\rho_l^l + \rho_{3-l}^l\right)^2 \sigma_\phi^2\right)$$

If $\sigma_{l,\xi}^2 = 0$ for $l=1,2$ (full disclosure to the private sector), then

$$\begin{aligned}\kappa_0^0 &= r\frac{\sigma_\phi^2 + \frac{\sigma_\phi^2\left(\sigma_\phi^2 + 4\sigma_\varepsilon^2\right)}{5\sigma_\phi^2 + 4\sigma_\varepsilon^2}\sum_{l=1}^2\left(\rho_l^l + \rho_l^{3-l}\right)}{r\sigma_\phi^2 + 4\sigma_\varepsilon^2} \\ \kappa_k^0 &= \frac{\left(r + (1-r)\kappa_0\right)}{r}\frac{2\sigma_\varepsilon^2}{\sigma_\phi^2 + 4\sigma_\varepsilon^2} + \frac{2\left(\sigma_\varepsilon^2 + \sigma_\phi^2\right)}{5\sigma_\phi^2 + 4\sigma_\varepsilon^2}\sum_{l=1}^2\left(\rho_l^l + \rho_l^{3-l}\right)\end{aligned}$$

and

$$E\left(W|\theta, \sigma_{l,\omega}^2 = 0, \sigma_{l,\xi}^2 = 0\right) = -\left(\left(\kappa_0^0\right)^2 \sigma_\varepsilon^2 + \frac{1}{2}\sum_{l=1}^2\left(1 - \kappa_0^0 + \rho_{3-l}^{3-l} + \rho_l^{3-l} - \rho_l^l - \rho_{3-l}^l\right)^2 \sigma_\phi^2\right)$$

Assume that the policymakers follow an ex-ante quasi-symmetric policy, that satisfy the condition

$$\rho_1^1 + \rho_2^1 = \rho_2^2 + \rho_1^2 = s$$

for some constant $s \in \mathbb{R}$. Then we can simplify the expected welfare to

$$\begin{aligned}
E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} \to \infty, s\right) &= -\left(\left(1+2s\right)^2 \sigma_\varepsilon^2 + 2s^2 \sigma_\phi^2\right) \\
E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} = 0, s\right) &= -\gamma\left(r^2\left(5\sigma_\phi^4 + 4\sigma_\varepsilon^2\sigma_\phi^2 + 2\sigma_\phi^2\left(\sigma_\phi^2 + 4\sigma_\varepsilon^2\right)s\right)^2 \sigma_\varepsilon^2\right. \\
&\quad \left. + \left(20\sigma_\varepsilon^2\sigma_\phi^2 + 16\sigma_\varepsilon^4 - 2r\sigma_\phi^2\left(\sigma_\phi^2 + 4\sigma_\varepsilon^2\right)s\right)^2 \sigma_\phi^2\right) \\
\gamma &= \left(\frac{1}{\left(r\sigma_\phi^2 + 4\sigma_\varepsilon^2\right)\left(5\sigma_\phi^2 + 4\sigma_\varepsilon^2\right)}\right)^2
\end{aligned}$$

The following proposition presents the main result of this subsection.

Proposition 1. *Let $\sigma_\varepsilon^2 > 0$, $\sigma_\phi^2 > 0$ be known constants.*

If $15r\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^3 > 32 + 16\left(7+r\right)\frac{\sigma_\phi^2}{\sigma_\varepsilon^2} + 2\left(65+4r\right)\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^2 + \left(50+4r^2\right)\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^3$, then for all $s \in \mathbb{R}$,

$$E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} = 0, s\right) > E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} \to \infty, s\right)$$

If the condition holds with equality, then the conclusion holds for all $s \in \mathbb{R}\setminus\{s^*\}$, with $s^* < 0$.

If $15r\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^3 < 32 + 16\left(7+r\right)\frac{\sigma_\phi^2}{\sigma_\varepsilon^2} + 2\left(65+4r\right)\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^2 + \left(50+4r^2\right)\left(\frac{\sigma_\phi^2}{\sigma_\varepsilon^2}\right)^3$, then there exist constants $s_1 < 0$ and $s_2 > s_1$ such that

$$E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} = 0, s\right) > E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} \to \infty, s\right)$$

for all $s \in (-\infty, s_1) \bigcup (s_2, \infty)$.

Proposition 1 gives us conditions under which full disclosure is better than no disclosure at all.

A case of interest occurs when $s = 0$. This is the inactive policymaker from Morris and Shin [2002]. It can be shown that, under full information sharing between policymakers,

$$E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} = 0, s=0\right) > E\left(W|\theta, \sigma^2_{l,\omega} = 0, \sigma^2_{l,\xi} \to \infty, s=0\right) \iff \frac{\sigma_\phi^2}{\sigma_\varepsilon^2} < \frac{2}{2-r}$$

This is a similar result to the one obtained by Baeriswyl [2011] (eq 14).

4 Active policymakers

4.1 No information sharing among policymakers

Consider the benchmark case of no information sharing among policymakers. Then there is no information contained in the signal w_k and the policy rule takes the form $g_l = \rho_l^l z_l$. The public signal produced by the l-policymaker is $y_l = z_l + \xi_l$ (because $\beta_\theta^l = 1$, or $E_l(\theta) = z_l$) and expected welfare simplifies to

$$E(W_\infty|\theta) = -\left(\kappa_0^2\sigma_\varepsilon^2 + \sum_{l=1}^{2}\left(\kappa_l - \rho_l^l\right)^2\sigma_\phi^2 + \sum_{l=1}^{2}\kappa_l^2\sigma_{l,\xi}^2\right)$$

where the coefficients κ_k are evaluated when $\sigma_{l,\omega}^2 \to \infty$.

Given σ_ε^2, σ_ϕ^2 and $\sigma_{l,\xi}^2$ for $l = 1, 2$, the l-policymaker chooses ρ_l^l to maximize expected welfare. Let ρ_l^{l*} denote the optimal policy. We can define the value function $W_\infty^*\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_{1,\xi}^2, \sigma_{2,\xi}^2\right) = E(W_\infty|\theta)|_{\rho=\rho^*}$ and choose $\sigma_{1,\xi}^2, \sigma_{2,\xi}^2$ to maximize welfare.

We now consider the symmetric equilibrium. If $\sigma_{1,\xi}^2 = \sigma_{2,\xi}^2 = \sigma_\xi^2$, then the optimal policy will be symmetric, and $\rho_1^{1*} = \rho_2^{2*}$.

Let $W_\infty^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right) = W_\infty^*\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2, \sigma_\xi^2\right)$. We can maximize $W_\infty^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right)$ with respect to σ_ξ^2, and we get the following result for the optimal selection of the degree of disclosure, for symmetric policymakers.

Proposition 2. *Let $\sigma_\varepsilon^2 > 0$, $\sigma_\phi^2 > 0$ be known constants. The function $W_\infty^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right)$ satisfies*

$$W_\infty^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right) \leq \lim_{\sigma_\xi^2\to\infty} W_\infty^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right) = -\frac{\sigma_\varepsilon^2\sigma_\phi^2}{\sigma_\phi^2 + 2\sigma_\varepsilon^2}$$

and thus is maximized when $\sigma_\xi^2 \to \infty$.

If $r > \frac{1}{2}$, then

$$\sigma_\xi^{2*} = \frac{\sigma_\varepsilon^2 + 2(1-r)\sigma_\phi^2 + \sqrt{\sigma_\varepsilon^4 + 2(1-r)\sigma_\varepsilon^2\sigma_\phi^2 + 4r\sigma_\varepsilon^2\sigma_\phi^2(1-r) + 2\sigma_\phi^4(1-r)}}{2r-1}$$

achieves the maximum value $-\frac{\sigma_\varepsilon^2\sigma_\phi^2}{\sigma_\phi^2+2\sigma_\varepsilon^2}$. Also, σ_ξ^{2*} is decreasing in r, and increasing in σ_ε^2, σ_ϕ^2.

Proposition 2 shows that zero disclosure is always optimal. But when the strategic complementarity in the private sector optimal action is weak, $r > \frac{1}{2}$, an interior optimum arises. We have $2\sigma_\varepsilon^2 < \sigma_\xi^{2*} < \infty$, and thus partial informative signals can globally maximize expected welfare as well. This breaks James and Lawler [2011]'s result. Note however that full disclosure is never globally optimal.

This result is similar to the one in Morris and Shin [2002]: when private signals are not that informative, increasing the accuracy of public signals can be welfare improving. In this model, the active policymakers act as "private" agents, maximizing their own objectives given their information sets. Thus, when $\sigma_\omega^2 \to \infty$, the precision of the private information in the economy is reduced, and Morris and Shin [2002]'s kind of result kicks in.

The intuition is as follows: when the policy authorities produce public signals, the weight that the private agents assign to their own private signals decrease, due to the strategic complementarity. That is, private information is less relevant for decision making. On the other hand, the public signals help to transmit the information content of policymakers' private signals. This information transmission is complemented, for the active policymakers, with the optimal choice of the policy coefficients. When the strategic complementarity is weak, the welfare cost of increasing the precision of public signals is reduced, and an interior optimum arises.

If $\sigma_\omega^2 < \infty$, this result does not hold. The reason is that private information is more precise, and it is optimal for the policymakers to transmit this information through the choice of the policy, avoiding the costs associated with partially informative public signals. In that case, the mechanisms highlighted by James and Lawler [2011] dominate.

4.2 Full information sharing among policymakers

Consider the case in which the policymakers choose to cooperate with each other. We assume that the cooperation consists in complete information sharing. In this case $\sigma_{l,\omega}^2 = 0$ for $l = 1, 2$. The policy rule takes the form $g_l = \rho_l^l z_l + \rho_l^{3-l} z_{3-l}$. The public signal produced by the l-policymaker is $y_l = \frac{z_l + z_{3-l}}{2} + \xi_l$ (because $\beta_\theta^l = \frac{1}{2}$) and expected welfare simplifies to

$$E\left(W_0|\theta\right) = -\left(\kappa_0^2\sigma_\varepsilon^2 + \sum_{l=1}^{2}\left(\frac{1}{2}\kappa_l + \frac{1}{2}\kappa_{3-l} - \rho_l^l - \rho_{3-l}^l\right)^2\sigma_\phi^2 + \sum_{l=1}^{2}\kappa_l^2\sigma_{l,\xi}^2\right)$$

where the coefficients κ_k are evaluated when $\sigma_{l,\omega}^2 = 0$.

We now consider the symmetric equilibrium. If $\sigma_{1,\xi}^2 = \sigma_{2,\xi}^2 = \sigma_\xi^2$, then the optimal policy will be symmetric, and $\rho_1^{1*} = \rho_2^{2*}$ and $\rho_1^{2*} = \rho_2^{1*}$. Also, when $\sigma_{1,\omega}^2 = \sigma_{2,\omega}^2 = 0$ and $\sigma_{1,\xi}^2 = \sigma_{2,\xi}^2 = \sigma_\xi^2$ we have $\alpha_{z,l}^l = \alpha_{z,3-l}^l = \alpha_{w,l}^l = \alpha_{w,3-l}^l$. Imposing these constraints, it can be shown that the function $E\left(W_0|\theta\right)$ depends only on the sum $\rho_l^l + \rho_{3-l}^l$. The optimal policy in the symmetric case chooses the sum of the policy coefficients to maximize expected welfare. The value function for the symmetric equilibria is denoted by $W_0^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right)$.

Proposition 3. *Let $\sigma_\varepsilon^2 > 0$, $\sigma_\phi^2 > 0$ be known constants. The function $W_0^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right)$ satisfies*

$$W_0^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right) < \lim_{\sigma_\xi^2 \to \infty} W_0^s\left(\sigma_\varepsilon^2, \sigma_\phi^2, \sigma_\xi^2\right) = -r^2 \frac{\sigma_\varepsilon^2 \sigma_\phi^2}{r^2 \sigma_\phi^2 + 2\sigma_\varepsilon^2}$$

and thus is maximized when $\sigma_\xi^2 \to \infty$.

Proposition 3 shows that zero disclosure is always optimal, and it is the unique degree of disclosure that globally maximizes welfare.

Note that

$$-r^2 \frac{\sigma_\varepsilon^2 \sigma_\phi^2}{r^2 \sigma_\phi^2 + 2\sigma_\varepsilon^2} > -\frac{\sigma_\varepsilon^2 \sigma_\phi^2}{\sigma_\phi^2 + 2\sigma_\varepsilon^2}$$

and thus the full information sharing policy achieves a higher welfare level, with zero disclosure to the private sector, than the no information sharing policy. This, of course, is due to the fact that under information sharing the policymakers receive two informative private signals, instead of one.

James and Lawler [2011] show that the maximum welfare level with only one policymaker is $-\frac{\sigma_\varepsilon^2 \sigma_\phi^2}{\sigma_\phi^2 + 2\sigma_\varepsilon^2}$. Two policymakers are able to obtain a higher level of welfare. The reason is that adding up policymakers is adding up informative private signals in this simple model, increasing the precision of the information available to the policymakers. In particular, two policymakers playing a symmetric equilibrium achieve the same optimal expected welfare achieved by one policymaker facing one private signal with variance $\frac{\sigma_\phi^2}{2}$.

4.3 Dispersion

In the linear rational expectations equilibrium we have $a_i = \kappa_0 x_i + \sum_{l=1}^2 \kappa_l y_l$. After substituting for x_i, y_1 and y_2, we can compute the sample mean $\overline{a} = \int_0^1 a_i di$. The sample dispersion of actions is given by:

$$\begin{aligned} \int_0^1 \left(a_i - \overline{a}\right)^2 di &= \kappa_0^2 \int_0^1 \varepsilon_i^2 di \\ &= \kappa_0^2 \sigma_\varepsilon^2 \end{aligned}$$

where we used the appropriate law of large numbers. It can be shown as well that the sample dispersion of private sector's expectations about the average action is

$$\int_0^1 \left(E_i\left(\overline{a}\right) - \int_0^1 E_i\left(\overline{a}\right) di\right)^2 di = \kappa_0^2 \sigma_\varepsilon^2$$

Let $\kappa_0^{\sigma_\omega^2}$ denote the parameter κ_0 evaluated at the optimal symmetric policy ρ^*, under variance σ_ω^2.

When $\sigma_\omega^2 \to \infty$, we have that

$$\kappa_0^\infty = \frac{\sigma_\phi^2}{\sigma_\phi^2 + 2\sigma_\varepsilon^2}$$

When $\sigma_\omega^2 = 0$, we have that

$$\kappa_0^0 = \frac{r^2\sigma_\phi^2}{r^2\sigma_\phi^2 + 2\sigma_\varepsilon^2}$$

Note that for $0 < r < 1$,

$$\kappa_0^0 < \kappa_0^\infty$$

and thus the sample dispersion of equilibrium actions (and expectations about the average action), under optimal policy and disclosure, is lower when the policymakers share private information than when they don't. That is, more cooperation between active policymakers reduces the dispersion of actions and expectations in the economy.

4.4 Nonmonotonic behavior of expected welfare

Expected welfare is a highly nonlinear function of $\sigma_{l,\omega}^2$, $\sigma_{l,\xi}^2$, $l = 1, 2$. We analyze the shape of this function for the symmetric case using two numerical examples, to illustrate the nonmonotonicity of expected welfare with respect to the precision of the signals produced by the policymakers.

We set $\sigma_\varepsilon^2 = 1$ and $\sigma_\phi^2 = 0.75$. Thus policymakers receive more precise signals. The results, however, do not depend on this assumption.

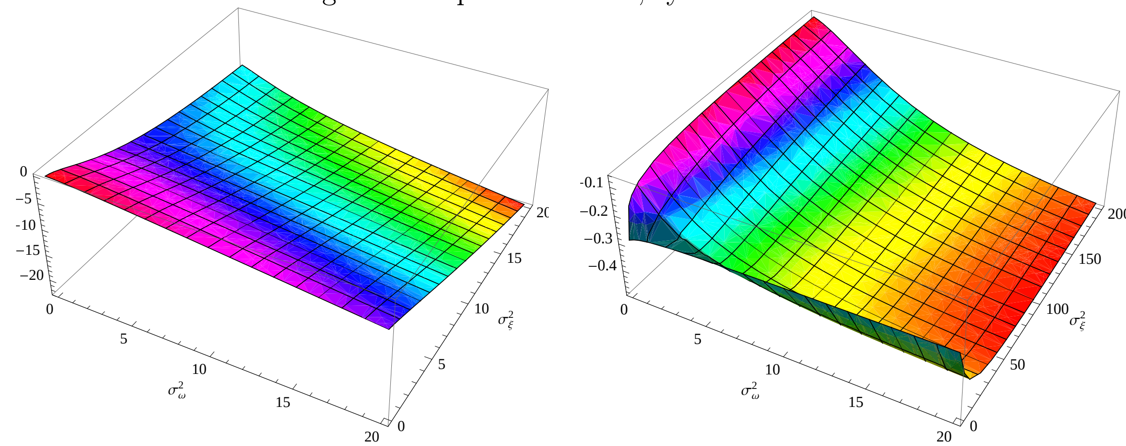
Figure 1 shows the expected welfare when $r = 0.4$.

The left panel presents expected welfare when the policymakers are passive. In particular, we set $\rho_l^l = -0.5$ and $\rho_{3-l}^l = -0.4$. Thus $s = \rho_l^l + \rho_{3-l}^l = -0.9$. We apply Proposition 1: for these parameter values, we have that $s < s_1$ and thus full disclosure is better than no disclosure, when $\sigma_\omega^2 = 0$. But expected welfare is a decreasing function of σ_ω^2, and therefore full information sharing and full disclosure globally maximize expected welfare.

The right panel of Figure 1 presents expected welfare when the policy is optimal. We see that increasing information sharing between policymakers (decreasing σ_ω^2) always improves welfare, because it is an improvement on the quality of private signals. More importantly, we note that when the information sharing is low (large σ_ω^2), then a reduction in σ_ξ^2 (more disclosure to the private sector) can be welfare improving, even when the policymakers set policy optimally. This breaks the con-transparency result of James and Lawler [2011].

Figure 2 shows the expected welfare for symmetric policymakers when $r = 0.6$ and $\sigma_\omega^2 \to \infty$. This Figure illustrates Proposition 2: expected welfare is globally maximized at $\sigma_\xi^{2*} = 16.3217$. For high levels of disclosure (small σ_ξ^2), more disclosure can locally improve the expected welfare. The right panel shows the function in more detail. There is a local minimum, and at the right of it the function is always increasing. As $\sigma_\xi^2 \to \infty$ we also achieve the maximum expected welfare.

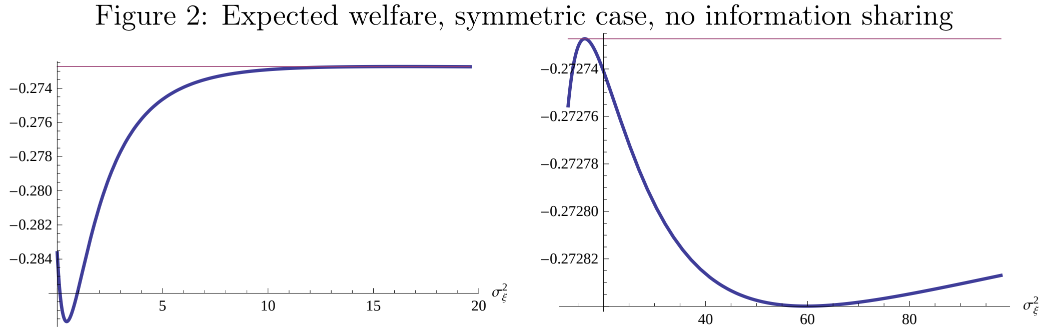
Figure 1: Expected welfare, symmetric case



Passive policy: $\rho_l^l = -0.5$, $\rho_{3-l}^l = -0.4$ Optimal policy

Parameters: $r = 0.4$, $\sigma_\varepsilon^2 = 1$, $\sigma_\phi^2 = 0.75$.

Figure 2: Expected welfare, symmetric case, no information sharing



Parameters: $r = 0.6$, $\sigma_\varepsilon^2 = 1$, $\sigma_\phi^2 = 0.75$.

We summarize the results in the following remark.

Remark 1. Expected welfare is increasing in the precision of the signals shared among policymakers. When the information sharing is low, more disclosure to the private sector can be welfare improving.

5 Conclusions

We show that when two policymakers cannot share information among them, and are choosing optimally a symmetric policy, then publication of partial informative signals can globally maximize expected welfare, when strategic complementarity is weak in the private sector.

We find that improving the quality of the signals transmitted to the private sector can increase the expected value of welfare, specially when the quality of the information received by the policymakers is low. In particular, when the information sharing among policymakers is low, more disclosure to the private sector can be welfare improving.

Finally, more information sharing is welfare improving, and more cooperation between active policymakers reduces the dispersion of actions and expectations in the economy.

These results weaken the con-transparency result of James and Lawler [2011], when there are two active policymakers. As most economies in the world have two or more policymakers, this analysis is relevant and of practical importance. The policy implications are clear: more information sharing between the policy authorities is welfare improving; and more transparency can be welfare improving, even if the policymakers play a Nash equilibrium in a noncooperative game. Political and social constraints might make impossible to play the no disclosure equlibrium that globally maximizes expected welfare. Thus, authorities subject to exogenous constraints could improve expected welfare choosing a local maximum. This makes a case for optimal partial transparency.

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