BANK RUNS IN MONETARY ECONOMIES

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Abstract. We combine a Diamond and Dybvig (1983) banking system with a Lagos and Wright (2008) dynamic general equilibrium monetary model. Equilibrium bank-runs are driven by sunpots and shocks to fundamentals. The expected frequency of these shocks affects ex ante bank portfolio decisions: investment is low in unstable economies. The Friedman rule does not eliminate bank-runs. A narrow-banking regime eliminates bank-runs, but at a welfare cost that may be worth paying in unstable regimes. Suspension of withdrawals eliminates bank-runs when they are driven by sunspots, but not when they are driven by fundamentals. When bank-runs are driven by fundamentals, monetary injections to the banking sector replaces a bank-run with an orderly partial default.

1. Introduction

As far as we know, this is the first attempt to build a model of bank runs with in an explicit monetary economy. As you will see, the paper below is incomplete and highly preliminary. We welcome the comments and criticisms of seminar participants at the Banco de la Republica.

2. The environment

Time, denoted t, is discrete and the horizon is infinite, $t = 0, 1, 2, ..., \infty$. Each time period t is divided into three subperiods: the *morning*, *afternoon* and *evening*. There are two permanent types of agents, each of unit measure, which we label *investors* and *workers*. All agents discount flow utility payoffs across periods with an identical subjective discount factor $0 < \beta < 1$.

Investors have preferences defined over three goods: a morning good, $x \in \mathbb{R}$ (transferable utility), an afternoon good $c_1 \geq 0$, and an evening good $c_2 \geq 0$. Investors have identical preferences for the morning good but have state-contingent preferences defined over the afternoon and evening goods. Let $\omega \in \{0, 1\}$ denote an investor's *type*–an idiosyncratic shock realized at the beginning of each afternoon. The expected utility payoff over afternoon and evening consumption is given by $E_\omega [u(c_1 + \omega c_2)]$, where u is an increasing and strictly concave function satisfying $-cu''(c)/u'(c) > 1$.

Let $\{c_1(\omega), c_2(\omega); \omega \in \{0, 1\}\}$ denote a state-contingent afternoon and evening allocation for an investor. Assume that the preference shock ω is an *i.i.d.* random

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variable, where $\pi = \Pr[\omega = 0]$. Then an investor's expected utility (flow) payoff can be written as,

$$x + \pi u\left[c_1(0)\right] + (1-\pi)u\left[c_1(1) + c_2(1)\right] \tag{2.1}$$

Assume that π also measures the known fraction of investors who want afternoon consumption only (there is no aggregate uncertainty).

Investors produce new capital goods in the morning. An investment of k units of capital yields Rk units of nonstorable output in the evening. We assume that $R > 1$ and that capital depreciates fully at the end of the evening. An investor bears the utility cost $-k$ (measured in morning output) associated with an investment level k. An investment undertaken in the morning can be interrupted in the afternoon, in which case k units of investment yields λk units of afternoon output ($0 < \lambda < 1 < R$) and zero units of evening output.[1]

Workers have linear preferences defined over the morning good $x \in \mathbb{R}$ (transferable utility) and produce nonstorable output $y \geq 0$ in the afternoon. The utility cost associated with producing y units of output is given by y.

An efficient allocation has the following properties. First, since $\lambda < 1$, it is never efficient to scrap investment. Second, since type $\omega = 0$ investors do not value consumption in the evening, efficiency dictates $c_2^*(0) = 0$. Third, since type $\omega = 1$ investors view consumption in the afternoon and evening as perfect substitutes, $R > 1$ implies that it is never efficient to satisfy their consumption in the afternoon: $c_1^*(1) = 0$. An efficient allocation will therefore satisfy the resource constraints $\pi c_1(0) = y$ and $(1-\pi)c_2(1) = Rk$. Consider the following welfare problem:

$$\max_{x,y,k}\left\{-x - k + \pi u\left(\frac{y}{\pi}\right) + (1-\pi)u\left(\frac{Rk}{1-\pi}\right) : x - y \geq v\right\}$$

where v denotes the minimum utility delivered to workers necessary to induce their participation. For simplicity, assume $v = 0$ so that $x = y$. In this case, the optimal allocation is completely characterized by a pair (y^*, k^*) satisfying,

$$u'(y^*/\pi) = 1 \tag{2.2}$$

$$Ru'\left[Rk^*/(1-\pi)\right] = 1 \tag{2.3}$$

3. A monetary economy

Assume that investors cannot commit to any promises made to workers, so that workers must be paid *quid-pro-quo* for output they produce in the afternoon. The lack of commitment implies a demand for an exchange medium, assumed here to take the form of a zero-interest-bearing government debt instrument (money), the total supply of which is denoted M_t at the beginning of date t. Assume that the initial money supply $M_0 > 0$ is owned entirely by workers. New money is created (destroyed) at the beginning of each morning at the constant rate $\mu > \beta$. New money $T_t = [M_t - M_{t-1}]$ is injected (withdrawn) as lump-sum transfers (taxes) bestowed (imposed) on workers.[2]

[1] Later on we should consider replacing this scrapping technology with a market in which the bank liquidates the investment for cash.

[2] Note that we could alternatively have held M fixed and paid interest on money in the afternoon (financed by a real lump-sum tax on workers) and also pay interest on money maturing the next

Trade of money-for-goods is assumed to take place in a sequence of competitive morning and afternoon spot markets, at prices p_t^m and p_t^a, respectively. We anticipate a sequence of spot trades that consist of investors selling their morning production for money, using the cash proceeds to purchase output in the afternoon.

We first consider a monetary equilibrium in the absence of intermediation. Since $\mu > \beta$, an educated guess tells us that investors will enter the morning with zero money balances, accumulated money in the morning, and spend all their money in either the afternoon or evening. Thus, the quantity

$$m_t = p_t^m x \tag{3.1}$$

represents both the nominal value of cash acquired and held by an investor in the morning.

For now, we assume that there is no secondary asset market in the afternoon that would permit patient and impatient investors to trade afternoon output for claims to evening output. In this set up then, the capital investment associated with an impatient investor needs to be scrapped or is otherwise lost.

If the investor turns out to be impatient, he faces the expenditure constraint

$$p_t^a c_1(0) \leq m_t + p_t^a \lambda k \tag{3.2}$$

If the investor turns out to be patient, he faces the expenditure constraints

$$p_t^a c_1(1) \leq m_t \tag{3.3}$$

$$c_2(1) \leq Rk \tag{3.4}$$

Combine (3.1) and (3.2) set to equality to form $c_1(0) = (p_t^m/p_t^a)\,x + \lambda k$. Similarly, (3.1) and (3.3) imply $c_1(1) = (p_t^m/p_t^a)\,x$. Next, set $c_2(1) = Rk$ and define $p_t \equiv (p_t^m/p_t^a)$. An investor's choice problem at the beginning of a period may then be stated as,

$$\max_{x,k} \{-x - k + \pi u\,(p_t x + \lambda k) + (1-\pi) u\,(p_t x + Rk)\} \tag{3.5}$$

The first-order necessary conditions characterizing an investor's optimal portfolio are given by,

$$\pi u'\,(p_t x + \lambda k) + (1-\pi) u'\,(p_t x + Rk) = 1/p_t \tag{3.6}$$

$$-\pi \lambda u'\,(p_t x + \lambda k) + R(1-\pi) u'\,(p_t x + Rk) = 1 \tag{3.7}$$

Consider now a worker who enters the morning with m_{t-1} units of money, supplemented with the transfer T_t. For every unit of output a worker sells in the afternoon, he receives p_t^a units of money, which he then sells for $1/p_{t+1}^m$ units of the morning good in the following period. Since his preferences in the afternoon and the following morning are linear, the following condition has to hold:

$$p_t = \beta / \Pi_{t+1} \tag{3.8}$$

where $\Pi_{t+1} = p_{t+1}^m / p_t^m$.

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morning (financed by a lump-sum tax on investors and workers). I think it would be possible in this set-up to enhance the rate of return on money heading into the afternoon to the point that it equals R. This would mean equating afternoon and evening consumption for investors AND eliminate bank-runs. But of course, equating consumption like this is not optimal. So there's potentially a nice trade-off here. Inefficient risk-sharing (weird, because consumption is equated) eliminates bank run equilibria.

In a stationary monetary equilibrium, all nominal prices grow at the same rate μ as the aggregate stock of money. Thus, $\Pi_{t+1} = \mu$ and so from (3.8) $p_t = \beta/\mu$. Furthermore, market clearing in the afternoon requires that $\pi[c_1(0) - \lambda k] + (1-\pi)\, c_1(1) = p_t x = y$ which implies that $x = y\mu/\beta$. Use these expressions to rewrite equations (3.6) and (3.7) as follows:

$$\pi u'\left(y + \lambda k\right) = \frac{1}{R+\lambda}\left(\frac{R\mu}{\beta} - 1\right) \tag{3.9}$$

$$R(1-\pi)u'\left(y + Rk\right) = 1 + \frac{\lambda}{R+\lambda}\left(\frac{R\mu}{\beta} - 1\right) \tag{3.10}$$

Definition 1. *A stationary monetary equilibrium is (y,k) that satisfy (3.9) and (3.10).*

4. Market for projects in the afternoon

Suppose that in the afternoon market investors can trade (unmatured) projects for money. Let p_t^k be the afternoon price of a unit of invested capital that matures in the evening. Impatient investors will want to sell all their investments in exchange for money, which they can then use to buy consumption goods from workers. We anticipate (and later verify) that impatient investor do not scrap any of their investments and that patient investors set $c_2(1) = 0$.

An impatient investor faces the following constraint in the afternoon:

$$p_t^a c_1(0) \leq m_t + p_t^k k \tag{4.1}$$

Let κ be the amount of capital bought by a patient investor in the afternoon. The patient investor faces the constraints:

$$p_t^k \kappa \leq m_t \tag{4.2}$$

$$c_2(1) \leq R(k+\kappa) \tag{4.3}$$

Note that we still have $m_t = p_t^m x$ in the morning. Recall $p_t = p_t^m/p_t^a$ and define $\rho_t = p_t^m/p_t^k$. Given that expenditure constraints are satisfied with equality, we obtain

$$c_1(0) = p_t(x + k/\rho_t) \tag{4.4}$$

$$c_2(1) = R(k + \rho_t x) \tag{4.5}$$

The problem of an investor at the beginning of the morning is

$$\max_{x,k}\ \{-x - k + \pi u[p_t(x + k/\rho_t)] + (1-\pi)u[R(k+\rho_t x)]\}$$

The first-order conditions imply:

$$-1 + \pi p_t u'\left[c_1(0)\right] + (1-\pi)R\rho_t u'\left[c_1(1)\right] = 0 \tag{4.6}$$

$$-1 + \pi(p_t/\rho_t)u'\left[c_1(0)\right] + (1-\pi)Ru'\left[c_1(1)\right] = 0 \tag{4.7}$$

Clearly, $\rho_t = 1$. Since $\lambda < 1$, we verify that an impatient investor would rather sells his unmatured project than scrapping it. We also verify that a patient investors sets $c_1(1) = 0$. To see this note that if he instead buys afternoon consumption in exchange for money, total consumption in the afternoon and evening equals

$(\beta/\mu)x + Rk$, which is less than his consumption in equilibrium, $R(k+x)$, as shown in (4.5).

Given $\rho_t = 1$, the investor's problem can be defined over $z \equiv x + k$. From the worker's problem, in a stationary equilibrium, $p_t = \beta/\mu$. Thus, z solves

$$\pi(\beta/\mu)u'[(\beta/\mu)z] + (1-\pi)Ru'(Rz) = 1. \tag{4.8}$$

Market clearing in the afternoon implies $c_1(0) = y/\pi$ and $(1-\pi)\kappa = \pi k$. Thus, $c_2(1) = Rk/(1-\pi)$. Therefore,

$$y = \pi(\beta/\mu)z \tag{4.9}$$

$$k = (1-\pi)z \tag{4.10}$$

Definition 2. *A stationary monetary equilibrium with a market for projects in the afternoon is (y,k) that satisfy (4.9) and (4.10) where z satisfies (4.8).*

Finally, note that $x = \pi z$.

5. Money and banking

To exploit the gains associated with risk-sharing, we assume that investors coalesce to form a "bank." Investors deposit x units of real money balances and k units of capital. Money is acquired in the morning spot market, $m_t = p_t^m x$. Capital is constructed directly with investor labor. The bank uses its deposits of cash and capital (m_t, k) to issue demandable liabilities.[3] The bank's "demand-deposit liabilities" are redeemable for cash in the afternoon (for the moment, we abstract from bank runs). In the evening, bank liabilities can be spent (redeemed) directly for evening output (the output generated by the maturing investment).

We conjecture (and later verify) that a bank will enter each period with zero money balances. Thus, afternoon consumption is limited by $\pi p_t^a c_1(0) \leq m_t$, a constraint that we anticipate will bind (this will be the case when money is dominated in rate of return). Then, this later constraint and $m_t = p_t^m x$ together imply $c_1(0) = p_t x/\pi$. In the evening, consumption is limited by $(1-\pi)c_2(1) \leq Rk$. Again, this latter constraint will bind because capital depreciates fully at the end of the period. Since all wealth is consumed within the period, the bank carries zero wealth into the future, so that the choice problem is static and can be written as follows,

$$\max_{x,k} \left\{ -x - k + \pi u\left(\frac{p_t x}{\pi}\right) + (1-\pi)u\left(\frac{Rk}{1-\pi}\right) \right\} \tag{5.1}$$

The first-order necessary conditions that characterize the optimal portfolio are given by,

$$p_t u'\left(\frac{p_t x}{\pi}\right) = 1 \tag{5.2}$$

$$Ru'\left(\frac{Rk}{1-\pi}\right) = 1 \tag{5.3}$$

[3] Liabilities are made demandable because an investor's type is private information.

Note that for workers (3.8) still holds. In a stationary no-run banking equilibrium, $p_t = \beta/\mu$. Furthermore, market clearing in the afternoon requires that $\pi c_1(0) = p_t x = y$ which implies that $x = y\mu/\beta$. Use these expressions to rewrite and equations (5.2) and (5.3) as follows:

$$u'(y/\pi) = \mu/\beta \tag{5.4}$$

$$Ru'\left(\frac{Rk}{1-\pi}\right) = 1 \tag{5.5}$$

Note that $k = k^*$ and $y < y^*$ for $\mu > \beta$. Under the Friedman rule, $\mu = \beta$, $y = y^*$.[4]

Definition 3. *A no-run banking equilibrium is (y, k) that satisfy (5.4) and (5.5).*

6. Money and banking: unexpected run allocation

In this section, we derive the allocation that occurs when there is an unexpected run. Furthermore, we assume that the probability that another run occurs in the future is zero. The analysis here is an intermediate step before we derive the sunspot banking equilibrium further below. The analysis of an unexpected run is simplified by the fact that we can take the allocation in the morning market (x, k) as given and equal to the one derived in the no-run banking equilibrium. In contrast, in the sunspot banking equilibrium the value of money x and the optimal choice of k are affected by the probability of a bank run. Thus, to simplify matters, let us first derive the afternoon allocation after an unexpected bank run.

Assume that an unexpected run occurs in the afternoon. Unexpected means that the allocation in the morning market (x, k) is described in Definition 3. We assume that the bank in this case liquidates the capital stock. In that process, a fraction $(1 - \lambda)$ is destroyed and the nominal income from selling the reminder for money is $p_t^a \lambda k$. Thus, the nominal quantity of money that the bank has is

$$m_t + p_t^a \lambda k$$

We assume a sequential service rule. In the no-run equilibrium, the bank has promised to hand out m_t/π units of money to agents that show up in the afternoon. A sequential service rule means that the bank continues to honor this promise until it runs out of money.[5] Accordingly, it can pay out m_t/π units of money to $\tilde{\pi}$ agents, where $\tilde{\pi}$ satisfies

$$\tilde{\pi} = \frac{m_t + p_t^a \lambda k}{m_t/\pi} = \pi \left[1 + \left(p_t^a/m_t\right)\lambda k\right] = \pi \left[1 + \left(\frac{p_t^a/p_t^m}{m_t/p_t^m}\right)\lambda k\right] = \pi \left[1 + \frac{\lambda k}{p_t x}\right] \tag{6.1}$$

These $\tilde{\pi}$ agents face the budget constraint

$$p_t^a \tilde{c}_1 \leq m_t/\pi$$

[4]In order to show that a non-run equilibrium exists, we have to show that an investor is willing to work in the CM in order to acquire m units of money and k capital and then deposit at the bank in exchange for the contract specified above. The alternative is self-insurance. That is, the investor can work in order to produce k^A capital and acquire m^A units of money and hold on to it. The utility of doing so depends of what we assume about the market structure.

[5]The bank has a contractual obligation to pay out m_t/π to every agent that shows up in the afternoon. So any attempt to pay out less than m_t/π while the bank has still some money in its vault results in the immediate execution of the bank manager. If the bank manager stops paying out money because the vault is empty, he is celebrated.

Accordingly, production in the afternoon is

$$y = \tilde{\pi}\tilde{c}_1 - \lambda k = \tilde{\pi}\left(m_t/p_t^a\right)/\pi - \lambda k = \tilde{\pi} p_t x/\pi - \lambda k$$

Using (6.1) to replace $\tilde{\pi}$ and the first-order condition of the sellers $p_t = \beta/\mu$, we can write this expression as follows

$$x = y\mu/\beta$$

Thus, sellers work the same hours as in the no-run equilibrium since y solves (5.4). The intuition for this result is that the sellers' linear production technology allows for no price level effects in the afternoon. Note also $\tilde{c}_1 = c_1(0)$; that is consumption for the $\tilde{\pi}$ agents that are able to redeem their deposits is equal to the consumption of impatient agents in the no-run equilibrium. The intuition for this finding is again that there are no price effects of a run in the afternoon because of the linear production technology of the sellers. Finally, such an run equilibrium exists since running yields expected utility $\tilde{\pi}u(\tilde{c}_1) + (1-\tilde{\pi})u(0)$ while non-running yields $u(0)$.

7. Sunspot banking equilibrium

Suppose sunspots determine whether there is a run or not. Denote θ the probability that in a period no run takes place. The potential of runs will affect the ex-ante choice of capital and money. The bank takes this into account when deriving the optimal allocation (x, k) as follows

$$\max_{x,k}\left\{-x - k + \theta\left[\pi u\left(\frac{p_t x}{\pi}\right) + (1-\pi)u\left(\frac{Rk}{1-\pi}\right)\right] + (1-\theta)\left[\tilde{\pi}u\left(\frac{p_t x}{\pi}\right) + (1-\tilde{\pi})u(0)\right]\right\}$$

The first-order necessary conditions that characterize the optimal portfolio are given by[6]

$$\theta p_t u'\left(\frac{p_t x}{\pi}\right) + (1-\theta)(\tilde{\pi}/\pi)p_t u'\left(\frac{p_t x}{\pi}\right) = 1 \tag{7.1}$$

$$\theta R u'\left(\frac{Rk}{1-\pi}\right) = 1 \tag{7.2}$$

Note that for workers (3.8) still holds. In a stationary sunspot equilibrium, then $p_t = \beta/\mu$. Thus, we can write (7.1) and (7.2) as follows:

$$\left[\theta + (1-\theta)(\tilde{\pi}/\pi)\right]u'\left[\frac{(\beta/\mu)x}{\pi}\right] = \mu/\beta \tag{7.3}$$

$$\theta R u'\left(\frac{Rk}{1-\pi}\right) = 1 \tag{7.4}$$

Definition 4. *A sunspot banking equilibrium is (x, k) that satisfy (7.3) and (7.4).*[7]

There are many interesting results. First, note that optimal capital stock $k(\theta)$ is independent of x since it is determined by (7.4). Furthermore, $k(\theta)$ is increasing in θ since

$$\frac{dk}{d\theta} = \frac{k}{\theta\Omega} > 0$$

[6] Here we assume that the bank does not take into account how its choices of x and k affect the probability $\tilde{\pi}$.

[7] Here, I deviate from the previous definitions by defining the equilibrium in terms of (x, k). We should switch to define all equilibria in terms of (x, k).

where $\Omega = \frac{-cu''(c)}{u'(c)}$ is relative risk aversion.

Second, if $\lambda = 0$, then $\tilde{\pi}/\pi = 1$ and the first-condition (7.3) yields $u'\left[\frac{(\beta/\mu)x}{\pi}\right] = \mu/\beta$ which is equal to (5.4) since market clearing in the afternoon implies $y = (\beta/\mu)\,x$. Note that

$$\frac{dx}{d\,(\beta/\mu)} = \frac{x\,(1-\Omega)}{\Omega}$$

Thus, the effect of inflation on the real value of money that is deposited in the bank depends on relative risk aversion and can be positive or negative. Since this is the same expression as in the banking equilibrium, the result also applies there.

Third, if $\lambda > 0$, then $\tilde{\pi}/\pi > 1$ and k and x interact as follows

$$\left\{\theta + (1-\theta)\left[1 + \frac{\lambda k}{(\beta/\mu)\,x}\right]\right\} u'\left[\frac{(\beta/\mu)\,x}{\pi}\right] = \mu/\beta \tag{7.5}$$

$$Ru'\left(\frac{Rk}{1-\pi}\right) = 1/\theta \tag{7.6}$$

Suppose R increases. Then, k increases. A larger k then implies that x increases: money and capital are complements. The same is true for an increase in θ.

8. Sunspot banking equilibrium (refinement)

In the previous formulation, we assumed that the bank took $\tilde{\pi}$, the measure of investors that could be serviced in the event of a run, parameterically. Consider now the case when the bank internalizes the effects of its decisions on this measure. To simplify exposition define $\Delta(x) = u(p_t x/\pi) - u(0) > 0$. The problem of the bank can be written as

$$\max_{x,k}\left\{-x - k + \theta\left[\pi u\left(\frac{p_t x}{\pi}\right) + (1-\pi)u\left(\frac{Rk}{1-\pi}\right)\right] + (1-\theta)\,\pi\left[1 + \frac{\lambda k}{p_t x}\right]\Delta(x)\right\}$$

where we dropped the constant term $(1-\theta)u(0)$ from the objective. After imposing $p_t = \beta/\mu$, the first order conditions imply

$$\{\theta(\beta/\mu) + (1-\theta)\,[1 + (\lambda k/x)]\}\,u'\left(\frac{(\beta/\mu)x}{\pi}\right) - (1-\theta)\pi\mu\lambda k/(\beta x)[\Delta(x)/x] = 1$$

$$\theta Ru'\left(\frac{Rk}{1-\pi}\right) + (1-\theta)(\pi\mu\lambda/\beta)[\Delta(x)/x] = 1$$

Clearly, from the second condition, the bank invests more capital relative to the case when it takes $\tilde{\pi}$ parameterically. The intuition is simple: more capital provides better insurance in case of a run, as more investors get serviced. Similarly, the demand for money is lower, so that its real value x drops.

9. Government intervention

Government intervention: lender of last resort. The benefits of providing liquidity in the afternoon.

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