Tax-Transfers Schemes, Informality and Search Frictions in a Small Open Economy

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Abstract In this paper we evaluate the impact of market-oriented structural reforms, in particular labor market policies (payroll and severance taxes), social assistance programs (subsidized health), and trade liberalization upon long run unemployment, wage inequality and the distribution of employment across sectors in a small open economy with search frictions and idiosyncratic productivity shocks. We build an equilibrium search and matching friction model of the labor markets, a substantial extension of the search and matching friction model developed by Mortensen and Pissarides (1994) with endogenous job destruction and stochastic job matching, in which an informal sector is incorporated and heterogeneous agents are subject to idiosyncratic productivity shocks. We solve the model numerically and calibrate the structural parameters of the model using micro data. We simulate the model for the Colombian economy by performing a set of experiments consistent with the structural reforms implemented in the 1990's and 2000's. Changes in relative prices that negatively affect the relative profitability of the formal sector have quite sizable compositional and distributional effects. These produce more long run unemployment and informality, and potentially widen the income gap. Changes in labor taxes have small effects if workers do not value social security services, but may have more sizable effects if workers associate high payroll taxes with more valuable and efficient social security services. An expansion of public health insurance to informal sector workers has minor aggregate and distributional effects.

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JEL Classification Numbers: J21, J32, J64, C15

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1 Introduction

In this paper we build an equilibrium search and matching model of a labor market in a developing economy with a large informal sector to understand the impact of policy reforms and market shocks on steady state unemployment, wage dispersion and distribution of employment across sectors in an economy with a persistently large informal sector.

The central idea behind modeling the labor market with a search model (instead of a neoclassical one) is that, given the existence of heterogeneity, frictions or imperfect information, search is a costly activity for both firms and workers who must spend resources before job creation and job destruction occurs. In these types of models equilibrium unemployment in the steady state emerges naturally as a result of the transitions in and out of unemployment, since some existing jobs break up before new matches are made. None of these properties characterize Walrasian labor markets.

The model is particularly relevant for developing economies, in particular Latin American (LA) economies, where wage inequality, high unemployment, job instability and large informal sectors are longstanding concerns, particularly in the last two decades (Inter-American Development Bank, 2004) [1].

After the 1990's, many LA countries followed a sequence of market-oriented structural reforms, including changes in labor market legislation, social security programs, and changes in the degree of trade and financial openness. The extent to which these countries' labor market, trade and social security reforms contribute to deteriorating labor market conditions is still a debatable question in the literature. Some other factors including aggregate and idiosyncratic market shocks, demographic changes affecting the size of the labor force, or skill biased-technological change may also play an important role[2].

We simulate the model for the Colombian economy, a country that in the 1990's and 2000's implemented substantial market -oriented reforms whose main goal was to deregulate labor and financial markets.

There are some previous empirical studies focusing on the effects of Colombian structural reforms on labor markets: Cardenas, Gutierrez (1996); Cardenas, Kugler and Bernal (1998); Kugler (1999); Camacho, Conover, Hoyos (2009); Eslava, Haltinwager, Kugler & Ku-

[1] The informal sector is particularly high in Latin American economies (from 30 to 70 % of GDP) according to some studies (Maloney, 2004). Other studies (Schneider, 2005) suggest also that the size of the informal sector in other economies is not negligible. They estimate that, for the period 1999-2000, the average size of the informal economy (as % of GDP) is 41.2 % in African countries, 26.3% in Asian countries, 37.9% in transitional economies and 16.7% in OECD countries

[2] For a survey of the impact of labor market regulations in Latin America see Heckman and Pages (2004), on the impact of trade liberalization and job turnover, see Haltinwager et al(2004). For a survey of the effect of trade on wage inequality and informality in Latin America and other developing economies see Goldberg and Pavcnik (2007).

gler (2010). Most of these studies employ reduced form estimation and none of them build a search model as an analytical framework to understand incentives behind flows across formal and informal sector. We expect to provide new lenses to study the impact of the reforms by building and calibrating a structural model.

Some studies have looked at informality in Latin American economies using a flow approach to unemployment: Bosch & Maloney (2007a, 2007b, 2008); Bosch, Goni & Maloney(2007); Fiess, Fugazza & Maloney (2008). Our model contributes to a growing recent literature that combines informality with labor market search frictions in emerging economies: Albrecht, Navarro & Vroman (2009); Bosch and Esteban-Pretel (2009); Cosar, Guner & Tybout (2011), Meghir, Narita and Robin (2012).

We develop a substantial modified version of the search and matching frictions model with endogenous job destruction by Mortensen and Pissarides (1994)(MP1994 hereafter). The main differences between this model and MP1994 are the following: a) An informal sector is added; b) There is a continuum of worker types; c) The idiosyncratic productivity shock process is modeled differently. While in MP1994 matches starts as "state of the art" or, at the fixed maximum productivity level, in this model they start with a draw from a distribution (stochastic job matching).

The model is similar in spirit to the one developed by Albrecht, Navarro, Vroman (2009) (ANV2009 hereafter) with an informal sector, but the modeling of the informal sector is different, capturing the idea that the informal sector is a 'disadvantaged' sector of a dualistic or segmented labor market [3], instead of an unregulated self-employment sector where agents stay or go as a matter of choice. Even though the informal sector has close linkages to the formal sector, it is still a 'disadvantaged' sector characterized by low entry barriers in terms of skills, and therefore, populated by workers with low productivity levels that are excluded or segregated from the formal economy.

The main differences with ANV2009 are the following: 1) In this model there are direct flows from formal to informal, while in their model there are no flows across sectors[4]; 2) In this model there is ex-post match-specific heterogeneity, so a workers' type (individual specific characteristics relevant to the labor market such as human capital level) is not the same as match-specific productivity; workers of a given type are not certain of whether they will be 'good' or 'bad' at a specific job, so we assume initial productivity in a match is a

[3]There is some supporting empirical evidence of a segmented labor market for the Colombian case. Most of the arguments are supported by the fact that the informal sector is countercyclical, absorbing labor during downturns. See Fiess, Fugazza, Maloney (2008) and Mondragon, Pena, Wills (2010).

[4]As shown in the next section, the estimated transition probability of flowing from formal to informal is not negligible: 41.41 % for the period June 2002 to June 2003. When considering the flow from informal to formal the estimate is smaller: 23.81 %.

draw from a distribution and not a fixed value (stochastic job matching); 3) Workers in both sectors have productivity shocks that may be 'good' or 'bad' with respect to the match productivity, while in their model, only workers in the formal sector are subject to shocks, and productivity shocks are always 'bad' shocks; 4) To understand the impact of social assistance programs and adapt the model to the Colombian case, workers in this model contribute to social security in the formal sector and have access to subsidized health in the informal sector, features that are not present in ANV2009.

The paper is organized as follows. The next section introduces some stylized facts of the Colombian labor markets, and a brief summary of the reforms. In section 3 and 4 we present the model and the empirical strategy. In section 5 we discuss the main conclusions.

2 Context: The Colombian Labor Markets and the Structural Reforms

Some stylized facts of the Colombian urban labor markets in the last two decades include a higher and more volatile long-run unemployment and informality rates, a higher (overall) wage inequality, and a rise in relative earnings favoring formal sector workers.

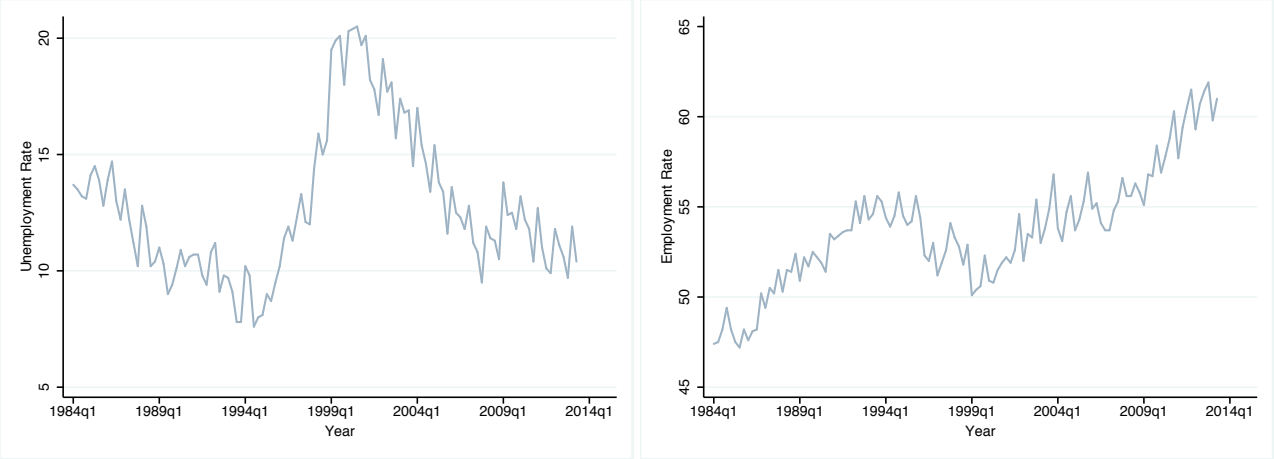
In the period 1984:Q1 to 2013:Q2, the unemployment rate is countercyclical, more volatile than real GDP, and highly persistent[5]. Increasing levels of unemployment associated with an economic recession were observed after the mid-1990s, reaching a peak in 2000:Q3, at 20.5%. Then, unemployment started a downward trend until reaching 10.4% in 2013:Q2. When looking at long-run levels, average unemployment went up from 11.81 % in the period 1984: Q1 to 1999: Q4 to 13.91 % in the period 2000:Q1-2013:Q2, and its volatility also increased [6].

Average long run employment rates follow an upward trend, increasing from 51.97 % in the period 1984: Q1 to 1999: Q4, to 55.77 % in the period 2000:Q1-2013:Q2.

[5] Autocorrelation coefficient of 67.3%

[6] Standard deviation went up from 2.83 to 3.23.

Figure 1. Evolution of Unemployment and Employment Rates
Total, Seven Metropolitan Areas [7]



Informality has been a persistent phenomenon in the last two decades: regardless of the definition of informality used, approximately one every two workers are informal[8]. The average informality rate using the FSO definition increased from 54.25% in the period 1984-1999, to 60.47% in 2000-2010, but its volatility decreased [9]. Even though average informality rate based on the health criterion diminished from 53.74% in 1984-1999 to 52.44% in 2000-2010, informality based on pension rose significantly from 65.02% in 1996-1999 to 66.72% in 2000-2010[10].

Figure 2. Evolution of Informality Rate, Seven Metropolitan Areas [11]

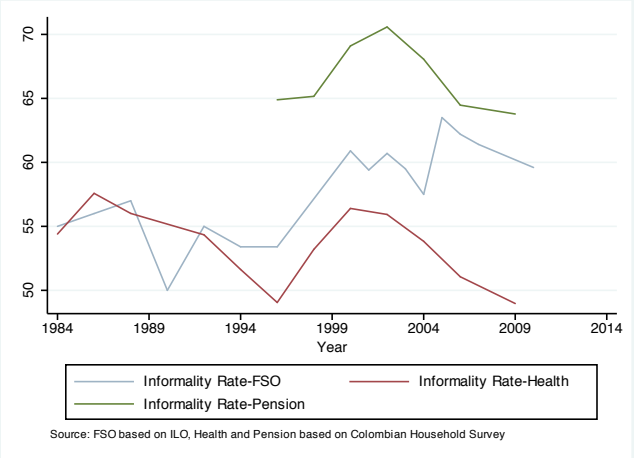
[7]The metropolitan areas included are Bucaramanga, Barranquilla, Bogota, Cali, Medellin, Manizales and Pasto. Source: Colombian Central Bank

[8]There are two alternative definitions of informality in Colombia: one based on social security (SS) contributions (health and pensions) and another one, set by the Colombian Statistics Department (DANE), based on firm size and occupation (FSO), consistent with the ILO definition.

[9]Standard deviation went down from 2.28 to 1.88.

[10]There is no information on pension contributions before 1996.

[11]Source: ILO and Colombian Household Surveys



When looking at the changes in earnings distribution pre and post-reforms, four facts are worth emphasizing[12].

Table 1: Decomposition of Variance in Log Real Hourly Wages

	June 1990	June 2003
	Informal Sector	Informal Sector
Mean of Log Wages (M_1)	7.40	7.11
Variance of Log Wages (σ_1)	1.92	1.78
Proportion of Employed in Sector (P_1)	0.46	0.44
	Formal Sector	Formal Sector
Mean of Log Wages (M_2)	8.10	8.06
Variance of Log Wages (σ_2)	1.13	1.36
Proportion of Employed in Sector (P_2)	0.54	0.55
	Economywide	Economywide
Mean of Log Wages $\left(\frac{P_1M_1+P_2M_2}{P_1+P_2}\right)$	7.78	7.63
Sum of Within-Sector Variance $\left(\frac{P_1\sigma_1+P_2\sigma_2}{P_1+P_2}\right)$	1.49	1.55
Between-Sector-Variance $\left[\frac{P_1P_2(M_1-M_2)^2}{(P_1+P_2)^2}\right]$	0.12	0.23
Total Variance	1.62	1.78

Author's calculations based on ENH, June 1990, 7 Metropolitan areas and ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using SP (health) definition. Wages deflated using CPI.

First, there are substantial differences in mean relative earnings, and after the 1990's the

[12]This table contains information of the first two moments of the real earning distributions in the overall economy and in the two sectors (in particular, sectoral and economy-wide means and variances of log wages) for June 1990 and June 2003. The economy-wide variance is broken down into two components: a) Variability within sectors; b) Variability between sectors. The proportion of population employed in sector does not correspond to informality and formality rates since the ratio is computed as a proportion of employed population who report wages (there is no wage information for all occupied). Total variance is computed by adding within-sector variance and between-sector variance.

gap increased substantially favoring formal sector workers. The gap between the mean log wages of the informal and formal sector was 0.70 in June 1990 and increased to 0.96 in June 2003 [13].

Second, overall wage inequality increased, driven by important changes in the means of sectoral log wages (total variance of log wages increased by 9.8%). Between-sector variance increased dramatically by 91.6% while within-sector inequality rose only by 4%. This small increase is explained by the fact that the reduction in within-sector inequality in the informal sector compensate the increase in the formal sector.

Third, most variance in log wages is due to within sector variability and not to between-sector variability both in the pre and post-reform periods. In June 2003, despite the substantial differences in relative earnings, 91.9% of variance in log wages is explained by within-sector variance and not by between-sector variance[14].

Fourth, average real wages fall in both sectors but at a higher rate in the informal sector, suggesting a drop in living standards in addition to the worsening income gap.

So overall, higher informality and unemployment rates, widening inequality and falling real wages worsen labor market conditions after the 1990's. In this period a sequence of structural reforms were implemented. Let's look at the most important aspects of the reforms.

In terms of government reforms, several market-oriented structural reforms were implemented in the 1990's and 2000's: the labor reforms of 1990, 2002 and 2012, the social security reforms of 1993, 2003 and 2007, and the trade liberalization reform that started in 1991.

Prior to the labor reforms, the focus of employment protection in Colombia was promoted by labor regulations that imposed high severance payments and early retirement and restrictions on temporary work, affecting labor markets flexibility, with the potential adverse effects on employment. In this context, the recent labor and social security reforms were intended to contribute to the flexibility and efficiency of the labor markets, but still maintaining some level of worker's protection.

The labor reform of 1990 was primarily aimed to stimulate job creation due to a more flexible system of hiring and layoffs by allowing short-term contracts, promoting a more

[13]This means that, while in June 1990 an informal sector worker earned on average 0.80 what a formal worker earns, in June 2003 the ratio decreased to 0.36. We construct the statistics using the Informality module in the Colombian household surveys, available every two years before 1996, and yearly afterwards. Here we use the social security definition of Informality, constructed using health contributions for both years since there is no information for pensions in 1990. The measure of earnings in the survey not only include monetary wages for workers (including tips, commissions) but also remuneration for self-employed so there is some measurement error in wages.

[14]It would be interesting to see how these results would changed if workers would be assigned randomly to sectors, so if selection reduces or increases inequality . In other words, what would be the aggregate wage variability if sectoral labor force quality were held constant(see Heckman & Sedlacek, 1985).

flexible wage regime, and more importantly, decreasing severance payments. The reform established that formal sector firms had to make an annual contribution to a private severance fund (including interest payments), instead of paying severance at the time the match ended[15].

The social security reform of 1993, implemented in 1994, introduced major changes in the health and pension system, monopolized by the government until then. The reform increased pension and health contributions and thereby non-wage labor (hereafter NWL) costs, with the main goal of expanding social security coverage. Aiming to reach universal health coverage, the Reform also created two coexisting health regimes: a contributive regime (CR) and a subsidized regime (SR). In the CR employers must provide health insurance, regardless of occupation[16], and the cost is shared between employers and employees. In the SR, 'poor' individuals who meet certain poverty criteria had access to subsidized health [17], where 'poor' was determined by a poverty index score based on the Census to the Poor (SISBEN). This system is financed with transfers from the contributive regime.

The labor reform of 2002 created a system of social protection, aimed to protected unemployed workers and to promote employment in recessionary periods but with limited impact on job creation, since most of the measures applied to a very limited group of workers and NWL costs were unaffected.

The pension reform of 2003 increased the age requirements for retirement, the length of service required for pension, and the pension contribution rate (gradually until 2008), thus increasing employer and employee NWL costs.

The health reform of 2007 increased the employer health contributions, raising employer NWL costs.

After three decades of rising NWL costs [18], the more recent labor reform of 2012 reduced employer NWL costs substantially by 8.96%, through the elimination of parastatal contributions (training and family allowances), with the main goal of stimulate employment.

All changes in social security contributions caused changes in employer and employee NWL costs, important factors that may affect job creation and job destruction in the formal and the informal sector.

Figure 3. Evolution of Employer and Employee NWL Costs(as % Wage)[19]

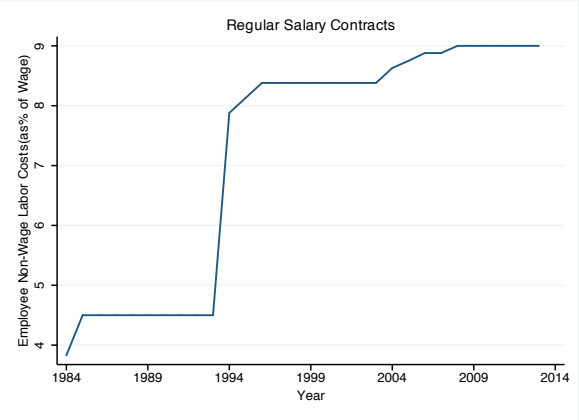
[15]However, the law applies only to workers hired after 1991, which means that the reduction in severance payments depend on turnover rate, and the amount of voluntary workers to the new regime. Hence the reduction in severance payments was mostly gradual rather than a single impact.

[16]According to the law, self-employees must also contribute to the system.

[17]The criteria are the following: being part of a SISBEN level 1 or 2 household, not being affiliated to the CR, not having an employment relationship, not being a retiree and not being a beneficiary of the CR.

[18]With the exception of the reduction in severance payments introduced by the Reform of 1990.

[19] See Appendix 6.5 for a detailed construction of these costs. It is standard to include severance costs in



Source: Author's Calculations based on Statutory Values

In the period 1985-1990, employer NWL costs were 47.08% of the wage. The reduction in severance payments introduced by the reform of 1990 caused a reduction in the average NWL cost paid by the employer from 1991 to 1993, distributing this payment linearly over time. The implementation of this change has been gradual, as discussed before. As a result, NWL costs were reduced to 45.98% in 1993. In the period 1994-1996, these costs began to rise to reach a level of 53.41 % in 1996, since the increasing pension and health contributions implemented by the social security reform of 1993 more than compensate the gradual reduction of severance payments. In the period 1996-2003, they remained constant . Finally, in the period 2004-2012, these costs continued to climb primarily because of increasing social security contributions, until they reached a level of 55.78% in 2012. The reform of 2012 counteracted the trend by reducing them significantly to 50.87%.

The employee NWL costs were constant at 4.5% during the period 1990-1993, since they were not affected by the labor reform of 1990. From 1994 to 1996, they started to increase driven primarily by increasing pension contributions introduced with the social security reform. During the period 1996-2003 they were stable, and since 2004 they began to climb until reaching a level of 9% in 2013.

In recent years, investment in social assistance programs (cash transfers, in-kind transfers and subsidized health) has been growing in Colombia. In particular, since 1993, there has been an expansion of subsidized health (Subsidized Regime) to workers who are not part of the contributive regime (informal under the SP definition), who are part of poor households [20]. The government is expanding access to health with the main goal of achieving universal healthcare in the near future. While in June 2003, approximately 11.84% of the total em-

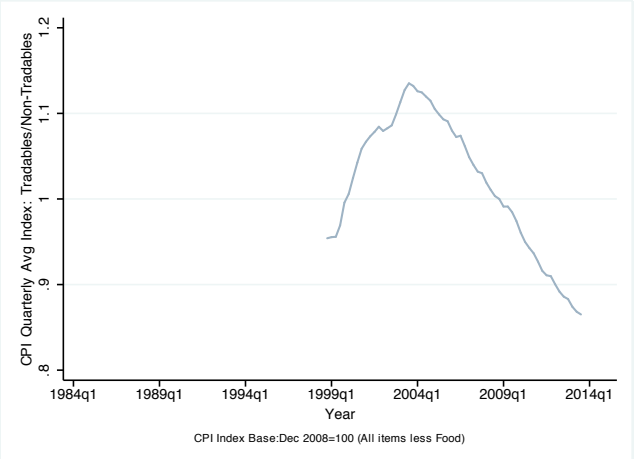
these calculations, since after the labor reform of 1990, severance is not a one-time fixed cost but an annual percent cost just as any payroll tax.

[20]"Poor" is determined by a poverty index determined by the Census of the poor(SISBEN).

ployed population were affiliated to the subsidized regime, in June 2012 was 23.0 % [21].

Colombia also started a dramatic process of trade liberalization and financial openness in 1991[22]. This may have caused important changes in the real exchange rate, defined as the price of tradables relative to non-tradables. Since the informal sector is mainly comprised of non-tradables [23], the real exchange rate is a key relative price in this small open economy, and can be quite relevant when it comes to explain movement of labor across sectors. A sharp increase is observed before 2003, followed by a real appreciation afterwards.

Figure 4. Evolution of Relative Prices[24]
CPI Quarterly Average Index- Tradables relative to Non-Tradables



Source: Colombian Central Bank- Banco de La Republica

Higher payroll taxes, expanding social assistance programs to informal workers, and changes in relative prices affecting the relative profitability of the two sectors [25] may explain the deteriorating labor market conditions observed in recent years.

[21] Author's calculations based on Colombian Household Survey

[22] Interest rate ceilings were eliminated as well as requirements to invest in government securities, exchange rate controls were abolished, restrictions to foreign direct investment were eliminated and international trade was largely liberalized, due to significant reductions in import tariffs)

[23] According to a recent study of Informality by the IMF (2010), 87 percent of self-employed and informal workers in Colombia are concentrated in non-tradables (i.e commerce and services)

[24] There is no data availability before 1998.

[25] An inflexible wage structure in the formal sector may also be a factor since previous literature (Maloney, Nunez, 2001) suggests that minimum wages in Colombia are high and binding. Here we don't consider any source of wage rigidity.

3 Model

3.1 Workers

In this economy agents live forever, discount the future at a constant rate r, and live in a stationary environment where there are no dynamic changes to the structural parameters.

The labor force, L , is assumed to be constant and normalized to unity.

In equilibrium, each agent can be in one of three states: unemployed, employed in the formal sector, or employed in the informal sector. We want to allow flows from and to each possible state (except from informal to formal-employment), so we have a total of five transitions in the model.

There is worker heterogeneity ex-ante and ex-post. Ex-ante, workers differ in individual-specific characteristics relevant to the labor market such as human capital level, which we call their type. Ex-post, workers differ in their labor market status and their initial productivity at the job (ex-post match-specific heterogeneity), as well as their future productivity since they are also subject to idiosyncratic productivity shocks.

Let y be the worker's type, where y is an iid [26] draw from a distribution $F(y)$ with support on the range $[0 \leq y \leq \infty]$. Let y' be the initial productivity of the job (match-specific), where y' is a draw from a distribution $H_i(y' \mid y)$, for $i = F, I$[27].

Let u be the measure of unemployed workers, v_i be the measure of vacancies in sector i, and n_i be the measure of workers in sector i [28]. Let θ and ϑ be the parameters that measures labor market tightness in the formal and informal sector respectively, where[29]

$$\theta = \frac{v_F}{u}; \vartheta = \frac{v_I}{u + n_F}$$

Let $U(y)$ and $N_i(y', y)$ be the present-discounted value of the expected income stream of, respectively, an unemployed worker of type y and an employed worker of type y with match-specific productivity y' in sector i.

While unemployed, workers enjoy real returns b[30] and receive offers from both sectors,

[26] This is the same assumption than in MP1994. Alternatively we could assume that y is a Markov process

[27] We assume first-order stochastic dominance, i.e. if $y_1 > y_2$, then $H_i(y' \mid y_1) < H_i(y' \mid y_2)$. In the calibration, we assume these distributions are log-normal with conditional mean varying linearly with $log(y)$, so the first-order stochastic dominance assumption is satisfied.

[28] Notice that v_i is the not total measure of vacancies in the economy, but only the vacancies in sector i. Also, since the labor force is normalized to 1, u is also equal to the aggregate unemployment rate.

[29] The tightness measure the number of vacancies relative to job seekers en each sector. While job seekers in the formal sector are only unemployed workers, job seekers in the informal are unemployed and formal sector workers affected by a 'bad' shock that look into informal sector opportunities.

[30] This is also usually interpreted as unemployment insurance benefit but there is no such insurance in Colombia so b is just the opportunity cost of leisure.

regardless of their type[31]. Let α be an exogenous Poisson rate at which informal sector offers arrive to unemployed[32], and $m(\theta)$ be the endogenous rate at which formal sector offers arrive [33]. Once a contact is made between a worker of type y and a potential employer in the formal sector, a productivity for the prospective match, y', is drawn.

The details of the job creation process are as follows.

Because of the existence of a productivity distribution for new matches, not all meetings create a match. Let $R_{UF}(y)$ and $R_{UI}(y)$ be the the minimum productivities below which neither the firm nor the worker want to start a match (endogenous reservation productivities).

If the realization of the productivity draw for a worker of type y is sufficiently 'high' [34], the worker and the firm in sector i decide to match, and the worker gets a capital gain of $N_i(y', y) - U(y)$; otherwise, the worker returns to the pool of unemployed, and the job remains vacant[35].

The flow value of unemployment for a worker of type y is:

$$rU(y) = b + \alpha Emax[N_I(y', y) - U(y), 0] + m(\theta)Emax[N_F(y', y) - U(y), 0]$$

Given the assumptions on the match-specific productivity this gives:

$$rU(y) = b + \alpha \int_{R_{UI}(y)}^{\infty} [N_I(y', y) - U(y)]dH_I(y' \mid y) + m(\theta) \int_{R_{UF}(y)}^{\infty} [N_F(y', y) - U(y)]dH_F(y' \mid y) \tag{1}$$

While employed in sector i, a worker of type y and current productivity y' enjoys flow utility $u_i(y', y)$. Then, the match that started at productivity y' may continue or be destroyed. Job destruction rate is endogenized by introducing idiosyncratic shocks to job productivities[36]: we assume that a productivity shock, x, arrives to jobs in sector i at Poisson rate λ.

[31] Alternatively, we could have assumed that workers with 'low' or 'high' type receive only offers from a particular sector.

[32] We are assuming no congestion effects in the informal sector (α is not a function of ϑ), so the measure of job seekers does not make it harder for an individual to find an informal sector opportunity. It may be the case that while job seekers are eager to find a formal sector job, they are not eager to find an informal sector job.

[33] We assume the matching function has standard properties, so $m(\theta)$ is increasing and concave in θ. In the calibration we assume a Cobb Douglas matching function given by $m(\theta) = A\theta^{1-\alpha_m}$.

[34] Sufficiently 'high' means $y' \geq R_{UF}(y)$ for the formal sector and $y' \geq R_{UI}(y)$ for the informal sector.

[35] Another way of modeling this choice is by assuming that workers choose whether or not to accept jobs based on a reservation wage. This is analogous to the reservation productivity concept.

[36] There are two reasons why the productivity of a job may fall below the reservation value: idiosyncratic or aggregate shocks. Previous evidence for Colombia using plant-level data from the Annual Manufacturing survey estimate that the actual impact of reforms through factor adjustment on aggregate productivity was modest (Eslava,Haltinwager, Kugler % Kugler, 2010), so introducing idiosyncratic shocks instead of aggregate shocks in the model seems more reasonable.

These shocks are iid draws from conditional distributions $H_i(x \mid y)$[37].

The same mechanism that governs the job creation process applies to job destruction: a match ends when it is in the mutual interest of the worker and the firm to do so, i.e., when a sufficiently bad draw of x is realized [38]. The threshold productivities for match dissolution in the informal and the formal sector are $R_{IU}(y)$ and $R_{FU}(y)$ respectively.

We introduce a feature in the model that captures the view of the informal sector is a disadvantage sector of a segmented labor market that expands during downturns to absorb displaced workers from the formal sector. When affected by a 'bad' shock, formal-sector workers may choose whether to take an informal sector opportunity already available for them, or to become unemployed. Alternatively, informal-sector workers don't have the option to move directly to the formal sector, they must become unemployed. Also, while formal-sector workers must make contributions to the social security (henceforth SS) system, informal-sector workers receive some subsidized health without incurring in any cost.

The flow value of taking a formal-sector job for a worker of type y and current productivity y' (new hire) [39] is:

$$rN_F(y',y) = u_F(y',y) + \lambda H_F(R_{FU}(y) \mid y) Emax[N_I(x,y) - N_F(y',y), U(y) - N_F(y',y)] \quad (2)$$

$$+\lambda \int_{R_{FU}(y)}^{\infty} [N_F(x,y) - N_F(y',y)] dH_F(x \mid y)$$

where

$$Emax[.] = \int_0^{R_{FI}(y)} [U(y) - N_F(y',y)] dH_I(x \mid y) + \int_{R_{FI}(y)}^{\infty} [N_I(x,y) - N_F(y',y)] dH_I(x \mid y)$$

When affected by a 'good' shock, the formal-sector worker stays in his job and get the capital gain $N_F(x,y) - N_F(y',y)$. When affected by a 'bad' shock, he may decide to transition to the informal-sector to get a capital gain of $N_I(x,y) - N_F(y',y)$, or to become unemployed and get $U(y) - N_F(y',y)$.

We assume that $u_F(y',y)$ depends on effective current labor income, net after paying SS contributions and adjusted by a subjective valuation of the total (employer and employee) contributions to the system [40]. It can be expressed as:

[37] Notice that we assume that productivity shocks affect both sectors symmetrically: workers receive shocks at the same rate, regardless of the sector in which they are in. Also, we assume that the idiosyncratic productivity that is draw after the shock arrives is independent of the initial productivity y' and is irreversible (the firm must produce at the new productivity or shut down), where $x \in [0, \infty]$.

[38] This means $x < R_{IU}(y)$ for the informal sector, and $x < R_{FU}(y)$ for the formal sector.

[39] This is the flow for a worker not affected yet by a shock so current productivity is still match-specific.

[40] We assume the utility function is linear in income and total SS benefits constitute a linear function of the total contributions (employer and employee).

$$u_F(y', y) = w_F(y', y)[(1 - \delta_2) + \tau(\delta_1 + \delta_2)] \tag{3}$$

where $w_F(y', y)$ is the hourly wage in the formal sector, δ_2 and δ_1 are the employee and employer NWL costs as a percentage of the wage [41], τ is a parameter that measures workers valuation of total social security contributions (including employer and employee contributions), $0 \leq \tau \leq 1$.[42]. Let $\hat{\delta}_2$ be the non-wage labor costs as a percentage of the wage adjusted by the worker's valuation of the benefits that he/she receives as a result of the total contributions to the social system [43], so:

$$\hat{\delta}_2 = \delta_2(1 - \tau) - \tau\delta_1 \tag{4}$$

We can rewrite the instantaneous utility more compactly as:

$$u_F(y', y) = w_F(y', y)(1 - \hat{\delta}_2) \tag{5}$$

An old hire in the formal sector must have a different value function than a new hire since, as we will explained later, the wage (and therefore utility) negotiated in a bilateral bargaining is different. When the firm negotiates with an old hire it must pay severance in case they mutually decide to discontinue the match, which weaken its bargaining position.

Let $w_F^s(x, y)$ and $u_F^s(x, y)$ be, respectively, the wage and utility of a worker of type y and current productivity x (old hire). The flow value of continuing in a formal-sector job for this worker is:

$$rN_F(x, y) = u_F^s(x, y) + \lambda H_F(R_{FU}(y) \mid y) Emax[N_I(x', y) - N_F(x, y), U(y) - N_F(x, y)] + \tag{6}$$

$$+\lambda \int_{R_{FU}(y)}^{\infty} [N_F(x', y) - N_F(x, y)] dH_F(x' \mid y)$$

where x' is a another draw from the distribution $H_F(. \mid y)$ and

$$u_F^s(x, y) = w_F^s(x, y)(1 - \hat{\delta}_2) \tag{7}$$

The flow value of taking an informal-sector job for a worker of type y and match specific

[41] These costs are made of social security (SS) contributions -health and pension- and non-SS contributions (professional risks)

[42] The valuation of these contributions reflects the value of these contributions and the efficiency of the services provided.

[43] Notice that $\hat{\delta}_2 < 0$ if $\tau(\delta_1 + \delta_2) > \delta_2$. Therefore, if the value of the SS services received is higher than the cost of contributing to the system, $\hat{\delta}_2$ works as a transfer and not as a tax.

productivity y' is:

$$rN_I(y',y) = u_I(y',y) + \lambda H_I(R_{IU}(y) \mid y)[U(y) - N_I(y',y)] + \lambda \int_{R_{IU}(y)}^{\infty} [N_I(x,y) - N_I(y',y)] dH_I(x \mid y) \tag{8}$$

We can express $u_I(y',y)$, the flow utility for a worker of type y and current productivity y' in the informal sector, as:

$$u_I(y',y) = w_I(y',y)[1 + \hat{\delta}_3] \tag{9}$$

$$\hat{\delta}_3 = 1 + \mu\delta_3 \tag{10}$$

where $w_I(y',y)$ is the hourly wage in the informal sector for a worker of type y and match-specific productivity y', δ_3 is the amount of social assistance (subsidized health) that workers receive from the government as percentage of the wage in the informal sector [44], and μ is a parameter that measures the workers valuation of the social assistance benefits received, where $0 \leq \mu \leq 1$. [45]

So informal workers are "vulnerable" population in the sense that, even if they may have access to partial insurance against health shocks due to a government subsidized health program, they do not have any insurance against unemployment shocks (i.e. severance payments) or aging(i.e mandatory pension or retirement accounts).

3.2 Firms

The small economy produces two composite goods: tradables and non-tradables. There are two productive sectors in this economy: formal and informal. The formal sector is assumed to produce tradables, while the informal produces non-tradables. Each sector has a continuum of small firms in the unit interval, which are identical in all respects within each sector. Each firm has one job and maximizes the present discounted value of profits and chooses whether to open a job vacancy and hire a worker or not, so the number of jobs/firms is endogenous. Since the profit maximization condition requires that the marginal value of

[44] We assume the benefit from social assistance is proportional to the informal-sector wage. For the case of Colombia, δ_3 corresponds to allocated health expenditures in the Subsidized Regime program as percentage of nominal wage. However, not all workers receive the subsidy: only those workers whose families are categorized as "poor" as determined by a Poverty Index Score, using the Census of the Poor (SISBEN).

[45] We are assuming that employee valuations of these services may be lower, equal or higher than in the SS system ($\mu < \tau$, $\mu = \tau$, $\mu > \tau$), depending on the perception of the efficiency of the services provided by the public sector. In the case where $\mu = 0$, informal workers do not value the services offered by the government. In this case, any change in subsidized health expenditures, δ_3, wont change the flow income in the informal sector. If $\mu > 0$, an expansion in subsidized health expenditures will cause a high flow income in the informal sector, with a consequent behavioral impact.

a vacancy must be zero, this is exactly equivalent to a zero-profit condition for firm entry.

Firms can only adjust to meet demand through changes in the extensive margin (number of jobs offered/employed people), but not through the intensive margin [46].

The main differences between firms across sectors is that informal-sector firms are not affected by labor market regulations such as social security contributions and severance payments. Distributions of productivity and wages are also different.

Let V_i be the present-discounted value of expected profit from a vacant job in sector i, and $J_i(y', y)$ be the present-discounted value of expected profit from a filled job in sector i with a worker of type of type y and match-specific productivity y'.

Firms in sector i open vacancies and search among the pool of job seekers , which involves a hiring cost, c, assumed to be constant [47]. They also face some uncertainty when meeting a job seeker, since don't know what type of worker will meet, and given that, how productive will be on the job.

The flow value of having a vacancy in the formal sector is:

$$rV_F = -c + \frac{m(\theta)}{\theta} Emax[J_F(y', y) - V_F, 0]$$

Formal sector vacancies meet searching workers (only unemployed workers) at the rate $\frac{m(\theta)}{\theta}$. If the job is filled, the firm get the corresponding expected capital gains, $J_F(y', y) - V_F$.

Given the assumption on the match-specific productivity this is equivalent to [48]:

$$rV_F = -c + \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF}(y)}^\infty [J_F(y', y) - V_F] dH_F(y' \mid y) dF_U^*(y) \tag{11}$$

where $F_U^*(y)$ is the distribution of y among the unemployed. Using Bayes rule we get:

$$dF_U^*(y) = \frac{u(y)f(y)}{\int_0^\infty u(y)f(y)dy} dy,$$

where $u(y)$ is the unemployment rate in the informal sector conditional on y.

In the other hand, informal-sector firms meet not only unemployed but also formal-sector workers affected by a 'bad' shock. The rates at which informal-sector vacancies meet unemployed and formal-sector workers are $\frac{\alpha}{\vartheta}$ and $\frac{\lambda}{\vartheta}$, respectively.

The flow value of having a vacancy in the informal sector is:

$$rV_I = -c + \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F} \right] Emax[J_I(y', y) - V_I, 0] + \frac{\lambda}{\vartheta} \left[\frac{n_F}{u + n_F} \right] Emax[J_I(x, y) - V_I, 0] \tag{12}$$

[46] In the appendix, we analyze the case where hours of work are chosen by workers.

[47] This can also depend on productivity or wages, capturing some business cycle effects.

[48] Notice also that the value of the vacancy does not depend on y.

Given the assumption on the productivity shock this is equivalent to:

$$rV_I = -c + \frac{\alpha}{\vartheta}\left[\frac{u}{u+n_F}\right]\int_0^\infty \int_{R_{UI}(y)}^\infty [J_I(y',y) - V_I]dH_I(y' \mid y)dF_U^*(y) +$$

$$+\frac{\lambda}{\vartheta}\left[\frac{n_F}{u+n_F}\right]\int_0^\infty \int_{R_{FI}(y)}^\infty H_F(R_{FU}(y) \mid y)[J_I(x,y) - V_I]dH_I(x \mid y)dF_F^*(y)$$

where $F_F^*(y)$ is the distribution of y among the job seekers in the formal sector that can be expressed as:

$$dF_F^*(y) = \frac{n_F(y)f(y)}{\int_0^\infty n_F(y)f(y)dy}dy.$$

A firm matched with a worker of type y and match-specific productivity y' receives some net return for a job, $\pi_F(y',y)$, given by the market value of output minus the net cost of labor (after paying SS contributions).

A positive or negative productivity shock arrives at rate λ, and two possible cases arise: if the shock is 'good', the match continues with the capital gain $J_F(x,y) - J_F(y',y)$; if the shock is 'bad', the match ends, the firm pays firing cost, s, and posts a new vacancy, so the capital loss suffered is $V_F(y) - J_F(y',y) - s$.

The flow value of a filled job in the formal sector with a worker of type y and match-specific productivity y' (new hire) is:

$$rJ_F(y',y) = \pi_F(y',y) + \lambda H_F(R_{FU}(y) \mid y)[V_F - J_F(y',y) - s] + \lambda \int_{R_{FU}(y)}^\infty [J_F(x,y) - J_F(y',y)]dH_F(x \mid y) \tag{13}$$

where $\pi_F(y',y)$ is the nominal value of a job's output in the formal sector, which can be defined as:

$$\pi_F(y',y) = p_F y' - w_F(y',y)(1+\delta_1) \tag{14}$$

and p_F is the price of formal sector good (price of tradable good).

The flow value of a filled job in the formal sector with a worker of type y and productivity x (old hire) is:

$$rJ_F(x,y) = \pi_F^s(x,y) + \lambda H_F(R_{FU}(y) \mid y)[V_F - J_F(x,y) - s] + \lambda \int_{R_{FU}(y)}^\infty [J_F(x',y) - J_F(x,y)]dH_F(x' \mid y) \tag{15}$$

where

$$\pi_F^s(x,y) = p_F x - w_F^s(x,y)(1+\delta_1) \tag{16}$$

Notice that $rJ_F(x,y)$ differs from $rJ_F(y',y)$ because the wage associated with each func-

tion is different[49].

The flow value of a filled job in the informal sector with a worker of type y and current productivity y' is:

$$rJ_I(y',y) = \pi_I(y',y) + \lambda H_I(R_{IU}(y) \mid y)[V_I - J_I(y',y)] + \lambda \int_{R_{IU}(y)}^{\infty} [J_I(x,y) - J_I(y',y)] dH_I(x \mid y) \tag{17}$$

where the nominal value of a job's output in the informal sector, $\pi_I(y',y)$ is defined as:

$$\pi_I(y',y) = p_I y' - w_I(y',y) \tag{18}$$

and p_I is the price of the informal sector good (non-tradable good).

The informal-sector job yields net return for the firm firm $\pi_I(y',y)$. The match may break (without any firing cost involved) because of the arrival of a 'bad' productivity shock (x below some productivity threshold $R_{IU}(y)$), and the firm must post a new vacancy, so the capital loss suffered is $[V_I - J_I(y',y)]$. Otherwise, the match continues and the firm get the corresponding capital gains, $J_I(x,y) - J_I(y',y)$.

3.3 Wage Determination

3.3.1 Formal Sector

Formal-sector wages in the steady state are determined by workers and firms using Nash bargaining, given exogenous worker bargaining parameter, β.

An unemployed worker of type y with match-specific productivity y' and a formal sector firm decide to form a match if it is worth it for both, that is if the joint surplus is positive, or equivalently, if $R_{UF}(y) \leq y' \leq \infty$. In the case they match, they decide how to split the surplus and negotiate a wage contract using Nash bargaining given exogenous worker bargaining parameters, β.

The initial wage [50] is given by:

$$\max_{w_F(y',y)} [N_F(y',y) - U(y)]^{\beta} [J_F(y',y) - V_F]^{1-\beta}$$

So the corresponding standard sharing rule using the free entry condition ($V_F = 0$)

$$(1-\beta)(1+\delta_1)[N_F(y',y) - U(y)] = \beta(1-\hat{\delta}_2) J_F(y',y) \tag{19}$$

[49] We will see later that $w_F^s(x,y) > w_F(x,y)$.

[50] This is the wage for new hires in the formal sector that haven't been affected by the shock

After doing some algebra, the corresponding wage equation for the formal sector for a worker of type y and current productivity y' is:

$$w_F(y',y) = \beta\left[\frac{p_F y'}{1+\delta_1} - \frac{\lambda s}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y')}^{\infty} N_I(x,y) - U(y)dH_I(x \mid y)}{(1-\hat{\delta}_2)}\right] \tag{20}$$

The wage negotiated in the formal sector is a weighted average between the productivity of the worker (adjusted by the expected severance cost) and the worker's continuation value (adjusted by a term that captures the flows from formal to informal).

The severance cost reduces the benefits the firm gets if it accepts the bargain with the new hire (and a 'bad' shock arrives), improving its bargaining power in the negotiation, and therefore reducing the negotiated wage.

The continuation value reflects not only the value for the worker if he doesn't accept the bargain (the flow value of unemployment), but also the benefits for the worker if he accepts the job in the formal, including the possibility of moving to the informal sector. After being employed in the formal, the worker may be affected by a 'bad' productivity shock, and either flow to unemployment or to the informal sector. The expected gains of these two possible states worsen the worker's bargaining position. If the worker expect to get greater gains after being affected by a shock while working in the formal, this reduces the continuation value and therefore the worker is willing to accept a lower wage in the formal sector.

In general, if the worker is low productive, has a low continuation value, low bargaining power and has to pay low non-wage labor costs as a formal employee, both worker and formal-sector firm are willing to accept a lower wage in the negotiation.

Since it is assumed that the wage is renegotiated every time a productivity shock arrives, wages for employed workers of type y and current productivity x, where $R_{FU}(y) \leq x \leq \infty$[51] , are determined by workers and firms using Nash bargaining, given exogenous worker bargaining parameters, β.

$$\max_{w_F^s(x,y)} [N_F(x,y) - U(y)]^{\beta}[J_F(x,y) - (V_F - s)]^{1-\beta}$$

So the corresponding standard sharing rule using the free entry condition ($V_F = 0$) is:

$$(1-\beta)(1+\delta_1)[N_F(x,y) - U(y)] = \beta(1-\hat{\delta}_2)[J_F(x,y) + s] \tag{21}$$

The wage equation for the formal sector for a worker of type y and current productivity

[51]This is the wage for old hires that have been affected by the shock

x is:

$$w_F^s(x,y) = \beta\left[\frac{p_F x}{1+\delta_1} + \frac{rs}{(1+\delta_1)}\right] + (1-\beta)\left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y)}^{\infty} N_I(x',y) - U(y)dH_I(x' \mid y)}{(1-\hat{\delta}_2)}\right] \tag{22}$$

Notice that $w_F^s(x,y) \geq w_F(x,y)$ since the severance tax now worsen the firm's bargaining position. The severance cost reduces the benefits the firm gets if it does not accept the bargain with the old hire, reducing its bargaining strength in the negotiation, and therefore increasing the negotiated wage.

3.3.2 Informal Sector

We assume there is bargaining over wages in the informal sector[52].

The wage for a worker type y solves:

$$\max_{w_I(y',y)} [N_I(y',y) - U(y)]^{\beta}[J_I(y',y) - V_I]^{1-\beta}$$

The F.O.C (assuming interior solution), gives us the following standard sharing rule:

$$(1-\beta)[N_I(y',y) - U(y)] = \beta(1+\hat{\delta}_3)[J_I(y',y) - V_I] \tag{23}$$

The wage equation for the informal sector is:

$$w_I(y',y) = \beta\left[p_I y'\right] + (1-\beta)\left[\frac{rU(y)}{(1+\hat{\delta}_3)}\right] \tag{24}$$

In this case, neither the productivity nor the continuation value term need to be adjusted since there is no severance tax nor flows from informal to formal. However, high benefits in the form of better access to subsidized health (high $\hat{\delta}_3$) worsen the worker's bargaining position, which leads to a lower wage in the bargaining process.

[52] We conceive the informal sector as a 'disadvantaged' countercyclical sector in which low-skilled workers are negotiating wages with small firms, rather than as unregulated procyclical self-employment. In Colombia, lack of compliance with social security is also a small firm phenomenon: approximately 87.2% of those informal under the SP definition are in small firms($\leq$ 10 employees).

3.4 Optimal decision rules and Reservation Productivities

Optimal decision rules are characterized by a reservation value property. Reservation productivities are obtained by:

$$\begin{aligned}
R_{FU}(y) : N_F(R_{FU}(y), y) = U(y) &\iff J_F(R_{FU}(y), y) = V_F - s = -s \\
R_{UF}(y) : N_F(R_{UF}(y), y) = U(y) &\iff J_F(R_{UF}(y), y) = V_F = 0 \\
R_{IU}(y) : N_I(R_{IU}(y), y) = U(y) &\iff J_I(R_{IU}(y), y,) = V_I = 0 \\
R_{UI}(y) : N_I(R_{UI}(y), y) = U(y) &\iff J_I(R_{UI}(y), y) = V_I = 0 \\
R_{FI}(y) : N_I(R_{FI}(y), y) = U(y) &\iff J_I(R_{FI}(y), y) = V_I = 0
\end{aligned}$$

The Nash surplus sharing rule guarantees mutual agreement between the worker and the firm. Notice that $R_{IU}(y)$=$R_{UI}(y)$=$R_{FI}(y)$. In the other hand $R_{UF}(y) \neq R_{FU}(y)$ since when the match breaks, the formal sector firm has to pay a severance cost.

The corresponding reservation wages can be obtained through the corresponding Nash Bargaining wage rule which maps productivities into wages, conditioning on type.

3.4.1 Reservation Productivity, $R_{FU}(y)$

A match in the formal sector is continued if it is a mutual interest of the worker and the firm to do so, so a necessary condition for match continuation is that the joint surplus must be non-negative. In equilibrium, the surplus is an increasing function of x [53], and by definition $R_{FU}(y)$ is the threshold productivity above which the joint surplus is never negative. Therefore, $R_{FU}(y)$ is defined by the zero surplus condition:

$$N_F(R_{FU}(y), y) - U(y) + J_F(R_{FU}(y), y) + s = 0 \tag{25}$$

Using the sharing rule (28) combined with the free entry condition we get:

$$J_F(R_{FU}(y), y) = -s \tag{26}$$

[53] See section 3.5 for proof.

Substitution and integration by parts gives [54]:

$$R_{FU}(y) = \frac{A_{FU}(y)}{(1-\hat{\delta}_2)P_F} - \frac{\left[1-\beta(1-\hat{\delta}_2)\right]rs}{(1-\beta)P_F} - \frac{\lambda}{r+\lambda}\int_{R_{FU}(y)}^{\infty} 1 - H_F(x' \mid y)dx' \qquad (27)$$

The term $A_{FU}(y)$ captures the worker's outside option adjusted by a term that captures flows from formal to informal as follows:

$$A_{FU}(y) = (1+\delta_1)\left[rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y)}^{\infty} N_I(x', y) - U(y)dH_I(x' \mid y)\right] \qquad (28)$$

This corresponds to a modified version of the standard upward-sloping job destruction curve, in which, for a given y, higher θ implies higher $U(y)$ (better worker's outside opportunities) , and therefore higher $R_{FU}(y)$ (more marginal jobs are destroyed). There is a secondary effect in play caused by the movement from formality to informality, since higher $R_{FU}(y)$ means higher probability of discontinuing the formal sector match, which affects negatively $A_{FU}(y)$. This equilibrium effect mitigates the impact of θ on $R_{FU}(y)$.

Given $U(y)$, the reservation productivity when transitioning from formal to unemployment is an increasing function in δ_1 and $\hat{\delta}_2$, and a decreasing function in $\hat{\delta}_3$[55], s and P_F, conditional on y, as expected.

The higher the firm's non-wage labor costs, the lower the price of the formal sector good or the severance cost, the higher the reservation productivity firms require to maintain the match after a productivity shock hits (become "pickier"). The lower the expected utility of the worker in the current match, either because of higher non-wage labor costs (adjusted by valuations), or lower $\hat{\delta}_3$(receiving less benefits if moving to the IS), the higher the minimum productivity workers require to maintain a match (becoming "pickier" as well) .

Also, intuitively $R_{FU}(y)$ should be increasing in y if the outside option term $A_{FU}(y)$ adjusted grows at a faster rate than the integral term in equation (27) ('labor hoarding effect') for a particular set of parameter values.

[54]Conditions for existence of fixed point are pretty straightforward. Let T: $R_{FU}(y) \to R_{FU}(y)$ be an operator on a metric space ($\mathbb{R}^n$, d_∞) where $\mathbb{R}^n$ is a space of functions. Then $T(R_{FU}(y))$ is a contraction if r+$\lambda H_F(R_{FU}(y) \mid y) > 0$. By Contraction Mapping Theorem , there is a unique fixed point $R_{FU}(y) \in \mathbb{R}^n$ such that $T(R_{FU}(y)) = R_{FU}(y)$.

[55]Only if flows from formal to informal are allowed.

3.4.2 Reservation Productivity, $R_{UF}(y)$

Analogously, the reservation productivity $R_{UF}(y)$ is:

$$R_{UF}(y) = \frac{A_{UF}(y)}{(1-\hat{\delta}_2)P_F} + \frac{\left[1-\beta(1-\hat{\delta}_2)\right]\lambda s}{(1-\beta)P_F} - \frac{\lambda}{r+\lambda}\int_{R_{FU}(y)}^{\infty} 1 - H_F(x' \mid y)dx' \tag{29}$$

where

$$A_{UF}(y) = (1+\delta_1)\left[rU(y) - \lambda H_F(R_{FU}(y) \mid y)\int_{R_{FI}(y)}^{\infty} N_I(x,y) - U(y)dH_I(x \mid y)\right] \tag{30}$$

So we get that:

$$R_{UF}(y) = R_{FU}(y) + \frac{\left[1-\beta(1-\hat{\delta}_2)\right](\lambda+r)s}{(1-\beta)P_F} \tag{31}$$

Notice that $R_{UF}(y) \geq R_{FU}(y)$ if the second term is weakly positive (assuming λ and $r > 0$, we just need $s \geq 0$). Formal sector firms are "pickier" when hiring a worker than when laying off since in the first case they don't have to pay a firing cost.

3.4.3 Reservation Productivity $R_{IU}(y)$

Informal sector matches are destroyed when idiosyncratic productivity $x < R_{IU}(y)$, so the reservation productivity $R_{IU}(y)$ is defined by the condition:

$$J_I(R_{IU}(y), y) = 0 \tag{32}$$

Substitution [56] gives:

$$R_{IU}(y) = \frac{A_{IU}(y)}{(1+\hat{\delta}_3)P_I} - \frac{\lambda}{r+\lambda}\int_{R_{IU}(y)}^{\infty} 1 - H_I(x \mid y)dx \tag{33}$$

where $A_{IU}(y) = rU(y)$.

Given $U(y)$, the reservation productivity when transitioning from informal to unemployment is a decreasing function in $\hat{\delta}_3$ and P_I. The higher the worker social assistance benefits (adjusted by valuations), the lower the minimum productivity workers require to maintain a match with an informal-sector firm (becoming less "picky"). Firms are also less picky when profitability is high (high P_I).

[56] T: $R_{IU}(y) \to R_{IU}(y)$ is a contraction as long as r+$\lambda H_I(R_{IU}(y) \mid y) > 0$

3.5 Total Surplus, Worker and Firm Surplus

Since most equilibrium objects are expressed as functions of worker or firm's surpluses, it is useful to derive expressions for these surpluses as functions of the reservation productivities.

3.5.1 Formal Sector

In order to get the firm surplus in the formal sector as a function of the reservation productivity $R_{UF}(y)$, we use the value function $J_F(y', y)$ and the wage equation $w_F(y', y)$. Then, we use the Nash sharing rule to obtain total surplus and worker surplus.

The total surplus, worker and firm surplus for a worker of type y and productivity y' in the formal sector are, respectively, given by[57]

$$S_F(y', y) = \frac{p_F \left(1 - \hat{\delta}_2\right) (y' - R_{UF}(y))}{(1 + \delta_1)(r + \lambda)} \tag{34}$$

$$N_F(y', y) - U(y) = \frac{\beta p_F \left(1 - \hat{\delta}_2\right) (y' - R_{UF}(y))}{(1 + \delta_1)(r + \lambda)} \tag{35}$$

$$J_F(y', y) - V_F = \frac{(1 - \beta) p_F (y' - R_{UF}(y))}{(r + \lambda)} \tag{36}$$

The total surplus, workers' and firms' surplus for a worker of type y and productivity x in the formal sector are, respectively, given by:

$$S_F(x, y) = \frac{p_F \left(1 - \hat{\delta}_2\right) (x - R_{FU}(y))}{(1 + \delta_1)(r + \lambda)} \tag{37}$$

$$N_F(x, y) - U(y) = \frac{\beta p_F \left(1 - \hat{\delta}_2\right) (x - R_{FU}(y))}{(1 + \delta_1)(r + \lambda)} \tag{38}$$

$$J_F(x, y) - V_F = \frac{(1 - \beta) p_F (x - R_{FU}(y))}{(r + \lambda)} - s \tag{39}$$

[57] Notice that workers always get a fixed proportion of the total surplus, so $N_F(y', y) - U(y) = \beta S_F(y', y)$. Substituting this expression in the Nash sharing rule we get $J_F(y', y) - V_F = \frac{(1-\beta)(1+\delta_1)}{(1-\hat{\delta}_2)} S_F(y', y)$. So the total surplus $S_F(y', y) = N_F(y', y) - U(y) + \frac{(1-\hat{\delta}_2)}{(1+\delta_1)} J_F(y', y)$.

3.5.2 Informal Sector

The corresponding surpluses for a worker of type y, current productivity y' in the informal sector are given by:

$$S_I(y', y) = \frac{p_I \left(1 + \hat{\delta}_3\right) (y' - R_{IU}(y))}{r + \lambda} \tag{40}$$

$$N_I(y', y) - U(y) = \frac{\beta p_I \left(1 + \hat{\delta}_3\right) (y' - R_{IU}(y))}{r + \lambda} \tag{41}$$

$$J_I(y', y) - V_I = \frac{(1 - \beta) p_I (y' - R_{IU}(y))}{r + \lambda} \tag{42}$$

Therefore, in both sectors the surpluses depend positively on the market value of the gap between current and minimum productivity at a particular match (adjusted by the relevant policy parameters δ_1, $\hat{\delta}_2$ and $\hat{\delta}_3$), and negatively on the rate of arrival of the productivity shock, since this increases the turnover rate.

3.6 Job Creation Conditions

In this section, we derive the job creation condition in the formal and the informal sector that allow us to pin down equilibrium labor market tightness, θ and ϑ respectively.

In equilibrium, formal sector firm open vacancies until rents are exhausted so free entry implies $V_F = 0$.

From (11) and the free entry condition we get:

$$\frac{c\theta}{m(\theta)} = \int_0^\infty \int_{R_{UF}(y)}^\infty J_F(y', y) dH_F(y' \mid y) dF_U^*(y) \tag{43}$$

Equilibrium θ is such that the expected cost of hiring a worker in the formal sector is equal to the expected benefit of hiring an unemployed worker.

When expressing $J_F(y', y)$ as a function of reservation productivity, $R_{UF}(y)$ we get:

$$c = \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF(y)}}^\infty \frac{(1 - \beta) p_F [y' - R_{UF}(y)]}{r + \lambda} dH_F(y' \mid y) dF_U^*(y) \tag{44}$$

Let's derive the same condition for the informal sector. From (12) and free entry we get:

$$c = \frac{\alpha}{\vartheta} \left[\frac{u}{u + n_F}\right] \int_0^\infty \int_{R_{UI}(y)}^\infty J_I(y', y) dH_I(y' \mid y) dF_U^*(y) + \tag{45}$$

$$+\frac{\lambda}{\vartheta}\left[\frac{n_F}{u+n_F}\right]\int_0^{\infty}\int_{R_{FI}(y)}^{\infty} H_F(R_{FU}(y)\mid y)J_I(x,y)dH_I(x\mid y)dF_F^*(y)$$

Expressing the firm's value function in terms of reservation productivities we get:

$$c=\frac{\alpha}{\vartheta}\left[\frac{u}{u+n_F}\right]\int_0^{\infty}\int_{R_{UI}(y)}^{\infty}\frac{(1-\beta)p_I\left(y'-R_{IU}(y)\right)}{r+\lambda}dH_I(y'\mid y)dF_U^*(y)+ \tag{46}$$

$$+\frac{\lambda}{\vartheta}\left[\frac{n_F}{u+n_F}\right]\int_0^{\infty}\int_{R_{FI}(y)}^{\infty} H_F(R_{FU}(y)\mid y)\left[\frac{(1-\beta)p_I\left(x-R_{IU}(y)\right)}{r+\lambda}\right]dH_I(x\mid y)dF_F^*(y)$$

3.7 Steady State Conditions

In this section we want to derive the equilibrium natural rate of unemployment, given the other equilibrium objects.

Let $u(y)$ be the fraction of the labor force in unemployment, $n_I(y)$ be the fraction in informal-sector employment and $n_F(y)$ be the fraction in formal-sector employment, so that $u(y)+n_I(y)+n_F(y)=1$.

In the steady state, the mean rate of unemployment is constant, so two steady state conditions apply.

First, the sum of the flows into unemployment must equal the sum of the flows out of unemployment [58].

$$\lambda H_I(R_{IU}(y)\mid y)n_I(y)+\lambda H_F(R_{FU}(y))H_I(R_{UI}(y)\mid y)n_F(y)= \tag{47}$$

$$=\alpha[1-H_I(R_{UI}(y)\mid y)]u(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]u(y)$$

Second, the analogous condition for the formal sector:

$$\lambda H_F(R_{FU}(y))n_F(y)=m(\theta)[1-H_F(R_{UF}(y)\mid y)]u(y) \tag{48}$$

When solving we get:

$$u(y)=\frac{\lambda H_F(R_{FU}(y)\mid y)H_I(R_{IU}(y)\mid y)}{\alpha[1-H_I(R_{UI}(y)\mid y)]L(y)+m(\theta)[1-H_F(R_{UF}(y)\mid y)]K(y)+\lambda H_F(R_{FU}(y))H_I(R_{IU}(y)\mid y)} \tag{49}$$

[58] Alternatively, we could have used the conditions that the flow into unemployment out of sector i must equal the flow out of unemployment into state i, for all i. These conditions are sufficient for SS to hold but not necessary.

$$n_F(y) = \frac{m(\theta)[1 - H_F(R_{UF}(y) \mid y)]K(y)}{\alpha[1 - H_I(R_{UI}(y) \mid y)]L(y) + m(\theta)[1 - H_F(R_{UF}(y) \mid y)]K(y) + \lambda H_F(R_{FU}(y))H_I(R_{IU}(y) \mid y)} \tag{50}$$

$$n_I(y) = \frac{\alpha[1 - H_I(R_{UI}(y) \mid y)]L(y)}{\alpha[1 - H_I(R_{UI}(y) \mid y)]L(y) + m(\theta)[1 - H_F(R_{UF}(y) \mid y)]K(y) + \lambda H_F(R_{FU}(y))H_I(R_{IU}(y) \mid y)} \tag{51}$$

where $L(y)$ and $K(y)$ are

$$L(y) \equiv H_F(R_{FU}(y) \mid y) \tag{52}$$

$$K(y) \equiv H_F(R_{FU}(y) \mid y) + H_I(R_{IU}(y) \mid y)\,[1 - H_F(R_{FU}(y) \mid y)] \tag{53}$$

Equation (49) corresponds to a modified version of the Beveridge curve, a negative relation between labor market tightness and unemployment, or alternatively, between vacancies and unemployment. When θ increases, $m(\theta)$ increases as well, encouraging job creation and reducing $u(y)$ ('job creation' effect). Stochastic job matching and endogenous job destruction gives us an additional counteracting effect ('reservation productivity' effect): an increase in θ also increases $rU(y)$ (more outside opportunities), which increases $R_{UF}(y)$, $R_{UI}(y)$, and $R_{FU}(y)$. The first two discourage job creation and the latter encourage job destruction, both of which increase unemployment[59].

Aggregating across types we get aggregate unemployment:

$$u = \int_0^\infty u(y)f(y)dy \tag{54}$$

Intuitively, in equilibrium there will be some imperfect sorting of workers among sectors based on their types, y. 'High' type workers are more likely to take formal-sector jobs while 'low type' workers are more likely to take informal-sector jobs, and medium type workers will take both[60].

In addition to compositional effects, changes in policy parameters affect the steady state

[59]We assume that the first effect dominates the second effect (since empirical evidence supports a downward-sloping Beveridge curve. Stochastic job matching means that changes in labor taxes, subsidized health or prices may shift the relationship between vacancies and unemployment, and that the effect of a change in the relevant policy parameter on u via job offers is mitigated, since there are additional effects that offset this demand-side effect.

[60]This is more or less consistent with the empirical fact that the formal sector is skilled labor intensive while the informal sector is unskilled labor intensive in Colombia: average years of schooling for a FS worker is 14.45 , while for an IS worker is 9.46 years (Source: Author's calculations based on ECH, June 2003)

distributions of productivities among sectors, affecting also the distributions of wages across formal and informal employment.

3.8 Determination of the Value of Unemployment

The only thing needed to solve the model is to determine $U(y)$ for all values of y, as a function of reservation productivities and labor market tightness parameters.

From equation (1), together with equation (35) and (41) we get:

$$rU(y) = b + \alpha\beta(1+\hat{\delta}_3)\int_{R_{UI}(y)}^{\infty} \frac{p_I\left(y' - R_{IU}(y)\right)}{r+\lambda} dH_I(y' \mid y) + \tag{55}$$

$$+m(\theta)\beta(1-\hat{\delta}_2)\int_{R_{UF}(y)}^{\infty} \frac{p_F\left(y' - R_{UF}(y)\right)}{(1+\delta_1)(r+\lambda)} dH_F(x \mid y)$$

3.9 Steady State Equilibrium

Definition 1 *Given a vector of parameters $\{b, \alpha, \beta, b, c, \lambda\}$, a vector of prices $\{p_F, p_I, r\}$, a vector of taxes and subsidies $\{\delta_1, \delta_2, \delta_3, s\}$, a vector of valuation of social security and social assistance services $\{\tau, \mu\}$, matching function $m(.)$, and cumulative density functions $F(y)$, $H_i(y' \mid y)$ and $H_i(x \mid y)$ (for i=I, F) , a* ***Steady State Equilibrium with an Informal sector*** *is a vector formed by the unemployment rate $u(y)$, sector-i employment rates $n_i(y)$, the value of unemployment $U(y)$, the reservation productivities $R_{UF}(y)$, $R_{FU}(y)$, $R_{UI}(y)$, $R_{IU}(y)$ and $R_{FI}(y)$, sector-i wages $w_i(y', y)$ (for i=I, F)and $w_i^s(x, y)$ (for i=F), and labor market tightness in the formal and informal sector θ and ϑ, such that:*

1. Given $R_{UF}(y)$, $R_{IU}(y)$, and θ, the flow value of unemployment $U(y)$ satisfies equation (55).

2. Given $U(y)$ and $R_{FI}(y)$, the reservation productivity schedule $R_{FU}(y)$ satisfies equation (27).

3. Given $R_{FU}(y)$, the reservation productivity schedule $R_{UF}(y)$ satisfies equation (31).

4. Given $U(y)$, the reservation productivity schedule $R_{IU}(y)$ satisfies equation (33).

5. Given $R_{UI}(y)$,$R_{UF}(y)$, $R_{FU}(y)$ and θ, the unemployment rate and the sector-i employment rates satisfy equations (49), (50) and (51).

6. Given $R_{UF}(y)$ and $u(y)$, the labor market tightness parameter θ satisfies equation (44).

7. Given $R_{FU}(y)$, $R_{IU}(y)$, $u(y)$ and $n_F(y)$, the labor market tightness parameter ϑ satisfies equation (46).

8. Given $U(y)$, $R_{FU}(y)$, $R_{FI}(y)$, $R_{IU}(y)$, formal-sector wages $w_F(y', y)$ and $w_F^s(x, y)$

satisfy equations (20) and (22).

9. Given $U(y)$, informal-sector wages $w_I(y', y)$ satisfies equation (24).

Existence and Uniqueness of Equilibrium- Proof.

The Steady State Equilibrium with an Informal sector exists if there is a unique θ that satisfies the job creation condition in the formal sector, equation (44), since all the other equilibrium objects are uniquely determined by θ.

Given that:

$$\lim_{\theta\to\infty} \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF(y)}}^\infty \frac{p_F(1-\beta)[y' - R_{UF}(y)]}{r+\lambda} dH_F(y' \mid y) dF_U^*(y) = 0$$

$$\lim_{\theta\to 0} \frac{m(\theta)}{\theta} \int_0^\infty \int_{R_{UF(y)}}^\infty \frac{p_F(1-\beta)[y' - R_{UF}(y)]}{r+\lambda} dH_F(y' \mid y) dF_U^*(y) = \infty,$$

A straightforward application of the Intermediate Value Theorem prove that a solution to equation (44) exists.

To establish uniqueness we need to show that the above expression is strictly monotone. First, $m(\theta)/\theta$ is monotonically decreasing by assumption. Second, $R_{UF}(y)$ is monotonically increasing in θ (since higher θ means higher $U(y)$), so $J_F(y', y)$ is monotonically decreasing in θ. Finally, $u(y)$ should be decreasing in θ due to the dominant negative impact of formal-sector job creation on unemployment (as explained before), and the aggregate unemployment rate u should also be decreasing in θ. Further assumptions on $H_i(y' \mid y)$ are required to prove that the ratio $u(y)/u$ is monotonically decreasing as well, so uniqueness is not guarantee. ■

Once we solve the model numerically [61], the empirical strategy consists in calibrating the model using micro data, and then simulate how labor market policies, subsidized health programs and relative prices affect the mix of worker types in two sectors (compositional effects) and the distribution of wages (distributional effects)[62].

[61] For details on computational algorithm to solve the model see Appendix, Section 6.2.

[62] Instead of simulating the steady state distributions of equilibrium productivity and wages in both sectors, we derived them analytically. See Appendix 6.1 for derivations

4 Empirical Strategy

4.1 Data

To calibrate the model, we use data from the Colombian Household Surveys [63], repeated cross-sections carried out by the Colombian Statistics Department (DANE) on employed and unemployed individuals in thirteen metropolitan areas, for the second quarter of 2003 (June), since we have an informality module for this period [64]. The sample size for this period is 111,082 observations representing 56,4 million people.

In addition to standard demographic socio-economic variables (age, gender, marital status, educational attainment, etc), the sample is described by the following labor market variables:

$$\left(\{W_s\}_{s\in E_j}; \{t_{es}\}_{s\in E_j}; \{t_{nes}\}_{s\in E_j}; \{t_{us}\}_{s\in U}; \{h_s\}_{s\in E_j}\right)_{j=F,I}$$

W_s: Accepted wages for individual s, where $s \in E_j$, so each individual can be employed in sector j (formal or informal) [65]

t_{es}: Employment duration for individual s, where $s \in E_j$.

t_{nes}:Non-Employment duration [66] of previous employment for individual s, where $s \in E_j$.

[63]These surveys include: Encuesta Nacional de Hogares (ENH) for the period 1984 qI to 2000 q2, Encuesta Continua de Hogares (ECH) for the period 2000 q2 to 2003 q2, and Gran Encuesta Integrada de Hogares (GEIH) for the period 2007m1-2010m12

[64]An informality module is available the second quarter of every year after 2001, and every two years before 2001.

[65] The following operational definitions of informality are used for the construction of statistics:

"Social Protection" (SP) Informality Definition: A worker is considered informal if any the following two conditions hold:

- *Health Affiliation*: Is not affiliated to health, or if is affiliated does not make any contributions to the system (part of subsidized regime but not contributive regime)
- *Pension Affiliation*: Not affiliated to a pension fund

This variable is primarily a proxy for non-compliance to labor regulations in Colombia.

"Firm Size and Occupation" (FSO) Informality Definition: A worker is considered informal if any of the following two criteria hold:

- *Firm Size*: Works in firm with less than 10 or fewer employees
- *Occupation*: Works as domestic employee, self-employment, employer or unpaid family worker.

This definition is the one used by the Colombian Statistics Department, DANE, and is consistent with the one used by the ILO, but it does not include any criteria related to non-compliance with regulations.

[66]These variable measures months without employment between the current job and the previous job (retrospective question), so we cannot identify whether it refers to unemployment or inactivity duration. The exact unemployment duration can be obtained from the unemployed population, however, this variable is right censored, while for the occupied population we don't have the same problem.

t_{us}: Unemployment duration for individual s, where $s \in U$ (incomplete spells for unemployed -right censored)

h_s: Weekly hours of work for individual s

Even if the data is not longitudinal, retrospective questions about previous unemployment and employment status for both employed and unemployed individuals are available, allowing to construct transition flows across the different states. There is no retrospective information on social security contributions, so to determine whether a worker was considered formal or informal before being unemployed or employed in the actual job we have to use the definition of informality based on firm size and occupation.

4.2 Descriptives

The following table shows the descriptive statistics for the occupied population, by sector.

Table 2: Descriptive Statistics for Occupied

	All Occupied	Informal	Formal	I/F Ratio
N	44,930	30,019	14,911	2.01
Population	23,001,253	14,182,143	8,819,110	1.61
E($lnW_s \mid s \in E_j$)	7.52	6.96	7.97	0.87
SD($lnW_s \mid s \in E_j$)	1.28	1.40	0.97	1.44
E($t_{es} \mid s \in E_j$)	78.60	74.58	85.08	0.88
SD($t_{es} \mid s \in E_j$)	103.80	107.6	96.90	1.11
E($t_{nes} \mid s \in E_j$)	10.12	11.93	7.66	1.56
SD($t_{nes} \mid s \in E_j$)	18.92	21.04	15.19	1.38
E($h_s \mid s \in E_j$)	47.48	46.03	49.81	0.92
SD($h_s \mid s \in E_j$)	19.44	21.72	14.78	1.47

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Nominal wages in hourly rates, employment and non-employment duration in months. Informality using SP definition (except for non-employment durations).

When using the SP definition, the estimated informality rate for 2003 is closed to 62% (s.e. of .486) , or the informal sector is 1.6x the size of the formal sector.

In comparison with the formal sector, the informal sector is characterized by:

- *Lower and More Disperse Wages* [67]: Log hourly wages are on average, lower, and more disperse than in the formal sector. On average, an informal sector worker earns 0.3

[67]The lower tail of the hourly wage distribution is excluded (wages below 1 peso) to minimize the effects of measurement error. These measure of wages includes tips, commissions but exclude non-monetary payments

what a formal worker earns.

- *Less Job Stability*: average employment duration in the informal sector is 74.58 months (6.2 years) while in the formal sector is 85.08 months (7.09 years)
- *Higher Non-Employment Duration*: if unemployed/inactive in the previous year, informal sector workers faced higher unemployment/inactivity duration on average than their formal sector counterparts (11.9 vs. 7.7 months)
- *Fewer hours of work*: informal sector workers work, on average, 46 hours per week, while their formal sector counterparts work 50 hours per week.

The following table show the descriptive statistics for the unemployed population.

Table 3: Descriptive Statistics for Unemployed

	All Unemployed
N	9,704
Population	4,766,695
$P(s \in U)$	17.17
$E(t_{us} \mid s \in U)$	50.16
$SD(t_{us} \mid s \in U)$	51.96

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. Unemployment duration in weeks

The estimated unemployment rate for the year 2003 is quite high: 17.17% (s.e. of 0.0016).

Average unemployment duration is quite high (50.16 weeks or 11.5 months), and slightly higher than the mean non-employment duration for the occupied population (10.2 months), even if the estimate is downward biased since the variable is right-censored.

4.3 Labor Market Dynamics

Standard static analysis of labor markets that analyze stocks of workers in different states does not tell us anything about where those workers arrived from, how long they will stay, or where they will go next.

Here we use retrospective questions from our cross-section database of June 2003 to construct a set of statistics that allows us to analyze labor market dynamics and derive some stylized facts about movement across sectors in the Colombian labor markets.

4.3.1 Annual Discrete Transition Matrix

We need empirical evidence supporting the assumption in the model of direct transitions from formal to informal, so we want to have a decent estimate of the flows between the formal and informal sector. For this, we want to condition on the number of workers who switch jobs in a 12-month period, and see how many of them change sector and how many stay in the same sector.

Let p_n^{ML} be the estimated probability of transitioning from state i to state j in a 12-month period. We estimate the annual discrete transition matrix using maximum likelihood as follows:

$$p_n^{ML} = \frac{\text{Number of transitions from state } i \text{ to state } j}{\text{Total number of observations in state } i} \tag{56}$$

The ML estimates are shown in the next matrix:

Table 4: Annual Discrete Transition Matrix

		June 2003		
		Formal	Informal	Total
June 2002	Formal	58.59	41.41	100
		(0.011)	(0.0066)	
	Informal	23.81	76.19	100

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

We found that despite there is a very high persistence, especially in the informal sector, the estimated flows across sectors are not negligible: 41.41% of workers who are in FS Employment in June 2002 become employed in the IS in June 2003, and 23.81 % of workers who were informal in June 2002 became formal in June 2003.

This empirical evidence supports the assumption in the model of direct transitions from formal to informal. Transitions from informal to formal are less important. It is important to note that these probabilities are not transition probabilities, since there are many workers who change status within a year whose transitions are not captured here.

4.3.2 Intensity Matrices and Hazard Rates

We estimate intensity matrices by estimating gross worker flows across states and use these flows to compute a maximum likelihood estimator [68]. Even if the data is not longi-

[68] Another approach may be to think about an underlying continuous Markov process that generates the discrete time mobility process, and therefore, estimate the transition probabilities accordingly. See Maloney and Bosh, 2007.

tudinal, we have retrospective questions in the survey that allow us to identify job-to-job transitions[69].

Let each element of the intensity matrix be $q_{ij}(t)$, the conditional probability of moving from state i to state j in period t [70]. Then $h_{ii}(t) = 1 - q_{ii}(t)$ corresponds to the hazard rate (separation rate) out of sector i, conditional of being in state i up to period t.

The following tables show the estimated worker flows across states and the corresponding intensity matrix [71].

Table 5: Gross Worker Flows across States (Population Values)

From \To	Unemployment	Formal Sector	Informal Sector
Unemployment	3,787,491	928,875	2,170,920
Inactive	449,701		
Formal Sector	231,801	7,059,177	491,916
Informal Sector	297,702	440,750	11,909,615
Total	4,766,695	8,428,802	14,572,451

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas.
All statistics weighted using expansion factors. Informality using Firm size and Occupation definition.

In 2003, we estimate that 79% of workers who are unemployed in June 2003 were also unemployed in June 2002, suggesting a hazard rate out of unemployment conditional of being unemployed of 0.21. Approximately 5% and 6% of workers who are unemployed in June 2003 were employed, respectively, in the formal and informal sector in June 2002. About 9% of unemployed workers in June 2003 came from an inactivity state.

When conditioning on working in the formal sector, even if there is some persistence in the formal sector (84 % of formal sector workers stay formal over the course of a year), approximately 11 % of formal workers have transited from the non-employment state (unemployment or inactivity) and 5 % have transited from informal sector jobs. These figures imply a hazard rate out of the formal sector of 0.16, which suggests low labor mobility from this sector to other sectors.

When conditioning on working in the informal sector, persistence is also quite high (82 %

[69]In the survey, we have the following questions for occupied: 1)How long have you been employed continuously in the current job?; 2) How many months were you without employment before your current job and the previous job? If the answer is less than 1 month, this can be classified as a job-to-job transition, while if it is more than 1 month it's a transition from u or inactive (cannot identify the original state) to a job. A similar question is asked to unemployed, so we can identify job-to-unemployed transitions.

[70]Here, t = June 2003

[71]Time aggregation bias may still arise because the monthly measurement may combine multiple transitions into a single aggregate transition, so transitions that occur at higher frequencies than a month are not captured here.

Table 6: Intensity Matrix

From \To	Unemployment	Formal Sector	Informal Sector
Unemployment	0.79	0.11 [a]	0.15 [b]
	(0.004)	(0.0026)	(0.0020)
Inactive	0.09		
	(0.0029)		
Formal Sector	0.05	0.84	0.03
	(0.0021)	(0.0030)	(0.0010)
Informal Sector	0.06	0.05	0.82
	(0.0024)	(0.0018)	(0.0022)

[a] Transition from non-employment state (unemployment or inactivity) to formal.
[b] Transitions from non-employment state (unemployment or inactivity) to informal.
Author's calculations based on ECH, June 2003, 13 Metropolitan Areas.
All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

of informal sector workers stay informal over the course of a year). Only approximately 15 % of informal workers have transited from from the non-employment state (unemployment or inactivity) and 3 % have transited from formal sector jobs. These figures suggest a hazard rate out of the informal sector of 0.18, higher than in the formal sector.

Table 7: Estimated Hazard Rates

	Unemployment	Formal Sector	Informal Sector
Hazard Rates	0.21	0.16	0.18
	(0.0041)	(0.0030)	(0.0022)

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors.
Informality using Firm size and Occupation definition. Standard errors in parentheses

4.3.3 Expected Durations

Let t_i be the duration spell for state i. Using a partial-partial model of expected unemployment duration where the hazard h_{ii} is constant, we can deduce that:

$$t_i \sim exp\left(h_{ii}\right) \tag{57}$$

so we can also back out average duration in state i from the separation rates as follows:

$$E(t_i) = \frac{1}{h_{ii}} \tag{58}$$

This means that if we compare the sample mean length of employment or unemployment spells and the inverse of the estimated mean hazard rate out of state i conditional of being in state i, these estimates should be quite closed.

In the data, we have information for non-employment duration (complete spells for occupied), and unemployment duration (incomplete spells for unemployed -right censored). We also have employment duration for workers in the formal and informal sector (incomplete spells). The following table shows the average duration in each state, using hazards as well as sample means for different definitions of informality.

Table 8: Estimated Mean Duration

		Unemployment (Weeks)	Formal Sector (Months)	Informal Sector (Months)
Estimated using hazards	FSO Definition	253	73.8	65.6
		(0.0041)	(0.0030)	(0.0022)
Estimated from Duration Data		50.16		
		(0.537)		
	SP Definition		85.08	74.58
			(0.795)	(0.623)
	FSO Definition		73.34	81.65
			(0.763)	(0.632)

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition. Standard errors in parentheses

Mean unemployment duration is 50 weeks using right censored-duration data, much lower than using estimates from hazards, which gives us an estimate of 253 weeks. When using the FSO definition, a formal sector job lasts 73 months on average, an estimate consistent when using duration data and hazards rates. Mean length of informal employment (FSO definition) varies more when using the two methodologies: 65 vs. 81 months.

4.3.4 Propensity Matrices

The intensity matrices tell us the probability of a worker moving across sectors but do not tell us anything about how easy or desirable is to move to a particular sector, so we cannot compare workers flowing into a particular sector from different states. For instance, it may be the case that the intensity of transition from state i (formal) into sector

j (informal) is higher for low-skilled workers, so we may think that low skilled workers are more willing to move to informal sector jobs. However, it may be also that more low-skilled workers are leaving the formal sector at a given point in time (higher separation rates).

To do the right comparison across groups, we would have to create a statistic that condition on the separation rates of that sector, the propensity, as follows:

$$r_{ij} = \frac{q_{ij}}{h_{ii}}, \forall i \neq j \tag{59}$$

This can be interpreted as follows: If all workers were to leave sector i at the same rate (conditioning on having the same separation rate h_{ii}), what would be the probability of ending in sector j. The following table shows the estimated propensities:

Table 9: Propensity Matrix

From \To	Unemployment	Formal Sector	Informal Sector
Unemployment		0.54	0.73
Formal Sector	0.30		0.21
Informal Sector	0.34	0.29	

Author's calculations based on ECH, June 2003, 13 Metropolitan Areas. All statistics weighted using expansion factors. Informality using Firm size and Occupation definition.

So conditioning on separation rates, unemployed workers are more willing to move to informality, formal sector workers are more willing to move to unemployment, and informal sector workers are also more willing to move to unemployment.

So for the previous example, higher hazard for low skilled workers means higher turnover, and therefore lower propensity. This 'corrected' probability will somehow capture the the predisposition of a particular type of worker to flow to another sector[72].

4.4 Calibration

In the calibration, we are particularly interested in matching aggregate unemployment and employment rate figures, getting reasonable labor market tightness parameters for international standards[73], and matching selected moments of the wage distribution in both sectors.

We partition the parameter space of the benchmark model in two groups. In the first group, parameters are calibrated based on previous results from micro studies or data. This

[72] See Maloney and Bosh, 2007.

[73]There are no reliable estimates for the Colombian case since there is no data on job vacancies.

group includes λ, b, β, the parameters of the matching function, the parameters of the distribution $f(y)$, and the location parameter of the distributions $h_i(y' \mid y)$ and $h_i(x \mid y)$. In the second group, we choose parameters to match the division of the labor force among unemployment, informal and formal-sector employment, and chosen moments of the earnings distributions. This includes c, α, and the scale parameters of distributions $h_i(y' \mid y)$ and $h_i(x \mid y)$.

The parameter values are chosen with a year as an implicit unit of time.

Table 10: Fixed Parameters -Based on data or previous micro studies

	Description	Value
PARAMETERS		
λ	rate of arrival productivity shock	0.3
b	opportunity cost of leisure	2.25
β	worker's Nash bargaining power	0.72
A	technological parameter, matching function	3
α_m	elasticity matching function	0.72
μ_y	mean, worker types	11.08
σ_y^2	variance, worker types	26.11
$\mu_{y'_F}$	FS match-specific productivity, conditional mean	$log(y)$
$\mu_{y'_I}$	IS match-specific productivity, conditional mean	$0.9log(y)$
μ_{x_F}	FS idiosyncratic productivity shock, conditional mean	$log(y)$
μ_{x_I}	IS idiosyncratic productivity shock, conditional mean	$0.9log(y)$
PRICES		
r	interest rate	0.04
P_F	price formal sector good	0.84
P_I	price informal sector good	0.76
WORKERS' VALUATIONS		
τ	FS worker, valuation of social security services	0
μ	IS worker, valuation of social assistance services	0.5

We choose the rate of arrival of the productivity shock, λ, equal to 0.3. Using the hazard rates out of the formal and informal sector, h_{FF} and h_{II} we can express λ as follows:

$$\lambda = \frac{h_{FF}}{H_F(R_{FU}(y) \mid y)} \tag{60}$$

$$\lambda = \frac{h_{II}}{H_I(R_{IU}(y) \mid y)} \tag{61}$$

Using duration data (FSO definition), we know that a formal-sector job lasts on average 6.1 years while an informal-sector job lasts 6.8 years, which gives us estimated hazard rates out of formal and informal-sector employment of 0.16 and 0.147, respectively. Given that

$0 \leq H_F(R_{FU}(y) \mid y) \leq 1$ and $0 \leq H_I(R_{IU}(y) \mid y) \leq 1$, we have that $0.147 \leq \lambda < \infty$ and $0.16 \leq \lambda < \infty$. In ANV2009, λ is set to 0.5, while in Hornstein, Per and Violante (2005)(hereafter HPV2005), λ is set to 0.1[74], so assuming 0.3 shocks per year is within an admissible range in the model and in the previous literature [75].

We set the opportunity cost of leisure, b, equal to 2.25. In Colombia there are no unemployment benefits, so we have to think about what the monetary compensation for the unemployed should be, relative to the average wage, or the average 'replacement rate'. According to HPV2005, such rate is 0.7 for European countries where benefits are relatively high, and 0.2 for the U.S (at most). In Shimer (2005) this rate is about 0.4, since $b = 0.4$ and the average wage is closed to 1. For the Colombian case, it may be reasonable to assume that this ratio is between 0.2 and 0.4. Considering that the average wage in the model is closed to 6.77, the chosen b gives us a replacement rate of 0.33, close to Shimer (2005) estimate.

Following Shimer (2005), we set $\beta = 0.72$, so shares are not necessarily split equally between workers and firms[76].

The parameters of the matching function are set in the following way. We assume a standard Cobb-Douglas function given by $m(\theta)$=$A\theta^{1-\alpha_m}$. We choose $A = 3.0$ and $\alpha_m = 0.72$. Since there is no data on vacancies for Colombia, we follow standard results for the U.S.

Following Shimer (2005), the elasticity of the matching function for the U.S. is about 0.72[77]. Also, job-finding rates in the U.S. are estimated to be 0.45 per month, or 5.4 per year, which gives us an upper bound for $m(\theta)$. We assume job-finding rates in Colombia in the formal-sector are closer to 3 per year. Assuming a reasonable labor market tightness, θ, ranging between 0.8 and 1.25, gives us a technological parameter A ranging between 2.81 and 3.64. In ANV2009 this parameter is set to 4.0, so setting $A = 3.0$ is quite reasonable.

We assume the following functional forms for the distributions of types and productivity:

$$f(y) = logN(\mu_y, \sigma_y^2)$$

$$h_i(y' \mid y) = logN(\mu_{y_i'}, \sigma_{y_i'}^2) \quad \text{, where} \quad \mu_{y_i'} = B_i log(y), \quad \text{for i=F,I.}$$

$$h_i(x \mid y) = logN(\mu_{x_i}, \sigma_{x_i}^2) \quad \text{, where} \quad \mu_{x_i} = B_i log(y), \quad \text{for i=F,I.}$$

The parameters are chosen in the following way: μ_y, σ_y^2 are chosen to coincide with the

[74]Using as unit of time one quarter.

[75]When using hazards, we have that $0.16 \leq \lambda < \infty$ and $0.18 \leq \lambda < \infty$, so choosing λ=0.3 is also within a permissible range.

[76]This is because of the Hosios condition for an efficient search, which requires $\beta = \alpha_m$.

[77] Alternative estimates for α_m include Mortensen and Nagypal (2007) and Brugemann (2008). The first paper estimate this elasticity at 0.45 and the latter between 0.54 and 0.63

corresponding empirical moments[78]. We assume $B_F = 1.0$ and $B_I = 0.9$, so the mean of y' varies linearly with y in both sectors, but the conditional mean of the match-specific productivity is less sensible to y in the informal than in the formal sector.

Prices r, P_F and P_I are chosen to match the real interest rate and the price of tradables and non-tradables in Colombia in the year 2003. The policy parameters δ_1, δ_2 and s correspond to our estimated 2003 values: $s = 0$, $\delta_1 = 0.534$, $\delta_2 = 0.0837$, and δ_3 is set at 0.118 [79].

With regards to worker's preferences, in the benchmark case we assume that workers do not value social security services ($\tau = 0$) as in a model without valuations, but they value social assistance benefits partially ($\mu = 0.5$), so a change in δ_3 affects worker's behavior[80].

Table 11: Calibrated Parameters (Moment Simulation)

	Description	Value
PARAMETERS		
c	cost of posting a vacancy	0.6
α	rate of arrival IS opportunities	2.2
$\sigma^2_{y'_F}$	FS match-specific productivity, log variance	0.25
$\sigma^2_{y'_I}$	IS match-specific productivity, log variance	0.28
$\sigma^2_{x_F}$	FS idiosyncratic productivity shock, log variance	0.25
$\sigma^2_{x_I}$	IS idiosyncratic productivity shock, log variance	0.28

We still need to choose the cost of posting a vacancy, c, and the rate of arrival of IS opportunities, α. We set $c = 0.6$ and $\alpha = 2.2$ to match the aggregate unemployment and employment rates in the two sectors, and relative mean log-wages.

Standard deviations are chosen to generate more dispersion in the informal sector so $\sigma^2_{y'_I} = \sigma^2_{x_I} = 0.28$, and $\sigma^2_{y'_F} = \sigma^2_{x_F}$=0.25.

The model can match pretty well the aggregate unemployment rate, formal and informal-sector employment rates, mean unemployment duration and relative mean log-wages. The model is producing shorter employment durations [81] and more dispersion in log-wages within the informal sector relative to the formal.

The following table summarizes the main results of the calibration.

[78] We use educational attainment as a proxy for worker types.

[79] To get s, severance in the formal sector is one monthly wage per year plus 1%interest (or 8.33% plus 1% interest=9.33% of average wage), but since it's paid every year and not when employment terminates, it's included in the total figure of δ_1. To get δ_3, we need to calculate the expected value of the services received in the subsidized regime as a % of the wage. We multiplied the value of the services offered in the contributive regime (we assume their value is the same) by the corresponding probability of getting those services, so $\delta_3 = P_i(\delta_1 + \delta_2)$, where P_i is the proportion of IS workers who receive subsidized health in the data. See detailed tables of employer and employee NWL costs in Appendix

[80] We will see in the policy experiments how results change with change in valuations.

[81] Here we compare with the mean estimated duration using hazards.

Table 12: Calibration: Data-based vs. Simulated Statistics

Variable	Model	Data
POLICY PARAMETERS		
s	0	0
δ_1	0.534	0.534
δ_2	0.083	0.083
δ_3	0.118	–
AGGREGATE UNEMPLOYMENT AND EMPLOYMENT RATES		
u	0.172	0.172
n_F	0.310	0.317
n_I	0.518	0.510
LABOR MARKET TIGHTNESS		
θ	1.018	n.a.
ϑ	0.598	n.a.
MEAN EMPLOYMENT AND UNEMPLOYMENT DURATIONS [a]		
t_U	12.3	11.5
t_{e_F}	47.2	73.8
t_{e_I}	56.6	65.6
MEASURES OF WAGE DISPERSION		
$\mu_{ln(W_I)}/\mu_{ln(W_F)}$	0.904	0.873
$\sigma_{ln(W_I)}/\sigma_{ln(W_F)}$	2.09	1.16

[a] All expected durations are in months. Durations from data are estimated using hazards.

Appendix 6.4 contains the the simulated distributions and equilibrium objects for the benchmark case.

Figure 1 and 2 show the assumed distribution of worker types, y, and the conditional distribution of match-specific productivities, y', for given levels of y, where the shape reflects the first -order stochastic dominance assumption. Figure 3 shows the steady state distribution of types across workers employed in both sectors, where as expected, the formal sector distribution is biased to the right compared to the one in the informal sector. These densities are 'contaminated' , i.e. only incorporate a restricted pool of workers in each sector.

Figures 4 to 6 show the relevant surplus functions in both sectors, decreasing in types for a given productivity value.

Figure 7 presents the reservation productivity schedules. We see that $R_{UF}(y) > R_{UI}(y)$ for all y, and both are strictly increasing in y. Conditional on type, workers in the formal sector are 'pickier' and less likely to start a match (and more likely to discontinue a match[82]) than workers in the informal sector. 'High' type workers are 'pickier' than 'low' type workers since they have more outside opportunities, an effect that for this parametrization is stronger

[82]Here, $R_{FU}(y) = R_{UF}(y)$ since it is assumed in the simulations that $s = 0$.

than the 'labor hoarding' effect.

Figure 8 and 9 show job destruction and creation rates, higher in the informal than in the formal sector, conditional on type. Job destruction and job creation rates are non-monotone in y, reflecting the non-monotonicity of $H_I(R_{IU}(y) \mid y)$ and $H_F(R_{FU}(y) \mid y)$. When workers are more educated they become 'pickier' (higher $R_{IU}(y)$ and $R_{FU}(y)$) since they have more outside opportunities, but when the shock arrives they are also more likely to draw a high productivity draw x (making them more likely to continue in the match), counteracting the first effect. For some range of y the second effect dominates, suggesting that the probability of discontinuing the match in both sectors is decreasing in y. The same argument applies to the job creation rates.

Figure 10 shows unemployment and informal-sector employment rates that are non monotone in y, and a formal-sector employment rate that is increasing in types. Regardless of type, all workers are more likely to be in the informal sector. 'Low' type workers are more likely to be informal or unemployed, and less likely to be in the formal sector, while 'high' type workers are more likely to be in the informal or formal sector, and less likely to be unemployed.

Figure 11 and 12 show non-monotone unemployment durations (but decreasing over a wide range of y), and non-monotone employment durations. Regardless of type, informal-sector jobs last more than formal-sector jobs.

Figure 13 to 15 show wage functions in both sectors increasing in types and productivity.

Figure 16 and 17 show simulated steady state distributions of wages and productivities, biased to the right in the formal sector.

4.5 Simulations

4.5.1 Labor Tax Experiments

We simulate an increase of 4.46% and 7.53% in employer and employee NWL costs, δ_1 and δ_2, respectively[83]. Four scenarios are analyzed under different worker's valuation of social security services ($\tau = 0$, $\tau = 0.1$, $\tau = 0.5$, $\tau = 1.0$). If workers do not value SS contributions ($\tau = 0$), an increase in δ_2 is seen as a higher net cost from the worker's perspective, affecting negatively flow utility in the formal sector, while an increase in δ_1 will not affect directly the worker's value function. If workers do value exaggeratedly these contributions ($\tau = 1$), an increase in δ_2, will not affect the worker's value function because, from the workers perspective the net benefit/cost is zero, while an increase in δ_1 increase the flow income in the formal sector, since workers associate this increase with more valuable and efficient services.

[83]This is the actual increase from 2003 to 2012.

Scenario 1: No valuation of SS services ($\tau = 0$)

Higher δ_1 and δ_2 makes continuing in a formal-sector match more costly both for the firm and the worker, which entails a higher reservation productivity $R_{FU}(y)$ and a greater probability of discontinuing the formal-sector match. From the job creation side, workers and firms are also less likely to start a formal-sector match (higher $R_{UF}(y)$), so the expected benefit of a formal-sector firm from hiring an unemployed worker decreases, which implies less vacancy creation in the formal sector (θ goes down from 1.01 to 0.8).

There is also a positive impact on vacancy creation in the informal sector driven by the allowed flows in the model from the formal to the informal sector. Since there is a higher probability that a formal-sector worker ends up in unemployment, the expected benefits of an informal-sector firm from hiring a formal-sector worker affected by a 'bad' shock are bigger, which involves more vacancy creation in the informal sector (ϑ goes up from 0.59 to 0.68). In other words, the informal-sector acts as a 'bumper' sector, absorbing some displaced workers from the other sector.

Given that there is more job destruction and less job creation in the formal sector, the formal-sector aggregate employment rate decreases from 0.310 to 0.262 but the level of absorption in the informal sector is quite high, since the informal-sector aggregate employment rate increases from 0.518 to 0.559. As a result, aggregate unemployment increases slightly from 0.172 to 0.178.

This policy increases unemployment duration slightly (in less than a month), and decreases the hazard rate out of unemployment by shifting up the reservation productivity schedule, $R_{UF}(y)$. It also decreases formal-sector job duration (in about 3 months), increasing slightly the hazard rate out of the formal sector.

Given the compositional effects of this policy and the upward shifts of the reservation productivity schedule, the density of productivities in the formal sector shifts to the right, causing an average formal-sector productivity rise. Also, when δ_1 increases, the benefits of the formal-sector firm from negotiating with the worker are reduced, improving firm's bargaining power in the negotiation, and causing a reduction in the formal-sector wage. From the worker's perspective, a higher δ_2 implies less benefits from the negotiation agreement with the firm, an improved bargaining power and therefore, a higher formal-sector wage.

There are also some additional effects that need to be considered, since a higher reservation productivity $R_{UF}(y)$ implies a lower continuation value in the wage negotiation, and therefore a lower formal-sector wage. The flow value of unemployment $U(y)$ is also affected in equilibrium affecting the continuation value of the negotiation in both sectors.

Finally, the average formal-sector wage increases when considering all equilibrium effects, so the mean log wage of the informal relative to the formal sector decreases from 0.90 to

0.879, leading to an important increase in between-sector variance that drives up variance of log wages slightly, from 0.131 to 0.1439.

Scenario 2: Low valuation of SS services ($\tau = 0.1$)

When workers value social security services somehow, $\tau > 0$, qualitative results change since workers may perceive this policy either as a policy that reduces employee NWL costs (or a reduction in the payroll tax), or, in a more extreme case, as a subsidy from the government.

In this case, formal-sector firms are affected by higher NWL costs (higher δ_1), and therefore are more willing to end the match. The parameter $\hat{\delta}_2$ is actually reduced (from 0.083 to 0.025), which means that, from the workers' perspective, the higher benefits more than compensate the higher costs. Workers perceive this policy exactly as a policy that reduces employee NWL costs in 71.9%, which means they are more willing to continue in a match, counteracting the initial upward impact on the reservation productivity. The latter effect dominates so the probability of discontinuing the formal-sector match goes down.

From the job creation side, workers are also more willing to start a formal-sector match, causing a lower $R_{UF}(y)$, affecting job creation positively in the formal sector (θ goes up from 1.01 to 1.97). There is less vacancy creation in the informal sector (ϑ goes down from 0.59 to 0.464).

Overall, this policy will shift employment from the informal to the formal: the informal-sector aggregate employment rate decreases from 51.8 to 43.13 and the formal-sector aggregate employment rate increases from 31.0 to 39.6 . As a result, overall unemployment slightly changed from 17.2 to 17.26 percentage points.

Total variance of log-wages decreased from 0.131 to 0.126, driven by a reduction in within-sector variance, despite the fact that relative mean wages are improved.

Scenario 3: Medium valuation of SS services ($\tau = 0.5$)

When workers value these services even more, the quantitative results obtained in the previous case are magnified.

In this case $\hat{\delta}_2$ is reduced from 0.083 to -0.23, so workers now value these contributions as purely net benefits ($\hat{\delta}_2 < 0$). This policy acts exactly as a subsidy that increases formal-sector wages by 23%. In this case the formal sector is expanded significantly (formal-sector employment rate changes from 31.0 to 72.2), informality is reduced dramatically (informal-sector employment rate changes from 51.8 to 15.86), and unemployment is greatly reduced from 17.2 to 11.91%. The hazard rate out of unemployment increases substantially, decreasing average unemployment duration from 12 months to 8 months. Average job duration in the formal sector increases from 47 to 55 months, while in the informal sector average job duration is reduced from 56 to 47 months.

Total inequality rises driven by an increase in within-sector variance not compensated by a reduction in between sector variance.

Scenario 4: High valuation of SS services ($\tau = 1.0$)

The same qualitative analysis than in the previous case applied. Quantitatively, results change dramatically as seen in the table.

4.5.2 Subsidized Health Experiments

We simulate an expansion of social assistance services to informal sector workers. While in 2003 only 19.2% of informal workers have access to subsidized health, we simulate an increase in coverage up to 50% of informal workers, that is an increase in δ_3 by 159.74 % [84].

Two scenarios are made for partial and full valuation of these services.

Scenario 1: Medium valuation of SA services ($\mu = 0.5$)

An expansion of subsidized health (increase in δ_3), contrary to what some previous literature has found, has a minuscule impact on the size of the informal sector (from 0.518 to 0.522), even if the percent change in δ_3 is quite big (159.75%).

This is explained by the fact that the initial size of δ_3 is quite small, given that a very small percentage of informal-sector workers are currently covered by the subsidy in June 2003. The job destruction curve in the informal sector shifts down (a lower reservation productivity $R_{IU}(y)$ for each labor market tightness ϑ), since workers have more benefits when continuing employment in the informal sector, and therefore are less likely to discontinue the match. This suggests an initial expansion of vacancies in this sector.

However, the assumption that formal-sector workers may move to the informal sector if there is a 'bad' shock introduces additional equilibrium effects which may counteract the initial positive effect on informal-sector vacancies. If formal-sector workers have a 'bad' productivity shock and have the option to move to the informal sector, they will receive more continuation value (benefits) from the current match when δ_3 rises. This means a higher δ_3 implies a lower $R_{FU}(y)$, so workers in the formal sector are less likely to end the match and more likely to start a formal sector match (lower $R_{UF}(y)$). Firms in the formal sector will react to that by creating more vacancies (θ increased from 1.01 to 1.06). If workers in the formal sector are less likely to end the match, the expected benefits of the informal-sector firm from hiring a formal-sector worker affected by a 'bad' shock are lower, affecting negatively vacancy creation in the IS and counteracting the initial positive effect (ϑ decreases).

The aggregate unemployment rate increases from 0.172 to 0.185 and the employment rate in the formal sector slightly declines from 0.31 to 0.293, while employment rate in the

[84] It is assumed that the value of the SA services is the same as the value of SS services (0.617 in benchmark case). This generates a change in δ_3 from 0.1185 (0.192*0.617) to 0.3085 (0.5*0.617).

informal sector barely changes (from 0.518 to 0.522). Employment and unemployment durations are barely changed.

Even though this policy has no important compositional changes, it has a more sizable impact on wage inequality, by inducing less between and within-sector inequality, and reducing variance of log wages from 0.131 to 0.115.

Scenario 2: Full valuation of SA services ($\mu = 1.0$)

The compositional effects of this policy change very little with respect to the previous case.

Distributional effects differ: total inequality increases driven by a higher between-sector variance since relative wages in the informal sector fall. Workers in the informal sector have less bargaining power when δ_3 rises, and this effect dominates in this case the other equilibrium effects.

4.5.3 Relative Price Experiments

We simulate a decrease in the price of tradables relative to non-tradables by 5% and 10%[85].

When the formal sector is less profitable due to trade policies or other factors that reduce relative prices (formal relative to informal), there is more destruction and less creation in the formal sector, pushing a higher reservation productivity $R_{UF}(y)$ and making vacancy creation in the formal sector less attractive.

In the informal sector the impact is quite the opposite, since a lower $R_{IU}(y)$ induces more job creation. A fraction of the workers displaced in the formal sector will be absorbed in the informal sector, and the rest will join the pool of unemployed. The rise in the number of job seekers in the informal (unemployed and formal-sector workers affected by bad shocks) is more than offset by an increase in vacancy creation in the informal sector, so ϑ rises. The reduction in formal-sector vacancies as well as the higher number of unemployed job seekers make θ fall.

A reduction in relative prices has similar qualitative compositional effects than the labor taxes policy with $\tau = 0$: there is a shift of resources from formality to informality but in this case, the level of absorption in the informal sector is lower, inducing a rise in overall unemployment. A lower hazard rate out of unemployment implies an increase in mean unemployment duration, while job duration in the formal-sector decreases and in the informal sector increases.

The quantitative impacts are much more sizable than in the case of labor taxes since there are two effects that come into play: the direct effect of the change in prices on worker and firm surpluses, as well as the equilibrium impact due to changes in reservation produc-

[85]Magnitudes of real appreciation occurred after 2003.

tivities and market tightness.

These two forces may affect inequality in two different ways: a lower relative price of formal-sector goods entail a lower wage rate in the bargaining negotiation, but the upward changes in reservation productivities shift the distribution of types and productivities in the formal-sector to the right, pushing the average wage rate up. If the first effect dominates (case when prices fall by 5%), mean relative wages in the informal sector increase substantially, suggesting less between-sector inequality, which together with a drop in within-sector variance, improve inequality. If the second effect dominates (case when prices fall by 10%), an important rise in between-sector inequality is observed.

The size of the distributional effects are also magnified since prices affect also the worker's marginal value of productivity, in addition to affecting the other equilibrium objects.

The next table summarizes the results of the simulations.

Table 13: Compositional and Distributional Effects of Labor Market Policies, Social Assistance and Relative Prices

	Valuations	Compositional Effects			LM Tightness		Distributional Effects				Durations [b]		
	τ, μ	u	n_F	n_I	θ	ϑ	μ_{lnW_I}/μ_{lnW_F}	σ_{WS}	σ_{BS}	σ	t_u	t_{n_F}	t_{n_I}
Benchmarck [a]	$\tau = 0$, $\mu = 0.5$	0.172	0.310	0.518	1.018	0.598	0.904	0.120	0.011	0.131	12.3	47.2	56.6
Δ Labor Taxes[c]													
$\delta_1 \uparrow 4.46\%$, $\delta_2 \uparrow 7.53\%$													
	$\tau = 0$	0.178	0.263	0.559	0.8	0.684	0.879	0.126	0.017	0.144	12.7	44.6	57.4
	$\tau = 0.1$	0.172	0.396	0.431	1.979	0.46	1.02	0.126	0.000	0.126	12.2	49.3	54.2
	$\tau = 0.5$	0.119	0.722	0.158	3.33	0.137	1.03	0.147	0.000	0.147	8.0	55.9	47.2
	$\tau = 1.0$	0.147	0.823	0.029	4.89	0.03	1.34	1.14	0.21	1.35	0.83	4.78	3.45
Δ SA [d]													
$\delta_3 \uparrow$ by 159.74%													
	$\mu = 0.5$	0.185	0.293	0.522	1.06	0.604	0.931	0.109	0.005	0.115	13.1	45.5	55.6
	$\mu = 1$	0.187	0.288	0.524	0.678	0.593	0.88	0.120	0.017	0.137	13.4	46.9	55.6
Δ Relative Prices[e]													
$P_F/P_I \downarrow$ by 5%		0.182	0.203	0.616	0.349	0.745	0.935	0.107	0.004	0.111	13.6	46.4	59.3
$P_F/P_I \downarrow$ by 10%		0.191	0.157	0.652	0.379	0.927	0.705	0.569	0.169	0.739	14.2	45.8	58.9

[a] Simulation results are robust to changes in initial conditions (guessed initial vectors for approximating ss equilibrium) and multiple repetitions of the sampling process.
[b] All durations are in months.
[c] In these set of experiments $\mu = 0.5$.
[d] In these set of experiments $\tau = 0$.
[e] In these set of experiments $\tau = 0$, $\mu = 0.5$

5 Concluding Remarks

In this paper we build a search and matching friction model of the labor market to understand the impact of labor market policies and social assistance programs on steady state unemployment, informal sector size and wage distribution in a small open economy with search frictions and idiosyncratic productivity shocks.

Our contributions to the literature are in two fronts: within the literature of reforms, we provide new lenses to assess the impact of the reforms using a structural model; within the search and matching frictions literature with an informal sector, we generalize previous settings by building the informal sector as a "disadvantaged' sector of a dualistic labor market (consistent with previous empirical evidence), and add extra dimensions of heterogeneity that allows us to better fit the wage distribution.

We solve the model numerically and calibrate the structural parameters using Colombian household-level data. The model accounts for the division of the labor force among unemployment, informal and formal sector employment, selected moments of the earning distribution, and unemployment duration.

We then simulate the model for the Colombian economy and perform a set of policy experiments consistent with the market-oriented structural reforms implemented in the country in the last two decades. An expansion of public health insurance to informal sector workers, high payroll taxation and changes in relative prices (tradables vs non-tradables) may affect the mix of workers in two sectors (compositional effects) as well as the distribution of wages (distributional effects). From a policy perspective, it is critical to quantify these effects.

Even though social assistance transfers and subsidized health offered to individuals who are employed in the informal sector may theoretically affect workers' occupational choice, this paper suggests that recent programs expanding subsidized health to informal sector workers in Colombia may only have a minuscule compositional effects and mild distributional effects, even if the simulated increase in coverage to workers unregistered in this subsidize regime is quite significant. This is explained by the fact the the proportion of IS workers receiving these benefits in Colombia is still quite low. The impact is even lower when workers in the IS do not value these services much, since the quality may not be comparable to the benefits offered in the contributive regime (individuals registered in social security).

Changes in labor taxes have small effects if workers do not value social security services, but may have more sizable compositional and distributional effects if workers associate high payroll taxes with more valuable and efficient social security services. This perception of

labor taxes as net transfers may actually induce a shift of resources from the informal to the formal sector and reduce unemployment, but may worsen overall income inequality by introducing more wage dispersion within each sector. The more workers value SS services, the more regressive these labor market policies become.

Changes in relative prices that affect negatively the relative profitability of the formal sector have quite sizable compositional and distributional effects, producing more long run unemployment and informality, and potentially widening the income gap. The increase in average formal-sector productivity may outweigh the lower relative price, augmenting the average formal-sector wage and inducing more between-sector inequality [86].

An interesting recent macro literature have embedded a search model of the labor markets in real business cycle models or neo-keynesian models with price and wage rigities: Andolfatto(1996); Walsh(2005); Shimer(2005 & 2007); Krause & Lubik (2007); Faia(2008); Gertler, Sala & Trigari(2008); Trigari(2006 & 2009); De Walque, Olivier, Sneessens & Wouters, Raf (2009); Elsby & Solon(2009), Gertler & Trigari(2009). These studies however are thought for developed economies so an informal sector is not present. A small group of studies have incorporated an informal sector in RBC frameworks: Conesa et al.(2002), Fugazza & Jacques(2003), Ihrig & Moe(2004). This motivates an interesting extension of this paper that allows us to understand out-of-steady state dynamics, in which this search model of the labor markets with formal and informal sector is embedded in a fully specified medium scale open economy DSGE model[87].

[86]It is important to note, however, that since this is not a DSGE model changes in relative prices do not affect consumers, and therefore consumption decisions. This also implies that idiosyncratic productivity shocks are mitigated only through social security, social assistance and labor supply decisions but not through consumption/savings decisions.

[87]This extension allows to perform not only steady state analysis, but also model dynamics and shocks propagation mechanisms, as well as identify the differential impact of permanent vs. temporary shocks or anticipated vs. non-anticipated.

6 Appendix

6.1 Derivation of Steady State Productivity Distributions

We want to know how policies affect the marginal distributions of types and productivities in each sector, which also affect the sectoral wage distributions.

6.1.1 Formal Sector

First, we want to compute the joint steady state distribution of current productivity and worker types across workers in the formal sector, $f_F(x, y)$.

$$f_F(x, y) = f_F(x \mid y) f_F(y) \tag{62}$$

We use Bayes rule to compute $f_F(y)$ as follows:

$$f_F(y) = \frac{n_F(y) f(y)}{\int_0^\infty n_F(y) f(y) dy}$$

Now we need $f_F(x \mid y)$.

Let N be the number of shocks that the worker has experienced to date (in the current spell of unemployment in the FS). If $N = 0$ current productivity x equals match-specific productivity y' with prob 1. If $N > 0$, x is a draw from a truncated density $h_F(x \mid y)/(1 - H_F(R_{FU}(y) \mid y))$, for $R_{FU}(y) \leq x \leq \infty$.

So we have:

$$P(N = 0) = P(x = y' \mid y) \quad \text{for } x{=}y'. \tag{63}$$

$$f_F(x \mid y) = \frac{h_F(x \mid y)}{(1 - H_F(R_{FU}(y) \mid y))} (1 - P(N = 0)) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y' \tag{64}$$

Let t be elapsed duration of employment in the current formal-sector job. Let N_t be the number of shocks that the worker has experienced to date t. Let's assume that:

$$t \sim exp\left(\lambda H_F(R_{FU}(y) \mid y)\right) \tag{65}$$

$$Nt \sim Poisson\left(\lambda(1 - H_F(R_{FU}(y) \mid y))t\right) \tag{66}$$

So we have:

$$P(N_t = 0) = exp\left(-\lambda(1 - H_F(R_{FU}(y) \mid y))t\right) \tag{67}$$

$$P[N=0] = \int_0^\infty exp\left(-\lambda(1 - H_F(R_{FU}(y) \mid y))t\right) \lambda \left(1 - H_F(R_{FU}(y) \mid y)\right) exp\left(-\lambda H_F(R_{FU}(y) \mid y)t\right) dt \tag{68}$$

So we get:

$$P[N=0] = H_F(R_{FU}(y) \mid y)) \tag{69}$$

The density of current productivity x given worker type y in the formal sector is given by:

$$P(x = y' \mid y) = H_F(R_{FU}(y) \mid y)) \quad \text{for } x{=}y'. \tag{70}$$

$$f_F(x \mid y) = h_F(x \mid y) \quad \text{for } R_{FU}(y) \leq x \leq \infty \text{ and } x \neq y' \tag{71}$$

We can now compute the steady state joint distribution of types and productivity in the formal sector, and the marginal distribution of current productivity in the formal sector as follows:

$$f_F(x, y) = f_F(x \mid y) f_F(y) \tag{72}$$

$$f_F(x) = \int f_F(x, y) dy \tag{73}$$

6.1.2 Informal Sector

Doing the analogous exercise for the informal sector we get:

$$f_I(x) = \int f_I(x, y) dy \tag{74}$$

$$f_I(x, y) = f_I(x \mid y) f_I(y) \tag{75}$$

where $f_I(y)$ and $f_I(x \mid y)$ are as follows:

$$f_I(y) = \frac{n_I(y) f(y)}{\int_0^\infty n_I(y) f(y) dy}$$

$$P(x = y' \mid y) = H_I(R_{IU}(y) \mid y)) \quad \text{for } x{=}y'. \tag{76}$$

$$f_I(x \mid y) = h_I(x \mid y) \quad \text{for } R_{IU}(y) \leq x \leq \infty \text{ and } x \neq y' \tag{77}$$

6.2 Derivation of Steady State Wage Distributions

To compute the impact of policies on wage inequality in both sectors, we need to derive the distribution of wages across formal and informal sector employment, $m_F(w)$ and $m_I(w)$.

6.2.1 Formal Sector

There are two types of workers currently employed in the formal sector: the workers that have not received any shock whose current productivity is match-specific ($x = y'$ with probability 1), and those who received a shock and continue in the match ($x \geq R_{FU(y)}$).

This suggests that the distribution of wages in formal-sector employment (conditional on y) consists of a smooth density for $w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)]$, and a mass point at $w_F = w_F(y', y)$.

So with P(N=0) the worker of type y receives:

$$w_F(y', y) = \beta \left[\frac{p_F y'}{1+\delta_1} - \frac{\lambda s}{(1+\delta_1)} \right] + (1-\beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} N_I(x, y) - U(y) dH_I(x \mid y)}{(1-\hat{\delta}_2)} \right] \tag{78}$$

and with $[1 - P(N = 0)]$ the worker of type y receives:

$$w_F^s(x, y) = \beta \left[\frac{p_F x}{1+\delta_1} + \frac{rs}{(1+\delta_1)} \right] + (1-\beta) \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} N_I(x', y) - U(y) dH_I(x' \mid y)}{(1-\hat{\delta}_2)} \right] \tag{79}$$

Let's first calculate calculate $m_F(w/y)$.

To compute the conditional density of a transformed variable (productivity as a function of wages conditional on y), we know that: $m_F(w/y) = h_F[x = S(w_F^s, y) \mid y)] \frac{dS(w,y)}{dw}$.

Inverting (81) we get $x = S(w_F^s, y)$ as follows:

$$x \equiv S(w_F^s, y) = \left[\frac{1+\delta_1}{\beta P_F} \right] w_F^s - \frac{rs}{P_F} \tag{80}$$

$$- \frac{(1-\beta)(1+\delta_1)}{\beta P_F} \left[\frac{rU(y) - \lambda H_F(R_{FU}(y) \mid y) \int_{R_{FI}(y)}^{\infty} N_I(x', y) - U(y) dH_I(x' \mid y)}{(1-\hat{\delta}_2)} \right]$$

$$\frac{dx}{dw} = \frac{dS(w_F^s, y)}{dw} = \frac{1+\delta_1}{\beta P_F} \tag{81}$$

The conditional distribution of wages in formal-sector employment is:

$$P[w_F = w_F(y', y)] = H_F(R_{FU}(y) \mid y) \quad \text{for } w_F = w_F(y', y). \tag{82}$$

$$m_F(w/y) = \left[\frac{1+\delta_1}{\beta P_F}\right] h_F(x \equiv S(w_F^s, y) \mid y) \quad \text{for } w_F \in [w_F^s[R_{FU}(y), y), w_F^s(\infty, y)] \text{ and } w_F \neq w_F(y', y) \tag{83}$$

We finally compute can compute $m_F(w)$ by using:

$$m_F(w) = \int m_F(w/y) f_F(y) dy \tag{84}$$

6.2.2 Informal Sector

Doing the analogous exercise for the informal sector we can get $m_I(w)$ as follows:

$$m_I(w) = \int m_I(w/y) f_I(y) dy \tag{85}$$

where $m_I(w/y)$ is the steady state conditional distribution of wages in IS employment and $f_I(y)$ is steady state density of types among IS employment.

The mapping from wages to productivity, conditional on y is:

$$x \equiv S(w_I, y) = \left[\frac{w_I}{\beta P_I}\right] - \left[\frac{(1-\beta)}{\beta P_I}\right] \left[\frac{rU(y) - m(\theta)[\int_{R_{IF(y',y)}}^{\infty} N_F(x, y, y') - U(y) dH_F(x \mid y)}{(1+\hat{\delta}_3)}\right] \tag{86}$$

$$\frac{dx}{dw} = \frac{dS(w_I, y)}{dw} = \frac{1}{\beta P_I} \tag{87}$$

After receiving a shock, the worker continues in the match and receives a salary only if $x \geq R_{IU(y)}$. The conditional distribution of wages in formal-sector employment is:

$$P[w_I = w_I(y', y)] = H_I(R_{IU}(y) \mid y) \quad \text{for } w_I = w_I(y', y). \tag{88}$$

$$m_I(w/y) = \left[\frac{1}{\beta P_I}\right] h_I(x \equiv S(w_I, y) \mid y) \quad \text{for } w_I \in [w_I^s[R_{IU}(y), y), w_I^s(\infty, y)] \text{ and } w_I \neq w_I(y', y) \tag{89}$$

6.3 Computational Algorithm

Following the definition of Steady State Equilibrium, we write the following computational algorithm to approximate the steady state of the model numerically:

1. Guess values for θ^0. Start an outer loop. Guess values for $R_{UF}(y)^0$, $R_{UI}(y)^0$ and $R_{FU}(y)^0$. Start an inner loop, for fixed values of θ^0. Substitute these values in equation (55) to calculate $rU(y)$.

2. Given $rU(y)$ and $R_{FI}(y)^0$, iterate the Bellman equation (27) to find the fixed point on $R_{FU}(y)$. Named the solution $\hat{R_{FU}}(y)$.

3. Given $\hat{R_{FU}}(y)$, use equation (29) to calculate $\hat{R_{UF}}(y)$.

4. Given $U(y)$, iterate the Bellman equation (33) to find the fixed point on $R_{IU}(y)$.
   Named the solution $\hat{R_{IU}}(y)$. Notice that $\hat{R_{IU}}(y)$=$\hat{R_{UI}}(y)$=$\hat{R_{FI}}(y)$.
   If the following conditions are met:
   $|| R_{UF}(y)^0 - \hat{R_{UF}}(y) ||< \epsilon_{UF}$, and
   $|| R_{UI}(y)^0 - \hat{R_{UI}}(y) ||< \epsilon_{UI}$, and
   $|| R_{FU}(y)^0 - \hat{R_{FU}}(y) ||< \epsilon_{FU}$, and
   Then stop inner loop. Otherwise update as follows:
   $R_{UF}(y)^{new} = R_{UF}(y)^0 + \nu_{UF}(R_{UF}(y)^0 - \hat{R_{UF}}(y))$
   $R_{UI}(y)^{new} = R_{UI}(y)^0 + \nu_{UI}(R_{UI}(y)^0 - \hat{R_{UI}}(y))$
   $R_{FU}(y)^{new} = R_{FU}(y)^0 + \nu_{FU}(R_{FU}(y)^0 - \hat{R_{FU}}(y))$
   where ϵ_{Ij} and ν_{Ij} are the tolerance levels and step sizes respectively.

5. Once convergence is reached in the inner loop, use $\hat{R_{UI}}(y)$, $\hat{R_{UF}}(y)$, and $\hat{R_{FU}}(y)$ and θ^0 in equations (49) - (51) to calculate $u(y)$, $n_F(y)$ and $n_I(y)$. Aggregate over y to get u, n_F and n_I.

6. Given $\hat{R_{UF}}(y)$ and $u(y)$, solve equation (44) to get the equilibrium labor market tightness parameter $\hat{\theta}$. If the following conditions is met, $\mid \theta^0 - \hat{\theta} \mid< \epsilon_\theta$, then stop outer loop. Otherwise update as follows: $\theta^{new} = \theta^0 + \nu_\theta(\theta^0 - \hat{\theta})$.

7. Once convergence is reached in the outer loop, use equilibrium reservation productivities,$R_{FU}(y)$ and $R_{IU}(y)$, $u(y)$ and $n_F(y)$, to get the labor market tightness parameter ϑ that satisfies equation (46).

8. Given equilibrium $U(y)$, equilibrium reservation productivities $R_{FU}(y)$, $R_{FI}(y)$, $R_{IU}(y)$, get formal sector wages $w_F(y', y)$ and $w_F(x, y)$ that satisfy equations (20) and (22).

9. Given equilibrium $U(y)$, get informal-sector wage $w_I(y', y)$ that satisfies equation(24).

6.4 Equilibrium Objects: Benchmarck Case

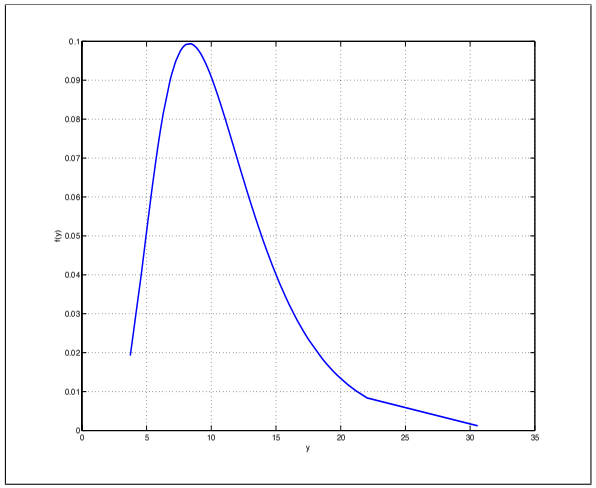


Figure 1: PDF worker types

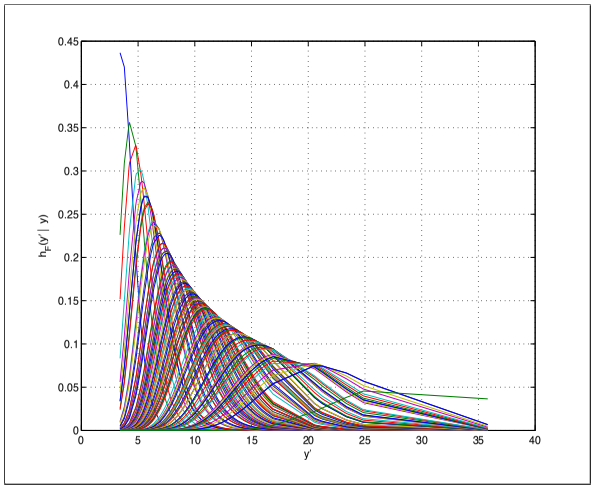


Figure 2: PDF match-spec productivity, FS

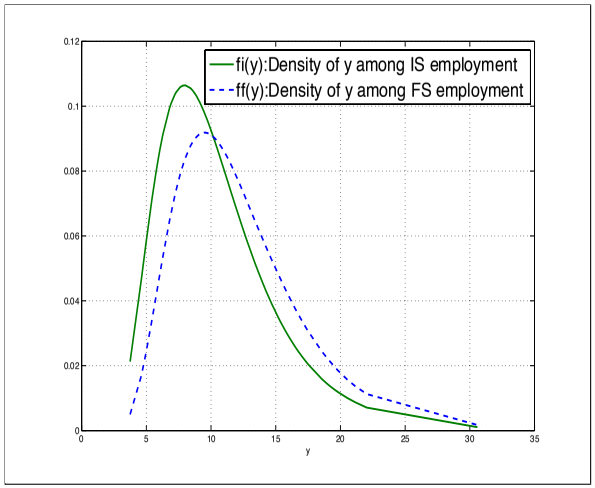


Figure 3: PDF worker types, by Sectors

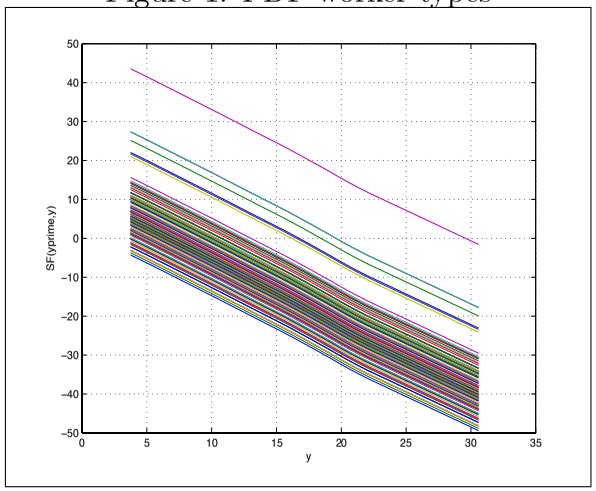


Figure 4: Total Surplus, new hires, FS

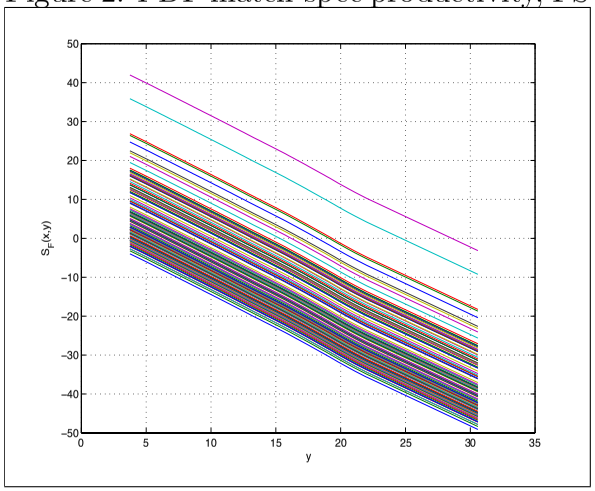


Figure 5: Total Surplus, old hires, FS

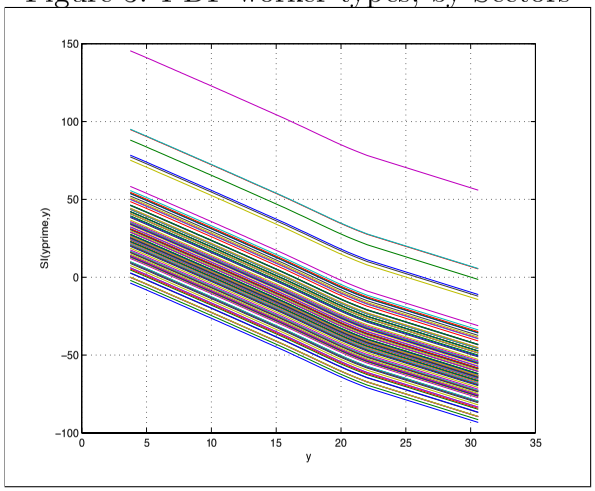


Figure 6: Total Surplus, new hires, IS

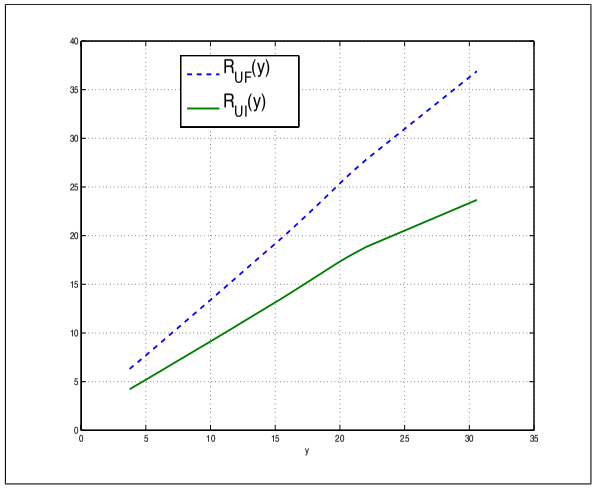


Figure 7: Reservation Productivities, FS and IS

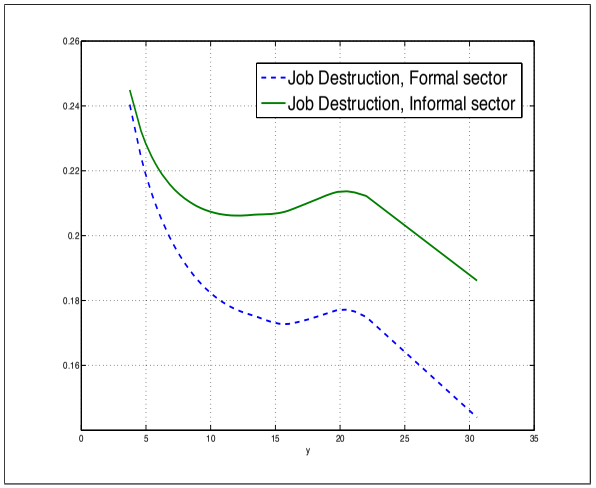


Figure 8: Job Destruction Rates, FS and IS

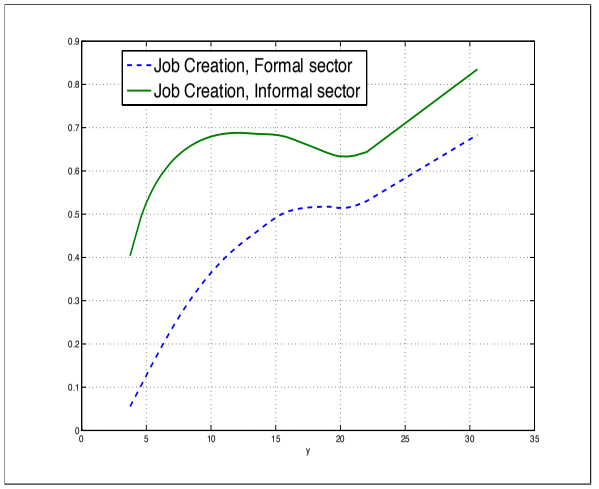


Figure 9: Job Creation Rates, FS and IS

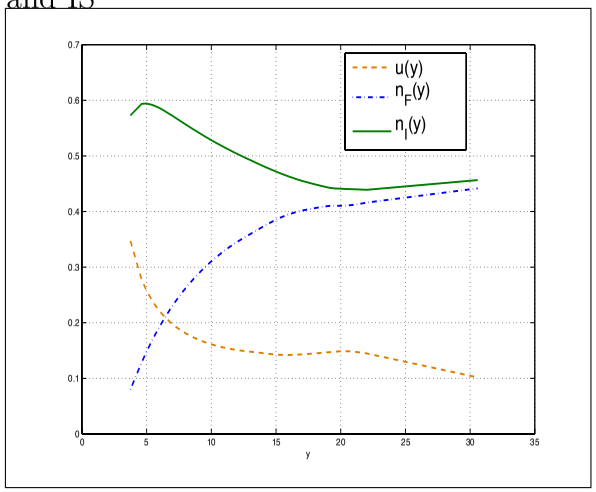


Figure 10: Unemployment Rate, FS and IS Employment Rates

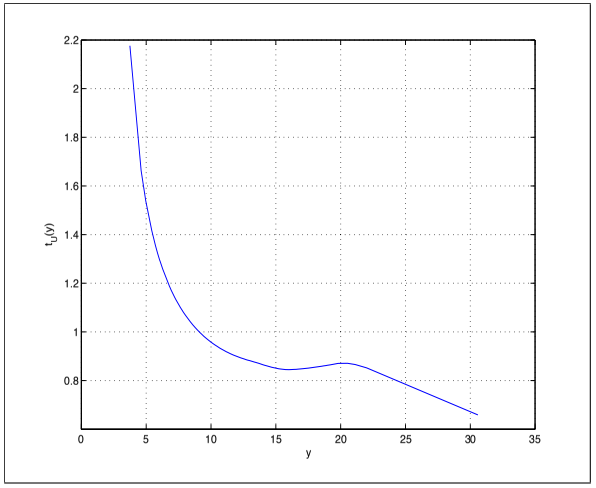


Figure 11: Unemployment Duration (years)

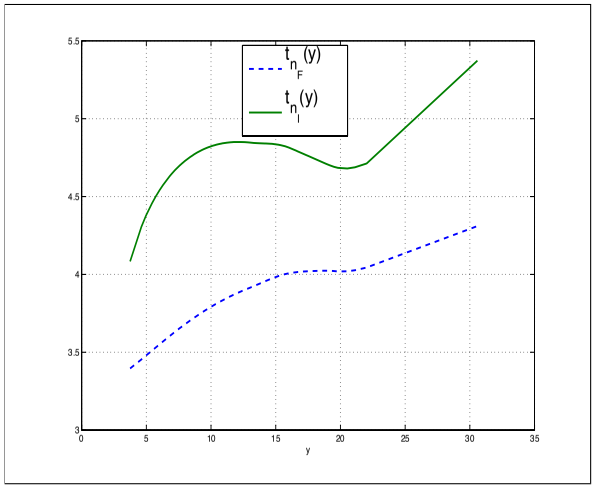


Figure 12: Employment duration, FS and IS (years)

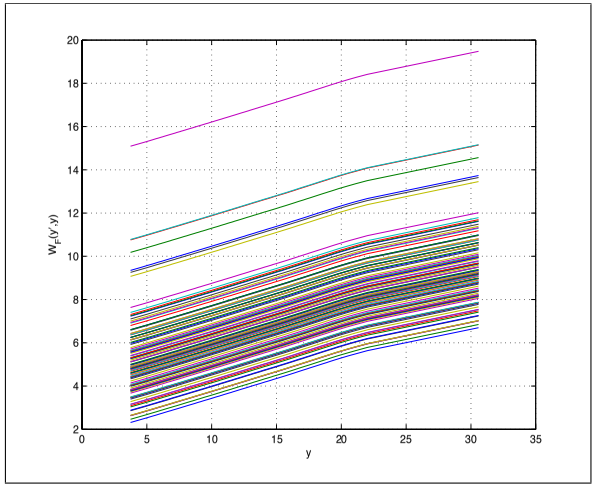


Figure 13: Wage Function, New Hires, FS

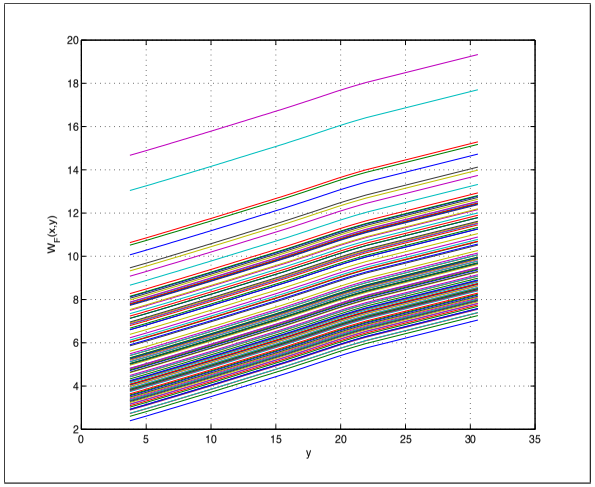


Figure 14: Wage Function, Old Hires, FS

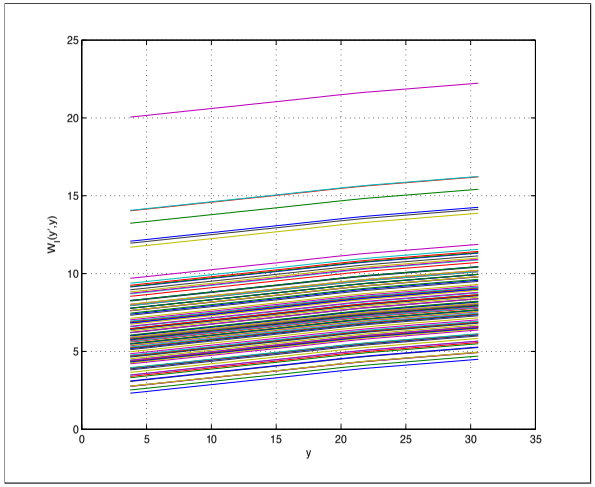


Figure 15: Wage Function, New Hires, IS

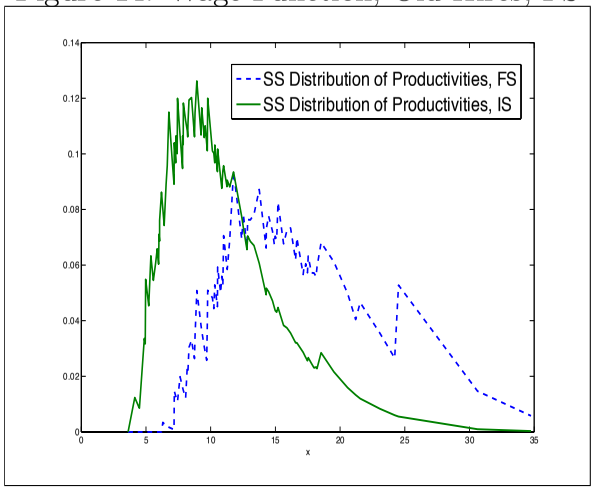


Figure 16: PDF Current Productivity, FS and IS

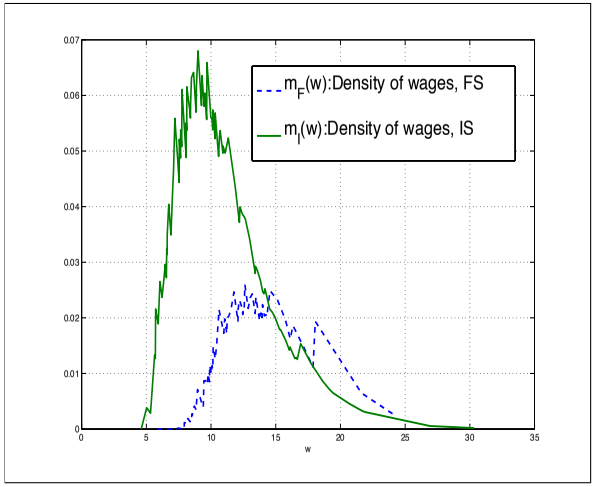


Figure 17: PDF Wages, FS and IS

6.5 Detailed Employer and Employee Non-Wage Labor Costs

Table 14: Detailed Employee Non-Wage Labor Costs (as % Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012 Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	1.50	2.17	2.17	2.17	2.17	2.875	3.125	3.375	3.625	3.75	3.875	3.875	4.0	4.0
Health	2.33	2.33	2.33	2.33	2.33	4.00	4.00	4.00	4.00	4.00	4.00	4.0	4.0	4.0
Subtotal Social Security	3.83	4.50	4.50	4.50	4.50	6.875	7.125	7.375	7.625	7.75	7.875	7.875	8.00	8.00
OTHERS														
Solidarity Fund		0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Subtotal Others		0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
TOTAL EMPLOYEE NWLC (AS % OF WAGE)	3.83	4.50	4.50	4.50	4.50	7.875	8.125	8.375	8.625	8.75	8.875	8.875	9.00	9.00

Author's Calculations based on statutory values established in the Labor Codes. See Law 100 of 1993, Law 797 of 2003, and Law 1122 of 2007.

Table 15: Detailed Employer Non-Wage Labor Costs (as % Wage)-Statutory Values (1984-2013)

	1984	1985-1990	Post90 Reform			Post93 Reform			Post2003 Reform			Post2007 Reform		Post2012Reform
	1984	1985 – 1990	1991	1992	1993	1994	1995	1996-2003	2004	2005	2006	2007	2008-2012	2013
SOCIAL SECURITY														
Pension	3.00	4.33	4.33	4.33	4.33	8.63	9.38	10.12	10.87	11.25	11.62	11.63	12.00	12.00
Health	4.67	4.67	4.67	4.67	4.67	8.00	8.00	8.00	8.00	8.00	8.00	8.50	8.50	8.50
Work Injury	2.0	2.00	2.00	2.00	2.00	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.4
Subtotal Social Security	9.67	11.00	11.00	11.00	11.00	19.03	19.78	20.53	21.28	21.65	22.03	22.53	22.90	22.90
PARA-STATAL														
Training	2.0	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.00
In-Kind Tranfers	2.0	2.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.00
Family Allowances	4.0	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
Subtotal Para-statal	8.0	8.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	4.0
OTHERS														
Mandatory Bonuses	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33	8.33
Paid Vacations	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25	6.25
Severance Pay	13.5	13.5	12.8	12.1	11.4	10.70	10.0	9.30	9.30	9.30	9.30	9.30	9.30	9.30
Subtotal Others	28.08	28.08	27.38	26.68	25.98	25.28	24.58	23.88	23.88	23.88	23.88	23.88	23.88	23.88
TOTAL EMPLOYER NWLC (AS % OF WAGE)	45.75	47.08	47.38	46.68	45.98	53.31	53.36	53.41	54.16	54.53	54.91	55.41	55.78	50.78

Author's Calculations based on statutory values established in the Labor Codes. See Law 50 of 1990, Law 100 of 1993, Law 797 of 2003, Law 1122 of 2007, and Law 1607 of 2012.
Work injury compensations vary with the degree of risk in the occupation, ranging from degree 1 (lowest risk) to degree 5 (highest risk), Here, we take the values of a level 3 risk.
Severance pay was paid upon employment termination prior to the 1990 Labor Reform, but it was turned into a payroll tax afterwards since firm have to deposit payments on a monthly basis in a financial account.
Training refers to SENA, In-kind Transfers to ICBF, and Family Allowances to Cajas de Compensacion Familiar.
The reduction in parastatal contributions in 2012 only applied to workers earning less than 10x the minimum wage.

6.6 Extension of the Model: Intensive Margin Decision

Lets' analyze how adding an intensive margin decision would change the results of the basic model.

Let's modify the utility function $u_F(y', y)$ as follows

$$u_F(y', y) = w_F(y', y)(1 - \hat{\delta}_2)h_F(y)\phi(1 - h_F, y);$$

where ϕ captures utility from leisure with standard properties [88].

Since in this specification the utility function is linear in income and non linear in hours of work, we are assuming complementarity between consumption and leisure.

Let's assume now that formal sector wages are determined by Nash bargaining, given exogenous worker bargaining parameter, β. Then, hours of work are determined unilaterally by workers, once in employment, given the wages already negotiated in the bargaining [89].

Using backward induction, let's solve first for maximization problem of the workers who choose hours of work. Now from (2) we get

$$(r + \lambda)N_F(y', y) = u_F(y', y) + \Xi(y', y)$$

where $\Xi(y', y)$ is a function that does not depend on $h_F(y)$:

$$\Xi(y', y) \equiv \lambda \int_{R_{FI}(y)}^{\infty} N_F(x, y)dH_I(x \mid y) + \lambda H_F(R_{FU}(y) \mid y) \left[H_I(R_{FI}(y) \mid y)U(y) + \int_{R_{FI}(y)}^{\infty} N_I(x, y)dH_I(x \mid y) \right]$$

Therefore, hours of work for a worker type y solves:

$$\begin{aligned} \underset{h_F(y)}{\text{Max}} \quad & N_F(y', y) = \frac{u_f(y', y) + \Xi(y', y)}{r + \lambda} \\ \text{s.t.} \quad & u_f(y', y) = w_F(y', y)(1 - \hat{\delta}_2)h_F(y)\phi(1 - h_F, y); \end{aligned}$$

Workers want to maximize the flow utility in the formal sector, which is a non linear function of hours of work, and they don't care about anything else since they revise the choice when they get a new productivity shock.

Optimal hours of work are determined by the The F.O.C (assuming interior solution):

[88] Notice that we assume that valuation for leisure only depends on workers' type, and not on current productivity. Also $\phi(1 - h_F(y), y) > 0$, $\phi\prime(., y) > 0$, $\phi\prime\prime(., y) \leq 0$, $0 \leq h_F \leq 1$ and the length of the day has been normalized to 1

[89] As analyzed in Pissarides (2000), this solution is not efficient since workers will choose to work fewer hours. Alternatively, hours of work may also be determined with wages by a bargain between a firm and a worker, leading to an efficient solution. We will cover this case as an extension of the benchmark model.

$$\frac{\phi'(1-h_F(y),y)}{\phi(1-h_F(y),y)}h_F(y)=1$$

Equilibrium hours of work depend only on the type-specific preferences for leisure described by $\phi(1-h_F(y),y)$, and do not interact with the rest of the model, since they are not affected by the wage rate, $w_F(y',y)$ or by employee non-wage labor costs, $\hat{\delta}_2$, since the substitution and the income effect cancels out (not an unreasonable assumption in the steady state) [90]. Labor market tightness, θ, doesn't affect this choice either since workers choose hours after having found the job, and again, they can re-optimize when changing jobs [91].

Therefore, results of the model would not change significantly by adding an intensive margin decision.

[90] The linear correlation coefficient between total hours of work and hourly wages in is quite small: -0.1343 for urban salaried workers in June 2003 (author's calculations based on the Colombian Household Survey).

[91] See Pissarides, 2000.

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