Bankruptcy and Aggregate Productivity

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Abstract

I develop a model of financial intermediation to study the link between bankruptcy efficiency —the amount a lender can recover from bankrupt borrowers— and aggregate productivity. The theory implies that countries with low bankruptcy efficiency are characterized by a low fraction of large (productive) firms, and low aggregate productivity. These implications are supported by the empirical evidence. I then use the model to evaluate the quantitative implications of the model. I find that differences in bankruptcy efficiency generate large aggregate productivity differences, close to those observed in the data for European countries.

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1 Introduction

Income differences across countries are large. The current consensus is that differences in physical capital, human capital and labor across countries can account at most for half of the differences in GDP per capita (Hsieh and Klenow, 2010). The remaining is accounted for by differences in Total Factor Productivity (TFP), which is the efficiency with which these inputs are converted into output. Consequently, understanding why TFP levels vary so much across countries is crucial for understanding the drivers of income differences. Yet, our understanding of the determinants of aggregate TFP remains vague. In this paper, I use evidence from firm-size distributions across European countries to propose a theory of aggregate TFP based on the efficiency of bankruptcy procedures.

The starting point of this paper is to note that aggregate TFP is an average of firm-level productivities. Therefore, aggregate TFP can be decomposed into two components: firm-level productivities (level component) and the measure of firms of each productivity type (composition component). Evidence from Europe suggests that the composition component is an important driver of TFP differences. I document five observations from four European countries: the United Kingdom, Germany, Spain, and Italy. The first observation is that Germany and the United Kingdom have higher aggregate productivity than Spain and Italy. Second, in all these countries larger firms (in terms of employees) are more productive than smaller firms. Third, German and British firm-size distributions have a larger proportion of large firms than Spain and Italy. Fourth, conditional on firm size, British and German firms are not more productive than Spanish and Italian firms. These observations together suggest that the composition component is an important driver of TFP differences. To illustrate the importance of the composition component, I perform an accounting exercise using the above data that suggests that aggregate productivity in Spain and Italy would be about 10% higher if they had the firm composition of Germany and the UK but kept its own firm productivities.

The last observation is that the efficiency of bankruptcy procedures, as measured by the percentage of a loan a lender can recover from a bankrupt borrower, is higher in Germany and the UK than in Spain and Italy.[1] I use these observations to propose a

[1] Aggregate bankruptcy efficiency for a country is measured by the World Bank Doing Business database using a methodology developed by Djankov, Hart, McLiesh and Shleifer (2008). I discuss the bankruptcy efficiency data in detail in Section 2.

new source of cross-country productivity differences: bankruptcy efficiency. Countries with less efficient bankruptcy procedures (tend to) have lower aggregate productivity because it induces lenders to allocate funds to smaller (less productive) firms. I formalize this idea in a model of financial intermediation with entrepreneurs of heterogenous productivities. The model allows for a closed form expression for TFP, which makes transparent the contribution of bankruptcy efficiency to TFP through the composition of firms. I then investigate whether differences in bankruptcy efficiency can generate large differences in aggregate productivity.

I consider a static model that features a competitive lender and households with either high or low entrepreneurial productivity. If households fail to secure funding to become entrepreneurs and operate their technology then they become workers and rent their labor for a wage. The lender wants to allocate resources to high-productivity entrepreneurs, but is constrained by two frictions. First, entrepreneurial productivity is private information of the entrepreneurs at the time when loans (resources) are allocated. However, productivity can be inferred by lenders after production has taken place. Hence, the lender could achieve first best allocations if it could impose unlimited penalties to entrepreneurs who misrepresent their type. The second friction limits the penalty the lender can impose for false productivity reports. An exogenous limit on the penalty for false reports can be mapped to bankruptcy efficiency in equilibrium, as it dictates the percentage of the loan that can be recovered by lenders on borrowers who misrepresent their type and do not have resources to pay back the full amount of the loan.

I show that, as the level of bankruptcy efficiency decreases, two forces decrease aggregate productivity. First, the average firm quality decreases. This effect is due to the tightening of the incentive compatibility constraint of low-productivity entrepreneurs. As low-productivity agents face lower punishments for misrepresenting their type, the lender deters them from lying by improving their relative chances of obtaining a loan. In the aggregate, the composition of firms involves a higher proportion of low-productivity firms than before. Second, the total number of firms decreases. This effect comes from the lender's feasibility constraint. With linear utility, the level of bankruptcy efficiency also caps the amount the lender can extract from profitable projects to finance other profitable projects. Hence, with lower bankruptcy efficiency, a lower number of projects are funded.

In equilibrium, the lender does not distort the firm-size margin so that firms produce at their efficient level. This implies that there is a one-to-one mapping between firm productivity and firm size (measured in number of employees). Hence, large firms are more productive than small ones. By construction, firm TFP levels are exogenous and therefore invariant with bankruptcy efficiency. Aggregate TFP then varies with bankruptcy efficiency solely through the composition of the firm-size distribution.

I then proceed to endogenize wages. Endogenous wages reinforce the differences in TFP. Lower bankruptcy efficiency decreases total production and wages. With a worsening outside option of working for a wage, private information frictions are reinforced by increasing the incentives of low-productivity agents to misreport in order to improve their chances of operating their technology.

I calibrate the model to the firm-size distribution and bankruptcy efficiency of the United States. I then vary the level of bankruptcy efficiency to levels of bankruptcy efficiency observed for other countries. I find that differences in bankruptcy efficiency are able to generate large differences in TFP among high-income countries, similar to those observed for European countries.

Empirical support for the mechanism proposed in my paper comes from Ponticelli (2013), which explores the impact of a reform in Brazilian bankruptcy law which increased Brazil's aggregate recovery rate by 12 points. The author exploits variation in the application of the bankruptcy reform across Brazilian judicial districts due to congestion of local courts. Crucially, Brazilian laws do not allow creditors or firms to choose the district in which to file a bankruptcy case. The author finds that bankruptcy reform led to higher probability of exit by small firms and higher firm growth, which would lead to the type of shift in the firm-size distribution proposed in my paper.

This paper is related to the existing literature on misallocation. Much of the literature has focused on the misallocation of capital across existing firms (Banerjee and Moll (2010), Buera, Kaboski and Shin (2011), Amaral and Quintin (2010), Midrigan and Xu (2014), Moll (2014), Steinberg (2013), Greenwood, Sanchez and Wang (2013)). I instead focus on the extensive margin of capital misallocation: not all high-productivity entrepreneurs are able to operate their technology, but once they do, they obtain sufficient capital and labor to operate at their efficient level. I abstract from capital misallocation at the intensive margin because I am interested in capturing the observation that the largest firms are also the most productive. With capital misallocation the relationship between size and productivity might be inverted (capital and labor could be mostly held by the unproductive firms).

2 European Productivity and Firm-Size Distributions

This section presents evidence motivating the theoretical approach of the article. The first observation is that Germany and the UK are more productive than Spain and Italy.

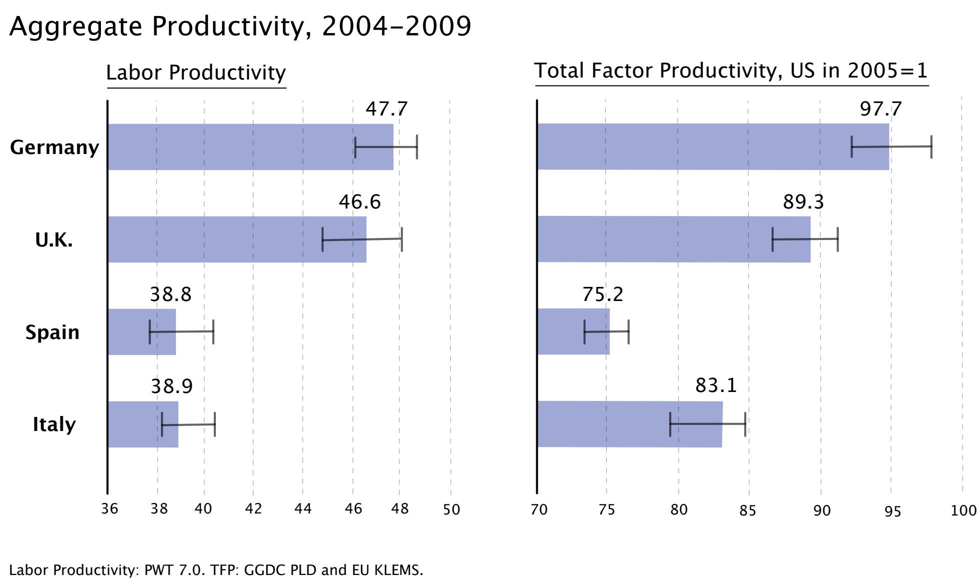


Figure 1: Germany and UK are more productive than Spain and Italy

Figure 1 plots GDP per hour worked (labor productivity) for the years 2000-2009.[2] The story is similar if one instead looks at TFP measurements in Figure **??**, which account for differences in capital intensity.[3] According to both figures, the UK and Germany are more productive than Spain and Italy (Italy since the year 2001).

A more disaggregated view of these economies reveals that an important driver of

[2]From Penn World Tables 7.0, variable rgdpl2th: PPP Converted GDP Laspeyres per hour worked by employees at 2005 constant prices.

[3]I construct the TFP series by combining cross-country TFP level rankings in the year 2005 from the GGDC PLD with TFP growth rates from EU KLEMS. TFP levels in GGDC PLD are TFP Value Added, Single Discounted. TFP growth rates in EU KLEMS come from sectoral data.

these TFP differences is the composition of each country's firm-size distribution.[4]

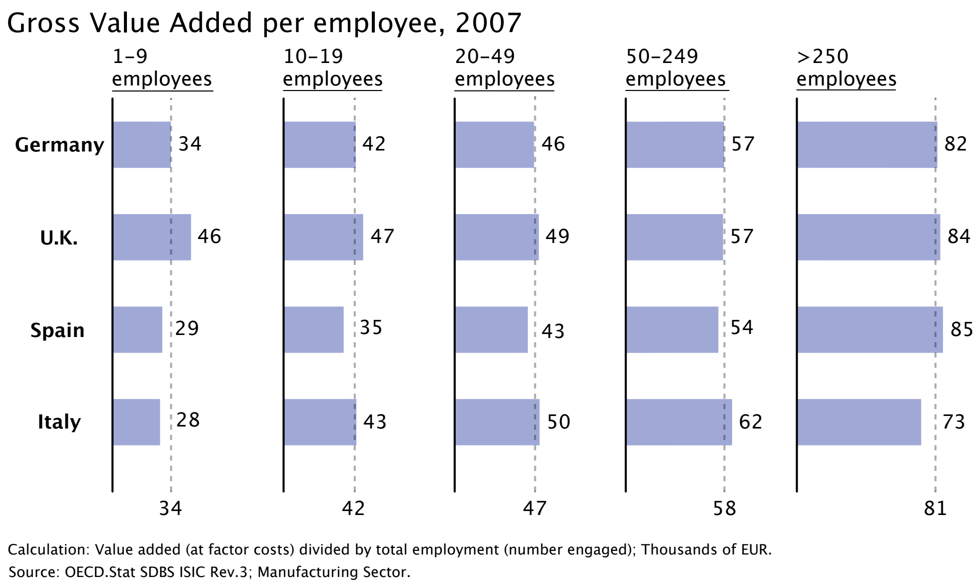


Figure 2: Large firms are more productive and firms of the same size are similarly productive across countries

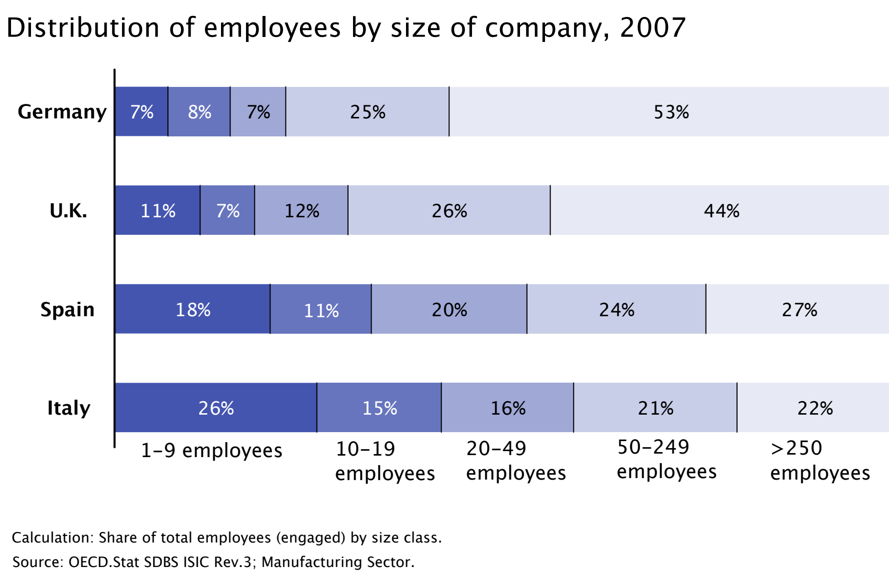


Figure 3: Germany and the UK have a higher proportion of workers in large productive firms

Figure 2 illustrates two observations. First, larger firms are more productive than

[4]Others have noticed this before. For example, see Mackenzie...

smaller firms across all four countries (observation 2) . Second, there are no striking differences between the productivity of firms of similar size across all four countries (observation 3). Italy, for example, is less productive than Germany and the UK in large and small firms, but Italy's medium sized firms are more productive than German and British firms. Spanish large firms are slightly more productive than German and British large firms.

Figure 3 illustrates observation 4. It shows that the firm composition is quite different across Northern and Southern European countries. In Germany and the UK, 57% and 44% of workers are employed by the largest firms[5]. In contrast, in Spain and Italy only 24% and 21% of employees work for the largest firms.

Productivity Levels vs Composition

In this section I use an accounting exercise to assess the relative importance of firm productivity levels versus the composition of the firm-size distribution for TFP.

The GDP identity from national accounts is,

$$Y = AK^{\alpha}N^{\gamma}$$

where Y is output, K is capital with share α, N is labor with share γ, and A is TFP.

Suppose firm i produces ouput y_i using capital k_i, labor n_i, and productivity a_i with a Cobb-Douglas technology,[6]

$$y_i = a_i k_i^{\alpha} n_i^{\gamma}.$$

We can connect firm productivity levels and composition of the firm-size distribution to aggregate productivity by the following identity,

$$A = \left(\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}} \right)^{1-\alpha-\gamma} \tag{1}$$

[5]The size brackets are those reported by the Eurostat Structural and Demographic Business Statistics.

[6]In order to sustain a non-degenerate distribution of firm sizes in an equilibrium model one needs to assume either a single good and decreasing returns to scale production technologies, or differentiated products and constant returns to scale technologies. Hsieh and Klenow (2009) (Appendix 1) show that both formulations are isomorphic.

where π_i is the measure of firms of productivity a_i. Hence, π_i's control the composition of firms whereas a_i's control firm productivity levels. Expression (1) is the main result of Proposition 2.

Firm labor productivity is a good proxy of firm total factor productivity if capital deepening is roughly similar across firms and across countries. Given the lack of availability of cross-country firm-level capital data,[7] I assume in this accounting exercise that capital-labor ratios, $k_i^\alpha / n_i^{1-\gamma}$, are constant and equal to one for all firms.

Table 1 shows how aggregate productivity would change in each country if it had other countries' set of firm level productivities and firm composition (a_i's and π_i's in equation 1).[8] For example, keeping Spain's firm-size productivities constant and moving to Germany's composition would yield *higher* productivity (78.31) than Germany itself (75.75). On the other hand keeping Germany's firm-size productivities constant and moving to Spain's composition would yield *lower* productivity (67.18) than Spain itself (69.12). The same is true for Spain and the UK; e.g. the change in composition by itself generates larger productivity gains and losses than those seen in the data.

The impact of composition on aggregate productivity is not as stark for the combinations of Italy with Germany and the UK, but it is still substantial. Italy's productivity would increase from 59.75 to 67.83 if it had the German firm composition, and to 65.69 if it had the British firm composition.

Table 1: Implied aggregate labor productivity from combinations of firm productivity levels and firm composition

	Germany Composition	UK Composition	Spain Composition	Italy Composition
Germany Productivities	**75.75**	73.09	67.18	65.90
UK Productivities	77.58	**74.87**	68.92	67.68
Spain Productivities	78.31	75.46	**69.12**	67.76
Italy Productivities	67.83	65.69	60.87	**59.75**

[7]The best known cross-country firm-level datasets, Amadeus and Orbis (published by Bureau van Dijk), do not typically include the smallest firms and the quality of data (percentage of firms covered) varies widely from country to country.

[8]The level of decreasing returns to scale is set to $\alpha + \gamma = 0.85$, a standard value in the literature, as in Basu and Fernald (1995).

Bankruptcy efficiency

A strand of literature argues that an important factor affecting the firm size distribution is the financing constraints faced by startups (for example Evans and Jovanovic (1989), Cabral and Mata (2003), and Kerr and Nanda (2009)). This paper proposes that one particular factor affecting lender's funding decisions — the ability to recover the loan in case of business failure — can by itself generate firm-size distributions consistent with those of Europe. Figure 4 shows that UK and Germany have higher recovery rates — the World Bank's measure of bankruptcy efficiency — than Spain and Italy. In Germany and the UK a bank can recover more than 80 cents on each dollar loaned to a bankrupt company, whereas this same figure is in the low 70's for Spain and in the low 60's for Italy.

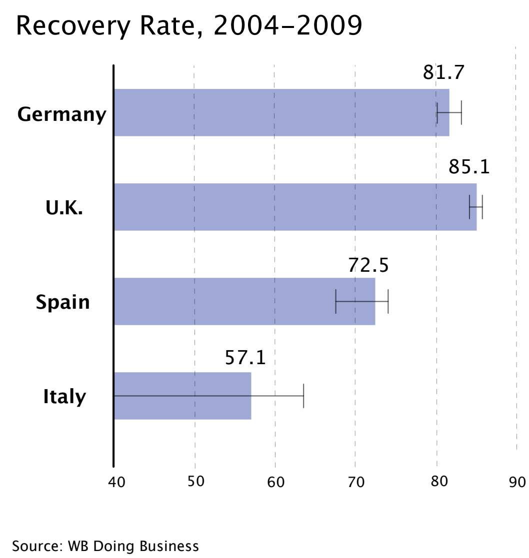


Figure 4: Germany and UK have better bankruptcy efficiency than Spain and Italy

The methodology used to collect the statistic was developed by Djankov, Hart, McLiesh and Shleifer (2008) and adopted by the World Bank. The way the statistic is collected is the following. The World Bank contacts bankruptcy lawyers in the country of interest and sends them a case scenario of a business that is facing insolvency. The scenario is made as specific as possible, detailing type of company, size, location, and other factors to allow for cross-country comparability. The company has a 10-year loan agreement with a domestic bank, and due to insolvency, is forced to default on its loan. The company also has real estate property valued at exactly the same amount outstanding under the loan agreement. Given that the bank wants to recover as much as possible of its loan, the bankruptcy lawyers are asked for their opinion on the fraction of the outstanding loan that can be recovered by the bank through reorganization, liquidation, or foreclosure. The recovery rate accounts for the cost and time of proceedings. If a country has not experienced more than one bankruptcy case go through its courts in the last four years no data is collected.

The following section sets up a model where firm composition is endogenously generated by differences in bankruptcy efficiency.

3 Model

I consider a standard version of the Lucas (1978) span of control model, augmented along the lines of Erosa and Hidalgo-Cabrillana (2008) to introduce financial intermediation. The economy is populated by a continuum of financial intermediaries and a continuum of households of mass one.

Preferences and Endowments

Households are endowed with initial resources $\hat{f}$, one unit of labor and a production technology. Some households become entrepreneurs and operate their technology. Entrepreneurs employ capital and labor, produce a final good, and consume the profits. The remaining households become workers and rent their labor to entrepreneurs. Households have linear utility over consumption, $u(c) = c$.[9]

Technology

Households are heterogeneous in entrepreneurial productivity a_i, where $i \in \{low, high\}$, and have a production function of the form,

$$y_i = a_i k^{\alpha} n^{\gamma}, \quad \alpha, \gamma \in (0, 1), 0 < \alpha + \gamma < 1, \tag{2}$$

[9]The assumption of linear utility is helpful in obtaining a closed-form solution to the contract. With linear utility, entrepreneurs can be thought of as firms who are maximizing profits rather than utility.

with capital k and labor n. A fraction ν of households are endowed with low entrepreneurial productivity a_ℓ and a fraction $1-\nu$ are endowed with high entrepreneurial productivity a_h. The costs of operating the technology include a fixed cost f. The need for financing arises because entrepreneurs cannot self-finance the fixed cost of production, $f > \hat{f}$.

Frictions

In the first-best scenario all agents pool their resources. Funding goes to the high-productivity agents first and any remaining funding goes to the low-productivity agents, given that they are profitable (their profits are higher than the outside option of working). There are two frictions that make the first-best allocation unachievable. First, entrepreneurial productivity is private information of the agents. Hence, low-productivity agents have an incentive to pretend to be high-productivity so as to improve their chances of obtaining funding.

Production occurs at the end of the period and is observable. Therefore, productivity levels can be inferred by common knowledge of the production function and observable inputs. Agents could still obtain the first best allocations if they write contracts that include infinite punishments for false reports. However, there is a second friction that limits repayments to a fraction ϕ of the loan. Bankruptcy in this model is the scenario where entrepreneurs are unable to repay their expected amount because they misrepresent their type. I label, ϕ, the level of bankruptcy efficiency.

Financial Intermediaries

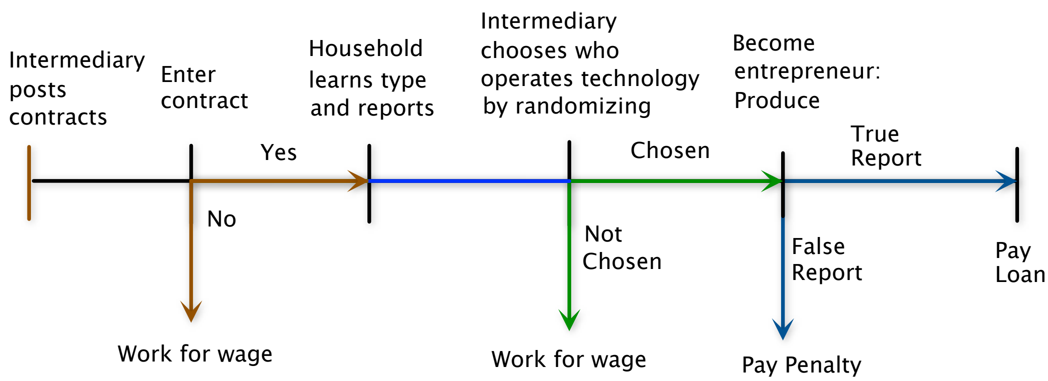
Financial intermediaries operate in a perfectly competitive market and lend to a set of homogeneous households (agents do not learn their type until after they have entered the contract). Competitive markets imply that intermediaries offer contracts that maximize the expected welfare of their pool of borrowers.[10] Ex-ante homogenous agents imply that there exists a representative financial intermediary.[11]

[10] The assumption of perfect competition might seem strong in this setting, but it could be replaced by an assumption of a few lenders engaging in Bertrand competition in contracts.

[11] If agents are heterogenous before they enter the contract, then a pooling Nash equilibrium - where no lender can deviate from the representative financial intermediary and offer a better contract in order to attract a better pool of agents - might not exist (see Prescott and Townsend (1984)).

The timing of events - see Figure 5 - is as follows:

Figure 5: Timing



1. Financial intermediaries post contracts. A contract is a 6-tuple $\{(e_\ell, L_\ell, L_\ell^F), (e_h, L_h, L_h^F)\}$. For each productivity type i, the contract specifies the fraction of entrepreneurs who will operate their technology, e_i. The rest (fraction $1 - e_i$) work for a wage. For entrepreneurs who are chosen to operate their technology, the contract specifies the repayment schedule for true reports, $L_i \equiv L(a_i|a_i)$, and for false reports, $L_i^F \equiv L(a_i|a_{-i})$.

2. Households decide whether to enter the contract with the financial intermediary. Households who do not enter the contract work for a wage, rent out their capital, and consume $w + \eta$ at the end of the period, where $\eta = (1 + r)\hat{f}$ is the household's initial endowment after earning interest.

3. Households learn their type and report it to the financial intermediary.

4. The financial intermediary chooses the households who operate their technology for each type (through a randomization device).

5. Households who are chosen to operate their technology become entrepreneurs. They are allocated capital k_i, labor n_i, and fixed cost f. The households who are not chosen to operate their production technology supply labor, earn the market wage rate and consume w.

6. Entrepreneurs produce and all information is revealed. If an entrepreneur reported her productivity truthfully she consumes $y_i - L_i$. If an entrepreneur misreported her productivity she consumes $y_i^F - L_i^F$.

Financial intermediaries maximize households' expected consumption subject to incentive compatibility, enforcement, participation, and resource feasibility constraints, as described below.

The Intermediary's Problem

The revelation principle allows us to focus, without loss of generality, on allocations in which households report their type truthfully.

The objective of the financial intermediary is to choose allocations $(k_\ell, n_\ell, k_h, \ell_\ell)$ and terms of contract $(e_\ell, L_\ell, L_\ell^F, e_h, L_h, L_h^F)$, given prices w and r, such that

1. Entrepreneur's expected consumption is maximized (before they learn their type),

$$\max \mathbb{E}[c] = \nu c_\ell + (1-\nu) c_h \tag{3}$$

   where $c_i = e_i(y_i - L_i) + (1-e_i)w$.

2. Incentive Compatibility:

$$e_i(y_i - L_i) + (1-e_i)w \geq e_-i(y_i^F - L_i^F) + (1 - e_-i)w, \quad \forall i \in \{\ell, h\} \tag{4}$$

   A type i entrepreneur who falsely claims to be type $-i$ will operate his productive technology with probability e_{-i} and be assigned capital k_{-i}, labor n_{-i}, and fixed cost f. With these inputs, type i entrepreneur will produce $y_i^F = \frac{a_i}{a_{-i}} y_{-i}$ (derivation in B).

3. Imperfect Enforcement:

$$\begin{aligned} L_i &\leq \phi y_i \;\; \forall i & (5) \\ L_i^F &\leq \phi y_i^F \;\; \forall i & (6) \end{aligned}$$

4. Participation Constraint: If a household declines to enter a contract, he gets wage w, and consumes his wage plus his net worth for a total consumption of $w + \eta$. The participation constraint is therefore

$$\nu c_\ell + (1-\nu)c_h \geq w + \eta \tag{7}$$

Since intermediaries can achieve any allocation that households can achieve on their own, and since the intermediary maximizes household's expected utility, households are weakly better off contracting with the intermediary.[12]

5. Feasibility:
The financial intermediary faces a known fraction ν of low-productivity entrepreneurs and a fraction $(1-\nu)$ of high-productivity entrepreneurs. Let $\kappa_i = rk_i + wn_i + f$ stand for the cost of producing output y_i. The feasibility constraint requires that the resources disbursed by the financial intermediary to entrepreneurs (left hand side) cannot exceed collections plus initial resources (right hand side).

$$\nu e_\ell \kappa_\ell + (1-\nu)e_h \kappa_h \leq \nu e_\ell L_\ell + (1-\nu)e_h L_h + \eta \tag{8}$$

Notice that in the direct mechanism the intermediary allocates capital and labor contingent on the productivity report, and hence contingent output y_i and y_i^F, directly to the entrepreneurs.

3.1 Partial Equilibrium: Optimal Contract

Allocations

The intermediary allocates the profit-maximizing levels of capital and labor to each operating enterprise, given prices r and w. To see why this is so, notice that the objective of the intermediary is equivalent to a social planner problem, and as such, the intermediary will try to distort as few marginal decisions as possible. Any incentive compatibility benefits from deviating from profit-maximizing output can be replicated by changes in

[12]The intermediary can match the outside option by setting $e_h = e_\ell = 0$ and returning the initial endowment to households.

the terms of the contract that do not distort the intensive margin of production. Thus, $k_i = k_i^*$ and $n_i = n_i^*$. To simplify notation from here on, (y_i, k_i, n_i) stand for their profit maximizing levels (y_i^*, k_i^*, n_i^*).

It follows that there is no capital misallocation on the intensive margin. I say there is misallocation of talent if there exists a high-productivity entrepreneur who is not operating her technology while a low-productivity entrepreneur operates hers.

Terms of the contract

The following propositions state some partial equilibrium properties of the contract.

Proposition 1 *Suppose wages are low enough so that the no-private-information allocation is not incentive compatible,* $w < \frac{a_\ell}{a_h} y_h - \phi \kappa_\ell$.

i. ***Average Quality:*** *The ratio of high to low-productivity projects funded is given by the expression*

$$\frac{e_h}{e_\ell} = \frac{y_\ell - L_\ell - w}{(1-\phi)\frac{a_\ell}{a_h} y_h - w}. \tag{9}$$

ii. ***Quantity of High-Productivity Projects:*** *The quantity of high-productivity projects funded is given by the expression*

$$e_h = \frac{\eta}{\nu(\kappa_\ell - L_\ell)\left(\frac{(1-\phi)\frac{a_\ell}{a_h} y_h - w}{y_\ell - L_\ell - w}\right) + (1-\nu)\Big(\kappa_h - \phi y_h\Big)} \tag{10}$$

iii. Furthermore, suppose wages are high enough so that low-productivity projects are not profitable, $w > y_\ell - \kappa_\ell$. *Then* $L_\ell = 0$*, and average quality of projects (9) and the quantity of high-productivity projects (10) are strictly increasing in the recovery rate* ϕ.

Proof. For i. and ii. see Appendix C. For iii., if $L_\ell = 0$ then $\frac{\partial (e_h/e_\ell)}{\partial \phi} > 0$ and $\frac{\partial e_h}{\partial \phi} > 0$ in (9) and (10), respectively. ■

Expression (9) is obtained by combining the binding incentive compatibility constraint for low-productivity with maximum punishment for false reports, $L_\ell^F = \phi y_\ell^F$, and replacing $y_\ell^F = \frac{a_\ell}{a_h} y_h$. Expression (10) is obtained by combining expression (9) with the feasibility constraint and maximum collection from high-type profitable projects,

$L_h = \phi y_h$.[13] The assumption of an upper threshold for wages is simply a formalization of the initial premise that private information is a constraint on lender's funding decisions. If wages are too high, low-productivity entrepreneurs are better off renting their labor than they are operating their technology. In that scenario they have no incentive to misreport their type as they prefer a lower probability of operating their technology, and information constraints do not bind.

The first part of Proposition 1 expresses the ratio of high to low productivity projects. If low-productivity projects are unprofitable, then this ratio decreases as the level of contract enforceability decreases. The intuition is that as the ability of punishment for false reports shrinks, the lender has to increase the relative probability of acceptance of low-type in order to prevent low-types from reporting falsely. Notice this "average quality" effect goes solely through the incentive compatibility of low-type.

The second part of Proposition 1 shows the expression for the total quantity of high-productivity projects. If low-type projects are unprofitable, the expression increases with the recovery rate. This effect comes from the feasibility constraint. As recovery rates increase, the lender is able to extract higher rents from profitable projects in order to fund other profitable projects.

The assumption of unprofitability of low type is a sufficient, but not necessary, condition to obtain clean predictions for the change in average quality and total quantity with recovery rates. If low types are profitable, the average quality and quantity of high type projects could still increase, depending on model parameters. However, in the quantitative exercise in Section 5, I do not impose any restrictions on parameters and verify that all conditions and assumptions hold.

Notice that inital resources, η, is an important parameter. If agents are born with zero wealth then $\eta = 0$ and there are no resources to allocate among entrepreneurs, hence $e_\ell = e_h = 0$. Higher initial wealth will increase the number of projects funded and will play an important role in interpreting the results for low-income countries in Section 5. For these countries, differences in recovery rates seem to be of secondary importance for explaining cross-country differences in TFP. Of primary importance is the initial resources they have to distribute among entrepreneurs, η.

[13] Incentive compatibility for high-type might bind, and then L_h is set by the binding incentive compatibility constraint for high-type. This is in contrast to Erosa and Hidalgo-Cabrillana (2008); see C for details.

3.2 Total Factor Productivity

Define $\pi_\ell \equiv \nu e_\ell$ and $\pi_h \equiv (1-\nu)e_h$ as the measure of low and high-productivity projects operated in the economy, respectively. Let $Y \equiv \sum_i \pi_i y_i$, $K \equiv \sum_i \pi_i k_i$, and $N \equiv \sum_i \pi_i n_i$ be the aggregate ouput, capital and labor in the economy. The following proposition states that the aggregate production function and TFP have a closed-form solution.

Proposition 2 *i. The aggregate production function has a closed-form solution given by*

$$Y = AK^\alpha N^\gamma \tag{11}$$

where total factor productivity is

$$A \equiv \left(\nu e_\ell a_\ell^{\frac{1}{1-\alpha-\gamma}} + (1-\nu)e_h a_h^{\frac{1}{1-\alpha-\gamma}}\right)^{1-\alpha-\gamma} \tag{12}$$

Measured TFP in the competitive equilibrium is

$$\frac{Y}{K^{\frac{\alpha}{\alpha+\gamma}} N^{\frac{\gamma}{\alpha+\gamma}}} = Y^{\frac{\alpha+\gamma-1}{\alpha+\gamma}} A^{\frac{1}{\alpha+\gamma}} \tag{13}$$

ii. TFP is strictly increasing in quantity of high-productivity projects e_h. In addition, if $(1-\nu)a_h^{\frac{1}{1-\alpha-\gamma}} > \nu a_\ell^{\frac{1}{1-\alpha-\gamma}}$, TFP is strictly increasing in the average quality of projects e_h/e_ℓ, as long as $e_\ell + e_h$ is simultaneously non-decreasing.

Proof. For i., see Appendix A. For ii., it follows from (12). ■

Notice A is not the same as measured TFP. In level accounting exercises the aggregate production function is assumed to display constant returns to scale with capital share equal to 1/3. Since firms in the model have decreasing returns to scale, equation (13) is the correct expression to compare to the data, with $\frac{\alpha}{\alpha+\gamma}$ set to 1/3.

For a non-decreasing total quantity of projects as the recovery rate increases, all that is needed is that the quantity of low-productivity projects does not decrease faster than high-productivity projects increase. In the quantitative exercise, both high-type projects and low-type projects increase with the recovery rate.

3.3 Competitive Equilibrium

With the household and intermediary's problem specified, the competitive equilibrium in this economy can now be defined. Aggregate labor supply is determined by the measure of households who did not become entrepreneurs, $\nu(1-e_\ell)+(1-\nu)(1-e_h)$. Aggregate labor demand is given by the total amount of labor demanded by firms of both types, $\nu e_\ell n_\ell + (1-\nu)e_h n_h$. Similarly aggregate capital demand and aggregate final good supply are determined by $\nu e_\ell k_\ell + (1-\nu)e_h k_h$ and $\nu e_\ell y_\ell + (1-\nu)e_h y_h$, respectively. Finally, aggregate final good demanded is determined by the residual consumption from the household's problem, $c^e = \nu[e_\ell(y_\ell - L_\ell) + (1-e_\ell)w] + (1-\nu)[e_h(y_h - L_h) + (1-e_h)w]$.

Definition 1 *A competitive equilibrium consists of a financial contract $\{e_i, L_i, L_i^F\}_i$, allocations $\{y_i, k_i, n_i\}_i$ and prices w and r such that*

1. *Allocations k_i, n_i and y_i maximize firms' profits, given prices w and r for all i*
2. *The financial contract solves the financial intermediary's problem*
3. *Markets Clear:*
   - *w clears the labor market: $\nu e_\ell n_\ell + (1-\nu)e_h n_h = \nu(1-e_\ell)+(1-\nu)(1-e_h)$*
   - *Given r, the capital market clears: $\nu e_\ell k_\ell + (1-\nu)e_h k_h = K$*
   - *The final goods market clears: $\mathbb{E}[c] + f(\nu e_\ell + (1-\nu)e_h) = \nu e_\ell y_\ell + (1-\nu)e_h y_h$*

Notice that the general equilibrium effects reinforce the misallocation of resource at low levels of enforceability. With better contract enforceability, there is higher labor demand which drives up wages. This relaxes incentive compatibility constraint of low types by making the outside option of entrepreneurs more attractive (higher wages relax (4) since $(1-e_\ell) > (1-e_h)$).

4 Taking the Model to the Data: Multi-Sector Model

As the previous section showed, an important parameter in the calculations of TFP is the levels of firm productivities, a_ℓ and a_h. The first order conditions for the firm problem show a direct relationship between the employment distribution and the distribution of productivity. In particular,

$$\frac{n_h}{n_\ell} = \left(\frac{a_h}{a_\ell}\right)^{\frac{1}{1-\alpha-\gamma}} \tag{14}$$

In order to calibrate the distribution of productivities one could arbitrarily divide the U.S. firm size distribution into a representative large firm and a representative small firm and use equation (14). Alternatively, one can extend the model in some direction to obtain a distribution of firm sizes. This is the approach I pursue here. In particular this section extends the model to the case of multiple sectors and shows that the main results hold in this more general framework.

Preferences, production functions, and frictions are the same as in the one-sector economy. A sector is defined as a group of firms who produce an identical product. There is a unit mass of agents born into each sector. Agents are born with a sector-specific technology but they can work in any sector for wage w, same across sectors. Sectors differ in the level of the fixed cost to operate a technology. There is one intermediary per sector (no cross-subsidization across sectors), so that the contract of the one-sector economy can be viewed as the contract of a specific sector. Let subscript j be the sector subscript and i be the productivity level subscript. To redefine the problem with different sectors, one can rewrite individual production functions as

$$y_{ij} = a_i k_{ij}^{\alpha} n_{ij}^{\gamma} \tag{15}$$

An entrepreneur's problem is

$$\max_{k_{ij}, n_{ij}} p_j y_{ij} - r k_{ij} - w n_{ij} - f_j \tag{16}$$

where p_j are sector-specific output prices. Let $\pi_{\ell j} \equiv \nu e_{\ell j}$ and $\pi_{hj} = (1-\nu) e_{hj}$ stand for the measure of low and high-productivity projects in sector j, respectively. Using the aggregation in A, each sector has a representative firm with production function of the form

$$y_j = A_j k_j^{\alpha} n_j^{\gamma} \tag{17}$$

where, $y_j = \sum_i \pi_{ij} y_{ij}$, $k_j = \sum_i \pi_{ij} k_{ij}$, $n_j = \sum_i \pi_{ij} n_{ij}$, and $A_j \equiv \left(\sum_i \pi_{ij} a_i^{\frac{1}{1-\alpha-\gamma}}\right)^{1-\alpha-\gamma}$. The representative sector firm solves the following problem

$$\max_{k_j, n_j} p_j y_j - r k_j - w n_j \tag{18}$$

Notice the fixed cost shows up as a cost in the firm problem but not in the representative sector firm problem. Finally, I introduce a new parameter, θ, which determines the complementarities between sectors. In particular, assume a perfectly competitive representative firm produces a single final good by combining sector outputs with a CES technology, so that it solves the following problem

$$\max_{\{y_j\}} \left(\sum_j y_j^\theta\right)^{1/\theta} - \sum_j p_j y_j \tag{19}$$

where $\frac{1}{1-\theta}$ is the elasticity of substitution between sectors.

The main difference with the previous competitive equilibrium is that there are now J new markets and prices, one for each sector.

Definition 2 *A competitive equilibrium with sectors consists of financial contracts* $\{c_{ij}, e_{ij}, L_{ij}, L^F_{ij}\}_{ij}$, *allocations* $\{n_{ij}, k_{ij}, y_{ij}\}_{ij}$ *and prices* w, r, *and* $\{p_j\}_j$ *such that*

1. k_{ij} *and* n_{ij}, *and* y_{ij} *solve the firm's problem,* $\forall i, j$
2. *The financial contract solves the financial intermediaries problem,* $\forall j$
3. *Markets Clear:*
   - p_j *clears the sector market,* $y_j^{Supply}(p_j, r, w) = y_j^{Demand}(p_j, \{c^e\})\ \forall j$
   - w *clears the labor market,* $\sum_j \sum_i \pi_i n_{ij} = \sum_j [\nu(1-e_{lj}) + (1-\nu)(1-e_{hj})]$
   - *Given* r, *the capital market clears:* $\sum_j \sum_i \pi_i k_{ij} = K$
   - *The final goods market clears,* $\sum_j \mathbb{E}[c_j] + \sum_j f_j(\nu(1-e_{lj}) + (1-\nu)e_{hj}) = \left(\sum_j y_j^\theta\right)^{1/\theta}$

where y_j^{Supply} is output by sector j, y_j^{Demand} is demand by the final good producer.

Proposition 3 *TFP in the multi-sector economy with sectors is analogous to the single sector economy. In particular,*

$$\frac{Y}{K^{\frac{\alpha}{\alpha+\gamma}}N^{\frac{\gamma}{\alpha+\gamma}}} = \left(\sum_j \left(y_j^{\frac{\alpha+\gamma-1}{\alpha+\gamma}} A_j^{\frac{1}{\alpha+\gamma}}\right)^{\frac{\theta}{1-\theta}}\right)^{\frac{1-\theta}{\theta}} \tag{20}$$

where

$$A_j \equiv \left(\nu e_{\ell j} a_\ell^{\frac{1}{1-\alpha-\gamma}} + (1-\nu) e_{hj} a_h^{\frac{1}{1-\alpha-\gamma}}\right)^{1-\alpha-\gamma} \tag{21}$$

Proof. See Appendix D. ■

5 Quantitative Analysis

In this section I calibrate the model to data for the United States. In my calibration I treat the United States as an economy with private information and imperfect enforcement frictions, and with a recovery rate of 80%. With the calibrated economy in place, I vary the recovery rate as in the data to explore what fraction of TFP differences that can be attributed to bankruptcy efficiency.

5.1 Calibration and Measurement

Several of the model parameters are those of the growth model and I follow standard procedures for choosing those values. Relative to the growth model, what is new are the parameters that determine the distribution of firms in equilibrium. Tables 2 and 3 summarize the calibration.

Table 2: Benchmark Calibration to U.S. Data: Parameters Set Before Equilibrium

Symbol	Definition	Value	Target/Source
ϕ	Recovery Rate	80%	Data
α	Firm Capital Share	28.3%	Agg. Capital Share, Dec. Returns
γ	Firm Labor Share	56.7%	Agg. Capital Share, Dec. Returns
θ	Complementarity between Sectors	0.9	Markup of 11%
J	Number of Sectors	36	Rajan and Zingales (1998)
$[f_1, f_{36}]$	Range of Fixed Costs	$[1, 4.3]$	Rajan and Zingales (1998)
a_ℓ	Productivity of Low Type	1	Normalization

Table 3: Benchmark Calibration to U.S. data: Parameters Calibrated to Equilibrium Outcomes

Symbol	Definition	Value	Target Moments	U.S. Data	Model
r	Interest Rate	10%	Capital Output Ratio	~ 3	3.12
a_h	Productivity of High Type	1.695	Top 10% employment share	65%	65%
ν	Fraction of Low Type	63%	Skew of firm-size distribution	5.05	4.99
η	Initial endowment	0.376	Mean firm size	50.5	50.5

First I discuss the choice of parameters for which direct estimates are available and then I discuss the ones that are chosen to match equilibrium moments. The data on recovery rate is collected by World Bank Doing Business database. I use the average from 2004 (the earliest available data) to 2009 (to exclude the financial crisis). The recovery rate is $\phi = 80\%$ for the United States.

The extent of decreasing returns in the production function is an important parameter in my analysis. Direct estimates of firm-level production functions and different calibration procedures point to a value for $\alpha + \gamma = 0.85$.[14] The split between α and γ is done according to the income share of capital and labor, so I assign 1/3 to capital and 2/3 to labor, implying $\alpha = 0.283$ and $\gamma = 0.567$.[15]

Sector outputs are aggregated with a CES function with elasticity parameter $\frac{1}{1-\theta}$. TFP differences are magnified by the degree of complementarity between sectors, so I am conservative in the choice of this parameter and choose $\theta = 0.9$. In a model of monopolistic competition this would deliver a markup of 11%, a lower bound among the empirical estimates of markup costs.[16]

The number of sectors and the sector-specific fixed cost are based on values provided by Rajan and Zingales (1998). Using Compustat data, Rajan and Zingales calculate the need for external finance (defined as the fraction of capital expenditures not financed with cash flow from operations) for 36 sectors in the United States during the 1980s. The sector with lowest need for external finance is Tobacco, with a measure of -0.45 and the one with the highest need is Drugs at 1.49. I set $J = 36$ and normalize the lowest

[14] See for example Basu and Fernald (1995), Atkeson and Kehoe (2005), and Amaral and Quintin (2010), among others.

[15] Using labor shares, Gollin (2002) shows that capital shares are close to 1/3 for different countries and do not systematically vary with development levels.

[16] This value is in line with the choice in Atkeson and Kehoe (2005), and the evidence in Basu and Fernald (1995), Basu (1996) and Basu and Kimball (1997).

level of sector fixed cost to 1. Sector fixed costs are chosen to range uniformly from 1 to $(0.45 + 1.49)/0.45 = 4.3$. To place the magnitude of fixed costs in perspective, consider that the wage in the competitive equilibrium is around 1.05. Therefore, fixed costs of starting a firm are about 1 to 4.3 times the average annual salary.

Now I discuss the parameters calibrated to equilibrium moments. The interest rate determines the capital output ratio and it is set to 10% to match a capital output ratio of 3. This is consistent with evidence in Gomme and Rupert (2007). I am left with four parameters to match to equilibrium targets, a_ℓ, a_h, ν and η. Any choice of a_ℓ can be undone by rescaling a_h. Thus, I set $a_\ell = 1$ as a normalization. The other parameters are calibrated to moments of the U.S. firm-size distribution.

The data for the U.S. firm-size distribution is from the US Census Bureau.[17] The U.S. Census reports the number of establishments for certain employment ranges for all sectors at the three-digit level. I restrict the observations to manufacturing to make it compatible with Rajan and Zingales (1998) fixed cost data. I set a_h to match the share of employment by the top 10% largest firms. ν controls the skewness of the distribution, and I set it to match a skewness of 5.05 (defined here as mean divided by median). The endowment η shifts the firm-size distribution, so it is set to match the average firm size in U.S. manufacturing of 50.5.[18]

6 Findings

6.1 Variation in the recovery rate, ϕ

Figures 6 and 7 illustrate the change in average quality and total quantity of projects brought about by an exogenous change in the recovery rate. The figures display the results for the median fixed-cost sector, but the pattern is similar in all sectors. Figure 6 shows that the average quality is not only increasing in the recovery rate, but the increase is happening at an increasing rate. Two things are worth noting. First, low-productivity projects are not profitable, so Proposition 1 (iii) applies directly: the partial equilibrium

[17] I am grateful to Mark Wright for providing a specially tabulated dataset of the US firm size distribution for the year 2000.

[18] From U.S. Census data I only observe the number of establishment for certain employment ranges. The mean employment size for each range is used to compute the mean firm size.

effect on the optimal contract leads the average quality to increase as the recovery rate increases. Second, the general equilibrium effect on improving wages with the recovery rate leads to a further increase in the average quality. Figure 7 shows that the overall quantity of projects increases as well. What is worth noting here is that the increase is not only happening in the high-productivity projects, as expressed in Proposition 1 (iii), but also in low-productivity projects.

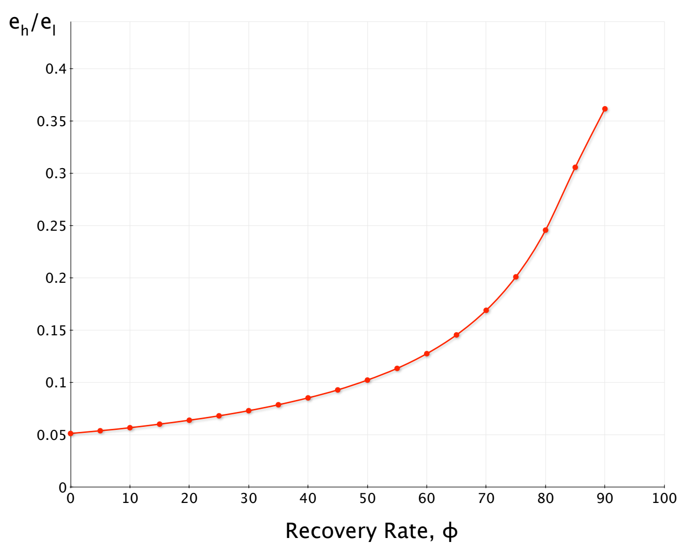


Figure 6: Average quality of projects

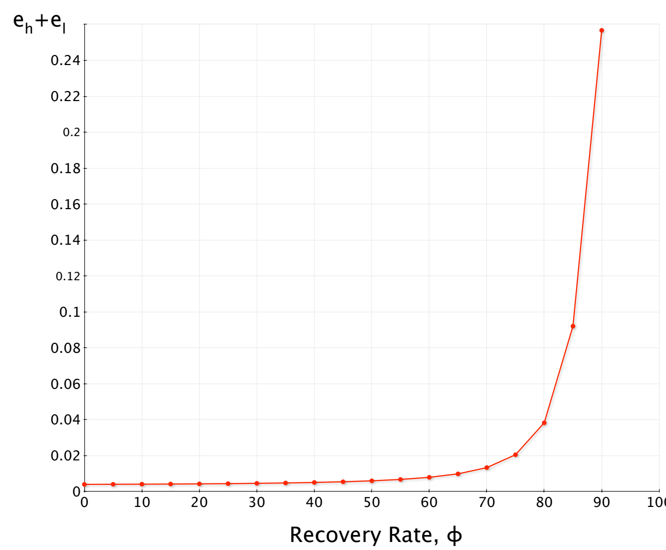


Figure 7: Quantity of projects

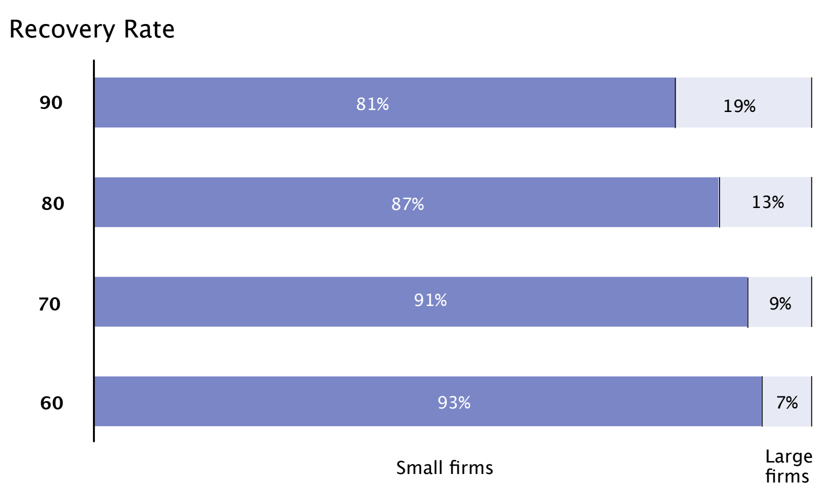


Figure 8: Firm size distribution. Small firms: $\pi_\ell/(\pi_\ell + \pi_h)$. Large firms: $\pi_h/(\pi_\ell + \pi_h)$

Figure 8 shows the overall effect in the firm-size distribution for four values of the recovery rate. As average quality and total quantity of projects increase, the firm-size distribution contains a lower percentage of small firms (low-productivity). The percentage of small firms is calculated as $\pi_\ell/(\pi_\ell + \pi_h)$ and the percentage of large firms as $\pi_h/(\pi_\ell + \pi_h)$.[19] This figure confirms that differences in bankruptcy efficiency can rationalize the observation that differences in bankruptcy efficiency alone generate variation of the firm-size distribution consistent with the European data in Figure 3 in Section **??**.

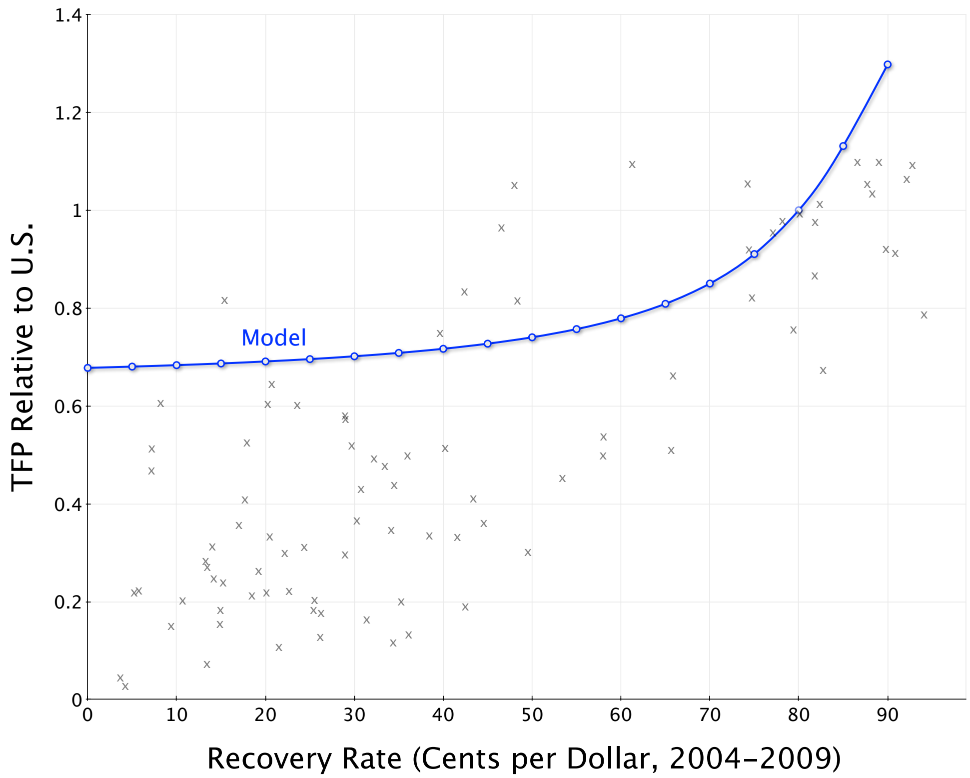


Figure 9: Model is calibrated to the United States at a recovery rate of 80% and TFP=1. The line traces the TFP predicted by the model from varying the recovery rate from 0 to 90%. The crosses map the combination of TFP and recovery rate of 90 countries in the data.

Figure 9 shows the implication of the changing firm-size distribution for TFP. The calibration to the United States is the point with TFP equal to 1 and recovery rate equal to 80%. Varying the recovery rate from 60 to 90% yields large variations in TFP. If the United States had a recovery rate of 60% as Italy does, its TFP would be about 20% lower than its current TFP. Alternatively, the TFP of the U.S. would be 20% higher if it improved it's recovery rate to 87%, as in the UK. However, recovery rates lose explanatory power

[19] π_i is the sum of π_{ij} across all sectors.

below recovery rates of 60. For recovery rates between of 0 to 60, recovery rates generates small variations in TFP.

6.2 Variation in the initial endowment, η

Motivated by the lack of variation of TFP in low recovery rate countries, I perform another experiment. In addition to varying the recovery rate, I ask what level of initial endowment would allow the model to match perfectly the TFP in the data (dashed line in Figure 10). I then hold recovery rate constant at the U.S. level and use the initial endowment from the perfect match to ask how much variation in initial endowment alone could explain TFP differences (red squares line in Figure 10).

Initial endowment alone (red squares line) explains less in high recovery rate countries, but a substantial amount of TFP variation in low recovery rate countries. One interpretation of this result is that improvements in bankruptcy procedures by itself has little impact in low-income countries, unless it is accompanied by improvements in initial endowments. Put differently, the misallocation of resources among entrepreneurs is of second order if the amount of resources to be allocated is low to begin with.

7 Conclusion

This paper documents that firm-size distributions are an important driver of productivity differences between European countries, and it proposes differences in bankruptcy efficiency as a driver of firm-size distributions. Low recovery rates, a measure of bankruptcy efficiency, affect productivity differences by reducing the average quality and total quantity of firms in an economy. The model calibrated to U.S. data suggests that differences in the recovery rate alone can generate TFP differences of similar magnitudes of those observed across European countries.

The quantitative exercise was also used to investigate the impact of initial endowments in explaining TFP differences. The results suggest that improvements in bankruptcy efficiency have the highest impact on middle and high-income countries. Other factors (such as corruption, wars, health) that are not explicitly modeled but might affect initial endowments are most important for low-income countries. The findings suggest that in

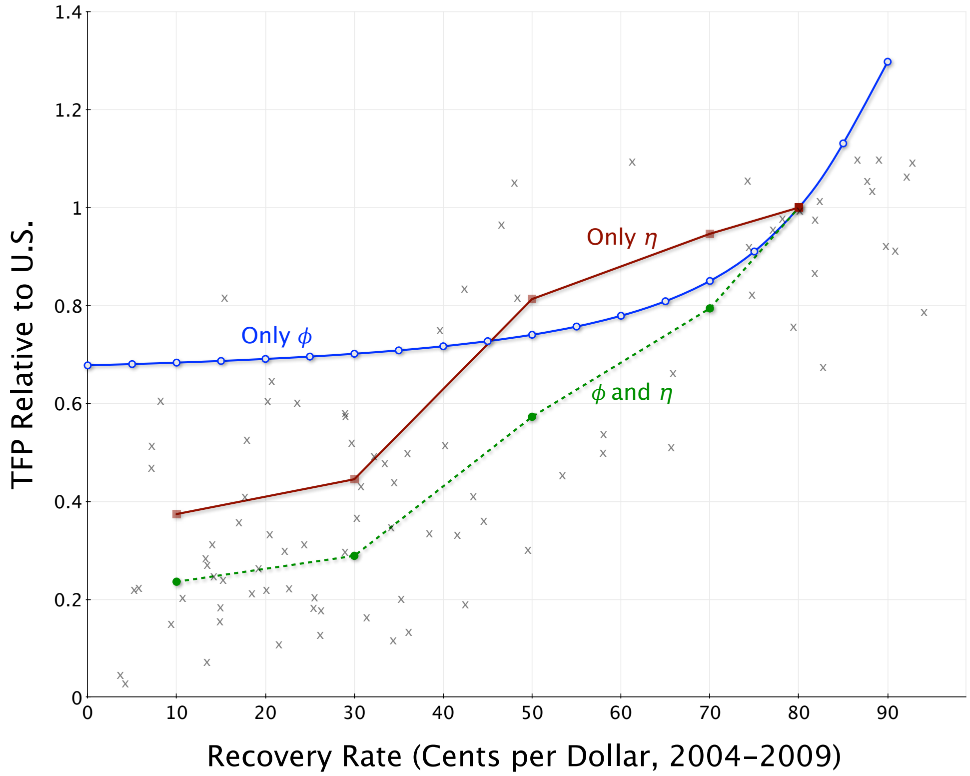


Figure 10: Crosses: Countries in data. Hollow circle (blue) line: Only recovery rate ϕ varies. Square (brown) line: Only initial endowment η varies. Dashed (green) line: Recovery rate and initial endowment vary simultaneously.

low-income countries reforms to improve bankruptcy efficiency must be accompanied by improvements in other factors that affect low endowments in order to generate significant progress in aggregate productivity and income. A unique focus on the improving the bankruptcy code seems inappropriate in low-income countries.

Appendix A Aggregate Total Factor Productivity and Firm Productivity

Individual i is endowed with production function y_i, which takes inputs k_i and n_i combines them with a Cobb-Douglas technology

$$y_i(k_i, n_i) = a_i k_i^{\alpha} n_i^{\gamma} \tag{A-1}$$

where $\alpha + \gamma < 1$. The first order conditions from the firm's problem are

$$\alpha \frac{y_i}{k_i} = r, \tag{A-2}$$

$$\gamma \frac{y_i}{n_i} = w. \tag{A-3}$$

The ratio between marginal products is

$$\frac{n_i}{k_i} = \frac{r\gamma}{w\alpha}. \tag{A-4}$$

Obtain expressions for unconditional factor demand by substituting (A-4) back into the first order conditions (A-2) and (A-3),

$$k_i(r, w) = B^{\frac{1}{1-\alpha-\gamma}} \frac{\alpha}{r} a_i^{\frac{1}{1-\alpha-\gamma}}, \tag{A-5}$$

$$n_i(r, w) = B^{\frac{1}{1-\alpha-\gamma}} \frac{\gamma}{w} a_i^{\frac{1}{1-\alpha-\gamma}}. \tag{A-6}$$

Where $B = \left(\left(\frac{\alpha}{r}\right)^{\alpha} \left(\frac{\gamma}{w}\right)^{\gamma} \right)$. Define $K = \sum_i k_i \pi_i$, $N = \sum_i n_i \pi_i$ and $Y = \sum_i y_i \pi_i$, where π_i is the fraction of projects of type i that are operated.

Aggregate equations (A-5) and (A-6) and divide both sides by $\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}}$ to obtain

$$K \cdot \left(\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}} \right)^{-1} = B^{\frac{1}{1-\alpha-\gamma}} \frac{\alpha}{r}, \tag{A-7}$$

$$N \cdot \left(\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}} \right)^{-1} = B^{\frac{1}{1-\alpha-\gamma}} \frac{\gamma}{w}. \tag{A-8}$$

Substitute back into (A-5) and (A-6),

$$k_i(K) = K \frac{a_i^{\frac{1}{1-\alpha-\gamma}}}{\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}}}, \tag{A-9}$$

$$n_i(N) = N \frac{a_i^{\frac{1}{1-\alpha-\gamma}}}{\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}}}. \tag{A-10}$$

Plugging back into the individual production function (A-1) yields

$$y_i = a_i^{\frac{1}{1-\alpha-\gamma}} \left(\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}} \right)^{-\alpha-\gamma} K^\alpha N^\gamma. \tag{A-11}$$

Aggregating one last time we find the expression for sector output,

$$Y = AK^\alpha N^\gamma \tag{A-12}$$

where

$$A \equiv \left(\sum_i \pi_i a_i^{\frac{1}{1-\alpha-\gamma}} \right)^{1-\alpha-\gamma}.$$

Appendix B Ouput of False Report

Substitute the marginal ratios (A-4) into the production function (A-1) and solve for conditional factor demand

$$y_i(w,r,k_i) = a_i B \left(\frac{r}{\alpha} \right)^{\alpha+\gamma} k_i^{\alpha+\gamma} \tag{A-13}$$

$$y_i(w,r,n_i) = a_i B \left(\frac{w}{\gamma} \right)^{\alpha+\gamma} n_i^{\alpha+\gamma} \tag{A-14}$$

with B as defined in A. Solving for factors, we get conditional factor demand,

$$k_i(w,r,y_i) = \frac{\alpha}{r} B^{-\frac{1}{\alpha+\gamma}} a_i^{-\frac{1}{\alpha+\gamma}} y_i^{\frac{1}{\alpha+\gamma}} \tag{A-15}$$

$$n_i(w,r,y_i) = \frac{\gamma}{w} B^{-\frac{1}{\alpha+\gamma}} a_i^{-\frac{1}{\alpha+\gamma}} y_i^{\frac{1}{\alpha+\gamma}} \tag{A-16}$$

Substituting into the cost function, $\kappa_i(w,r,y_i) = rk_i(w,r,y_i) + wn_i(w,r,y_i) + f$ and substituting in for B,

$$\kappa_i(w,r,y_i) = (\alpha+\gamma)\left(\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right)^{-\frac{1}{\alpha+\gamma}} a_i^{-\frac{1}{\alpha+\gamma}} y_i^{\frac{1}{\alpha+\gamma}} + f \tag{A-17}$$

Find output by plugging the cost function into the firm problem and maximize to get

$$y_i(w,r) = \left(\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\gamma}{w}\right)^{\gamma}\right)^{\frac{1}{1-\alpha-\gamma}} a_i^{\frac{1}{1-\alpha-\gamma}} \tag{A-18}$$

We can rewrite the cost function as

$$\kappa_i(w,r,y_i) = \psi_i y_i^{\frac{1}{\alpha+\gamma}} + f$$

where $\psi_i \equiv a_i^{-\frac{1}{\alpha+\gamma}}(\alpha+\gamma)\left(\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{\gamma}\right)^{\gamma}\right)^{\frac{1}{\alpha+\gamma}}$.

An entrepreneur who misrepresents his type is given funds $\psi_{-i} y_{-i}^{\frac{1}{\alpha+\gamma}} + f$ but his real production costs are $\psi_i (y_i^F)^{\frac{1}{\alpha+\gamma}} + f$. Making these two equal yields $y_i^F = \frac{a_i}{a_{-i}} y_{-i}$.

Appendix C Solution to the Contract

The solution begins with the no-private-information problem, and then adds private information.

Rewrite the problem as a linear programming problem, with $\tilde{L_i} \equiv e_i L_i$

$$\max_{e_\ell, e_h, \tilde{L_\ell}, \tilde{L_h}, \tilde{L_\ell^F}, \tilde{L_h^F}} \quad \mathbb{E}[c] = \nu(e_\ell(y_\ell - w) - \tilde{L_\ell}) + (1-\nu)(e_h(y_h - w) - \tilde{L_h}) + w \tag{A-19}$$

$$\text{s.t.} \quad e_\ell(y_\ell - w) - \tilde{L_\ell} \geq e_h(y_\ell^F - w) - \tilde{L_\ell^F} \tag{A-20}$$

$$e_h(y_h - w) - \tilde{L_h} \geq e_\ell(y_h^F - w) - \tilde{L_h^F} \tag{A-21}$$

$$\nu e_\ell(\kappa_\ell) + (1-\nu)e_h(\kappa_h) \leq \nu\tilde{L_\ell} + (1-\nu)\tilde{L_h} + \eta \tag{A-22}$$

$$0 \leq \tilde{L_i} \leq e_i \phi y_i \quad \forall i \tag{A-23}$$

$$0 \leq \tilde{L_i^F} \leq e_i \phi y_i^F \quad \forall i \tag{A-24}$$

$$0 \leq e_i \leq 1 \quad \forall i \tag{A-25}$$

No-private-information

In the no-private-information environment, (A-20) and (A-21) are absent. Notice the effect of $\tilde{L_\ell}$ and $\tilde{L_h}$ is neutral: Increasing either reduces (A-19) by the same amount it relaxes (A-22) so their values are indeterminate. The proportion of high ability projects funded is the maximum possible since it increases (A-19) by more than it reduces (A-22) because $y_h - \kappa_h - w > 0$. The amount of funded projects $e_h \geq e_\ell$ because $y_h - \kappa_h - w > y_\ell - \kappa_\ell - w$. The proportion of low projects funded is $e_\ell = 0$ if low type is unprofitable because it constraints (A-22) by more than it increases (A-19) $y_\ell - \kappa_\ell - w < 0$, and positive otherwise.

Private information

If $w > \frac{a_\ell}{a_h} y_h (1-\phi)$ then the no-private-information allocations are incentive compatible.

Recall the incentive compatibility constraint for low type,

$$e_\ell (y_\ell - L_\ell) + (1 - e_\ell) w \geq e_h (y_\ell^F - L_\ell^F) + (1 - e_h) w. \tag{A-26}$$

If low ability agent is unprofitable ($y_\ell - \kappa_\ell < w$) or there are sufficient high-ability agents ($e_h < 1$), then $e_\ell = 0$ in the no-private-information allocation. This allocation is incentive compatible if

$$w \geq e_h (y_\ell^F - L_\ell^F) + (1 - e_h) w. \tag{A-27}$$

A low-productivity entrepreneur who lies obtains output $y_\ell^F = \frac{a_\ell}{a_h} y_h$ (B). To deter lying, it is optimal to set the punishment for lying as high as possible, $L_\ell^F = \phi y_\ell^F = \phi \frac{a_\ell}{a_h} y_h$.

Substitute this expression in (A-27) to obtain,

$$w \geq e_h \frac{a_\ell}{a_h} y_h (1 - \phi) + (1 - e_h) w. \tag{A-28}$$

Substract $(1 - e_h) w$ and divide by e_h from both sides to obtain the threshold wage beyond which the no-private-information allocation is always incentive compatible.

$$w \geq \frac{a_\ell}{a_h} y_h (1-\phi). \tag{A-29}$$

Incentive compatibility binds: $w < \frac{a_\ell}{a_h} y_h (1-\phi)$

First, setting $\tilde{L_i^F} = e_i \phi y_i^F$ for all i is optimal since it relaxes (A-20) and (A-21) but has not other effects elsewhere.

Next, (A-20) binds and, rearranging, we obtain

$$\frac{e_h}{e_\ell} = \frac{y_\ell - L_\ell - w}{(1-\phi)\frac{a_\ell}{a_h} y_h - w}. \tag{A-30}$$

Feasibility always binds, so combining (A-30) with (A-22) one obtains

$$e_h = \frac{\eta}{\nu(\kappa_\ell - L_\ell)\left(\frac{(1-\phi)\frac{a_\ell}{a_h} y_h - w}{y_\ell - L_\ell - w}\right) + (1-\nu)\left(\kappa_h - L_h\right)} \tag{A-31}$$

and the amount of low-ability projects funded,

$$e_\ell = \frac{\eta}{\nu(\kappa_\ell - L_\ell) + (1-\nu)\left(\kappa_h - L_h\right)\left(\frac{y_\ell - L_\ell - w}{(1-\phi)\frac{a_\ell}{a_h} y_h - w}\right)}. \tag{A-32}$$

The remaining arguments of the contract are L_h and L_ℓ. We already argued that the direct effect of $\tilde{L_h}$ in the objective function and feasibility constraint is neutral. Yet, increasing L_h has another indirect effect on the objective function by increasing the quantity of projects e_h and e_ℓ. To see this, notice that $\frac{\partial e_h}{\partial L_h} > 0$ and $\frac{\partial e_\ell}{\partial L_h} > 0$ in expressions (A-31) and (A-32). Since $\frac{\partial c^e}{\partial e_\ell} > 0$ and $\frac{\partial c^e}{\partial e_h} > 0$ in (A-19), $L_h = \phi y_h$, unless (A-21) binds and then L_h is pinned down by (A-21) with equality.

$\tilde{L_\ell}$ is also directly neutral on the objective function and the feasibility constraint. Its indirect effect on the objective function through e_ℓ is positive, as $\frac{\partial e_\ell}{\partial L_\ell} > 0$. However, its indirect effect through e_h depends on the profitability of low-productivity projects. If low-type projects are profitable ($y_\ell - \kappa_\ell > w$) then $\frac{\partial e_h}{\partial L_\ell} > 0$ because $\frac{\partial}{\partial L_\ell} \frac{\kappa_\ell - L_\ell}{y_\ell - w - L_\ell} < 0$ in the denominator of expression (A-31). Hence increasing L_ℓ has an overall positive effect on the objective function, and $L_\ell = \phi y_\ell$. However, if low types are unprofitable, then $\frac{\partial e_h}{\partial L_\ell} < 0$ by the opposite argument. Since high-productivity projects are more beneficial to the objective function than low productivity projects ($\frac{\partial c^e}{\partial e_\ell} < \frac{\partial c^e}{\partial e_h}$), then the negative effect of increasing L_ℓ dominates and L_ℓ is driven to its lowest value, $L_\ell = 0$.

Appendix D Modified Model with Multiple Sectors

Recall the production function for the representative firm in sector j,

$$y_j = A_j k_j^\alpha n_j^\gamma$$

Take first order conditions of sector's j problem

$$p_j \alpha \frac{y_j}{k_j} = r$$

$$p_j \gamma \frac{y_j}{n_j} = w$$

Substitute back in the production function to get

$$p_j = y_j^{\frac{1-\alpha-\gamma}{\alpha+\gamma}} A_j^{\frac{-1}{\alpha+\gamma}} \left(\left(\frac{r}{\alpha}\right)^\alpha \left(\frac{w}{\gamma}\right)^\gamma \right)^{\frac{1}{\alpha+\gamma}} \tag{A-33}$$

Aggregate the first order conditions to get

$$rK = \alpha \sum_j p_j y_j \tag{A-34}$$

$$wN = \gamma \sum_j p_j y_j \tag{A-35}$$

By the assumption of zero profits for the final good producer, $Y = \sum_j p_j y_j$. Substitute in (A-34) and (A-35),

$$rK = \alpha Y \tag{A-36}$$

$$wN = \gamma Y \tag{A-37}$$

These expressions allow us to solve for prices and parameters in equation (A-33),

$$\left(\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{\gamma}\right)^{\gamma}\right)^{\frac{1}{\alpha+\gamma}} = Y\left(K^{\alpha}N^{\gamma}\right)^{-\frac{1}{\alpha+\gamma}} \tag{A-38}$$

Now take first order conditions from the final good producer problem (19) and aggregate to get

$$\left(\sum_j p_j^{-\frac{\theta}{1-\theta}}\right)^{-\frac{1-\theta}{\theta}} = 1 \tag{A-39}$$

Combining (A-33), (A-38) and (A-39) we get expression (20).

Appendix E Calculation of Total Factor Productivity in Data

Total factor productivity is calculated as in standard level accounting exercises. Measured TFP is

$$\text{Measured Productivity} = \frac{(Y/N)^{2/3}}{(K/Y)^{1/3}} \tag{A-40}$$

where $N = hL$ is number of workers adjusted for human capital differences. Data on capital, labor and output is from Penn World Tables (PWT), Mark 7.0. as described in Heston et al. (2011). Data for average human capital, necessary to measure TFP in the data, is taken from Caselli (2005). Capital stocks are calculated using the perpetual inventory method. Capital-output ratios, K/Y, are calculated by dividing the capital stock by its corresponding GDP per capita series.

The investment series is obtained by multiplying GDP per capita (RGDPL) in 2005 international prices by the corresponding investment share (KI). I follow Hall and Jones (1999) in guessing initial capital stocks are $\frac{I_0}{\delta+g_i}$, where I_0 is the level of investment of the first year of the sample and g_i is the geometric average of the growth rate of investment for the first 10 years of the sample. This initial guess is tantamount to assuming countries are originally at a balanced growth path. Since the sample is restricted to countries with more than 20 years of data, the initial guess should not be as important in calculating capital stocks in the last decade.

The initial sample includes countries reporting investment for at least 20 years leading up to 2009. Countries with no data for output per worker in the PWT or human capital in the Caselli database for the years 2004 to 2009 were dropped. Countries that do not report recovery rates are also dropped. The final sample consists of 90 countries.

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