

Designing an expert
knowledge-based Systemic
Importance Index for financial
institutions

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Designing an expert knowledge-based Systemic Importance Index for financial institutions[†]

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Abstract

Defining whether a financial institution is systemically important (or not) is challenging due to (i) the inevitability of combining complex importance criteria such as institutions' *size*, *connectedness* and *substitutability*; (ii) the ambiguity of what an appropriate threshold for those criteria may be; and (iii) the involvement of expert knowledge as a key input for combining those criteria.

The proposed method, a *Fuzzy Logic Inference System*, uses four key systemic importance indicators that capture institutions' *size*, *connectedness* and *substitutability*, and a convenient deconstruction of expert knowledge to obtain a Systemic Importance Index.

This method allows for combining dissimilar concepts in a non-linear, consistent and intuitive manner, whilst considering them as continuous –non binary- functions. Results reveal that the method imitates the way experts themselves think about the decision process regarding what a systemically important financial institution is within the financial system under analysis.

The Index is a comprehensive relative assessment of each financial institution's systemic importance. It may serve financial authorities as a quantitative tool for focusing their attention and resources where the severity resulting from an institution failing or near-failing is estimated to be the greatest. It may also serve for enhanced policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

Keywords: Systemic Importance, Systemic Risk, *Fuzzy Logic*, Approximate Reasoning, Too-connected-to-fail, Too-big-to-fail.

JEL Classification: D85, C63, E58, G28.

[†] The opinions and statements are the sole responsibility of the authors and do not necessarily represent neither those of Banco de la República nor of its Board of Directors. Results are illustrative; they may not be used to infer credit quality or to make any type of assessment for any financial institution. Authors are grateful to the officers and technical staff involved in the design of the expert knowledge base. Discussions with Alejandro Reveiz regarding the design of the model were essential for its implementation. Valuable comments and suggestions were provided by Joaquín Bernal, Ernesto Aguirre, Dairo Estrada, Orlando Chipatecua, Freddy Cepeda, Fabio Ortega and Constanza Martínez. Balance sheet data was processed with assistance from Jorge Cely and Linda Mondragón.

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Introduction

Defining whether a financial institution³ is systemically important or not may be decidedly intricate but key to the oversight, supervision and regulation of the financial system. To be able to identify systemic importance may serve the purpose of assisting financial authorities in focusing their attention and resources –the intensity of oversight, supervision and regulation- where the systemic severity resulting from a financial institution failing or near-failing is estimated to be the greatest. Identifying systemically important institutions may also serve financial authorities for enhanced policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

As a consequence of the global financial crisis that began in August 2007 the International Monetary Fund (IMF), the Bank for International Settlements (BIS) and the Financial Stability Board (FSB) developed a set of guidelines and recommendations on how national authorities can assess the systemic importance of financial institutions, markets or instruments. Among the recommendations and concerns of the document (hereafter referred as IMF et al., 2009) it is worthwhile emphasizing the following:

- Three key criteria that are helpful in assessing and identifying the systemic importance of financial institutions are: *size*, *connectedness* and *substitutability*.⁴
- A high degree of judgment founded in a detailed knowledge of the functioning of the financial system is required in any assessment of systemic importance.
- Assessing the systemic importance of an institution does not lend itself to binary outcomes.
- The assessment of systemic importance cannot be based simply on quantitative methods.
- From an operational point of view a qualitative framework could be used to integrate the different components of the assessment and help arrive at judgments of systemic importance.

Authors recognize that Engineering has faced similar challenges when designing practical solutions to complex multifactor and non-linear systems where human reasoning, expert knowledge and imprecise information are valuable inputs. One of the solutions provided by Engineering is the design and implementation of a *Fuzzy Logic Inference System (FLIS)*.

³ For this document the authors embrace the term “financial institution” as comprising depository institutions (e.g. banks or savings associations), brokers, dealers, investment companies (e.g. mutual funds), insurance companies, investment advisers and credit unions; this is, those that may not be regarded as a “financial market utility”, where the latter is defined as in the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act: “any person that manages or operates a multilateral system for the purpose of transferring, clearing or settling payments, securities, or other financial transactions among financial institutions or between financial institutions and the person”.

⁴ A consultative document by the Basel Committee on Banking Supervision (BCBS-BIS, 2011) suggests adding two criteria (i.e. cross-jurisdictional activity and complexity) in order to attain banks’ global systemically importance and the difficulty of resolving a systemic event. Because the herein document focuses on non-global banking and non-banking institutions’ systemic importance, and since derivatives and other complex instruments are rather scarce in the Colombian market, the criteria is limited to *size*, *connectedness* and *substitutability*, as originally suggested by IMF, BIS and FSB (IMF et al., 2009). However, as discussed below, the methodology would be able to consider these two or other criteria.

Fuzzy Logic has been extensively employed in the real world, mostly in an Engineering context, to control systems where the timing and level of inputs are at least to some extent uncertain (Cruz, 2002). Some of its most important applications include NASA's software design for safe and reliable autonomous landing of spacecrafts and rover navigation (Serrano and Seraji, 2007; Howard and Seraji, 2002 and 2000; Tunstel et al., 2001; Seraji, H., 2000), along with everyday applications to medicine, automotive industry, water treatment, air and ground traffic control, and home appliances design (Sivanandam et al., 2007; von Altrok, 2002 and 1996; Klir and Yuan, 1995; McNeill and Thro, 1994). Its application to Finance and Economics is related to insurance, credit card fraud detection, credit risk analysis, bond ratings and operational risk (Revez and León, 2010; Bojadziev and Bojadziev, 2007; Bundesbank, 1999; McNeill and Thro, 1994).

This paper presents the authors' proposal to address the previously mentioned recommendations and issues based on the design of a FLIS as in Revez and León (2010). The key criteria or inputs of the model are [A] volume of deposits and money market borrowing; [B] volume of financial assets under management; [C] contribution to the large-value payment system; and [D] *betweenness centrality*, which altogether aim to capture *size*, *connectedness* and *substitutability* for banking and non-banking financial institutions, as further described below.⁵ According to *fuzzy sets theory*, each input is modeled as consisting of a set of continuous and overlapping functions identifiable by linguistic terms (i.e. LOW, MEDIUM and HIGH).

The *expert fuzzy system*, which is entitled the task of mimicking expert reasoning capabilities, is designed by deconstructing financial authorities' officers and technical staff knowledge about what a systemically important financial institution is. This deconstruction is the result of a survey conducted within the central bank's officers and technical staff about how the chosen inputs may interact in the Colombian financial market as joint indicators of systemic importance. According to *fuzzy sets theory* and based on the *expert fuzzy system*, each combination of the inputs is modeled as resulting in an output set consisting of continuous and overlapping functions of systemic importance, identifiable by linguistic terms (i.e. VERY LOW, LOW, MEDIUM-LOW, MEDIUM, MEDIUM-HIGH, HIGH, VERY HIGH).

Finally, each institution's Systemic Importance Index results from simultaneously evaluating each institution's unique combination of inputs according to the expert knowledge of Colombia's central bank's officers and technical staff, and mapping the outputs' linguistic terms in an ordinal scale or index. This approach is analogous to NASA's research on designing a *Fuzzy Logic*-based *traversability index* for safe and reliable autonomous rover navigation (Howard et al., 2001; Seraji, 2000), where the inputs are the terrain's *slope*, *roughness* and *hardness*, and of a *Fuzzy Logic*-based *landing site quality index* for spacecrafts' autonomous landing on planetary surfaces (Serrano and Seraji, 2007; Howard and Seraji, 2000), where the inputs are the terrain's safety (i.e. *slope* and *roughness*), *fuel consumption* and *scientific return*.

The document is structured as follows: next section briefly introduces the systemic importance concept, focusing on the recommendations and concerns provided by IMF et al. (2009). Second section introduces *Fuzzy Logic* basics. Third section addresses the design of an expert knowledge-based Systemic Importance Index for financial institutions within *Fuzzy Logic* theory, and presents some aggregated results for the

⁵ Please note that the inputs capture the current developments regarding the need to complement the *too-big-to-fail* criteria with the *too-connected-to-fail* criteria, as in León et al. (2011), Chan-Lau (2010), Machado et al. (2010), ECB (2010), Clark (2010). As will be further discussed the choice of key indicators allow for applying the model to capture the importance of –otherwise ignored– non-banking institutions (i.e. the “shadow banking system”).

Colombian case. Based on the herein proposed model, the fourth section exhibits the systemic importance assessment for Colombian financial institutions participating of the large-value payment system (CUD) as of May 2011.⁶ The fifth section presents some final remarks. Exhibits provide further information about some of the methodological approaches herein implemented.

1 Systemic risk and systemic importance

As presented by IMF et al. (2009), G20 countries embrace the following –general– definition of systemic risk: the risk of disruption of financial services that (i) is caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy. Regarding payment systems, the Committee on Payment and Settlement Systems (CPSS-BIS, 2011) define it as the risk that the inability of a financial institution to meet its obligations could result in the inability of other system participants or of financial institutions in other parts of the financial system to meet their obligations as they become due.

Irrespective of which of these definitions is embraced, and despite there is no single definition of risk that can be completely satisfactory in every situation (Dowd, 2005)⁷, it is common to think of risk as a function based on two parameters: frequency and severity (Condamin et al., 2006), also referred as likelihood and impact, respectively (Gallati, 2003). Although academic effort has traditionally focused on systemic concerns based on the estimation of systemic risk (i.e. the product of frequency and impact, as in Norman et al. (2009)), there is a recent interest in focusing on systemic severity or importance.⁸

For example, Paul Tucker, Executive Director for Markets and member of the Monetary Policy Committee of the Bank of England, pointed out the following (Tucker, 2005):

[T]he interesting question is not whether or not risk will crystallize, as in one form or another risks crystallize every day. Rather, the important question is whether, in the event of nasty shocks, our capital markets can absorb them or whether they have developed characteristics which may, as some suggest, leave them vulnerable.

More recently, as a result of the most recent episode of global financial crisis, the consultative report “Principles for Financial Market Infrastructures” published by the Bank of International Settlements’ Committee on Payment and Settlement Systems (CPSS-BIS, 2011) includes several principles that aim to provide a high degree of confidence that a financial market infrastructure will continue operating and serve as a source of financial stability even in extreme market conditions. Principles 4 and 7 emphasize the importance of focusing on the severity of the systemic shocks; the latter addresses liquidity risk for financial market infrastructures:

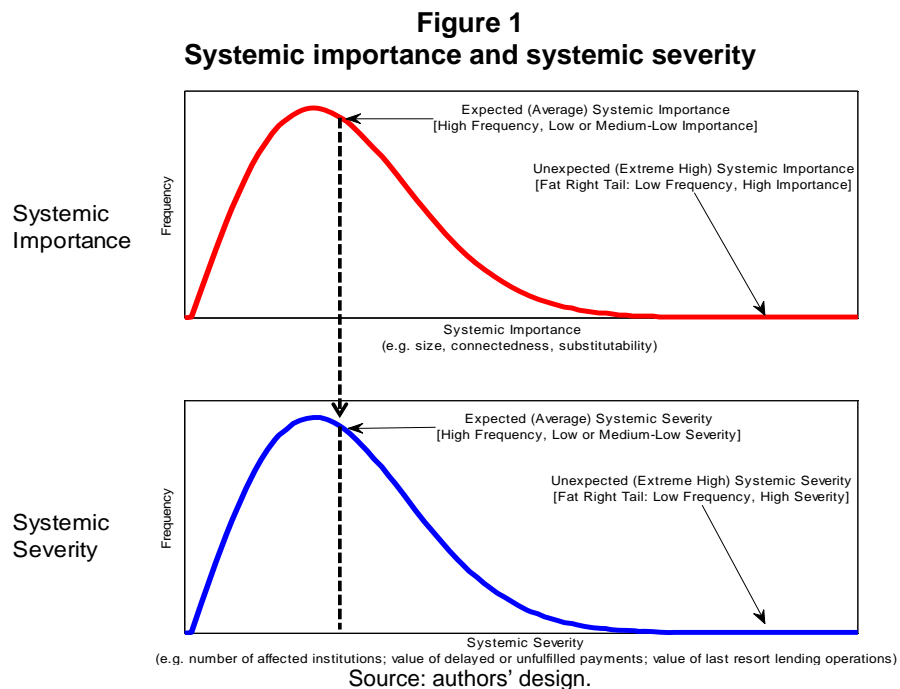
⁶ Colombia’s large-value payment system (CUD) is a direct participation system where any financial institution can maintain deposits and conduct transactions with other participants without the need for an agent or intermediary. As of May 2011 participating financial institutions surpassed 145.

⁷ A proper definition of risk is beyond the aim of this paper. Interesting reviews of risk definitions and their implications can be found in Hubbard (2009).

⁸ Some authors (Rebonato, 2007; Taleb, 2007) argue that models and techniques for estimating very low probabilities of very disastrous occurrences have demonstrated to yield poor results, and even question the usefulness of those models and techniques for capturing extreme adverse events not found in historical data. Rebonato (2007) also questions the convenience of regulators using VaR-type approaches (i.e. based on estimating low probabilities) to determine prudential capital since even a high percentile (e.g. 99%) would allow a firm to incur losses equal to its regulatory capital rather often (i.e. 2-3 times a year); not to mention if extreme losses cluster, as they do.

Principle 7 (Liquidity Risk): A financial market infrastructure should maintain sufficient liquid resources to effect same-day and, where appropriate, intraday settlement of payment obligations with a high degree of confidence under a wide range of potential stress scenarios that should include, but not be limited to, the default of [one/two] participant[s] and [its/their] affiliates that would generate the largest aggregate liquidity need in extreme but plausible market conditions.

Such increasing interest in the impact of systemic shocks –beyond the interest in their frequency- results from the intrinsic characteristics of the financial and payments systems. As pointed out by Haldane (2009) and León et al. (2011), financial and payments networks nowadays may be described as robust to random disturbances, but highly susceptible to targeted attacks.⁹ This results from the systemic importance of financial institutions (e.g. *size, connectedness, substitutability*) being distributed with a high degree of asymmetry (right skew) and excess kurtosis, where the average institution is of low systemic importance (Figure 1, upper panel) and the average default or failure-to-pay results in low systemic severity (Figure 1, lower panel); correspondingly, systemically important institutions and their consequent high systemic severities lurk in the extreme right tail of the distributions.



This means that traditional focus on estimating risk as the sum of multiplying each participant's estimated frequency of failure (or near failure) times its corresponding estimated impact may be dangerously diverting financial authorities from its aim of ever preserving financial stability and payment systems safety: on average the financial stability and payments system safety may be "guaranteed", but not when confronted with a systemically important participant failing. This is, focusing on estimating probabilities of systemic events happening financial authorities would be preparing

⁹ As mentioned by Haldane (2009), this explains why there exist long periods of apparent robustness, where peripheral –not systemically important- nodes are subject to random shocks, and short but severe episodes of systemic distress, where systemically important institutions endanger financial stability. Therefore Haldane's characterization of the current international financial network: "robust-yet-fragile".

themselves (i.e. overseeing, supervising, regulating) for a severe systemic shock based on the impact of a single –systemically modest- average institution.

Moreover, estimating systemic risk as the sum of multiplying each participant's estimated frequency of failure (or near failure) times its corresponding estimated impact assumes that failures or near failures by different participants do not come together (i.e. they are independent).¹⁰ As recently exhibited by the concurrent episodes of distress of AIG, Lehman and Bear Sterns, such independence is by no means guaranteed.

Therefore, as financial authorities should be prepared to confront a non-average but extreme threat to financial stability or payment systems' safety, the supervision, oversight and regulation should be designed to cope with one (or even two) systemically important institution(s) failing or near failing, as suggested by CPSS-BIS (2011) when formulating Principles 4 and 7 for measuring, monitoring and managing credit and liquidity risks for financial infrastructures. In this sense financial authorities' prudential supervision, regulation and oversight (i.e. policy-making) and decision-making rely on defining what systemic importance is, and identifying institutions that comply with such definition.

According to IMF et al. (2009), G-20 members state that an institution may be considered as systemically important if its failure or malfunction causes widespread distress, either as a direct or indirect impact (i.e. contagion), where the main criterion for assessing systemic importance relates to their potential to have a large negative impact on the financial system and the real economy. This overall criterion may be conveniently explained by three more concise criteria: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009).

1.1 Size

Some authors regard an institution as systemically important when exceeding an asset-*size* cutoff (Saunders et al., 2009), whilst others (IMF et al., 2009) prefer to gauge the amount of financial services it provides to the system. This is the traditional approach to systemic risk, where the systemic importance of a financial institution generally increases with its *size*, where systemically important institutions are labeled as *too-big-to-fail*.

When considering the amount of financial services provided to the system as the metric for *size* some intuitive and straightforward key indicators emerge. Because they belong to the traditional institution-centric approach to micro-prudential supervision, standard accounting data already contains relevant information, such as balance and off-balance sheet exposures (e.g. deposits, money market borrowing and lending) and volume of assets it warehouses or manages, etc.. Other relevant *size* indicators such as the volume of payments by individual institutions are not publicly disclosed, but are available for financial authorities via their involvement in large-value payment systems or via their oversight and supervision duties.

¹⁰ When estimating market risk this inconvenience is absent: it is impossible that two (or more) scenarios crystallize; there is a unique outcome (i.e. if return resulted to be 1.2% all other realizations are impossible), thus assuming independence of each realization is appropriate. For estimating systemic or credit risk, where simultaneous occurrence of outcomes is feasible (e.g. several firms may enter into default simultaneously or within a short period), this assumption may be inappropriate.

1.2 *Connectedness*

According to the European Central Bank (2010) the properties and behavior of an institution may be affected by institutions that have links to it, and also by other institutions that have no direct links, but are linked to its neighbors. Therefore, the larger the number –and volume- of the links an institution maintains with other market participants, the larger the contagion or spillovers it may generate; this is, the systemic importance of a financial institution generally increases with its degree of *connectedness*. Despite its intuitiveness this is a rather novel approach to systemic risk, where systemically important institutions are labeled as *too-connected-to-fail* (León et al., 2011; Machado et al. 2010; Chan-Lau, 2010; ECB, 2010; Clark, 2010; Zhou, 2009).

Unlike financial institutions' *size*, *connectedness* may be intricate to assess, with regulators and central banks currently lacking the resources to carry out this kind of analysis (Clark, 2010). Network theory¹¹ provides some concepts and metrics that may assist the assessment of *connectedness*. The most simple concept is the *in-degree* and *out-degree centrality*, which refer to the number of “neighbors” or “partners” an institution has within the network, where the former (latter) corresponds to incoming (outgoing) flows.

Traditional application of network theory for assessing systemic risk relies on balance sheet data such as interbank funding and lending, as in Garrat et al. (2011) or Chan-Lau (2010). Alternatively León et al. (2011) and Machado et al. (2010) use large-value payment system's databases. The choice of *connectedness* metric and of data source (i.e. balance sheet or large-value payment system) will be addressed in the third section.

1.3 *Substitutability*

If the absence of a financial institution distorts the system because it is difficult (or impossible) to find another institution able to provide the same (or similar) type and volume of financial services (e.g. settlement, payments, interbank lending, custody, brokerage), such institution is systemically important. As pointed out by Manning et al. (2009), the severity of the impact of a payment system failure, and hence the extent of systemic risk, depends critically on whether substitutes are readily available to allow payment flows to be rerouted via another system. Consequently, the systemic importance of a financial institution generally decreases with its degree of *substitutability*, where *connectedness* and *substitutability* are both related to the *too-connected-to-fail* criteria.

Unlike financial institutions' *size*, the degree of *substitutability* may be intricate to assess. Despite there are cases in which it is easy to determine that a participant or infrastructure is *non-substitutable* (e.g. if there is a sole infrastructure in charge of all

¹¹ Network theory (also referred as network topology or analysis) is a method used in Statistical Physics to understand and analyze the structure and functioning of complex networks. As acknowledged in authors' prior works (León et al., 2011; Machado et al., 2010), network theory provides appealing methods and techniques to cope with the need to change from an institution-centric to a systemic approach. The studies by Soramäki et al. (2006) and Bech and Garrat (2006) use network theory to characterize the United States (Fedwire) payment system, while Iannotta et al. (2004) apply it to the Japan case (BoJ-Net). Cepeda (2008) applies network theory to the Colombian large-value payment system (CUD) to quantify the impact of failures on its stability.

the market's clearing), it may be cumbersome to determine other participants' degree of *non-substitutability*.

For these cases network theory provides some concepts and metrics that may assist the assessment of *substitutability*. An interesting concept is *betweenness centrality* (Newman, 2010 and 2003; Buechel and Buskens, 2008; de Nooy et al., 2005), which is a measure of a network's resilience based on the assessment of the involvement of a participant in the –indirect- connection of all other participants.

As with *connectedness*, network theory for assessing *substitutability* could rely on data gathered from institutions' balance sheets (e.g. interbank funding and lending) or from large-value payment systems. The choice of *substitutability* metric and of data source (i.e. balance sheet or large-value payment system) will be addressed in the third section.

2 Designing an expert knowledge-based index with *Fuzzy Logic*¹²

The fundamental concept of *ordinary sets* is “membership”, which states that an element belongs or not to a set. This type of sets, described by an unambiguous definition and boundaries, is known as *ordinary* or *crisp sets*; these sets are characterized by discrete–bivariate membership (*yes* or *no*, 1 or 0, *true* or *false*) and classic, Boolean or Aristotelic logic.

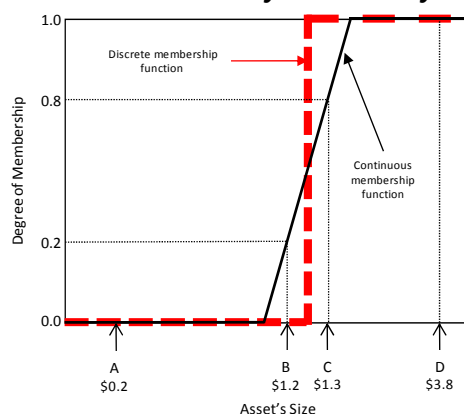
In contrast to *ordinary sets*, Lofti A. Zadeh (1965) acknowledged the fact that in reality there are elements characterized by *membership functions* which are not discrete, but continuous, where different degrees of membership exist between *yes* or *no*, 1 or 0, *true* or *false*; this type of sets have unclear boundaries, therefore Zadeh named them as *fuzzy sets*. As stated by Sivanandam et al. (2007), the main contribution of the *fuzzy set* concept is the ability to model uncertain and ambiguous information, the kind of information frequently found in real life.

For the case in hand it would be difficult to label a financial institution as *too-big-to-fail* based on a unique threshold related to the *size* of its assets, as suggested by Saunders et al. (2009). Figure 2 compares using a discrete *membership function* – typical of ordinary sets- with a *fuzzy sets'* continuous *membership function*, where the criteria is financial institutions' assets' *size*¹³.

¹² This chapter is based extensively on Reveiz and León (2010), where Fuzzy Logic theory and the design of a Fuzzy Logic Inference System are briefly explained. Several references were omitted for practical reasons. The familiar reader may skip this section.

¹³ A *membership function* is the line which defines the transition between sets, thus mapping the degree of membership of the elements of such sets. A continuous *membership function*, typical of *fuzzy sets*, recognizes that elements may belong to different categories in some degree, with this degree varying in a smooth and continuous manner.

Figure 2
Assets' size as an ordinary and a fuzzy variable



Source: authors design

The discrete *membership function* (dashed line) may yield non-intuitive and impractical results, which could seriously misguide financial authorities' analysis and decision making: (i) despite being clearly different, institution A and B are both regarded as non-systemically important; (ii) institution C's *size*, despite not being significantly different from B's, is considered as resulting in a *too-big-to-fail* institution; (iii) notwithstanding institution D is significantly bigger than C, they are both regarded as equally important because of their *size*.

In the other hand, a simple continuous *membership function* that replicates the same *too-big-to-fail* approach to systemic risk yields intuitive and practical results. Institution A is regarded as non-systemically important, with a null degree membership to the *size* criteria; institution B and C are systemically important to some extent, where B's degree of membership to the *too-big-to-fail* criteria (20%) is lower than C's (80%); and where D's *size* corresponds unequivocally (100%) to a systemically important institution due to its *size*.

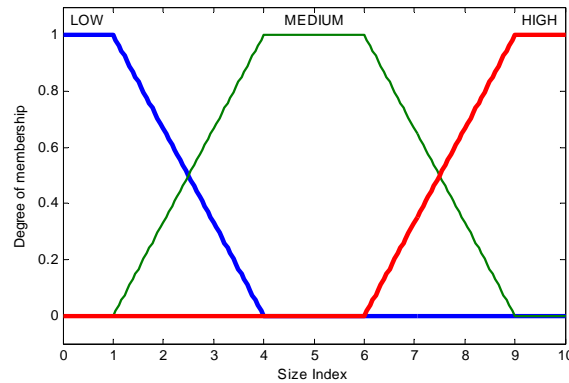
Hence, it can be seen that the membership of the elements to the *size* set is not clearly bounded, is a matter of degree, therefore it is better described by a *fuzzy set*; it is straightforward to apply the same rationale to other criteria, such as *connectedness* and *substitutability*. This is rather important since, as acknowledged by IMF et al. (2009), assessing the systemic importance of a financial institution does not lend to binary outcomes.

Additionally, in order to attain greater generality, higher expressive power, an enhanced ability to model real-world problems and, most importantly, a methodology for exploiting the tolerance for imprecision (Klir and Yuan, 1995), it is possible to use a mixture of continuous *membership functions* to further characterize the degree of membership.¹⁴ This is presented in Figure 3: three trapezoidal *membership functions* are used to evaluate the degree of membership with three categories for *size* (i.e. LOW, MEDIUM, HIGH) instead of using a single category (i.e. big), where the nominal value of *size* (e.g. US billion) is replaced by a *size index* from 0 to 10.¹⁵

¹⁴ The choice of the *membership function* is somewhat arbitrary but should be done with simplicity, convenience, speed and efficiency in view (Mathworks, 2009). Cox (1994) emphasize that special attention should be drawn to the overlapping between *membership functions*: the overlapping is a natural result of fuzziness and ambiguity associated with the segmentation and classification of a continuous space.

¹⁵ The *size index* consists of a typical standardization of the nominal values of assets' *size* for each institution considered; the biggest institution is assigned the maximum index value (10 in this case) and the

Figure 3
Assets' size as a fuzzy variable (input)



Source: authors' design

This procedure, consisting of the conversion of a crisp quantity to the appropriate *fuzzy sets* through the use of *membership functions*, is known as *fuzzification* (Sivanandam et al., 2007; Klir and Yuan, 1995; McNeill and Thro, 1994). An important property of this procedure is that *Fuzzy Logic* models are rarely sensitive to the choice of *membership function* (Cox, 1994), making them quite robust, which is an important property when models are initially prototyped.

Concerning the logic used to evaluate propositions, *ordinary sets* rely on *ordinary logic*; this type of logic, also known as classical, Aristotelic or Boolean logic, conceives the universe in terms of well-structured categories, where an item is either a member of a set or not. Using the logical operators AND, OR and NOT, which correspond to conjunction, disjunction and complement, respectively, propositions are evaluated as follows:

Figure 4
Ordinary logical operators

Conjunction			Disjunction			Complement	
A	B	A AND B	A	B	A OR B	A	NOT A
0	0	0	0	0	0	0	1
0	1	0	0	1	1	1	0
1	0	0	1	0	1		
1	1	1	1	1	1		

Source: León and Reveiz (2010), based on Mathworks (2009).

Ordinary sets can be regarded as a particular case of *fuzzy sets*, in which degrees of membership are restricted to two extreme alternatives (0 or 1). Due to this fact the choice of the *Fuzzy Logical operators* should be able to preserve the *ordinary logical operators* for bivariate memberships –as in Figure 4- and be capable of evaluating multivariate degrees of membership. This is conveniently and typically attained by using $\min(\cdot)$ instead of AND for conjunction, $\max(\cdot)$ instead of OR for disjunction and $1 - (\cdot)$ instead of NOT for complement.

The existence of these *Fuzzy Logical operators* allows for developing and evaluating *fuzzy inference rules*, which are rules for deriving truths from stated or proven truths (McNeill and Thro, 1994). The set of *fuzzy inference rules* or *knowledge base* that

rest is assigned an index value by means of linear interpolation. Such standardization is straightforward and makes comparisons and calculations easier. It is important to emphasize that the assessment obtained with the Index is not absolute, but relative to the most systemically important institution.

contains general knowledge pertaining to a problem domain connects antecedents with consequences, premises with conclusions, or conditions with actions (Klir and Yuan, 1995). If *A* and *B* are fuzzy sets, the simplest form of a *fuzzy inference rule* is the following:

if A, then B

For the case in hand, with the three criteria previously considered, the rules may look like the following:

*If SIZE is [LOW / MEDIUM / HIGH] AND ...
 SUBSTITUTABILITY is [LOW / MEDIUM / HIGH] AND ...
 CONNECTEDNESS is [LOW / MEDIUM / HIGH],
 THEN SYSTEMIC IMPORTANCE IS...
 [VERY LOW / LOW / MEDIUM LOW / MEDIUM / MEDIUM HIGH / HIGH / VERY HIGH]*

Inference rules result from expert knowledge and try to imitate human's reasoning capabilities. Cox (1994) claims that the process of building a *knowledge base* via the design of *fuzzy inference rules* forces experts to deconstruct their expertise into fragments of knowledge, which results in a significant benefit of fuzzy system modeling: to gain the ability to encode knowledge directly in a form that is very close to the way experts themselves think about the decision process¹⁶; this is commonly referred as "approximate reasoning" (Serrano and Seraji, 2007).

As stressed by Sivanandam et al. (2007), the Achilles' heel of a fuzzy system is its rules; smart rules give smart systems and other rules give less smart or even dumb systems. Bojadziev and Bojadziev (2007) emphasize the important role played by the experience and knowledge of human experts when developing the *knowledge base* because they are appointed to state the objective of the system to be controlled.

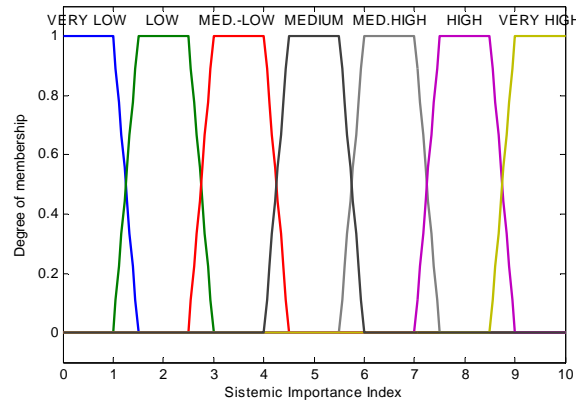
The evaluation of the *inference rules* is carried out by a *fuzzy inference processing engine*, which is based on the *Fuzzy Logical operators* previously introduced. The *fuzzy inference processing engine* is in charge of evaluating input's degree of membership to the fuzzy output sets (Figure 5) according to all the *inference rules*, where such evaluation is done simultaneously.¹⁷ As exhibited in Figure 5 the fuzzy output set consists of a mixture of seven trapezoidal *membership functions* for systemic importance [VERY LOW / LOW / MEDIUM LOW / MEDIUM / MEDIUM HIGH / HIGH / VERY HIGH].¹⁸

¹⁶ Cox (1994) emphasizes that conventional expert and decision systems fail because they force experts to crisply dichotomize rules, resulting in an unnecessary multiplication of rules and the inability to articulate solutions to complex problems.

¹⁷ According to Cox (1994) the main difference between conventional expert systems and a *fuzzy expert system* is the latter's simultaneous evaluation of inference rules, which compared to the serial evaluation of the former has the advantage of being able to examine all the rules and their impact in the output space.

¹⁸ The choice of the inputs' and output's number of *membership functions* follows two criteria. First, the number of *membership functions* should allow a detailed characterization and differentiation of what a systemically important institution is. Second, the number of membership functions should be limited in order to avoid unnecessary complexity for the model, and to facilitate deconstructing experts' knowledge.

Figure 5
Systemic importance as a fuzzy variable (output)

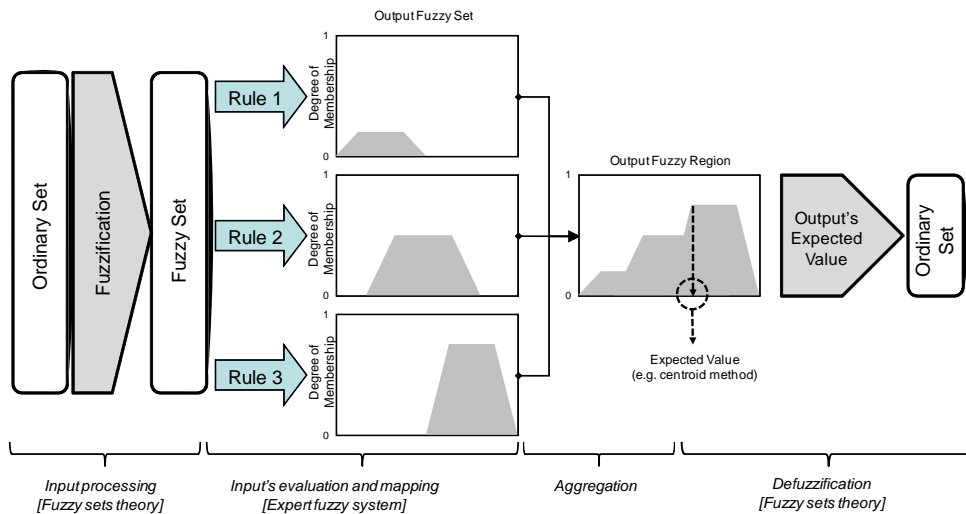


Source: authors' design.

Each time the *fuzzy inference processing engine* evaluates an input's degree of membership to the *inference rules* it maps each solution variable into its corresponding output *fuzzy set*, where the resulting number of output *fuzzy sets* matches the number of *inference rules* used to evaluate the inputs. For example, as in the left part of Figure 6, evaluating and mapping an input with three *inference rules* would result in three output *fuzzy sets*. The aggregation of these three *fuzzy sets* produces the final output fuzzy region, which contains the information of the degree of membership (or truth) of the inputs (or propositions) after the simultaneous evaluation of the *inference rules*.

Afterwards, because a single and crisp quantity is required (i.e. the Index), the best representative value –expected value- of the output fuzzy region has to be calculated; because of consisting in the conversion of fuzzy into ordinary quantities, this process is known as *defuzzification*, and corresponds to the calculation of the expected value of the output (Cox, 1994).

Figure 6
The Fuzzy Logic Inference System



Source: Reveiz and León (2010).

According to Sivanandam et al. (2007), Klir and Yuan (1995) and Cox (1994), the most used *defuzzification* method is the *centroid*, also known as the *center of gravity method* or *center of area method*.¹⁹ It is calculated as the weighted average of the output fuzzy region, and corresponds to the point in the x-axis which divides the output fuzzy region into two equal subareas (Figure 6).

For the case in hand the result of the *defuzzification* is a Systemic Importance Index level, as presented in Figure 5's x-axis. This Index level corresponds to a quantitative relative assessment of the systemic importance of each institution based on its inputs (criteria) and the expert knowledge embedded in the fuzzy rules set.

Finally, according to McNeill and Thro (1994), the combination of *fuzzy inference rules* and the *fuzzy inference processing engine* –based on *Fuzzy Logical operators*- results in an *expert fuzzy system*. Jointly, as in Figure 6, the use of an *expert fuzzy system* and *fuzzy sets theory* results in a *Fuzzy Logic Inference System* (FLIS).

3 Designing an expert knowledge-based systemic importance index

Based on the basic concepts and procedures introduced in the previous section, and based on the criteria defined by IMF et al. (2009) and Manning et al. (2009) (i.e. *size*, *connectedness* and *substitutability*), this section presents authors' proposal for designing an expert knowledge-based Systemic Importance Index. Following the procedure depicted in Figure 6, this section (i) describes the definition and *fuzzification* of the inputs according to the criteria; (ii) depicts the design of the *fuzzy inference rules*; and (iii) describes the resulting Systemic Importance Index as a product of the *defuzzification* process.

3.1 Defining and fuzzifying the inputs

According to recent literature on systemic importance for financial institutions and payment systems (IMF et al., 2009; Manning et al., 2009), the most relevant criteria are *size*, *connectedness* and *substitutability*, where the first two relate to their potential to have a large negative impact on the financial system and the real economy, whereas the latter relates to the magnitude of the indirect impact.

Consequently, the authors define four key indicators that aim to capture *size*, *connectedness* and *substitutability*, as exhibited in Table 1. Concurrent with IMF et al. (2009) the authors embrace the amount of financial services each institution provides to the system as a metric for *size*, with standard balance sheet data such as volume of deposits, money market borrowing and financial assets under management.

Concerning *connectedness* and *substitutability* the authors agree with recent literature that calls for network theory (ECB, 2010) as a mean to gain a better understanding of the financial system. Unlike standard application of network theory, authors avoid using

¹⁹ Cox (1994) highlights *centroid's* consistency and well-balanced approach, its sensitiveness to the height and width of the total fuzzy region and the smooth changes in the expected value of the output across observations, behaving similarly to Bayesian estimates; that is, it selects a value that is supported by the knowledge accumulated from each executed proposition. Cox also emphasizes that unless there are reasons to believe that the model requires a more advanced or specialized method of *defuzzification*, the model should be limited to either the *centroid* or the *max-membership-principle* method; therefore, this is the authors' choice. For other –less common- methods please refer to Sivanandam et al. (2007) and Klir and Yuan (1995) and Cox (1994).

balance sheet data as the input for network theory, and decide to use data from the large-value payment system as the primary source of information for assessing both criteria; this is also the choice of various authors for analyzing how financial institutions interact with each other (Leon et al., 2011; Machado et al., 2010; Cepeda, 2008; Soramäki et al., 2006; Bech and Garrat, 2006; Iannotta et al., 2004).

As suggested by León et al. (2011), using large-value payment system data has several advantages for assessing *connectedness* and *substitutability*: (i) it is not clear whether off-balance positions are being captured or not when using claims, whilst payments comprise all transactions between payments system's participants; (ii) unlike claims, relying on payments allow for considering liquidity as a key factor in systemic risk; (iii) as emphasized by Kodres (2009), failure or insolvency are not the only sources of systemic shocks, but mere failure-to-pay or non-payment of transactions can gridlock the entire financial system; and (iv) as acknowledged by Tumpel-Gugerell (2009), a particular institution might not only be systemically relevant because other institutions are financially exposed to it via balance sheet positions, but also because other market participants rely on the continued provision of its services.

It is important to emphasize that the choice of broad key indicators follows several considerations. First and most important, broad key indicators allow for assessing systemic importance of banking and non-banking financial institutions. Unlike most models on assessing systemic importance, which are focused on banking institutions (as in BCBS-BIS (2011)), the authors consider imperative to be able to consider non-banking institutions as relevant as banking institutions; as non-banking-related systemic events have demonstrated (e.g. LTCM, Lehman, AIG, Bear Sterns, Freddie Mac and Fannie Mae), and as put forward by Ötcher-Robe et al. (2011), it is essential to improve the understanding of the shadow banking system to prevent non-banking institutions gaining systemic importance in an unnoticed manner.²⁰

Second, broad key indicators allow for a parsimonious model, which would allow for continuous (e.g. monthly) monitoring of systemic importance. Third, broad key indicators are convenient for comparing results across different financial systems.

²⁰ Nevertheless each key indicator may be broken down into other –more specific- key indicators, as the consultative document by the BCBS-BIS (2011) suggests. Nevertheless, such decomposition may result in an implicit preference for assessing systemic importance of some types of financial institutions (e.g. commercial banks), whilst overlooking others (e.g. brokerage firms, hedge funds). Hence, authors suggest to use wide-ranging key indicators when initially implementing the proposed model, and subsequently increasing their specificity if necessary.

Table 1
Systemic importance key indicators

Key indicators	Description	Source / Estimation	Rationale (When facing a failing or near failing institution...)
[A] Volume of deposits and money market borrowing	Face value of liabilities a financial institution would fail to pay to the public and to other participants of the financial system in the short run.	Balance sheet data provided by the Banking Superintendence of Colombia.	... the larger the deposits and money market borrowing... <ul style="list-style-type: none"> • the larger the potential loss in confidence of the public. • the larger the potential impact on other institutions liquidity and solvency. • the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.
[B] Volume of financial assets under management	Market value of proprietary assets that may be sold in order to obtain liquidity in the short run, and the volume of assets from third parties which could be compromised or mismanaged in the short run in case of a failure or near failure.	Balance sheet data provided by the Banking Superintendence of Colombia.	... the larger the volume of financial assets under management... <ul style="list-style-type: none"> • the larger the potential impact on liquidity and solvency of other financial institutions via "liquidity spirals".²¹ • the larger the potential impact on the real economy via market prices and portfolios' mismanagement. • the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.
[C] Contribution to the payment system	Contribution to the total payments of the large-value payment system, weighted by the contribution to the total connections of the large-value payment system (CUD).	Large-value payments system statistics provided by Banco de la República (CUD).	... the larger the volume of payments and the number of connections... <ul style="list-style-type: none"> • the larger the number of potential institutions affected and the severity of the affectedness. • the larger the potential disruption in the money, capital and exchange markets. • the larger the potential monetary impact of central bank's liquidity supply to affected financial institutions.
[D] <i>Betweenness centrality</i>	Degree of involvement of a participant in the –indirect-connection of all other participants within the large-value payment system (CUD).	Estimated as the change in the average number of links necessary for each participant to be connected to all other participants; if removing an institution results in a major (minor or nil) increase in the average number of links all institutions require to remain connected as before, the removed institution is to be considered as of low (high) <i>substitutability</i> . ²² Data provided by CUD.	... the larger the betweenness centrality... <ul style="list-style-type: none"> • the higher the potential efficiency and safety losses for the system. • the larger the potential disruption in the money, capital and exchange markets.

Source: authors' design

²¹ "Liquidity spirals" refers to the internal amplifying process whereby a falling asset leads to more sales (deleveraging), which further drives down asset prices, financial intermediaries' profit and loss statements, and balance sheets' net worth (Brunnermeier et al. 2009).

²² Refer to Exhibit B for a brief introduction to the measurement of *connectedness* and *betweenness centrality*.

Under the *size*, *substitutability* and *connectedness* criteria proposed by IMF et al. (2009) and Manning et al. (2009), authors consider *size* is to be captured directly by key indicators [A] and [B], and indirectly by [C]; *connectedness* is to be captured directly by [C], and indirectly by [D]; and *substitutability* is to be captured directly by key indicator [D], and indirectly by [C] (Table 2).

Table 2
How the selected key indicators of systemic importance relate to criteria from IMF et al. (2009) and Manning et al. (2009)

Key indicators	Criteria to be captured		
	Size	Connectedness	Substitutability
[A] Volume of deposits and money market borrowing			
[B] Volume of financial assets under management			
[C] Contribution to the payment system			
[D] <i>Betweenness centrality</i>			

Directly captured	
Indirectly captured	
Non-captured	

Source: authors' design.

Moreover, as presented in Table 3, the four systemic importance key indicators concur with the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.²³

Table 3
How the selected key indicators of systemic importance relate to criteria from the 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act.

Key indicators	Criteria to be captured			
	Aggregate monetary value of transactions	Aggregate exposure to counterparties	Interdependencies and interactions with other participants	Effect on critical markets, institutions and the system
[A] Volume of deposits and money market borrowing				
[B] Volume of financial assets under management				
[C] Contribution to the payment system				
[D] <i>Betweenness centrality</i>				

Directly captured	
Indirectly captured	
Non-captured	

Source: authors' design.

²³ The 2010 Dodd-Frank Wall Street Reform and Consumer Protection Act (hereafter Dodd-Frank Act) is United States of America's legislative response to the most recent episode of international financial crisis. Its main objective is to promote financial stability of the United States, whereas Section 804 of the Act addresses the main considerations to designate what systemic importance is.

The Dodd-Frank Act considers that a financial market utility or payment, clearing or settlement activity may be labeled as systemically important under the following four considerations²⁴: (i) the aggregate monetary value of its transactions, which is to be directly captured by key indicator [C], and indirectly by [A] and [B]; (ii) its aggregate exposure of their counterparties, which is to be directly captured by key indicator [A], and indirectly by [C]; (iii) its relationship, interdependences, or other interactions with other participants, which is to be directly captured by key indicators [D] and [C], and indirectly by [A]; (iv) the effect that its failure or its disruption would have on critical markets, financial institutions, or the broader financial system, which is to be directly captured mainly by [A], [B] and [C], and indirectly by [D].

3.2 Designing the *fuzzy inference rules*

The *fuzzy inference rules* were designed in order to capture the expert knowledge of financial authorities' officers and technical staff. First, the authors calculated the possible combinations of the three different categories [*LOW / MEDIUM / HIGH*] for the four chosen key indicators; this is, what are all the possible types of financial institutions that may result from the combination of the key indicators, where each indicator has three possible outcomes?

The number of possible combinations is 81 (i.e. 3⁴), where each rule corresponds to the following syntax:²⁵

*If a financial institution XYZ's [A] is [LOW / MEDIUM / HIGH] AND ...
 XYZ's [B] is [LOW / MEDIUM / HIGH] AND ...
 XYZ's [C] is [LOW / MEDIUM / HIGH] AND ...
 XYZ's [D] is [LOW / MEDIUM / HIGH] ...
 THEN an XYZ's SYSTEMIC IMPORTANCE IS...
 [VERY LOW / LOW / MEDIUM LOW / MEDIUM / MEDIUM HIGH / HIGH / VERY HIGH]*

Afterwards, all combinations were included in a survey conducted within some of the central bank's officers and technical staff. The individuals taking part in the survey were chosen because they (i) are directly involved in the regulatory and decision making process of last-resort lending at Colombia's central bank; (ii) may be considered experts in the functioning of the payment systems; and (iii) may be considered experts in financial stability.²⁶

It is worth emphasizing three advantages resulting from using expert knowledge as herein proposed. First, deconstructing expert knowledge allows for recognizing the main characteristics of the financial system under analysis. It is most likely to find that two different financial systems result in two different sets of *inference rules*, even if the panel of experts is the same. Likewise, it is natural to find that the same financial system results in different sets of *inference rules* across time; the evolution of the institutional framework, participants, products, services and regulation would explain

²⁴ The Dodd-Frank Act does not limit the considerations to these four; the Act includes a fifth, which states that any other factors may be included because of their relevance.

²⁵ Please note that the inference rules were designed with the AND operator exclusively, which allows for effectively covering all possible combinations of financial institutions. Using the NOT operator may help reducing the number of rules, but may affect the process of deconstructing expertise into fragments of knowledge. Additionally, because all possible combinations of financial institutions may be attained with the AND operator, the OR operator was also discarded.

²⁶ Due to disclosure issues the herein results correspond to a limited sample of five individuals considered as experts willing to take the survey. Results are for illustrative purposes, and they do not necessarily represent opinions or statements neither of Banco de la República nor of its Board of Directors.

such finding. This is why expert knowledge gathered from Colombia's financial authorities' officers and technical staff is relevant for the Colombian case only.

Second, unlike a weighting approach²⁷, where the aggregated index results from the – linear- weighted sum of all key indicators, deconstructing expert knowledge allows for capturing non-linearities arising from accumulating key indicators. This is a convenient feature since it is intuitive that the systemic risk arising from merging two financial institutions is different (i.e. expectedly higher) than the mere sum of their systemic risk;²⁸ different from portfolio theory, where adding assets results in an equal or lower volatility of the portfolio, aggregating financial institutions (i.e. their key indicators) may result in higher systemic risk.

Third, unlike a fixed weighting approach, where weights remain constant across different combinations of key indicators, deconstructing expert knowledge allows for capturing non-linearities arising from combining key indicators. This is the case of *non-substitutability*: it is rather intuitive that the more connected or bigger a financial institution, the more important its degree of *substitutability*; *substitutability* may not be a significant factor when the institution's contribution to the payment system is low, but may turn decisive when it is high. This type of non-linearity is ignored when using a simple weighting approach, but may be captured by experts' judgment.

As presented in Exhibit A, the individuals taking part in the survey had to "tick" what level of systemic importance results for each of the 81 combinations. A single set of rules was obtained from observing the answer that occurred most often in the survey (i.e. the mode) for each of the 81 combinations.

Based on Sivanandam et al. (2007) the *inference rules* or propositions should comply with four desirable properties: completeness, consistency, continuity and non-interactivity. A set of rules is considered (i) complete when any combination of input values result in an appropriate output value; (ii) inconsistent if two same rules yield different output values; (iii) continuous if it does not have neighboring rules with output fuzzy sets that have empty intersections; (iv) non-interactive when inference do not relate to each other.

Because the survey do not consider propositions combining two different types of institutions (e.g. [IF institution X's A is HIGH ... THEN institution Z's systemic importance is HIGH]), the non-interactivity property is assured. Moreover, because the number of rules (81) strictly consists of the attainable rules, no rules are repeated and, therefore, consistency is also assured.

Completeness and continuity may be distinguished by visual inspection of the rules set and surface plots resulting from the survey; the surface should be continuous and

²⁷ Unlike the approach herein suggested, which uses expert knowledge to determine the importance of each key indicator and of all their possible combinations, the consultative document by the BCBS-BIS (2011) suggests an equal and fixed weighting approach (i.e. five major key indicators, each one assigned a 20% weight). Besides not being able to capture non-linearities arising from combining key indicators, the weighting approach may be oversimplifying: it is not clear that all criteria are equally important for all markets, or at all times. As suggested by IMF et al. 2009, a qualitative framework could be used to integrate the different components of the assessment and help arrive at judgments of systemic importance, where a high degree of judgment founded in a detailed knowledge of the functioning of the financial system is required in any assessment of systemic importance; this is the rationale behind the herein presented approach.

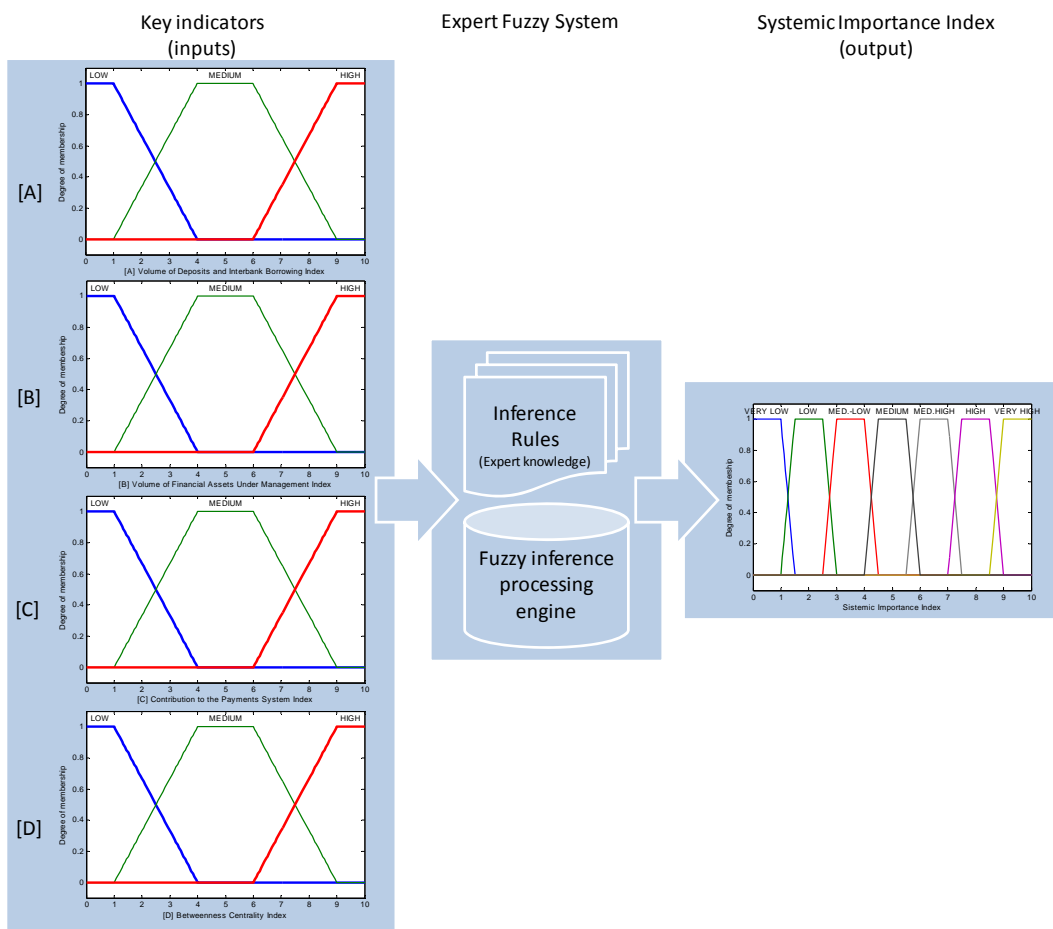
²⁸ Exhibit B displays a graphical representation of the results of the survey, where the non-linearities captured by deconstructing expert knowledge (i.e. the right skew of the distribution) contrast with the linearity (i.e. symmetry) of a purely quantitative weighting approach.

intuitive, where the latter means that for the issue in hand the higher the input (*size, connectedness and non-substitutability*), the higher or equal the systemic importance; as presented below (Figure 8 and Exhibit D), surface plots may be depicted as complete and continuous.

3.3 The Fuzzy Logic inference system and the systemic importance index

As in Figure 3, three trapezoidal *membership functions* are used to evaluate the degree of membership to the four key indicators ([A], [B], [C], [D]), where three *membership functions* or categories exist [*LOW / MEDIUM / HIGH*]. As in Figure 5, seven trapezoidal *membership functions* are used to evaluate the degree of membership to the output (i.e. systemic importance), where seven *membership functions* or categories exist [*VERY LOW / LOW / MEDIUM LOW / MEDIUM / MEDIUM HIGH / HIGH / VERY HIGH*]. After including the expert knowledge of Colombia’s financial authorities’ officers and technical staff (i.e. the *inference rules*), the FLIS would look like in Figure 7.

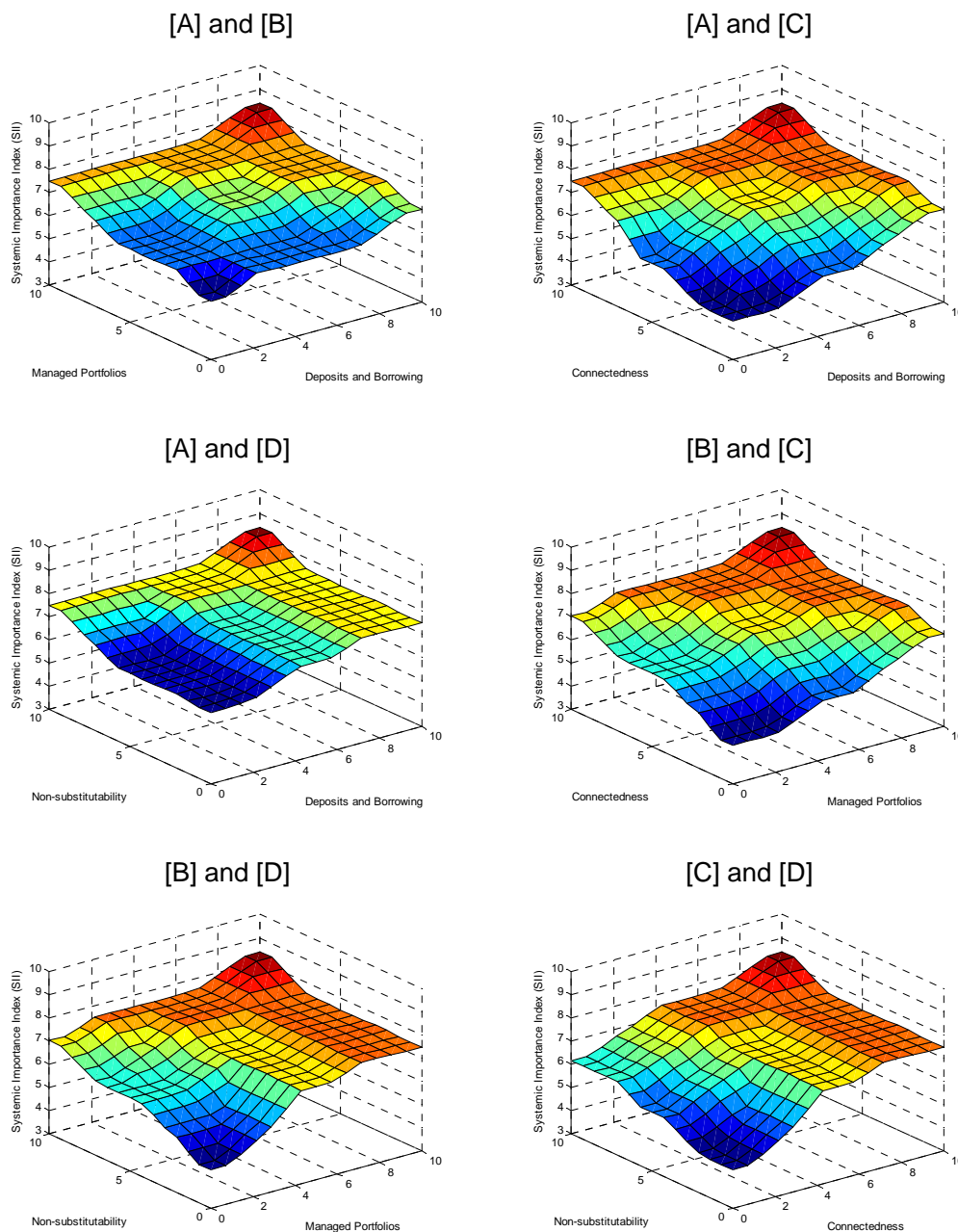
Figure 7
Systemic importance as a Fuzzy Logic Inference System



Source: authors' design.

The resulting systemic importance surfaces are displayed in Figure 8. They exhibit different combinations of key indicators and the Systemic Importance Index level they yield according to the expert knowledge deconstructed via the mentioned survey (Exhibit A), where non-displayed key indicators are set equal to 7.²⁹

Figure 8
Resulting systemic importance surfaces
(non-displayed key indicators set equal to 7)



Source: authors' calculations, based on the survey (Exhibit A).

²⁹ It is worth emphasizing that the displayed surfaces (Figure 8) correspond to the case in which the non-displayed key indicators in each plot are set equal to 7. If this assumption is modified the surfaces will vary; therefore, the displayed surfaces do not cover all the possible combinations of key indicators. Exhibit D presents the surfaces corresponding to non-displayed key indicators set equal to 5.

4 Systemic importance assessment³⁰

Based on the proposed method for obtaining the Systemic Importance Index, this section uses May 2011 data to calculate the individual systemic importance of the financial institutions participating in Colombia's large-value payment system (CUD). During May 2011 the main financial institutions directly participating in the CUD were 145, and they were classified as in Table 4.³¹

Table 4
Main Colombian market's financial institutions directly participating in CUD
(as of May 2011)^d

Class	Institution type	Main purpose ^c
Credit Institutions (CI) ^a	Commercial Bank (CB)	Provision of deposit and loans, including mortgages. [21]
	Commercial Financial Corporation (CFC)	Provision of deposit and loans focused on goods and services commercialization (e.g. leasing). [20]
	Financial Corporation (CF)	Provision of deposit and loans focused on medium term industrial financing; akin to an investment bank. [3]
Non-Credit Institutions (NCI)	Mutual Fund (MF)	Provision of investment vehicles with the purpose of investing in securities and other assets according to the risk profile of the investor. [27]
	Brokerage Firm (BF)	Provision of brokerage services with the purpose of buying and selling securities (e.g. stocks, bonds, currencies); allowed to trade for its own account. [26]
	Pension Fund Manager (PFM)	Provision of investment vehicles with the purpose of investing for retirement. [6]
	Other ^b	Insurance companies, financial cooperatives and other. [18]

^a Financial cooperatives pertain to Credit Institutions; due to their low contribution to the CUD and their *size* they were included in the "Other" class; CIs are the only institutions able to receive last-resort lending liquidity.

^b The "Other" class gathers financial institutions characterized by their particularly low (or nil) relevance for the key systemic importance indicators.

^c Only the main differencing feature appears; the number of institutions as of May 2011 appears in brackets.

^d Financial infrastructures and official financial institutions are excluded from this Table and were not analyzed. Source: León et al. (2011).

Each financial institution's systemic importance key indicator was estimated according to Table 1. Indicators [A], [B] were obtained from statistical data from the Banking Superintendence of Colombia. [C] and [D] were obtained based on large-value payment system's databases (Banco de la República-CUD), and calculated as described in Exhibit B.

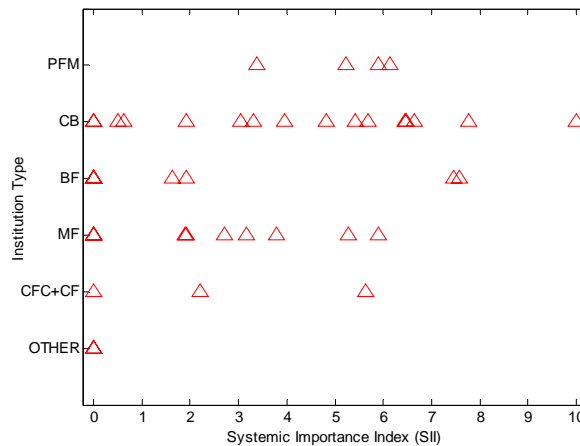
The main quantitative assessment of systemic importance is displayed in Figure 9. Each financial institution's Systemic Importance Index is mapped as a triangle in a 0 to 10 scale, where each level in the y-axis corresponds to a different type of institution. It is worth emphasizing that the Index provides a relative assessment of each institution's systemic importance: an Index equal to zero does not correspond to the –absolute–

³⁰ Results are illustrative. They may not be used to infer credit quality or to make any type of assessment for any financial institution. Results do not represent an opinion or statement of Banco de la República nor of its Board of Directors. The name of each institution is not revealed due to disclosure restrictions.

³¹ For a brief introduction to the functioning and characteristics of the Colombian large-value payments system (CUD), please refer to Banco de la República (2011), León et al. (2011), Machado et al. (2010) or Cepeda (2008).

absence of systemic importance for that institution, but a negligible importance with respect to the most important institution.

Figure 9
Systemic Importance Index
(as of May 2011)

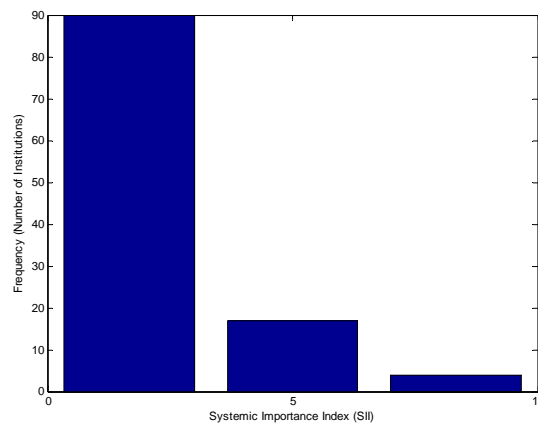


Source: authors' calculations.

The types which concentrate most systemic importance in the Colombian financial market are commercial banks (CBs) and brokerage firms (BFs), as in León et al. (2011) and Machado et al. (2010). According to the definition of systemic importance as a fuzzy variable (Figure 5) CBs and BFs are the only type of institutions pertaining to some degree to the HIGH and VERY HIGH categories (i.e. membership functions).

An important attribute of the Systemic Importance Index is its high level of skewness (Figure 10). Only a few financial institutions (4) pertain to some degree to the HIGH or VERY HIGH categories of systemic importance, whilst most of the participants (83) pertain to some degree to the LOW and VERY LOW categories.

Figure 10
Distribution of the Systemic Importance Index
(as of May 2011)

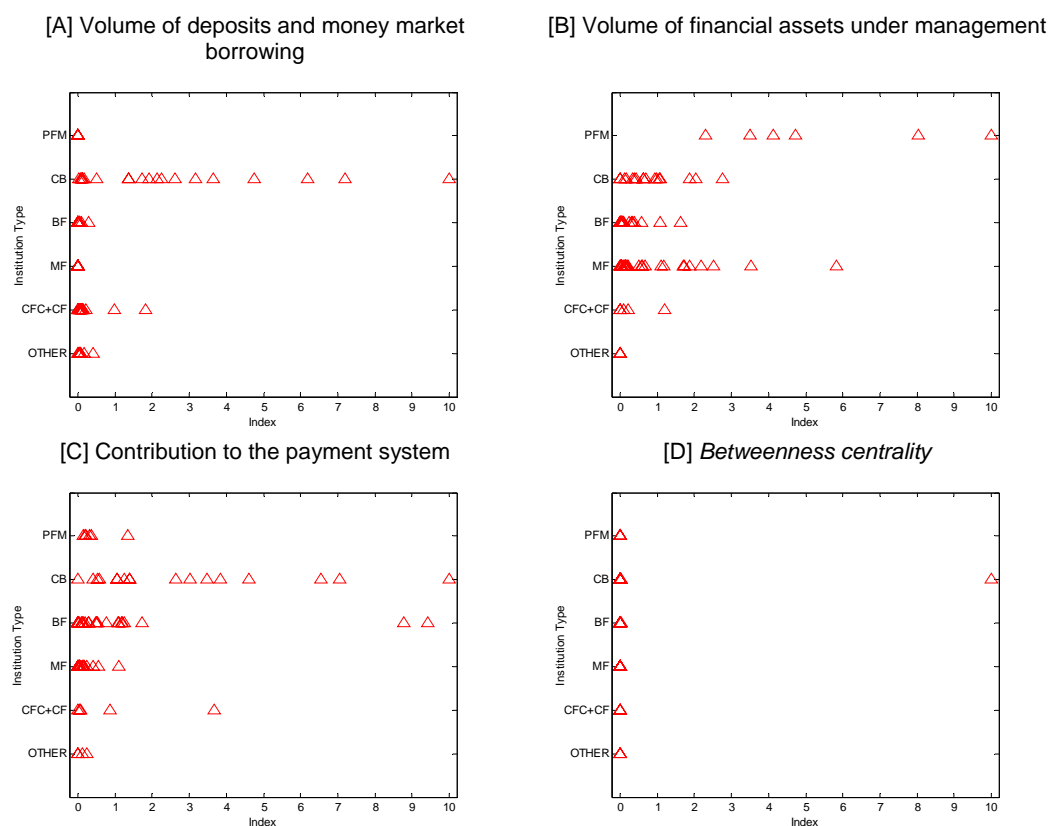


Source: authors' calculations.

Such skewness confirms the intuition regarding the high degree of asymmetry (right skew) of systemic importance, where the average institution is of low systemic importance and the average default or failure-to-pay results in low systemic severity. In this case the average financial institution's Systemic Importance Index is 1.33, and pertains to some degree to the LOW and VERY LOW categories; thus, relying on the systemic importance of the average financial institution would divert financial authorities from its aim of ever preserving financial stability and payment systems safety.

The overall view of systemic importance of Figure 9 may also be conveniently desegregated into its key indicators. Figure 11 displays the four systemic importance key indicators for all the analyzed financial institutions.

Figure 11
Systemic importance key indicators
(as of May 2011)



Source: authors' calculations.

As expected, CBs emerge as particularly important due to their volume of deposits and money market borrowing, along with their contribution to the large-value payment system (CUD). Regarding *betweenness centrality* (Figure 11, lower-right panel), a CB resulted as the only institution that would systemically endanger financial and payment system stability because its absence resulted in the disconnection of two participants of the CUD (an insurance company and a brokerage firm); therefore, the absolute

impossibility to substitute this CB for these two participants conceals other participant's degree of *non-substitutability*.³²

Two brokerage firms (BFs) display some degree of HIGH and VERY HIGH contribution to the large-value payment system, even significantly higher than most CBs; this is intuitive since their primary role is the provision of buying and selling services under a broker-dealer scheme. Since the main source of systemic importance of these two BFs is their contribution to the large-value payment system (Figure 11, lower-left panel), it is worthwhile noting that their overall systemic importance (Figure 9) suggests that the experts taking the survey already regard the *too-connected-to-fail* criteria as important as the *too-connected-to-fail* criteria, which concurs with developments after the most recent episode of global financial crisis.

It also important to highlight that network theory based on payment system data allowed identifying those two BFs as systemically important due to their *connectedness*. This is not the result of financial system's balance sheet being exposed to BFs, but the result of the financial system relying on these BFs' continuous provision of their services; as the upper-left panel of Figure 11 reveals, these two BFs' systemic importance would have been missed if using network theory based on traditional balance sheet data.

Finally, pension fund managers (PFMs) and mutual funds (MFs) display their systemic importance as the main portfolio managers of the financial system, with a –relative–minor involvement in the large-value payment system.

5 Final remarks

As pointed out by León et al. (2011), the most recent episode of global market turmoil exposed the limitations resulting from institution-centric metrics and the –resulting–traditional focus on *too-big-to-fail* institutions within an increasingly systemic-crisis-prone financial system. This has encouraged the appearance of the *too-connected-to-fail* concept, and has resulted in the emergence of several challenges regarding the estimation of financial institutions' systemic importance by financial authorities.

As previously documented, the main challenges relate to considering measures corresponding to *size*, *substitutability* and *connectedness*, and using methodologies able to capture judgment (i.e. detailed knowledge) of the functioning of the financial system, and capable of yielding non-binary results (IMF et al., 2009).

Despite these challenges are intricate by nature, there are methodologies that may overcome some of the difficulties. First, as has been recently discussed in systemic risk literature and included in the herein proposed model, network theory may effectively assess financial institutions' *connectedness* and *substitutability*. Second, as supported by literature on network theory application to financial systems' analysis (Leon et al., 2011; Machado et al., 2010; Cepeda, 2008; Soramäki et al., 2006; Bech and Garrat, 2006; Iannotta et al., 2004), large-value payment system data was used to better approximate the way financial institutions interact with each other. Third, analogous to the approach chosen by NASA for safe and reliable autonomous rover navigation (Seraji, H., 2000) and autonomous landing on planetary surfaces (Serrano and Seraji, 2007; Howard and Seraji, 2002), the proposed *Fuzzy Logic*-based systemic importance

³² Exhibit B also presents some illustrative estimations for February and June 2006, and September 2009, where several participants display various levels of *betweenness centrality*.

index allows for circumventing some shortcomings faced by traditional quantitative methods when a high degree of expert judgment is required.

As demonstrated throughout the document the proposed methodology allowed for (i) capturing ambiguous (non-binary) concepts such as *size*, *substitutability* and *connectedness*; (ii) simultaneously evaluating each financial institution's relative size, *substitutability* and *connectedness* in an intuitive manner that mimics human reasoning; (iii) considering the intrinsic functioning of the local financial system by means of expert knowledge; and (iv) obtaining a relative, non-binary, intuitive and practical quantitative assessment (i.e. the Index) for identifying systemic importance within Colombia's financial market.

Results obtained by the proposed methodology are straightforward and grant financial authorities with the ability to acquire a comprehensive relative assessment of each financial institution's systemic importance. This may serve the purpose of assisting financial authorities in focusing their attention and resources –the intensity of oversight, supervision and regulation- where the systemic severity resulting from a financial institution failing or near-failing is estimated to be the greatest. Moreover, identifying systemically important institutions may also serve financial authorities for enhanced policy-making (e.g. prudential regulation, oversight and supervision) and decision-making (e.g. resolving, restructuring or providing emergency liquidity).

It is important to emphasize that focusing on the systemic importance departs from the traditional (as in Norman et al. (2009)) focus on systemic risk. As previously explained, authors justify such shift after documenting that focusing on the level of risk (i.e. frequency times severity) may mislead financial authorities from closely overseeing financial institutions capable of generating extreme systemic disturbances; this is, involving the probability of systemic events occurring may result in authorities overseeing, supervising and regulating for a severe systemic shock based on the impact of a single –systemically modest- average institution.

It is also important to call attention to the importance of *connectedness* for the experts taking the survey. Two brokerage firms resulted as pertaining to some degree to the HIGH and VERY HIGH systemically important, only surpassed by two commercial banks. Since the main source of systemic importance of these two BFs is their contribution to the large-value payment system, it is worthwhile noting that their overall systemic importance (Figure 9) suggests that the experts taking the survey already regard the *too-connected-to-fail* criteria as important as the *too-big-to-fail* criteria, which concurs with developments after the most recent episode of global financial crisis.

Additional advantages from the proposed approach are parsimony and ease of calculation, which would allow for convenient continuous (e.g. monthly) monitoring of systemic importance, and for the ability to capture the dynamics of the financial market (e.g. financial innovations, regulatory changes, etc.) via periodic revisions (e.g. yearly) of the survey.

Nevertheless, it is important to emphasize that the proposed methodology is by no means a substitute for sound judgment by financial authorities, or the sole metric to use when deciding the systemic importance of a financial institution. Authors regard this methodology as providing a valuable and novel relative metric for assessing systemic importance by financial authorities, which conveniently complements existing methods.

Moreover, financial authorities are to decide whether an important/unimportant threshold is to be defined within the Systemic Importance Index. Despite authors consider that defining such threshold is intricate and outside the scope of this

document, they suggest considering (i) the purpose of defining the threshold (e.g. for defining a capital charge or deciding which institutions to follow closely); (ii) the degree of clustering of the Index (i.e. its skewness); and (iii) a detailed knowledge of the functioning of the financial system.

Regarding this proposal some challenges still remain. First, this document focuses on financial institutions, which may be conveniently characterized by the chosen set of key factors. Other participants of the financial system, such as those labeled as market utilities (e.g. central counterparty clearing houses, electronic payments networks, large and non-large value payment systems), may not be adequately characterized by financial institutions' set of factors. It is in the authors' research agenda to identify those key factors that may help characterize financial market utilities in a convenient and comprehensive manner, and to develop an appropriate methodology for identifying and assessing their systemic importance.

Second, financial institutions tend to pertain to a financial conglomerate. Because it is intuitive that the systemic importance arising from merging two financial institutions is different (i.e. expectedly higher) than the mere weighted sum of their systemic importance, it is of great importance to address financial conglomerates' systemic importance. Therefore, it is in the authors' research agenda to assess conglomerates' systemic importance based on the herein proposed approach.

Third, other key factors for assessing financial institutions' systemic importance may emerge from further research, as those proposed by BCBS-BIS (2011) regarding cross-jurisdictional activity and complexity. Despite it is rather uncomplicated to add a new key factor to the (four) herein proposed, the construction of the knowledge base may turn particularly dense. For example, the number of rules in the survey for four key indicators with three categories yields 81 rules, whereas five (six) key indicators with three categories yields 243 (729) rules. This may be tackled to some extent with the use of rules with the NOT operator, but could also complicate the survey.

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³³ Some preliminary versions of published documents (♠) are available online: (http://www.banrep.gov.co/economic_reports/er_draf.htm).

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8 Exhibit B: Assessing *connectedness* and *substitutability* with network analysis basics

As explained throughout the document, literature recognizes three criteria as key to measuring and identifying systemic importance: *size*, *connectedness* and *substitutability* (IMF et al., 2009; Manning et al., 2009). Because of their novelty and intricacy, the first two sections of this exhibit briefly describe the methodologies imported from network analysis in order to measure *connectedness* and *substitutability*³⁵, where the third section displays some results obtained with the described methodologies.

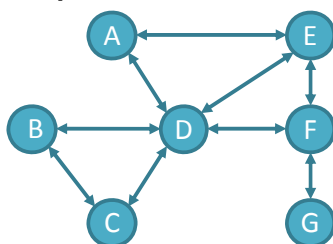
In order to make this exhibit comprehensible please acknowledge the following concepts (Table B1), which pertain to network analysis terminology.

Term	Description
Vertex	The fundamental unit of a network. Also referred as node, actor or participant.
Edge	The line connecting two vertices. Also referred as bond, link or tie. It may be directed if runs in only one direction or undirected if it runs in both directions.
Degree	The number of edges connected to a vertex. In-degree (out-degree) refers to the number of incoming (outgoing) edges.
Geodesic path	It is the shortest path through the network from one vertex to another. Note that there may be more than one geodesic path between two vertices, and that in a directed network the geodesic path may be different from one vertex to another and its reverse.
Distance	The number of links that is minimally needed to connect two vertices. Neighbors (directly connected) have distance equal to 1; neighbors of neighbors that are not directly connected are at distance 2, and so forth.

Source: authors' design, based on Buechel and Buskens (2008) and Newman (2003).

In Figure B1 the previous concepts could be applied as follows: the graph consists of seven vertices or nodes (i.e. A, B, C, D, E, F, G), where the A node is connected to nodes E and D via two undirected edges³⁶ (i.e. incoming/outgoing from/to E and D); thus, the in-degree and out-degree of A is two. There are two geodesic paths from nodes A to G, consisting of three edges (i.e. the path A-D-F-G and the path A-E-F-G).

Figure B1
A simple undirected network



Source: authors' design

³⁵ Network analysis provides many other metrics and measures related to centrality. This exhibit focuses on the approach chosen by the authors. For a comprehensive review and explanation of alternative metrics and measures please refer to Newman (2010).

³⁶ Because network analysis is to be applied to payments, where each payment is necessarily related to an immediate, previous or forthcoming transfer of a financial asset (e.g. a bond, a stock, money, etc.), please note that directed edges (i.e. with only one direction) will not be considered.

8.1 Connectedness

The main intuition of the *connectedness* criteria asserts that the larger the number – and volume- of the links an institution maintains with other market participants, the larger the contagion or spillovers it may generate; this is, the systemic importance of a financial institution generally increases with its degree of *connectedness*.

According to Newman (2010) and de Nooy et al. (2005), the simplest centrality measure in a network is the *degree* of a vertex; this is, the number of edges connected to it, or the number of its neighbors. This type of centrality metric assesses how intensely the vertex is connected to the network, which relates in our case to how easily can payments arrive to or spread from that vertex. For example, in Figure B1 the most central vertex (i.e. with most edges) is D, which has five of them.

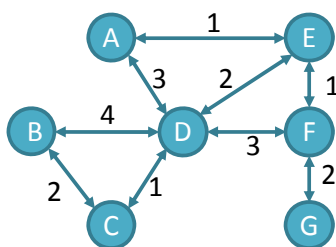
It is common to find that the *degree* of each vertex is normalized with respect to the highest *degree* attainable. Let N be the number of vertices in a network, and E_D the *degree* of vertex D, then $(N - 1)$ is the highest *degree* attainable, then the *degree centrality* of vertex D (C_D) may be expressed as in [F1]. Please note that if a vertex is connected to all the other vertices of the network its *degree centrality* will equal 1, whilst an isolated vertex (not connected to the network) will yield a *degree centrality* equal to 0.

$$C_D = \frac{E_D}{N - 1} \quad \text{[F1]}$$

Therefore, because of its documented simplicity and usefulness, authors rely on *degree centrality* as a customary metric for *connectedness*. Nevertheless, *degree centrality* of a vertex or node would suffice to assess its centrality only if all edges are judged as equally important; this is the standard case of network analysis applied to social relations or informational networks. In the case in hand, where edges represent payments, it is convenient to recognize the importance of each edge according to the value of the payments it intends to represent.

Consequently, as in other applications of network analysis³⁷, it is important to consider each edge's strength, weight or value. Let Figure B2 be a weighted version of Figure B1, where each edge's number represents the weight of the connection between the vertices. It is rather evident that vertex D remains as the most important regarding the weight of the edges it shares with other vertices, with weights adding up to 13.

Figure B2
A weighted network



Source: authors' design

³⁷ A common case of weighted networks is the bandwidth or the amount of data flowing between nodes within the world wide web (Newman, 2010).

Considering the edges' weights provides new information that was not apparent when using *degree centrality*. Vertices B and C in Figure B1 appear to be equally central, both with two undirected edges, both sharing connections between themselves and to vertex D; moreover, when calculating the *degree centrality* as in [F1], B and C yield the same result ($E_B/(N-1) = E_C/(N-1) = 2/6 = 0.33$). Despite sharing the same *degree centrality*, it is rather clear in Figure B2 that considering weights would signal vertex B as being more intensely connected than vertex C, with weights equal to 6 and 3, respectively.

Analogous to *degree centrality*, authors normalize the total weight of each vertex with respect to the sum of each vertex's edge weight.³⁸ Let W_j represent the total weight of all the edges of vertex j , each vertex's share of the network's weight (V_j) may be expressed as in [F2].

$$V_j = \frac{W_j}{\sum_{j=1}^N W_j} \quad [F2]$$

Vertex D, which displays the most weight, yields a 0.342 share of the network's weight. Calculating B's and C's share of the network's weight would yield 0.158 and 0.079, respectively, avoiding considering B and C as equally important within the network because of exhibiting the same number of edges. Unfortunately, comparing B's and F's share of the network's weight (both 0.158) would consider these two vertices as equally important to the network despite F has one more neighbor than B.

Because both approaches to assessing the intensity of the connection are valuable for the case in hand (i.e. payments), where the systemic importance increases with the number of connected institutions and with the share of the total payments, the authors use the product³⁹ of both metrics as an overall measure of the contribution to the payment system (K_j), as in [F3]:

$$K_j = C_j \times V_j \quad [F3]$$

Based on Figure B2, the result of employing such approach is presented in Table B2, where K_j corresponds to the j -vertex overall contribution to the network, and $Kindex_j$ corresponds to the standardized value of K_j in a 0 to 10 scale.⁴⁰

It is worth noticing that (i) the vertex most contributing to the network is D, receiving the highest score in the 0 to 10 scale, which is intuitive since is the vertex with most connections and with the highest share of the network's weight; (ii) the vertex less contributing to the network is G, receiving the lowest score; (iii) vertices B and C are no longer deemed as equally important as with *degree centrality* alone, where B is more important than C because of the latter's edges weights; and (iv) vertices B and F are no longer deemed as equally important as with the share to the network's total weight,

³⁸ Please note that the calculation of V_j as in [F2] counts separately the weights in either direction between each vertex pair, which results in counting each weight twice. As pointed out by Newman (2010), it is possible to compensate for this double-counting by dividing each weight by 2; nevertheless it makes little difference since the analysis focuses on the relative magnitudes and not the absolute values. Moreover, this notation allows for applying [F2] to directed networks as well.

³⁹ Please note that multiplying both metrics (i.e. the degree centrality and the share of the network's total weight) is analogous to using the AND –conjunction- operator in Fuzzy Logic (Cox, 1994). If a vertex displays both a high (low) level of centrality AND a high (low) level of contribution to the network's total weight, then the product of both levels will be high (low).

⁴⁰ As previously introduced (footnote 14), this standardization procedure assigns the maximum index value (10) to the most contributing vertex, and the rest is assigned an index value by means of linear interpolation. Such standardization is straightforward and makes comparisons and calculations easier.

where F is more important than B because of the former's number of edges . Hence, as displayed in Table B2, using the product of both metrics allows for comprehensively and intuitively assessing the importance of the network's vertices.

Vertex	C_j	V_j	K_j	$Kindex_j$
A	0,3333	0,1053	0,0351	1,2
B	0,3333	0,1579	0,0526	1,8
C	0,3333	0,0789	0,0263	0,9
D	0,8333	0,3421	0,2851	10,0
E	0,5000	0,1053	0,0526	1,8
F	0,5000	0,1579	0,0789	2,8
G	0,1667	0,0526	0,0088	0,3

Source: authors' calculations

8.2 Substitutability

The severity of the impact of a payment system failure, and hence the extent of systemic risk, depends critically on whether substitutes are readily available to allow payment flows to be rerouted via another system (Manning et al. 2009). Consequently, the systemic importance of a financial institution generally decreases with its degree of *substitutability*.

A key concept for assessing *substitutability* comes from network analysis' *betweenness centrality*. *Betweenness centrality* measures the brokerage role of a certain vertex within a network (Buechel and Buskens, 2008) or, as defined by de Nooy et al. (2005), it captures the extent to which a vertex is needed as a link in the chain of contacts that facilitate the spread of information within a network; the more a vertex is a go-between, the more central its position in the network.

The calculation of *betweenness centrality* relies on the *geodesic path* concept, which is the shortest path through the network from one vertex to another. It is calculated for vertex j as the proportion of all *geodesic paths* between pairs of other vertices that include such vertex (de Nooy et al., 2005). Let G_{pq} be the number of *geodesic paths* between vertices p and q , and G_{pq}^j the number of *geodesic paths* between p and q that go through vertex j , *betweenness centrality* of vertex j (B_j) is calculated as in [F4]:

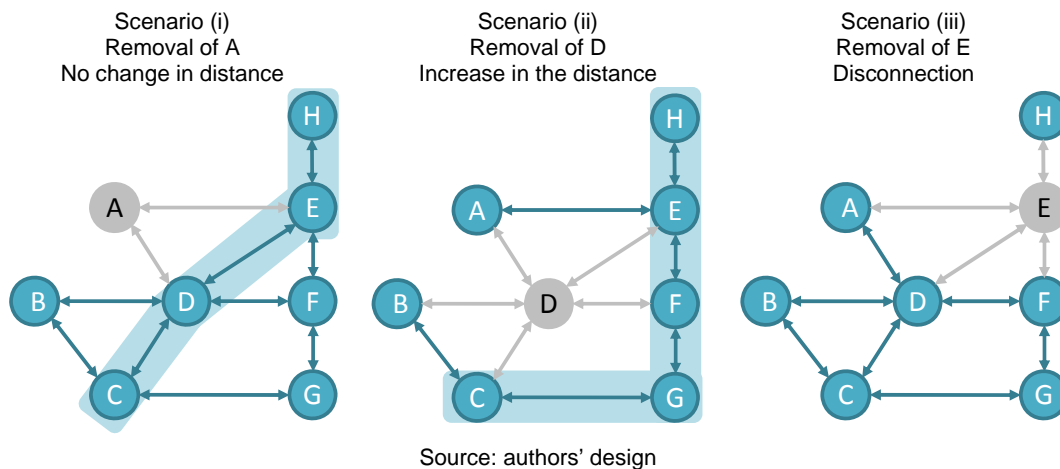
$$B_j = \sum_{pq} \frac{G_{pq}^j}{G_{pq}}, \forall G_{pq} \neq 0, p \neq j, q \neq j, p \neq q \quad [F4]$$

Nevertheless, because *substitutability* is related to the severity of the impact of a vertex being removed, which depends on whether substitutes are readily available to allow preserving the distances between vertices, *betweenness centrality* by itself fails to address this criteria in a proper manner. *Betweenness centrality* captures the importance of a vertex as an intermediary between all the others vertices that compose the network, but it does not capture the severity of the impact resulting from the removal of a vertex.

Because *substitutability* relates to the severity of the impact for the *geodesic path* between vertices p and q (G_{pq}) resulting from the removal of a j -vertex, which could be conveniently defined in terms of distance between the remaining vertices, please consider that there are three possible scenarios resulting from removing j from the network: (i) there is no change in the distance corresponding to the *geodesic path* between vertices p and q (G_{pq}) because other vertex or vertices could substitute the removed j -vertex, or because the j -vertex does not pertain to the *geodesic path* (G_{pq}); (ii) other vertex or vertices could substitute the removed j -vertex, but with an increase in the distance corresponding to the new *geodesic path* between vertices p and q (G_{pq}); and (iii) there is no other vertex or vertices which may serve the purpose of indirectly connecting vertices p and q , hence there would be no *geodesic path* between them ($G_{pq} = 0$), and the distance between vertices p and q would be defined as a number larger than any other possible distance in the network.⁴¹

These three scenarios are depicted in Figure B3. The vertex removed changes in each panel of the figure, and the *geodesic path* to be analyzed is the one corresponding to connecting vertices C and H:

Figure B3
Scenarios of substitutability
Geodesic path connecting vertices C and H



In Scenario (i) the impact resulting from the removal of vertex A is nil; the *geodesic path* between C and H (G_{CH}) comprises two vertices (D and E, with a total distance of 3), where vertex A is not present. In Scenario (ii) the impact from the removal of vertex D is an increase in the distance between C and H, which increases from 3 to 4 where vertices G, F and E belong to this new *geodesic path*. Scenario (iii) exhibits the third case, where the removal of vertex E results in the impossibility of connecting vertices C and H.

Consequently, for the relation between vertices C and H, vertex E is not *substitutable* (i.e. no other vertex or vertices are able to fulfill its role), vertex A is not relevant (i.e. the *geodesic path* does not include vertex A), and vertex D is *substitutable* but with an increase in the distance between C and H. This result is relevant and emphasizes the importance of considering both *connectedness* and *substitutability*. *degree centrality*

⁴¹ Buechel and Buskens (2008) suggest assigning a number larger than any possible actual distance in the network for those pairs of vertices that cannot reach each other, and choose to use N ; this is also the authors' choice.

alone would consider Figure B3's vertex E as of lower systemic importance than D (i.e. E has less edges than D), and would not recognize that the removal of D does not result in a major disruption in the network (i.e. it would be substituted by using other – longer- paths), whilst the removal of E would result in a vertex being disconnected from the network.

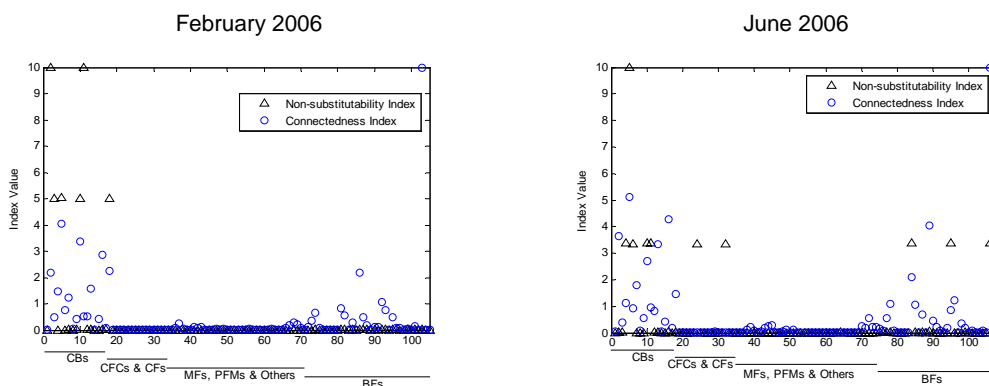
Accordingly, the approach chosen by the authors to assess the *substitutability* of each vertex comprising the network consists of an iterative procedure of the analysis just described. This is, (i) calculating the average distance of the *geodesic paths* for all the vertices within the network; (ii) removing a vertex from the network and recalculating the average distance of the *geodesic paths* for all the vertices within the network; (iii) calculating the increase in the average distance of all the *geodesic paths* after removing the vertex; (iv) repeating these steps for all the network's vertices.

This procedure will yield each vertex's effect on the average distance of the *geodesic paths* of the network, which is a distance-based metric for assessing the severity of the impact of a vertex being removed, and a useful metric for *substitutability*. It is expected that (i) removing a perfectly *substitutable* vertex (i.e. may be substituted without increasing the distance of the *geodesic paths*) will result in a constant average distance of the *geodesic paths* for all vertices within the network; (ii) removing a non-perfectly *substitutable* vertex will result in an increase in the average distance of the *geodesic paths*, whereas the magnitude of the increase is negatively related to the *substitutability* of the vertex; and (iii) removing a *non-substitutable* vertex will result in an increase in the average distance of the *geodesic paths* and the disconnection of one or more vertices of the network.

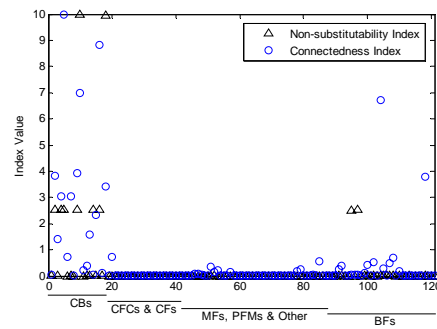
8.3 Comparison of *connectedness* and *non-substitutability* indexes

This section presents the results for the *connectedness* and *substitutability* indexes as of February and June 2006, and September 2009 for Colombian large-value payments system; as described in Leon et al. (2011) and Machado et al. (2010), this three periods correspond to low and high volatility of the Colombian local debt market, and to high liquidity of the large-value payment system, respectively.

Figure B4
Connectedness and non-substitutability indexes



September 2009



Source: authors' calculations

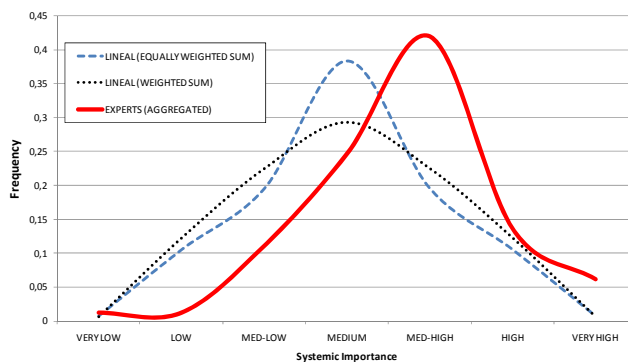
Despite this section's main purpose is solely to present the results of the usage of the *connectedness* and *non-substitutability* indexes as presented in this exhibit, the reader should be aware of the following:

- As argued in the first section of the document, it is evident that *connectedness* and *substitutability* indexes are distributed with a high degree of asymmetry (right skew) and excess kurtosis, where the average institution is of low *connectedness* and high *substitutability*. Therefore, focusing on the average institution for assessing systemic risk would be inappropriate.
- Despite *connectedness* and *non-substitutability* appear to be somewhat related (i.e. CBs and BFs both display high index levels in Figure B4), they capture different characteristics of financial institutions. Not all *too-connected* institutions are *too-non-substitutable* and vice versa.
- It is important to realize that the more connected (e.g. the more contributing to the payment system, the more money market counterparties) and the bigger (i.e. the more assets under management, the more deposits and money market liabilities) a financial institution is, the more relevant its degree of *substitutability*. This is a non-linear feature worth acknowledging when considering *connectedness* and *substitutability* as key indicators for systemic importance.
- As in León et al. (2011) and Machado et al. (2010), *connectedness* index displays that commercial banks (CBs) and brokerage firms (BFs) are the most connected institutions within the Colombian large-value payments system (CUD).
- As a complement to León et al. (2011) and Machado et al. (2010), the herein introduced *non-substitutability* and *connectedness* indexes concur with their centrality index, where systemically important financial institutions are circumscribed to commercial banks (CBs), brokerage firms (BFs) and financial corporations (CFs), in that order.

9 Exhibit C: A graphical display of the survey's results

This exhibit displays a graphical representation of experts' judgment concerning how the systemic risk builds up when combining the four key indicators (i.e. [A], [B], [C], [D]) as presented in the 81-rules survey (Exhibit A).

Figure C1
Survey results



Source: authors' calculations

When using a purely quantitative approach such as the equally weighted sum (dashed blue line) or the unequally weighted sum of the key indicators (dotted black line) the result is a symmetric distribution that peaks around the MEDIUM systemic importance level, where VERY LOW and VERY HIGH systemic importance is exclusive of all-LOW (rule #41) or all-HIGH (rule #1) combinations in Exhibit A, correspondingly.⁴² Therefore, most of the combinations are to be considered of MEDIUM systemic importance.

When using a qualitative approach based on expert knowledge the systemic importance distribution appears skewed to the right. Experts' aggregated judgment (red solid line) shows that they consider that systemic importance results from a non-linear combination of the key indicators, where accumulating systemic importance key factors within a single institution results in a rapidly increasing overall systemic importance. It is remarkable to find that experts' aggregated judgment consider of VERY-HIGH systemic importance other combinations besides the all-HIGH combination (rule #1 in Exhibit A). It is also remarkable that experts consider that most of the systemic importance key indicators combinations are to be considered of MEDIUM HIGH systemic importance, with VERY-LOW and LOW importance resulting from very few combinations.

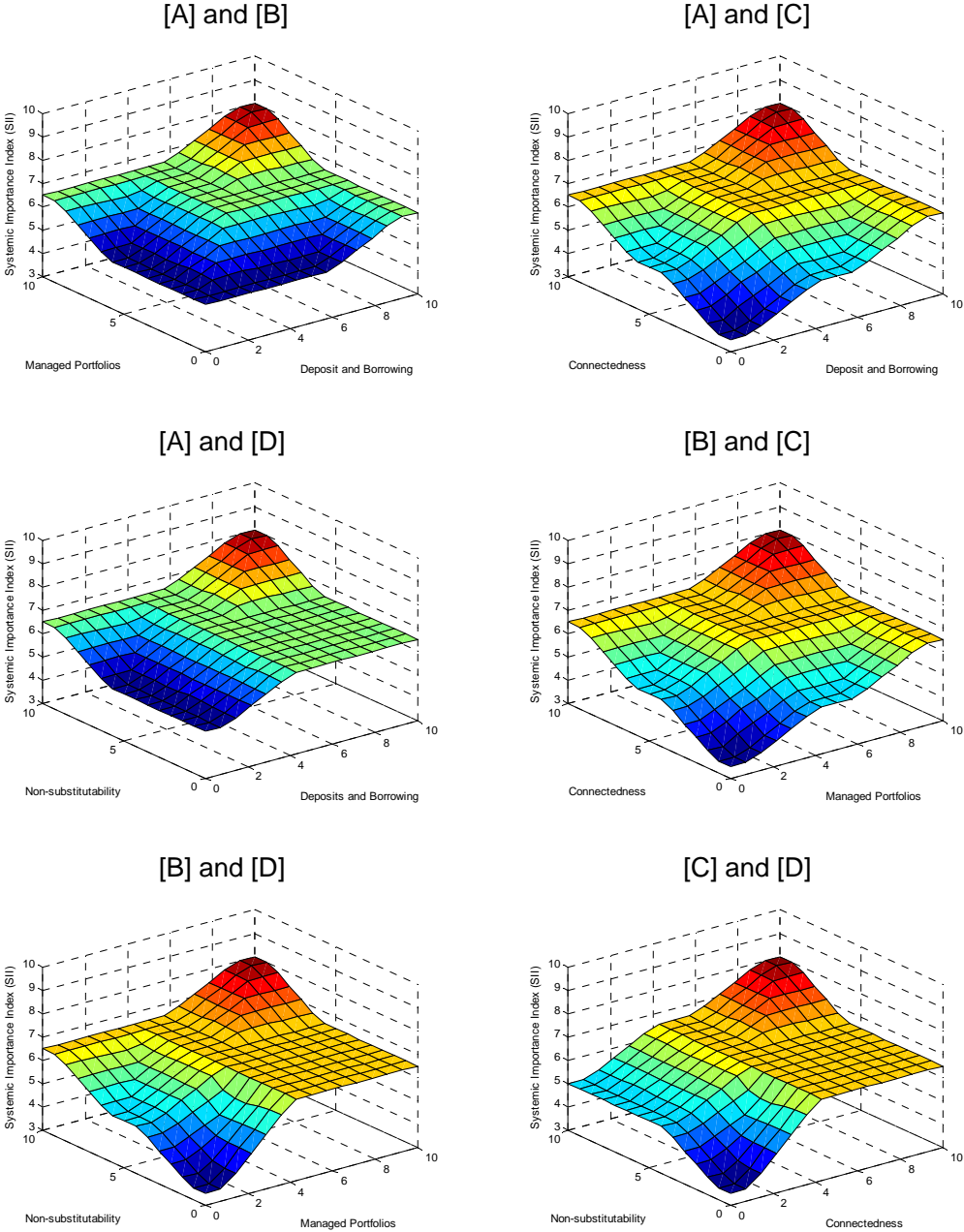
The rationale behind such judgment from the experts is rather simple. Systemic importance arising from merging two financial institutions is different (i.e. expectedly higher) than the mere weighted sum of their systemic importance.⁴³ Similarly, aggregating systemic risk key indicators within a single financial institution is expectedly higher than the mere weighted sum of its key indicators.

⁴² The equally weighted case corresponds to the BCBS-BIS (2011) consultative document. The unequally weighted consisted of the following weights: [A] 35%; [B] 20%; [C] 30%; [D] 15%, for illustrative purposes.

⁴³ In this sense aggregating systemic importance key indicators is the inverse of aggregating volatility in Portfolio Theory, where calculating the weighted sum of volatilities ignores diversification effects taking place; likewise, calculating the weighted sum of systemic importance key indicators may ignore systemic importance "concentration" effects taking place within a single financial institution.

10 Exhibit D: Resulting systemic importance surfaces

Figure D1
Resulting systemic importance surfaces
(non-displayed key indicators set equal to 5)



Source: authors' calculations, based on the survey (Exhibit A).