

Credit and Saving Constraints in General  
Equilibrium: Evidence from Survey Data

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# Credit and Saving Constraints in General Equilibrium: Evidence from Survey Data\*

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## Abstract

In this paper, we build a heterogeneous agents-dynamic general equilibrium model wherein saving constraints interact with credit constraints. Saving constraints in the form of fixed costs to use the financial system lead households to seek informal saving instruments (cash) and result in lower aggregate saving. Credit constraints induce misallocation of capital across producers that in turn lowers output, productivity, and the return to formal financial instruments. We calibrate the model using survey data from a developing country where informal saving and credit constraints are pervasive. Our quantitative results suggest that completely removing saving and credit constraints can have large effects on saving rates, output, TFP, and welfare. Moreover, we note that a sizable fraction of these gains can be more easily attained by a mix of moderate reforms that lower both types of frictions than by a strong reform on either front.

**Keywords:** saving constraints, credit constraints, financial inclusion, misallocation, saving, formal and informal financial markets.

**JEL Classification Numbers:** E21, E44, G21, O11, O16

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# 1 Introduction

Financial inclusion –broadly defined– has become a priority for economists and policy makers trying to advance the development agenda.<sup>1</sup> Indeed, in recent years the longstanding goal of improving access to credit in developing countries has been joined by a growing interest in the role that saving should have in a comprehensive financial inclusion strategy.

While the literature on credit frictions is well developed and includes both empirical and theoretical contributions, the literature on the causes and consequences of exclusion from formal (i.e., through financial institutions) saving markets mostly comprises field experiments in relatively small communities.<sup>2</sup> In fact, little is known about the general equilibrium effects of saving constraints or the way in which they may interact with other frictions, such as those found in credit markets. Our goal in this paper is to present a framework that can be used to quantify these effects and study these interactions.

More specifically, we develop a model of heterogeneous agents in which financial market frictions distort credit and saving decisions by households and firms. Households save for precautionary reasons using either a deposit contract with a bank (formal saving) or cash (informal saving). Saving constraints result from the fact that using the deposit contract is costly. Entrepreneurs can access credit markets when deciding on their capital input, but face collateral requirements due to limited enforcement problems. Saving constraints lead households to seek informal saving instruments (cash) and result in lower aggregate saving. Credit constraints induce misallocation of capital across producers, and in turn this lowers output, productivity, and the return to formal financial instruments.

To discipline the model parameters, we use data from the Colombian Longitudinal Survey, which contains income and occupational data as well as detailed information concerning financial decisions by households. In particular, we use data from this household survey to obtain saving rates and observe the incidence of informal saving. We complement these figures with data from credit markets, firm dynamics, and macroeconomic aggregates.

Our counterfactual exercises suggest that the production efficiency and welfare losses from financial market distortions can be substantial. In fact, completely eliminating saving and credit constraints can double the fraction of households who save and increase the average saving rate by over 60%. Moreover, it can increase total factor productivity (TFP) by 5.6%, output by 25%, and households welfare by 50%. While efficiency gains (TFP and output) are mainly due to

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<sup>1</sup>According to the [Alliance for Financial Inclusion \(2016\)](#), by 2015 over 35 countries had committed to implementing or had already implemented financial inclusion strategies.

<sup>2</sup>Reference studies from the credit frictions literature are [Kaplan & Zingales \(1997\)](#) and [Buera et al. \(2011\)](#), and from the saving constraints literature are [Dupas & Robinson \(2013\)](#) and [Karlan et al. \(2014\)](#).

better capital allocation, the welfare effects result from both higher average consumption and smoother consumption profiles made possible by the first-best saving policy. Interestingly, a sizable fraction of these gains can be attained by a mix of moderate policy reforms that address distortions in both credit and saving decisions.

This paper is related to a number of recent studies addressing the interaction between formal and informal financial markets in developing countries. In this respect, [Wang \(2014\)](#) develops and estimates a dynamic equilibrium model of borrowing and saving decisions that allows him to interpret Thailand's financial reform in 2001 as one that reduced formal borrowing interest rates, lowered costs of access to credit, and relaxed collateral constraints. This reform in turn led to an increase in the proportion of households borrowing formally and to a fall in informal interest rates. He finds that the welfare gains from these policies are smaller than suggested by previous studies that disregard informal saving options.

Furthermore, two streams of the financial development literature are relevant for the present study.<sup>3</sup> One stream has been looking into the determinants of access to and use of savings instruments and their effect on economic outcomes in recent years. This strand of the literature has been mainly focused on the extensive margin, and includes both cross-country studies ([Demirgüç-Kunt & Klapper, 2013](#); [Rojas-Suarez & Amado, 2014](#)) and field experiments inside villages or larger regions within a country (see [Dupas & Robinson, 2013](#); [Kast & Pomeranz, 2014](#); [Prina, 2015](#), to name a few).

Overall, this strand of the literature shows that the world's population –particularly in poor regions– often saves using formal or informal instruments that entail high risk, are costly, and have limited functionality. This leads to low saving rates, with significant welfare consequences: reduced consumption smoothing, low resilience to shocks, and foregone profitable investment opportunities. In a survey of this literature, [Karlan et al. \(2014\)](#) group constraints to saving into five categories; transaction costs, lack of trust and regulatory barriers, information and knowledge gaps, social constraints, and behavioral biases. This paper focuses on market frictions that hinder the supply of savings products.

The other stream has been devoted to quantitatively examining the impact of financial frictions on economic development. Within this strand of the literature, some studies argue that imperfect contract enforceability generates distortions in the allocation of capital across production units that in turn leads to aggregate productivity losses. These studies typically characterize imperfections in financial markets in the form of collateral constraints ([Buera et al., 2011](#); [Buera & Shin, 2013](#); [Midrigan & Xu, 2014](#)). The model developed in this paper, as in these studies,

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<sup>3</sup>For a recent survey of the financial development literature, see [Fernandez & Tamayo \(2017\)](#).

features financial frictions due to limited enforcement and allows studying their implications for capital misallocation.

In a similar vein, several studies focus on the effect of credit market imperfections on occupational choice. Among these, [Buera et al. \(2011\)](#) and [Buera & Shin \(2013\)](#) highlight that financial frictions trigger distortions in the allocation of entrepreneurial talent that result in low aggregate productivity and output. Somewhat similarly, [Antunes et al. \(2008\)](#) show that intermediation costs and contract enforcement may explain cross-country differences in entrepreneurship and other development indicators. Moreover, [Erosa \(2001\)](#) finds that costly intermediation, when individuals choose occupations, has nontrivial consequences for saving behavior and production efficiency.

Relying on this strand of the literature, recent studies have attempted to evaluate the impact of relaxing constraints to financial deepening and inclusion on growth and income inequality. In this regard, [Dabla-Norris et al. \(2015\)](#) analyze three types of financial frictions: participation costs, collateral requirements, and costly monitoring. Their results suggest that the effect of policies alleviating these frictions individually or jointly depends on country-specific characteristics. For Colombia, using the [Dabla-Norris et al. \(2015\)](#) approach, [Karpowicz \(2014\)](#) finds that lowering collateral requirements promises higher growth while inequality would be better tackled through reductions in participation costs.

Missing in the literature are studies quantifying the efficiency gains from ameliorating distortions in the allocation of credit and savings through formal financial instruments. Filling this void is important for at least two reasons. First, development experiments have revealed that general equilibrium effects from relocating savings toward formal financial instruments are important ([Flory, 2017](#)). Second, a significant determinant of the demand for formal savings instruments is its return, which is an endogenous outcome of the financial intermediation process thus affected by credit allocation.

The paper is organized as follows. In Section 2, we present some empirical regularities pertaining to barriers to financial inclusion and patterns of saving behavior in Colombia. The main aspects of the model economy are described in Section 3. Simulations and policy scenarios are presented in Section 4. Section 5 concludes.

## 2 Empirical Regularities

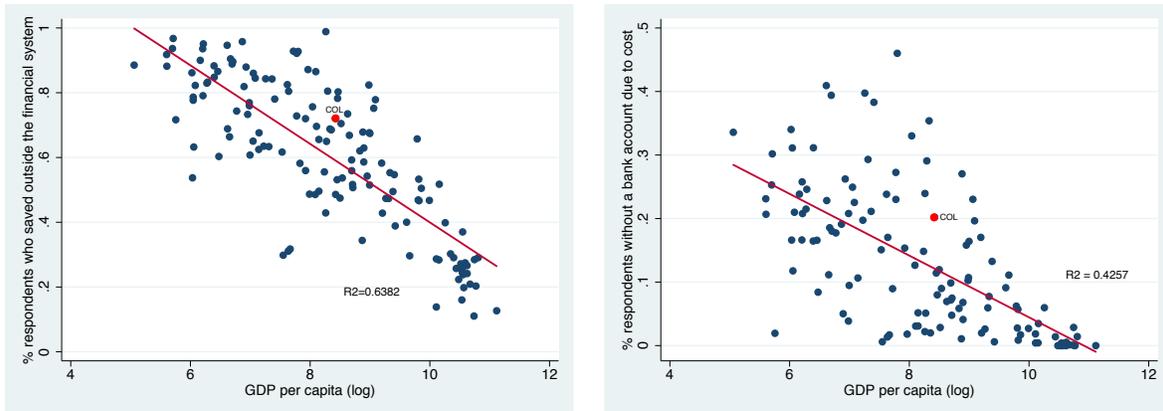
Increasing awareness of the role of the financial sector and the importance of financial inclusion have led to the construction of a growing number of databases and surveys, providing evidence on

access to and use of financial services by households and firms in developing countries. With the aim to lay the ground for our theoretical model, we use some of these data sources to build a set of empirical regularities regarding saving behavior in Colombia. The following lines reflect upon this exercise, while featuring some comparisons with other Latin American peers and countries with similar levels of development.

According to the Global Financial Inclusion Indicators (Global Findex), in Colombia about 72% of the respondents who were savers in 2014 reported that they save outside the financial system. This percentage places the country in the 60th position among 159 nations in terms of use of informal saving instruments such as cash or saving chains. Further, given its level of development (proxied by PPP GDP per capita), Colombia appears to be somewhat of an outlier in this respect (see Figure 1, left panel).

Moreover, Colombia is one of the countries where most respondents (about 20%) did not have an account at a financial institution because they found it too expensive.<sup>4</sup> In this sense, Colombia holds the 37th position among 142 countries, above some Latin American peers such as Brazil, Chile, Uruguay, and Costa Rica. As with informal savings, Colombia’s percentage of financially excluded for cost-related reasons is higher than would be expected given its income level (Figure 1, right panel).

Figure 1: Informal savings and the importance of cost in access to bank accounts



Source: Global Findex 2014.

In order to look at these issues in more detail, we use the financial chapter of the Colombian Longitudinal Survey (henceforth ELCA), which provides specific information on financial

<sup>4</sup>The Global Findex survey asks respondents if they own an account at a “bank or credit union” (or another financial institution, where applicable, like a cooperative in Latin America) (Demirgüç-Kunt & Klapper, 2013, p. 313).

inclusion and the use of financial services for saving.<sup>5</sup> It can be seen from this survey chapter that a considerable share of the respondents usually do not set aside a fraction of their earnings as savings, and hence have not developed saving habits (see Table 1). This share, over 77%, remains invariant between 2010 and 2013, thus suggesting that such behavior is pronounced and persistent.

As for the types of financial instruments used for saving, the figures that emerge from this survey chapter point in the same direction as those resulting from the Global Findex even though the questions asked by the ELCA are somewhat different.<sup>6</sup> According to these figures, also presented in Table 1, nearly 50% of the respondents who were savers in 2013 reported to save “mainly” outside the financial system (i.e., using informal financial instruments). This is a slight increase compared to the share observed in 2010.

Table 1: Incidence and composition of savings

	2010			2013		
	Total	Workers	Firms	Total	Workers	Firms
Does not save	77.2%	72.9%	77.3%	77.2%	73.3%	76.1%
Savers	22.8%	27.1%	22.7%	22.8%	26.7%	23.9%
Formal	52.3%	61.5%	42.8%	50.2%	62.2%	37.5%
Informal	47.7%	38.5%	57.2%	49.8%	37.8%	62.5%

Source: Authors’ calculations based on ELCA.

It is worth noting that this share is higher for households located in lower income deciles and decreases with income, yet informal saving is pervasive even among the highest income deciles. These features are consistent with evidence from an earlier study on access to and use of financial services in Bogotá showing that the “unbanked” –namely, those households and individuals who do not have deposits or savings in formal financial institutions such as commercial and social interest banks, regulated cooperatives, and credit unions– tend to be poor but also include individuals from high income groups and do not differ from the overall population in terms of home ownership (Solo & Manroth, 2006).

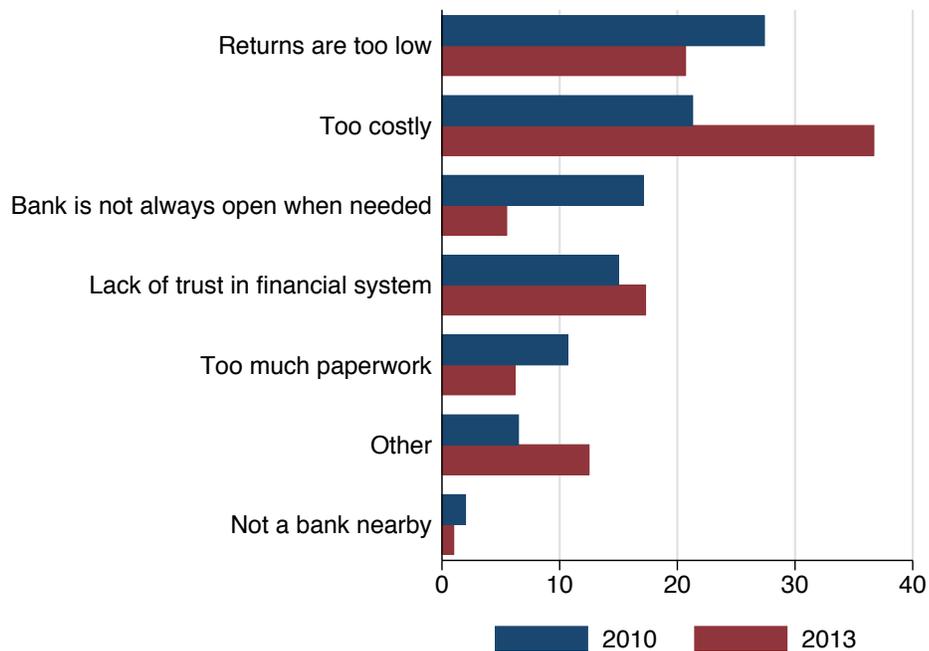
Likewise, recent studies using ELCA data suggest that labor formality is positively correlated with the likelihood of saving using formal financial instruments. Cadena Ordoñez & Quintero

<sup>5</sup>The ELCA is a household survey recently designed and implemented by the Universidad de los Andes. Data from two waves (2010 and 2013) of the survey have been published so far, and the third wave was rolled out in 2016. For methodological details, see Bernal et al. (2014). We use the urban module of the survey, as the rural module lacks comprehensive information on crucial variables such as occupation.

<sup>6</sup>The ELCA surveyors ask people where they “mainly” save. Respondents are given the options of (a) bank or financial institution, (b) cash, (c) employee funds, (d) saving clubs or chains, and (e) other instruments. We assume that all savings in employee funds is channeled through the financial system.

Salleg (2015) show that the use of banks and financial entities is concentrated among people with formal jobs. Similarly, Iregui-Bohórquez et al. (2016) find that household members who are employed in the informal sector save less and that a stable labor situation increases the probability of saving in banks and employee funds or cooperatives and reduces the probability of saving informally. One approach to the relationship between informality and saving behavior through the lens of a dynamic equilibrium model with heterogeneous agents is proposed in Granda & Hamann (2015).

Figure 2: Reasons to save outside the financial system



Source: Authors’ calculations based on ELCA.

In line with the Global Findex data, the ELCA also shows that a major reason why people in Colombia save outside the financial system has to do with the costs of using it. Indeed, Figure 2 suggests that taxes, fees and other charges constitute an important motivation for such a saving pattern.<sup>7</sup> However, it may not be the only reason as, alongside the mentioned costs, low returns

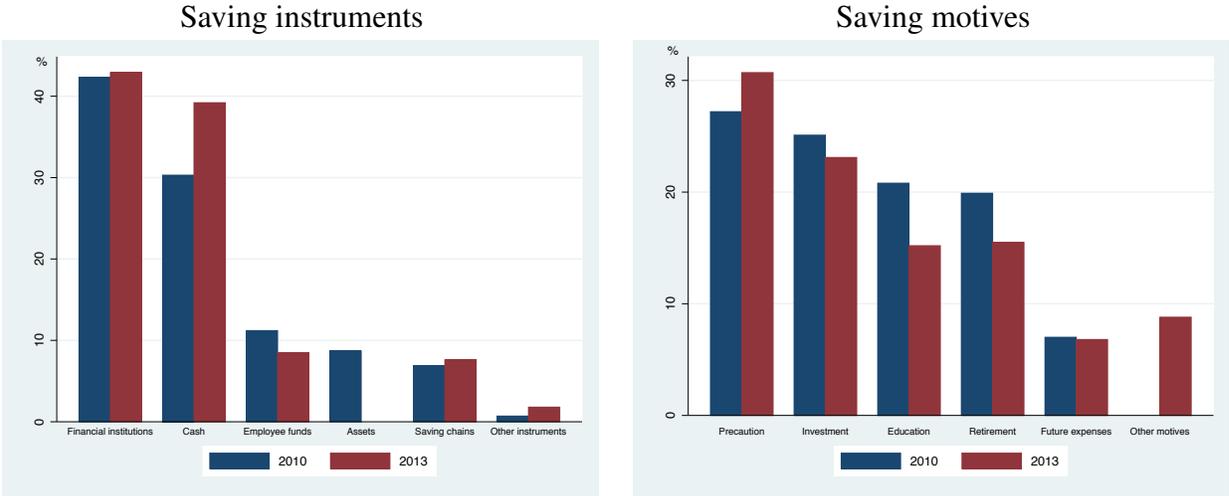
<sup>7</sup> Respondents in 2013 claimed not to save using the financial system mainly because they considered that there was too little money to save (32.4%); however, this reason was reported as one of the least important (around 2%) in 2010. In view of this apparent inconsistency, we opted for excluding this response for the construction of Figure 2.

Also, it is worth noting that the questionnaire of the 2013 wave included reasons for not saving in the financial system that were not explicitly considered in the 2010 wave and that we grouped in Figure 2 as follows: “too costly”

to savings appear to be a crucial factor keeping savers from resorting to banks and other financial institutions. Also, there are cultural and educational elements that may be reflected in lack of trust in the financial system and thereby contribute to encouraging informal saving.

That a significant fraction of saving takes place outside the financial system is observed not only at the extensive margin (i.e., the number of people saving “informally”), but it can also be seen along the intensive margin (the volume of savings channeled through informal instruments). In this regard, Figure 3 (left panel) shows that there was an increase in the fraction of total savings directed toward instruments different from financial institutions and employee funds between 2010 and 2013.<sup>8</sup> The relative share of cash holdings played a major role in such an increase by going up from 30.3% to 39.2%, which gives an idea of how pervasive the use of informal financial instruments is among Colombian households.

Figure 3: Saving instruments and motives



Source: Authors’ calculations based on ELCA.

To understand why Colombians find it appropriate to save outside the financial system, it is necessary to first identify their motivations for saving. As can be seen in Figure 3 (right panel), the ELCA shows in that the main reason people in Colombia save is precaution (i.e., buffering unexpected events), whose share increased from 27.2% in 2010 to 30.7% in 2013. Other im-

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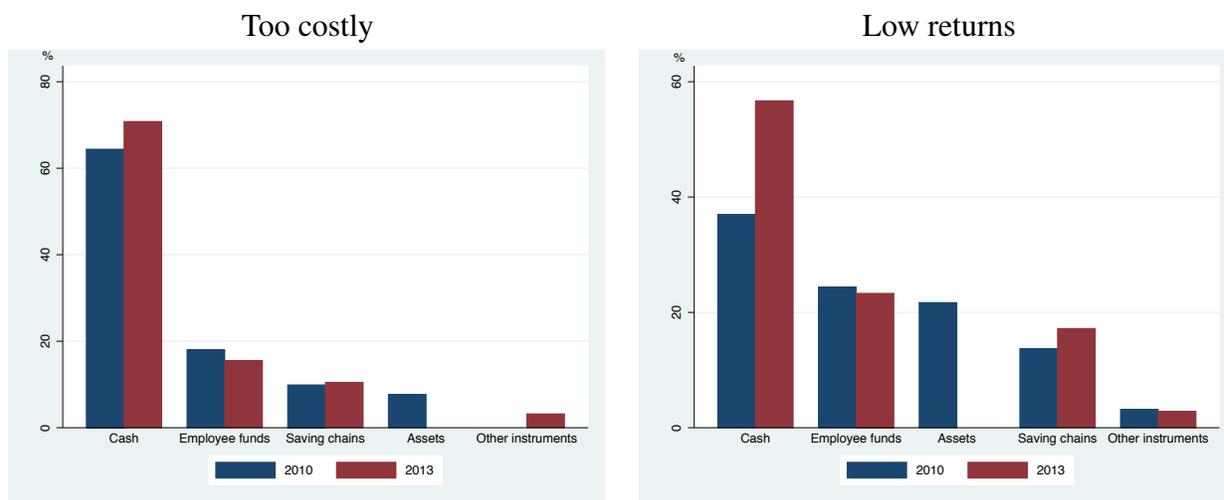
comprises “maintenance fees and use charges are high” and “4 x 1000 is too expensive”; “other” includes “does not know how to”, “believes bank would refuse to open account”, and “bank refused to open account”, among others.

<sup>8</sup>Although we lack direct measures of saving volumes by instruments, we construct a rough estimate of these using ELCA data by creating indicator variables for the different instruments considered in this survey (see footnote 6). Then, for every individual that reports positive savings, we multiply their stated savings amount by the indicator variable corresponding to their mainly used financial instrument. Thus we obtain an estimate of the savings that were deposited in each instrument, formal and informal.

portant motives that lost importance during the mentioned years are investment (accumulation of physical assets and business start-ups), for which the share went down from 25.1% to 23.1%; retirement (from 19.9% to 15.5%); and education (from 20.8% to 15.2%). These figures are somewhat consistent with those of the first nationally representative study on financial capability (see Reddy et al., 2013), which finds that 35.5% of its respondents who claimed to be savers report to save for unforeseen events.

When making their saving decisions, the precautionary motive is perhaps the only one in which returns to savings may be relatively unimportant. In this case, savers should exhibit a preference for liquid instruments that typically offer very low returns, in turn leaving transaction costs (fees, taxes, and other charges) as the main determinant in the choice of instrument. It is therefore not surprising that those reporting that the financial system is “too costly” as their reason for not saving through a financial institution save mostly by holding cash (Figure 4, left panel), while those claiming to save informally because the financial system offers “too low returns” save relatively less in cash and more through saving chains (or similar schemes) and other instruments (Figure 4, right panel).

Figure 4: Saving instruments according to the reason to save outside the financial system



Source: Authors’ calculations based on ELCA.

So far we have been able to establish that in Colombia a significant fraction of people save outside the financial system, and that a sizable fraction of the saving volume may be allocated to informal instruments such as cash or saving chains. Moreover, we have seen that high costs (as entry barriers) and low returns are both important in explaining these patterns of saving behavior.

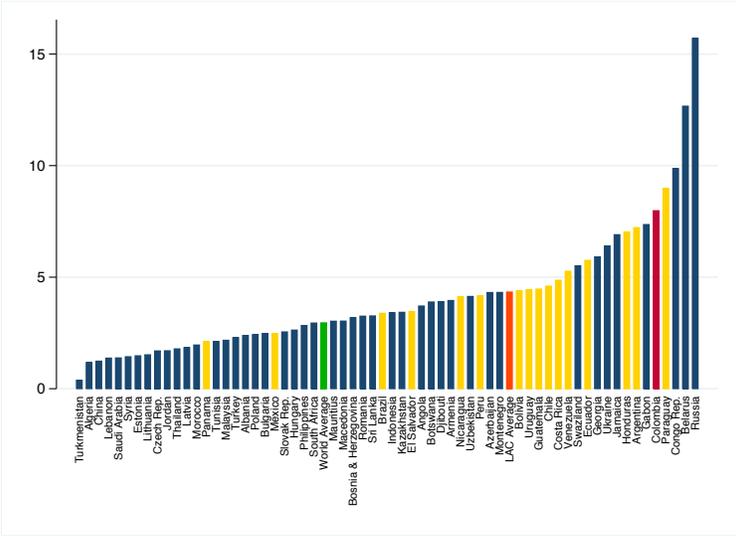
High costs of service typically result from a combination of fixed costs of infrastructure

and a small scale of operation. One can think of these factors as the underlying reason for the negative relationship between GDP per capita and people finding it too expensive to use the financial system (shown in Figure 1 above). That is, more developed countries have larger financial systems allowing providers to fully exploit economies of scale.

High operational costs induce intermediaries to increase the fees charged for extending loans and opening and maintaining accounts as well as lowering interest payments on savings and other deposits (Rojas-Suarez & Amado, 2014). One commonly used indicator of such costs, and thus of intermediation inefficiency, is the ratio of overhead costs to total assets (see Beck et al., 2010). Figure 5 presents data on this measure from the Global Financial Development Database.

In Figure 5, data from Latin American countries is displayed alongside a set of countries with comparable levels of real income per capita. It is worth noting the substantial variation in overhead costs even among countries with similar levels of development, which might be due to differences in the regulatory framework (compliance costs, distortionary taxes) or industrial organization issues (entry barriers, concentration, lack of innovation, etc.). Also, it can be seen that Colombia stands out among the sample because of its banking inefficiency, the second highest and about two times the average of the region.

Figure 5: Bank overhead costs to total assets (%)

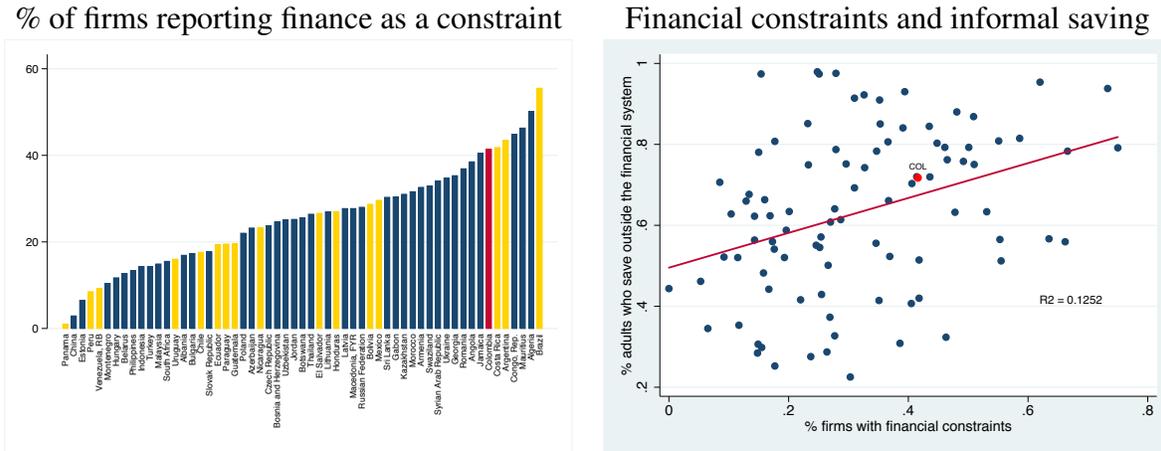


Source: Global Financial Development Database (2011).

Low returns on savings are more difficult to rationalize in an economy where capital is scarce and inflation is relatively low and predictable, as has been the case in Colombia for the last two decades. In fact, data from the International Monetary Fund show that real interest rates on

money market instruments were lower in Colombia during the period 2000–2014 than in Latin American peers like Peru, Mexico, Uruguay, and Brazil. Such low returns on savings can be explained either by costly intermediation or low returns on investment. In this latter respect, our calculations using the methodology of [Caselli & Feyrer \(2007\)](#) suggest that in recent decades the marginal productivity of aggregate capital has been lower in Colombia than in other Latin American countries such as Mexico, Chile, and Peru, as well as in emerging Asia.<sup>9</sup>

Figure 6: Financial constraints for firms



Source: World Bank Enterprise Survey and Global Findex.

What can lie behind such low returns to investment? Recent studies show that this phenomenon is usually associated with financial frictions distorting credit allocation. Such frictions make capital flow toward less profitable projects, which in turn results in aggregate productivity losses and a lower economy-wide return to capital (see [Buera et al., 2011](#); [Midrigan & Xu, 2014](#)). One natural place to look for potential capital misallocation issues lies in indicators of financial constraints. In this respect, data from the Enterprise Survey reveal that, in 2010, 41.5% of Colombian firms reported access to finance as being a major obstacle to their operations. This

<sup>9</sup>The aggregate marginal productivity of capital (MPK) is calculated as:

$$MPK_R = (1 - \theta)\alpha \left( \frac{Y}{K_R} \right) \left( \frac{P_Y}{P_{KR}} \right), \quad (1)$$

where  $\frac{P_Y}{P_{KR}}$  is the inverse of the relative price of capital obtained as the ratio of the investment deflator to the GDP deflator using data from the World Development Indicators;  $1 - \theta$  is the ratio of reproducible capital to total capital (including natural resources) obtained from the World Bank database “Wealth of Nations”; and  $\alpha$ ,  $K_R$ , and  $Y$  are the share of capital in output, reproducible capital, and GDP, respectively, all taken from the Penn World Tables. Using data for 2005, this methodology yields an MPK of 7.6% in Colombia, whereas it is 8.4% in a typical Latin American country, 9.3% in a typical country in emerging Asia, and 11.1% in a typical advanced economy.

places Colombia as the fourth country in Latin America in which firms find themselves more financially constrained (see Figure 6, left panel). Moreover, Figure 6 (right panel) suggests that there is a positive relation between the tightness of financial constraints and the likelihood of saving outside the financial system, presumably because of the low returns channel described above.

### 3 A Model of Credit and Saving Constraints

In this section, we develop a dynamic general equilibrium model with saving constraints at the household level and wherein entrepreneurs are credit constrained. Some of the key features of the model come directly from the evidence found in the ELCA and the World Bank databases: saving in banks is costly, people save mainly for consumption smoothing, the main alternative to bank saving is cash, and firms face borrowing constraints.

The economy is populated by a measure  $N$  of workers and a unit measure of entrepreneurs. Both workers and entrepreneurs are heterogeneous with respect to their productivity and seek to maximize lifetime utility given by

$$\max \left\{ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t) \right\}, \quad (2)$$

where period utility is of the constant relative risk aversion form:

$$u(C) = \frac{C^{1-\chi}}{1-\chi}, \quad (3)$$

with  $\chi > 0$ .

Entrepreneurs can borrow and save with financial intermediaries, but face a collateral requirement that constrains the amount they can borrow. Workers face uninsurable idiosyncratic labor income risk and have access to financial markets. There are two types of financial instruments available: a one-period risk-free asset (formal) and cash (informal).

**Entrepreneurs.** Entrepreneurs have access to a decreasing returns technology that uses labor  $L$  and capital  $K$  to produce a consumption good  $Y$ . Specifically:

$$Y_t = A_t [a \exp(z_t)]^{1-\theta} (K_t^\lambda L_t^{1-\lambda})^\theta. \quad (4)$$

Here,  $a$  is a permanent ability component,  $z$  is a transitory productivity component, and  $A$  is an aggregate efficiency component. Also, fraction  $0 < \lambda < 1$  of output corresponds to capital and

fraction  $1 - \lambda$  to labor, and  $\theta < 1$  is the degree of decreasing returns to variable inputs. Capital depreciates between periods at rate  $\delta$ .

An entrepreneur's idiosyncratic productivity consists of a permanent component  $a$  and a transitory component  $z$ . Permanent entrepreneurial ability (or talent)  $a$  is drawn at birth from a distribution  $\Gamma(\cdot)$ . In each period, a fraction  $1 - \eta$  of entrepreneurs die and is replaced by new ones, in which case the firms they own exit at zero market value.<sup>10</sup> Aggregate efficiency  $A_t$  grows deterministically at a constant rate  $g$ ,  $A_t = A_{t-1}g$ , while the transitory productivity component  $z_t$  evolves over time according to a finite-state Markov process with transition probabilities  $\pi(z', z) = Pr(z_{t+1}|z_t)$  and ergodic distribution  $\xi(z)$ .

Because of growth in  $A$ , most aggregates in this economy are non-stationary with a deterministic trend. Normalizing  $A_0 = 1$  and defining  $\gamma = g^{(1/(1-\alpha))}$ , such a trend can be found to be  $\gamma^t$ .<sup>11</sup> Throughout the paper, we deal with de-trended variables only. Moreover, because  $a$  is permanent and our problem is homogeneous as in [Midrigan & Xu \(2014\)](#), we can scale all variables in the entrepreneur's problem by her permanent productivity  $a$ . Henceforth,  $x$  denotes the de-trended, scaled value of any variable  $X$  (i.e.,  $x_t \equiv (X_t/a^t)$ ).

Entrepreneurs decide how much to borrow ( $d_t$ ) and save ( $b_{t+1}$ ). Since  $b_t = k_t - d_t$ , and  $b$  is pre-determined, choosing  $d_t$  amounts to choosing  $k_t$ . Further, we assume that entrepreneurs cannot fully commit to repaying loans because financial contracts are imperfectly enforceable. In particular, defaulting entrepreneurs keep a percentage  $1 - \phi$  of their capital stock; the remaining fraction  $\phi$  is recovered by the lender.<sup>12</sup> Finally, we assume that all saving by firms is done through the one-period bank deposit and that using a bank to save requires paying a per-period fixed cost  $\tau$ .<sup>13</sup>

Given prices,  $(r, w)$ , an entrepreneur's problem can be stated recursively as:

$$V(b, z) = \max_{b', k, l} \frac{c^{1-\chi}}{1-\chi} + \beta \eta \gamma^{1-\chi} \sum_{z'} V(b', z') \pi(z'|z) \quad (5)$$

subject to

$$c + \gamma b' + \tau = [\exp(z)]^{1-\theta} (k^\lambda l^{1-\lambda})^\theta - (r + \delta)k - wl + (1 + r)b \quad (6)$$

<sup>10</sup>This reflects the fact that firms exit the market for reasons not internalized by the model. It is well known that without exogenous exit some firms would eventually accumulate enough assets to overcome borrowing constraints and over time the mass of firms would grow without bound ([Quadrini, 2004](#)).

<sup>11</sup>For further details, see [Appendix 6.1](#).

<sup>12</sup>Although the collateral constraint ensures that all contracts are enforceable, if a firm were to default, it would exit the market irreversibly.

<sup>13</sup>This follows the evidence found in [Didier & Schmukler \(2014\)](#) that virtually all firms in Latin America own and use formal bank accounts.

and the collateral constraint

$$d \leq \phi k, \quad (7)$$

which can be rewritten as

$$k \leq \frac{b}{1 - \phi}. \quad (8)$$

Notice that our specification of the collateral requirement is virtually identical to that used in [Midrigan & Xu \(2014\)](#).

**Workers.** Each worker is endowed with a unit of labor which is supplied inelastically. Labor income, however, depends upon the worker's idiosyncratic efficiency, which is a composite of innate (permanent) ability  $v$  and transitory shocks  $\varepsilon_t$ . Permanent ability is drawn from a distribution  $\Omega(\cdot)$ , while  $\varepsilon_t$  evolves over time according to a finite-state Markov process with transition probabilities  $\psi(\varepsilon', \varepsilon) = Pr(\varepsilon_{t+1} | \varepsilon_t)$  and ergodic distribution  $\mu(\varepsilon)$ . Workers save using cash  $s$  or a one-period deposit contract with a bank  $q$ . While cash (the “informal” instrument) yields no interest, deposits (the “formal” instrument) yield a non-negative risk-free rate of return  $r$ .<sup>14</sup> Those workers who engage in deposit saving must pay a fixed cost  $\tau$  for every period they use the bank. As with the entrepreneurs, we write the workers' problem in terms of de-trended, scaled variables (i.e.,  $x_t \equiv (X_t / v \gamma^t)$ ).

Given prices  $(r, w)$ , a worker's problem can be stated recursively as:

$$W(q, s, \varepsilon) = \max_{c, q', s'} \frac{c^{1-\chi}}{1-\chi} + \beta \gamma^{1-\chi} \sum_{\varepsilon'} W(q', s', \varepsilon') \psi(\varepsilon' | \varepsilon) \quad (9)$$

subject to

$$c + \gamma q' + \gamma s' + \tau \mathbb{I}_{q' > 0} = w \exp(\varepsilon) + (1 + r)q + s \quad (10)$$

and the no-borrowing constraints

$$q \geq 0, s \geq 0, \quad (11)$$

where  $\mathbb{I}_{q' > 0}$  is an indicator variable that equals one if the worker saves using the formal instrument, and zero otherwise.

**Financial intermediaries.** Banks take deposits from workers and lend them to firms. Because all contracts are strictly enforceable (i.e., there is no default in equilibrium), firms pay and workers receive exactly the risk-free rate which is endogenously determined. Naturally, some

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<sup>14</sup>In this sense, one might think of cash saving as a storage technology with zero returns next period.

firms will face a higher shadow price of capital than others depending on whether the collateral constraint binds, and some workers will face a lower return once they account for fixed costs of deposit market participation.

**Equilibrium.** The scaled and de-trended economy has a stationary equilibrium that consists of a set of prices  $(w, r)$ , stationary distributions of workers  $g$  and entrepreneurs  $h$ , and decision rules  $\{c(q, \varepsilon), q_{+1}(q, \varepsilon), s_{+1}(q, \varepsilon), b_{+1}(b, z), k(b, z), l(b, z)\}$ , where “ $_{+1}$ ” stands for one period ahead, such that:

- All workers and entrepreneurs optimize, that is,  $l(b, z), k(b, z), b'(b, z)$  solve problem (4)-(7) and  $c(q, \varepsilon), q'(q, \varepsilon), s'(q, \varepsilon)$  solve (8)-(9);
- The labor market clears,

$$\sum_{b,z} h(b, z) l(b, z) = N \sum_{\varepsilon} \varepsilon \mu(\varepsilon); \quad (12)$$

and

- The asset market clears,

$$\sum_{b,z} h(b, z) k_{+1}(b, z) = \sum_{q,s,\varepsilon} g(q, s, \varepsilon) q_{+1}(q, s, \varepsilon) + \sum_{b,z} h(b, z) b_{+1}(b, z). \quad (13)$$

## 4 Quantitative Performance

In this section we describe how data are used to calibrate the model presented in Section 3. We also present a series of policy experiments that allow us to quantify the costs associated with saving and credit constraints in a developing economy such as Colombia.

### 4.1 Calibration

The model is calibrated to be consistent with a number of features of the Colombian economy. We divide the parameter vector into two groups. The first group includes preference and technology parameters that are difficult to identify using our data (see Table 2). We assign these parameters values that are common in the existing dynamic, general equilibrium (DGE) literature. Accordingly, the period is set to one year so that the discount factor is equal to 0.958. This is a common value in studies on emerging market economies. The risk aversion coefficient is set to 2.3, which is close to the value estimated for Colombia in Prada & Rojas (2010).

As for the technology parameters, [Zuleta et al. \(2010\)](#) obtain several estimates of the factor shares that we in turn use to get a measure of returns to scale. Accordingly, the capital income share  $\lambda$  is set equal to 0.46 and the share of variable inputs  $\theta$  sums to 0.85. Also, the depreciation rate  $\delta$  is set to 0.075 as in [Hamann & Mejía \(2013\)](#). Further, the survival rate of entrepreneurs  $\eta$  is set so that  $1 - \eta = 0.07$  in order to match the average firm exit rate in the manufacturing sector, as reported in [Eslava et al. \(2013, Table 1\)](#). Finally, the trend growth parameter  $\gamma$  corresponds to the long-run output growth rate and is estimated as the average annual growth rates of output from 1976 to 2012 using yearly data from the National Department of Statistics (DANE).

Table 2: Preference and technology parameters

Parameter	Value	Description	Source
$\beta$	0.958	Discount factor	DGE literature
$\chi$	2.300	Risk aversion coefficient	DGE literature
$\theta$	0.850	Share of variable inputs	<a href="#">Zuleta et al. (2010)</a>
$\lambda$	0.460	Capital share in output	<a href="#">Zuleta et al. (2010)</a>
$\delta$	0.075	Capital depreciation rate	<a href="#">Hamann &amp; Mejía (2013)</a>
$1 - \eta$	0.070	Firm exit rate	<a href="#">Eslava et al. (2013)</a>
$\gamma$	1.038	Trend output growth	DANE

Unlike the above, parameters in the second group are chosen to replicate certain moments of the Colombian data (see [Table 3](#)). First, the transitory productivity component of workers  $\varepsilon$  is assumed to evolve according to a first-order autoregressive process with Gaussian disturbances and is discretized into a 10-state Markov chain using the [Rouwenhorst \(1995\)](#) method. The autocorrelation coefficient  $\rho_\varepsilon$  and the standard deviation  $\sigma_\varepsilon$  are chosen to approximately match the saving rate and the fraction of non-savers obtained from the financial module of the ELCA. Similarly, the transitory productivity component of entrepreneurs  $z$  follows an AR(1) process discretized into a 15-state Markov chain so that the persistence parameter  $\rho_z$  and the standard deviation  $\sigma_z$  approximately mimic the entrepreneurial saving rate and the fraction of non-savers.

Second, the permanent skill component for workers  $v$  is assumed to follow a truncated and discretized version of a Pareto distribution with probability density  $\Omega(v) = \omega v^{-(\omega+1)}$  for  $v \geq 1$ . The tail parameter  $\omega$  is chosen to replicate the share of labor income generated by the top 1% of workers also obtained from the ELCA. Likewise, entrepreneurial ability  $a$  is assumed to be a truncated and discretized version of a Pareto distribution with probability density  $\Gamma(a) = \zeta a^{-(\zeta+1)}$  for  $a \geq 1$ , so that the tail parameter  $\zeta$  aims to mimic the share of total income generated by the top 1% of the population computed from the National Household Survey (GEIH).

Finally, we calibrate the parameters that govern the functioning of financial markets. In this

regard, the cost of using formal saving instruments  $\tau$  is set to match the fraction of savers that resort to formal financial instruments obtained from the ELCA. Also, the parameter that captures limited enforcement  $\varphi$  is chosen to replicate a proxy of the credit-to-output ratio that measures the ratio of credit to enterprises (corporate plus microcredit) to private value added computed using data from Banco de la República (the Central Bank of Colombia).

Table 3: Summary of calibrated parameters

Param	Value	Description	Target	Source
$\omega$	1.900	Tail param Pareto workers	Top 1% income share (workers)	ELCA
$\zeta$	2.010	Tail param Pareto firms	Top 1% income share (all)	GEIH
$\rho_\varepsilon$	0.675	AR(1) labor productivity	% of workers who do not save	ELCA
$\sigma_\varepsilon$	0.235	Std dev labor productivity	Workers saving rate	ELCA
$\rho_z$	0.150	AR(1) entrep productivity	% of entresps who do not save	ELCA
$\sigma_z$	0.560	Std dev entrep productivity	Entrepreneurs saving rate	ELCA
$\varphi$	0.165	% of pledgeable collateral	Credit-to-output ratio	Central Bank
$\tau$	0.020	Fixed cost of formal saving	% of formal savers	ELCA

The resulting economy, as can be seen in Table 4, resembles the targeted moments fairly well. Specifically, the model economy appropriately replicates key statistics such as the workers' saving rate, the fraction of households that saves using formal financial instruments, and the credit-to-output ratio. Also, the benchmark economy mimics the percentage of income owned by the top 1% of the workers' and the economy-wide income distributions very closely. Yet the model is not as successful in replicating all statistics. In particular, it overpredicts the share of entrepreneurs who are savers. In this sense, it must be noted that the ELCA surveys mostly small entrepreneurs that are not representative of the full entrepreneurial population that the model aims to portray.

Table 4: Calibration results

Targeted moment	Data	Model
% of workers who do not save	73.3%	62.9%
% of formal savers	62.2%	63.1%
Workers saving rate	12.1%	12.0%
% of entrepreneurs who do not save	76.1%	20.8%
Entrepreneurs saving rate	23.9%	19.4%
Credit-to-output ratio	0.318	0.312
% income in top 1% (workers)	7.2%	7.1%
% income in top 1% (economywide)	11.3%	11.1%

## 4.2 Counterfactual Analysis

In order to study the effects of alternative financial inclusion policies, we analyze the implications and limitations of a number of policy scenarios. The first experiment aims to measure the impact of eliminating all costs associated with formal saving. This is accomplished by reducing the costs of using the financial system  $\tau$  from its calibrated value to zero. The results of such a reduction are presented in the second column of Table 5, in which, to facilitate comparison, we reproduce the performance of the benchmark economy under the label “model Colombia”.

Table 5: Policy experiments

Statistic	Model “Colombia”	$\tau = 0$ $\varphi = \text{Colombia}$	$\tau = 0$ $\varphi = \text{Chile}$	Model “Efficient”
% of workers who do not save	62.9%	62.5%	33.1%	27.0%
% of formal savers	63.1%	100.0%	100.0%	100.0%
Workers saving rate	12.0%	11.5%	12.1%	19.2%
% of entrepreneurs who do not save	20.8%	20.7%	24.8%	50.1%
Entrepreneurs saving rate	19.4%	19.3%	19.6%	21.2%
Credit-to-output ratio	0.31	0.32	0.72	2.35
% income in top 1% (workers)	7.1%	7.1%	7.1%	7.0%
% income in top 1% (economywide)	11.1%	11.3%	10.5%	8.1%
% of capital financed by firms	83.6%	83.5%	65.4%	8.9%
Capital intensity (K/Y)	1.909	1.932	2.069	2.584
Aggregate output	31.540	31.860	33.324	39.504
Total factor productivity (TFP)	1.971	1.974	1.988	2.072
Net real interest rate	6.31%	4.66%	6.05%	7.59%
Real wage rate	0.390	0.392	0.410	0.488
Welfare (utilitarian)				
Households (workers)	-1,646.2	-1,613.2	-1,364.6	-657.2
Firms (entrepreneurs)	-248,663.1	-250,474.5	-208,193.9	-58.5
Income distribution				
% income in quintile 5	59.4%	60.0%	57.0%	42.8%
% income in quintiles 1 and 2	15.2%	15.1%	15.8%	22.1%
% income in quintiles 3 and 4	24.4%	24.9%	27.1%	35.1%

It can be seen that accomplishing  $\tau = 0$  entails some benefits for households, as an important fraction of these could now save using a formal financial instrument. An important metric for comparison of policy experiments is the so-called utilitarian welfare measure. This measure

assigns equal weights to each household’s welfare, which in turn is calculated as the present value of intertemporal utility when every household follows her optimal plan:

$$\mathbb{W}^* = \sum_{q,s,\varepsilon} W(q,s,\varepsilon)g(q,s,\varepsilon) \quad (14)$$

where  $W(q,s,\varepsilon)$  is as found in equation (9). Using this metric, it can be observed that eliminating the costs of formal saving increases household welfare by 2%. This effect takes place not only due to an increase in average consumption, but also because formal saving allows households to better smooth consumption in the face of income shocks. Also, it is noteworthy that a “policy” of free formal saving gives rise to an increase in aggregate output of 1% and in capital intensity of 1.2%.

It is important to note that, since the economy is closed, this policy affects the interest rate to influence the observed results. However, these interest rate changes have a moderate effect on the credit-to-output ratio, which increases by only 2.2%. This occurs because the policy is not accompanied by any reforms in access to credit whatsoever; that is, there are no changes in the financial constraints faced by entrepreneurs ( $\varphi$  remains constant). Indeed, the observed impact of reducing the costs of using the financial system on saving behavior is a combination of two countervailing effects. First, a higher percentage of households save in deposits and receive returns  $rq$ , which increases their (non-labor) income. Second, a general equilibrium effect ensues, as the higher supply of loanable funds –absent any significant change in the demand for credit– lowers the interest rate (discouraging saving).<sup>15</sup> In a small open economy, where the interest rate is more or less determined by the rate that prevails in international capital markets, the effect of reducing the costs of formal saving on welfare is much higher as the general equilibrium effect is absent. In this sense, this experiment must be considered as a lower bound for the impact that a policy incentivizing formal saving could bring about.

The next experiment we consider precisely addresses the issue discussed above. That is, we complement the “formal saving policy” with a financial reform that reduces –although it does not eliminate– enforcement problems in the credit market. In particular, we raise  $\varphi$  so that the resulting credit-to-output ratio increases to a level similar to the one observed in Chile, a country frequently used as a leading example for Colombia in terms of financial market development. The outcomes are presented in the third column of Table 5.

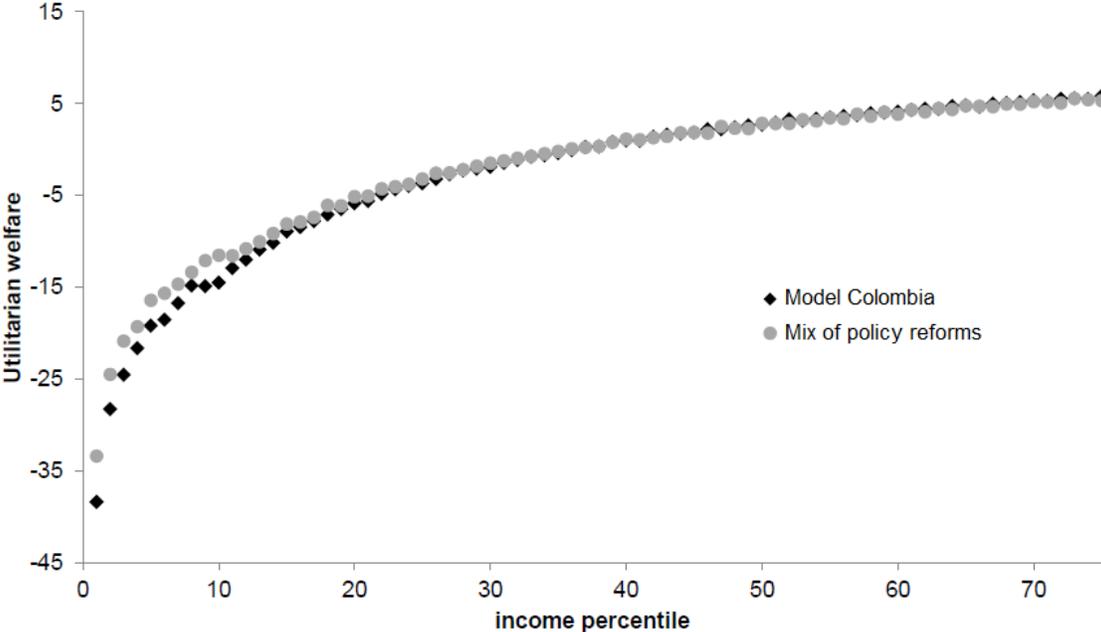
A number of results from this combination of reforms are worth noticing. First, although the saving rate of savers remains roughly constant, the fraction of households that save almost

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<sup>15</sup>Note the marginal fall in the saving rate and the fraction of non-savers.

doubles (from 37.5% to 66%), which in turn means that the aggregate saving rate increases substantially. This result is in line with the argument made recently by [Inter-American Development Bank \(2016, Chapter 11\)](#) that multi-faceted financial reforms are needed to promote higher domestic savings in Latin America. Given this increase in saving by workers, the fraction of firms that save and the share of capital financed by firms both fall. Capital intensity increases by 8.4%, while output rises by 5.6%. Most importantly, the welfare of both households and entrepreneurs increases substantially, by 15% and 17%, respectively. In this economy, the interest rate is higher because the productivity of capital increases, as is the wage rate. Finally, there is a moderate increase in aggregate TFP and a moderate decrease in income inequality. Figure 7 illustrates that the latter effect gives rise to a situation in which the increase in welfare that results from the combination of reforms is larger for the lowest percentiles of the income distribution.

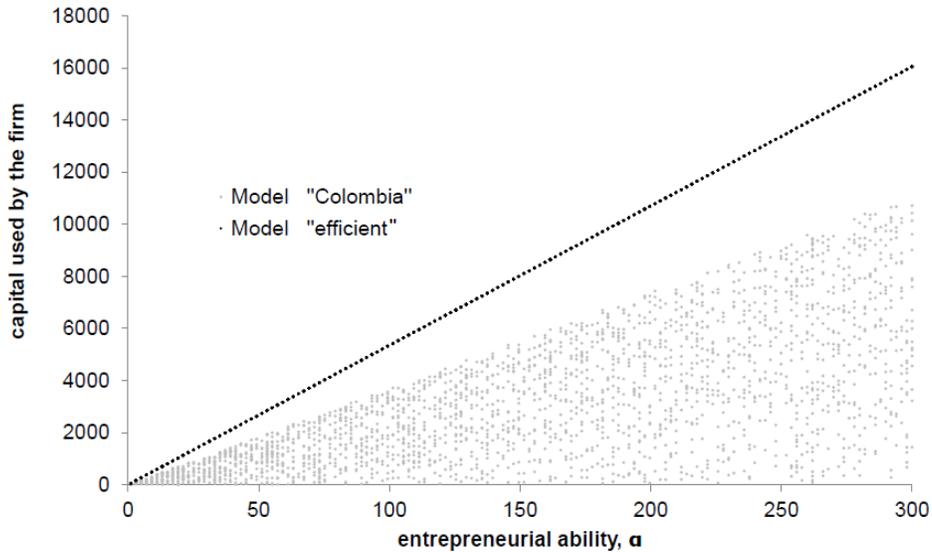
Figure 7: Financial reforms and welfare



The final experiment that we consider implies relaxing completely both saving and credit constraints. This is easily done by setting  $\tau = 0$  and  $\varphi = 1$ . The results from this experiment are presented in the last column of Table 5 under the “efficient” label. In this economy, over 70% of workers save and thus they finance virtually the entire capital stock. Most importantly, losses due to misallocation are eliminated as entrepreneurial talent becomes the only determinant of capital input (i.e., credit frictions do not constrain firm size). This point is illustrated by Figure 8, which plots the capital input for each level of entrepreneurial ability under both parameteriza-

tions (“Colombia” and “efficient”). Compared with the model calibrated to Colombia, workers’ saving rate is almost 60% higher, the capital-to-output ratio is 35% higher, output is 25% higher, and TFP is over 5% higher. Welfare increases by over 50% for workers and nearly 100% for entrepreneurs.

Figure 8: Entrepreneurial ability and capital allocation



## 5 Concluding Remarks

In this paper, we used recently collected survey data to study the costs associated with saving and credit constraints through the lens of an otherwise standard heterogeneous agents setting. In our model, the costs of using financial instruments distort saving decisions by households, leading to volatile consumption profiles. These constraints interact with credit frictions to generate a vicious circle of informal savings, capital misallocation, and low returns to formal saving instruments.

Our quantitative results point to potentially large gains to be made in terms of production efficiency and welfare by removing these constraints. These provide support to the importance of comprehensive strategies to develop financial markets, especially in developing countries. At the same time, our results suggest that this type of study could greatly complement the growing literature on small-scale field experiments associated with financial inclusion policies.

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## 6 Appendix

### 6.1 De-trending

Let  $\tilde{X}$  be the value of  $X$  after scaling by permanent ability components  $z, a$ . Next, let  $x$  stand for the de-trended value of  $\tilde{X}$ . The programming problems before de-trending can be written as follows. Workers:

$$\max_{\tilde{C}_t, \tilde{S}_{t+1}, \tilde{Q}_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\tilde{C}_t^{1-\chi}}{1-\chi} \quad (15)$$

subject to

$$\tilde{C}_t + \tilde{Q}_{t+1} + \tilde{S}_{t+1} + \tilde{\tau} \mathbb{I}_{\tilde{Q}_{t+1} > 0} = \tilde{W}_t \exp(\varepsilon_t) + (1+r_t)\tilde{Q}_t + \tilde{S}_t \quad (16)$$

Entrepreneurs:

$$\max_{\tilde{C}_t, \tilde{K}_t, \tilde{B}_{t+1}, l_t} E_0 \sum_{t=0}^{\infty} (\beta\eta)^t \frac{\tilde{C}_t^{1-\chi}}{1-\chi} \quad (17)$$

subject to

$$\tilde{C}_t + \tilde{B}_{t+1} = A_t \exp(z_t)^{1-\theta} (\tilde{K}_t^\lambda l_t^{1-\lambda})^\theta - \tilde{W}_t l_t - (r+\delta)\tilde{K}_t + (1+r)\tilde{B}_t - \tilde{\tau} \quad (18)$$

$$\tilde{K}_t \leq \frac{\tilde{B}_t}{1-\varphi} \quad (19)$$

To save on notation, first define  $\alpha = \lambda\theta$  and  $\vartheta = (1-\lambda)\theta$ , so that output is given by  $\tilde{Y}_t = A_t \exp(z_t)^{1-\theta} \tilde{K}_t^\alpha l_t^\vartheta$ . In a balanced growth path,  $\tilde{Y}, \tilde{B}, \tilde{W}, \tilde{S}, \tilde{Q}, \tilde{C}$ , and  $\tilde{K}$  exhibit a common trend. To find this common trend, recall that  $A_t = A_{t-1}g = A_0g^t$ . Normalizing  $A_0 = 1$ , the common trend can be found to be  $g^{(1/(1-\alpha))t}$ . To see this, divide by this factor:

$$\frac{\tilde{Y}_t}{g^{(1/(1-\alpha))t}} = \frac{g^t \exp(z_t)^{1-\theta} \tilde{K}_t^\alpha l_t^\vartheta}{g^{(1/(1-\alpha))t}}. \quad (20)$$

Now, re-write  $g^{(1/(1-\alpha))t}$  as  $g^{(\alpha/(1-\alpha))t} g^t$  so that

$$\frac{\tilde{Y}_t}{g^{(1/(1-\alpha))t}} = \exp(z_t)^{1-\theta} \left( \frac{\tilde{K}_t}{g^{(1/(1-\alpha))t}} \right)^\alpha l_t^\vartheta. \quad (21)$$

Hence  $\tilde{Y}_t = y_t g^{(1/(1-\alpha))t}$  and  $\tilde{K}_t = k_t g^{(1/(1-\alpha))t}$ . To save on notation, we define  $\gamma = g^{(1/(1-\alpha))}$  so that the common trend is  $\gamma^t$ , but recall that  $g$  (not  $\gamma$ ) is the rate at which TFP grows. While all trending variables are divided by  $\gamma^t$ , the problems for workers and entrepreneurs can be stated in terms of scaled, de-trended variables as in the main text.

## 6.2 Comparative statics

Below we present the full set of plots that result from the comparative statics with respect to the two main parameters of interest  $(\varphi, \tau)$ .

Figure 9: Comparative statics with respect to  $\varphi$

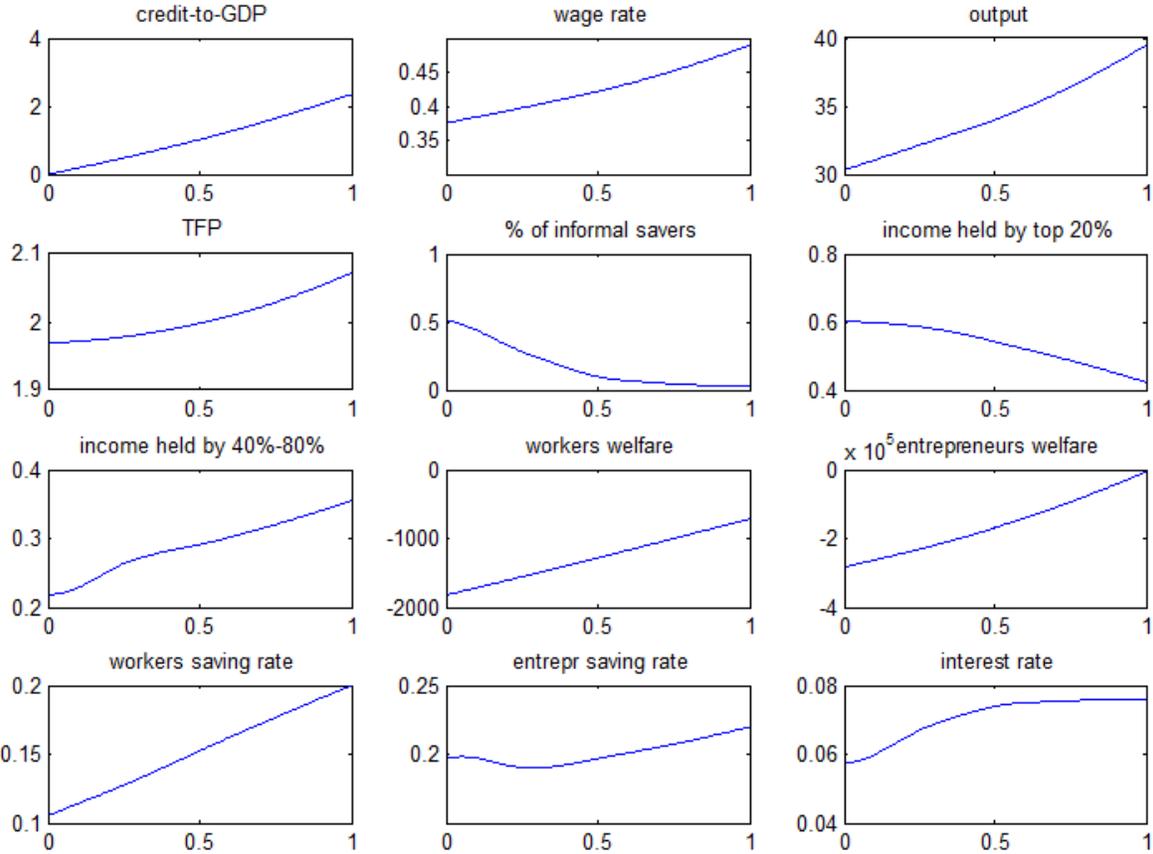


Figure 10: Comparative statics with respect to  $\tau$

