A Macro CGE Model for the Colombian Economy

Banco de la República’s Internal Seminar

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Motivation

- Macro CGE models acknowledge the links between National Accounts and *Balance of Payments* and *Fiscal Accounts*.
- Allow for Taxation and Sectoral analyses.
- CGEM are NOT intended for Policy Recommendations but are mostly used to present the economy’s outcomes after assessing different *alternative* scenarios.
### Social Accounting Matrix:

<table>
<thead>
<tr>
<th>Factors</th>
<th>Production and products</th>
<th>Distribution</th>
<th>Agents</th>
<th>Rents taxes and transfers</th>
<th>Bal.</th>
<th>TOT.</th>
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**Notes:**
- The matrix presents the flow of economic activities and transactions among different agents and sectors.
- Each entry represents a flow or stock variable in the social accounting matrix, with columns indicating the type of economic activity and rows indicating the agents or sectors involved.
- The values represent the magnitude of these flows, typically in monetary terms.
The Model: Supply Side

Production factors, Indirect taxes, Intermediate consumption and Imports are combined to create total supply of the representative good (Activity).

This process involves solving three different cost minimization problems:

▶ Factor Demand Problem.
▶ GDP - Intermediate Consumption Problem.
▶ Output - Imports Problem.

Firm also maximizes its revenue by optimally solving:

▶ FC and IC Distribution Problem.
▶ FC components Distribution Problem (link to demand side).
Factor Demand Block
Three factors combined + Production Taxes = Value Added.

Firms solve the Factors Demand Problem (minimizes expenditure subject to production):

\[
\min_{\{L, K, Z\}} p_L L + p_K K + p_Z Z,
\]

s.t.

\[
FAC = \theta_F \left( \pi_L L^{\frac{\sigma_F - 1}{\sigma_F}} + \pi_K K^{\frac{\sigma_F - 1}{\sigma_F}} + \pi_Z Z^{\frac{\sigma_F - 1}{\sigma_F}} \right)^{\frac{\sigma_F}{\sigma_F - 1}}
\]

Elasticity of substitution among factors satisfies \( \sigma_F > 0 \).
From the FOCs we derive the optimal demand of factors:

\[ L = \left( \theta_F \pi_L \frac{p_F}{p_L} \right)^{\sigma_F} \frac{FAC}{\theta_F}, \quad K = \left( \theta_F \pi_K \frac{p_F}{p_K} \right)^{\sigma_F} \frac{FAC}{\theta_F}, \]

\[ Z = \left( \theta_F \pi_Z \frac{p_F}{p_Z} \right)^{\sigma_F} \frac{FAC}{\theta_F}, \]

where the aggregated price of factors \( p_F \) is expressed as

\[ p_F = \frac{1}{\theta_F} \left( \pi_L^{\sigma_F} \frac{1 - \sigma_F}{p_L} \pi_K^{\sigma_F} \frac{1 - \sigma_F}{p_K} + \pi_Z^{\sigma_F} \frac{1 - \sigma_F}{p_Z} \right)^{\frac{1}{1 - \sigma_F}} \]
The Model: Indirect taxes and GDP

- Added value, $AV$, is completed once indirect production taxes are acknowledged (nominal terms):

$$p_{AV} AV = p_F FAC + TX_{va},$$

where tax revenue in production ($TX_{va}$) is given by

$$TX_{va} = tx_{va} p_F FAC.$$

- GDP ($Y$) supply is obtained by adding up $AV$, indirect (net) taxes over products ($TX_{yy}$) and import tariffs ($TR_{ff}$):

$$p_Y Y = p_{AV} AV + TX_{yy} + TR_{ff},$$

where indirect product taxes and tariffs are given by

$$TX_{yy} = tx_{yy} p_{AV} AV,$$

and

$$TR_{ff} = tr_{ff} p_{MM}.$$
Domestic Supply Block
The Model: Domestic Output Problem I

- GDP and IC combined yield total domestic supply of the representative good.
- The firm solves for optimal combination of Y and IC in the domestic output’s second-level cost minimization problem:

\[
\min_{\{Y, IC\}} p_Y Y + p_{IC} ICD
\]

s.t.

\[
OUT = \theta_O \left( \pi_Y Y^{\frac{\sigma_O - 1}{\sigma_O}} + \pi_{ICD} ICD^{\frac{\sigma_O - 1}{\sigma_O}} \right)^{\frac{\sigma_O}{\sigma_O - 1}}
\]

- Again, elasticity of substitution between Y and IC satisfy \(\sigma_O > 0\), however, these goods are more complementary than substitutes (0 < \(\sigma_O < 1\)).
The Model: Domestic Output Problem II

- From FOCs, optimal GDP and Intermediate Consumption demands are

\[ Y = \left( \theta_O \pi_Y \frac{p_O}{p_Y} \right)^{\sigma_o} \frac{OUT}{\theta_O} \text{, and } ICD = \left( \theta_O \pi_{ICD} \frac{p_O}{p_{IC}} \right)^{\sigma_o} \frac{OUT}{\theta_O}, \]

where the aggregated price of domestic output \( p_O \) is

\[ p_O = \frac{1}{\theta_O} \left( \pi^{\sigma_o} p_Y^{1-\sigma_o} + \pi^{\sigma_o}_{ICD} p_{IC}^{1-\sigma_o} \right)^{\frac{1}{1-\sigma_o}} \]

- With Y’s Demand and Supply equations, one can solve for the price of GDP, \( p_Y \):

\[ p_Y = \left[ \frac{p_{AV} AV + TX_{yy} + TR_{ff}}{\left( \theta_O \pi_Y p_O \right)^{\sigma_o} \frac{OUT}{\theta_O}} \right]^{\frac{1}{1-\sigma_o}} \]
Total Supply Block
The Model: Activity (total supply) I

- When The firm when it solves the first-level cost minimization problem given by:

$$\min_{\{OUT, M\}} \quad p_O \cdot OUT + p_M \cdot M$$

s.t.

$$ACT = \theta_A \left( \pi_O \cdot OUT^{\frac{\sigma_A - 1}{\sigma_A}} + \pi_M \cdot M^{\frac{\sigma_A - 1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A - 1}},$$

- Elasticity of substitution between OUT and M is $\sigma_A > 0$. 
The Model: Activity (total supply) II

- From FOCs, optimal domestic output and imports demands are, respectively

\[ OUT = \left( \theta_A \pi_O \frac{p_A}{p_O} \right)^{\sigma_A} \frac{ACT}{\theta_A}, \text{ and } M = \left( \theta_A \pi_M \frac{p_A}{p_M} \right)^{\sigma_A} \frac{ACT}{\theta_A} \]

with aggregated price of ACT, \( p_A \) given by

\[ p_A = \frac{1}{\theta_A} \left( \pi_O^{\sigma_A} p_O^{1-\sigma_A} + \pi_M^{\sigma_A} p_M^{1-\sigma_A} \right)^{\frac{1}{1-\sigma_A}} \]

- OUT demand and supply equations are solved for price of OUT, \( p_O \):

\[ p_O = \theta_A \pi_O \left[ \frac{ACT}{\theta_A} \right]^{\frac{1}{\sigma_A}} p_A \]

\[ \theta_O \left( \pi_Y Y^{\sigma_O-1} + \pi_{ICD} ICD^{\sigma_O-1} \right)^{\frac{\sigma_O}{\sigma_O-1}} \]
The Model: Activity (total supply) III

Additional considerations on ACT formation:

- The clearing market condition assures that

\[ p_{ACT} = p_{OUT} + p_{M}M. \]

- RW provides all demand for imports inelastically at the international price \( p^{*}_M \), and therefore

\[ p_{M} = \bar{e}p^{*}_M \]

where \( \bar{e} \) is the nominal exchange rate.
The Model: Supply Distribution I

Total supply, ACT, is distributed between intermediate and final consumption.

- The firm determines distribution of ACT between intermediate (IC) and final consumption (FC) by maximizing revenue from sales:

\[
\max_{\{IC, FC\}} p_{IC} ICS + p_{FC} FC
\]

s.t. a CET technology of distribution

\[
ACT = \theta_{AD} \left( \pi_{ICS} ICS \frac{\tau_A - 1}{\tau_A} + \pi_{FC} FC \frac{\tau_A - 1}{\tau_A} \right)^{\frac{\tau_A}{\tau_A - 1}}
\]

- The elasticity of transformation between intermediate and final consumption is \(\tau_A < 0\).
The Model: Supply Distribution II

- From the FOCs, optimal FC and IC supplies are

\[
ICS = \left( \theta_{AD} \pi_{ICS} \frac{p_A}{p_{IC}} \right)^{\tau_A} \frac{ACT}{\theta_{AD}}, \text{ and } FC = \left( \theta_{AD} \pi_{FC} \frac{p_A}{p_{FC}} \right)^{\tau_A} \frac{ACT}{\theta_{AD}}
\]

with aggregated price of activity, \( p_A \), given by

\[
p_A = \frac{1}{\theta_{AD}} \left( \pi_{ICS}^{\tau_A} p_{IC}^{1-\tau_A} + \pi_{FC}^{\tau_A} p_{FC}^{1-\tau_A} \right)^{1/(1-\tau_A)}
\]

- However, \( p_A \) is determined through the market clearing condition:

\[
p_A \cdot ACT = p_{IC} \cdot ICS + p_{FC} \cdot FC
\]

- With IC supply and demand equations, we have

\[
p_{IC} = \left[ \left( \theta_{O} \pi_{ICD} p_{O} \right)^{\sigma_o} \frac{OUT}{\theta_{O}} \right]^{\frac{1}{\sigma_o-\tau_A}} \left[ \left( \theta_{AD} \pi_{ICS} p_A \right)^{\tau_A} \frac{ACT}{\theta_{AD}} \right]
\]
The Model: Final Consumption Distribution I

- The firm determines distribution of FC supply between Consumption (C), Investment (I), Government Expenditure (G) and Exports (X). X are classified between traditional, $\bar{X}_T$ (which are assumed as exogenous); and non-traditional, $X_N$.

- Firm maximizes its revenue from selling final consumption:

$$\max_{\{C,I,G,X_N\}} p_C C + p_I I + p_G G + p_{\bar{X}_T} \bar{X}_T + p_{X_N} X_N$$

s.t.

$$FC = \bar{X}_T + \theta_{FC} \left( \pi_C C^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_I I^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_G G^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_{X_N} X_N^{\frac{\tau_{FC}-1}{\tau_{FC}}} \right)^{\frac{\tau_{FC}}{\tau_{FC}-1}}$$

- Elasticity of transformation between types of final consumption satisfies $\tau_{FC} < 0$. 
From FOCs we derive the optimal supply of each of the FC components:

\[
Z = \left[ \theta_{FC} \pi_Z \frac{p_{FC} FC - p_{XT} \bar{X}_T}{p_Z (FC - \bar{X}_T)} \right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}}, \text{ with } Z \in \{C, G, I, X_N\}
\]

With \( p_{FC} \) given by

\[
p_{FC} FC = \frac{1}{\theta_{FC}} \left( \sum \pi_Z^{\tau_{FC}} p_Z^{1-\tau_{FC}} \right)^{\frac{1}{1-\tau_{FC}}} (FC - \bar{X}_T) + p_{XT} \bar{X}_T
\]

FC supply or FC demand equations can be placed in the latter expression in order to solve for \( p_{FC} \).
Factor Remuneration Block
Factor supply is assumed to be exogenous (completely inelastic): $\bar{L}$, $\bar{K}$, and $\bar{Z}$.

Then it holds:

$$p_L \bar{L} = REM = REM_{HH} + F_L$$
$$p_K \bar{K} = EBE = EBE_{HH} + EBE_{FR} + EBE_{GV}$$
$$p_Z \bar{Z} = MIX = MIX_{HH}$$

Given the supplies of factors, and the demands (AV production), we derive the factor prices:

$$p_W = \theta_F \pi_W \left( \frac{FAC}{\theta_F \bar{W}} \right)^{1 / \sigma_F} p_F \quad \text{for } \ W \in \{L, K, Z\}.$$
The Model: Income Distribution II

Distribution of factor remunerations and rents in the model are paid according to fixed coefficients:

- **Factor remunerations:**
  - \( REM_{HH} = \pi^{REM}_{HH} REM \) and \( F_L = \pi^{REM}_{RW} REM \).
  - \( EBE_{HH} = \pi^{EBE}_{HH} EBE \), \( EBE_{FR} = \pi^{EBE}_{FR} EBE \) and \( EBE_{GV} = \pi^{EBE}_{GV} EBE \).
  - \( MIX = MIX_{HH} \).

- **Rents:**
  - **Payments:** \( R^{HH} = \pi^{HH}_R EBE_{HH} \), \( R^{FR} = \pi^{FR}_R EBE_{FR} \) and \( R^{GV} = \pi^{GV}_R EBE_{GV} \).
  - \( R = R^{HH} + R^{FR} + R^{GV} = R_{HH} + R_{FR} + R_{GV} + F_K \).
  - **Recipients:** \( R_{HH} = \pi^{R}_{HH} R \), \( R_{FR} = \pi^{R}_{FR} R \), \( R_{GV} = \pi^{R}_{GV} R \), and \( F_K = \pi^{R}_{RW} R \).
The Model: Direct Taxes

- Households’ income: 
  \[ Y_{HH} = REM_{HH} + EBE_{HH} + MIX_{HH} + (R_{HH} - R^{HH}) \].

- Firm’s income: 
  \[ Y_{FR} = EBE_{FR} + (R_{FR} - R^{FR}) \].

- Government’s income: 
  \[ Y_{GV} = EBE_{GV} + (R_{GV} - R^{GV}) \].

Assuming no tax evasion and perfect fiscal compliance, institutional agents pay direct taxes as a constant fraction of their income:

\[ TX_{hh} = tx_{hh}Y_{HH}, \quad TX_{ac_{FR}} = tx_{ac_{FR}}Y_{FR}, \]

\[ and \quad TX_{ac_{GV}} = tx_{ac_{GV}}Y_{GV}. \]

Total direct taxes are given by

\[ T = TX_{hh} + TX_{ac_{HH}} + TX_{ac_{GV}}. \]
The Model: Transfers I

There are four types of transfers: social contributions (SC), social benefits (SB), current transfers (CT), and product transfers (PT).

- We assume exogenous payments of social contributions $SC^{HH}$ by HH, which is distributed FR and GV:

  $$SC^{HH}_{FR} = \pi^{SC}_{FR} SC^{HH}, \text{ and } SC^{HH}_{GV} = \pi^{SC}_{GV} SC^{HH}.$$  

- HH receive exogenously assumed social benefits, $SB = SB^{HH}$, from FR and GV:

  $$SB^{FR}_{HH} = \pi^{SB}_{FR} SB^{HH}, \text{ and } SB^{GV}_{HH} = \pi^{SB}_{GV} SB^{HH}.$$  

- FR and RW pay CT exogenously, $CT^{RW} + CT^{FR} = CT$, which is distributed to HH and GV as:

  $$CT^{HH} = \pi^{CT}_{HH} CT, \text{ and } CT^{GV} = \pi^{CT}_{GV} CT.$$
We also assume exogenous product transfers from the Government to households, $\overline{PT}_{HH}$.

Net transfers are then represented by the following equations:

\[
\begin{align*}
NT_{HH} &= -SC_{HH} + SB_{HH} + CT_{HH} + \overline{PT}_{HH}^G \\
NT_{FR} &= SC_{FR} - SB_{HH}^{FR} - \overline{CT}^{FR} \\
NT_{GV} &= SC_{GV}^{HH} - SB_{HH}^{GV} + CT_{GV} - \overline{PT}_{HH}^G \\
NT_{RW} &= -\overline{CT}^{RW}
\end{align*}
\]
Demand Block
The Model: Domestic Demand I

- HH have standard well-behaved preferences (e.g. Cobb-Douglas with savings in utility), which yield final consumption demand and savings as:

\[ C = \alpha \frac{DY_{HH}}{p_C}, \text{ and } S_{HH} = DY_{HH} - p_C C \]

with \( DY_{HH} = Y_{HH} + NT_{HH} - TX_{hh} \). HH’s marginal propensity to consume (MPC) satisfies \( 0 < \alpha < 1 \).

- HH and FR investment form private investment, \( I_{PR} \), as

\[ I_{HH} = \beta I_{PR} \]
\[ I_{FR} = (1 - \beta) I_{PR} \]
\[ I_{PR} = I_{HH} + I_{FR} \]

with \( 0 < \beta < 1 \).
Accordingly, we have FR savings given by

\[ S_{FR} = Y_{FR} + NT_{FR} - TXac_{FR}. \]

Total demand for investment, \( I \), is

\[ I = I_{PR} + I_{GV} \]

which along with investment supply yields the price of investment equation, \( p_I \)

\[ p_I = \theta_{FC} \pi_I \frac{p_{FC} FC - p_{XT} \bar{X}_T}{(FC - \bar{X}_T)} \left[ \frac{FC - \bar{X}_T}{\theta_{FC} (I_{PR} + \bar{I}_{GV})} \right]^{\frac{1}{\tau_{FC}}} \]
The Model: Domestic Demand III

- We assume GV’s expenditure, $G$, and investment, $I_{GV}$, to be exogenous:

$$G = \bar{G}, \text{ and } I_{GV} = \bar{I}_{GV}$$

- GV expenditure price is jointly determined by its supply and demand functions

$$p_G = \theta_{FC} \pi_G \frac{p_{FC}}{\tau_{FC}} FC - p_{XT} \bar{X}_T \left( \frac{FC - \bar{X}_T}{\theta_{FC} \bar{G}} \right)^{\frac{1}{\tau_{FC}}}$$

- Accordingly, GV savings are given by

$$S_{GV} = Y_{GV} + NT_{GV} + Tx + T - TXac_{GV} - p_G \bar{G}$$

with indirect taxes $Tx = TX_{va} + TX_{yy} + TR_{ff}$.
The Model: External Demand

- RW demand for $X_{NT}$ is defined according to:

$$X_N = \left( \theta_{M^*} \pi_{COL} \frac{e \tilde{p}_{M^*}}{p_{XN}} \right)^{\sigma_p^*} \frac{\tilde{M}^*}{\theta_{M^*}}$$

where $\theta_{M^*}$ and $\pi_{COL}$ are scale and Colombian share parameter in the aggregation of RW imports, $\tilde{M}^*$ which are assumed to be exogenous, such as their price, $\tilde{p}_{M^*}$.

- Total exports quantities must satisfy $X = X_N + \tilde{X}_T$, and their price is determined by

$$p_X = \frac{p_{X_T} \tilde{X}_T + p_{X_N} X_N}{X}$$
The Model: External Demand II

- Price of $X_N$, $p_{X_N}$, is determined by its supply and demand equilibrium:

$$p_{X_N} = \begin{cases} \frac{\left(\theta_M^* \pi_{COL} e\bar{p}_{M^*}^*\right)\sigma_M^* \bar{M}^*}{\theta_{M^*}} \left(\theta_{FC} \pi_{X_N} p_{FC} FC - p_{XT} X_T \right) \frac{1}{\sigma_M^* - \tau_{FC}} & \text{if } \theta_{FC} p_{FC} FC - p_{XT} X_T \neq 0 \\ \theta_{FC} \pi_{X_N} p_{FC} FC - p_{XT} X_T \frac{1}{\theta_{FC}} & \text{if } \theta_{FC} p_{FC} FC - p_{XT} X_T = 0 \end{cases}$$

- RW demands $X_T$ at the international price $p_{X_T}^*$, which means that the internal price of $X_T$ is given by

$$p_{X_T} = e p_{X_T}^*$$
The Model: Closure Equations I

We set \( Y_{RW} = F_L + F_K - C\overline{T}^{RW} \).

Private Investment Closure

- Exogenous exchange rate: \( e = \bar{e} \).
- Exogenous \( I_{PR} \): \( I_{PR} = \bar{I}_{PR} \).
- Endogenous \( S_{RW} \):

  \[ -CC = S_{RW} = Y_{RW} + p_M M - p_X X \]

- S-I balance depends on Endogenous \( p_C \) (replacing \( S_{HH} \)):

  \[ p_l \bar{I} = S_{HH} + S_{FR} + S_{GV} + S_{RW} \]

  \[ p_C = \frac{DY_{HH} + S_{FR} + S_{GV} + S_{RW} - p_l \bar{I}}{C} \]
RW Savings Closure

- Exogenous consumption price: $p_C = \bar{p}_C$.
- Exogenous External Savings: $S_{RW} = \bar{S}_{RW}$.
- Endogenous exchange rate, $e$ (derived from the following equation):

$$\bar{S}_{RW} = Y_{RW} + p_M(e)M(e) - p_X(e)X(e)$$

- $I_{PR}$ is determined by the S-I balance:

$$I_{PR} = \frac{S_{HH} + S_{FR} + S_{GV} + \bar{S}_{RW}}{p_I} - \bar{I}_{GV}$$
Parameter Calibration: An example (I)

Using information from the Macro-SAM constructed for the model, we show an example of how share and scale parameters are calibrated. All parameters can be calibrated following the same steps.

- Share parameters: We have that

\[
\pi_K = \pi_L \frac{p_K}{p_L} \left( \frac{K}{L} \right)^{\frac{1}{\sigma_F}}, \quad \pi_Z = \pi_L \frac{p_Z}{p_L} \left( \frac{Z}{L} \right)^{\frac{1}{\sigma_F}}
\]

and

\[
\pi_L + \pi_K + \pi_Z = 1,
\]

which yields

\[
\pi_L = \frac{p_L L^{\frac{1}{\sigma_F}}}{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}
\]

all other parameters can be calibrated analogously.
Parameter Calibration: An example (II)

- Scale parameters: Using the share parameters and FAC, we have

\[ \theta_F = FAC \left( \frac{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}{p_L L + p_K K + p_Z Z} \right)^{\frac{\sigma_F}{\sigma_F - 1}} \]
Model Summary

A grand total of 99 variables:

- 73 endogenous variables.
- 22 exogenous variables.
- 4 closure variables:
  - 2 endogenous variables (depending on which closure we choose).
  - 2 exogenous remaining variables: i) A *nominal anchor*, and ii) a real quantity.
### Endogenous Variables

#### Endogenous Variables List (73)

<table>
<thead>
<tr>
<th>FAC</th>
<th>$p_F$</th>
<th>AV</th>
<th>$TX_{va}$</th>
<th>$p_{AV}$</th>
<th>$TX_{yy}$</th>
<th>$TR_{ff}$</th>
<th>Y</th>
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<tbody>
<tr>
<td>$IC_D$</td>
<td>$p_Y$</td>
<td>OUT M</td>
<td>$p_M$</td>
<td>$X_N$</td>
<td>$ACT$</td>
<td>$p_M$</td>
<td>$IC_S$</td>
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<td>FC</td>
<td>$p_A$</td>
<td>I</td>
<td>$p_I$</td>
<td>$REM_{HH}$</td>
<td>$F_L$</td>
<td>$REM$</td>
<td>EBE</td>
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<td>MIX</td>
<td>$p_L$</td>
<td>$Y_{HH}$</td>
<td>$p_Z$</td>
<td>$R_{HH}$</td>
<td>$R_{FR}$</td>
<td>$EBE_{HH}$</td>
<td>EBE</td>
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<td>$EBE_{GV}$</td>
<td>$R^{HH}$</td>
<td>$Y_{FR}$</td>
<td>$R^{GV}$</td>
<td>$TX_{hh}$</td>
<td>$TX_{ac_{FR}}$</td>
<td>$EBE_{FR}$</td>
<td>$F_K$</td>
</tr>
<tr>
<td>R</td>
<td>$Y_{HH}$</td>
<td>$Y_{FR}$</td>
<td>$Y_{GV}$</td>
<td>$CT$</td>
<td>$CT_{HH}$</td>
<td>$T$</td>
<td>$NT_{HH}$</td>
</tr>
<tr>
<td>$SC^{HH}_{FR}$</td>
<td>$SC^{HH}_{GV}$</td>
<td>$SB_{FR}^{HH}$</td>
<td>$SB_{GV}^{HH}$</td>
<td>$DY_{HH}$</td>
<td>$I_{HH}$</td>
<td>$I_{FR}$</td>
<td>$S_{FR}$</td>
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<tr>
<td>$NT_{FR}$</td>
<td>$NT_{GV}$</td>
<td>C</td>
<td>$S_{HH}$</td>
<td>$X$</td>
<td>$p_X$</td>
<td>$p_{X_N}$</td>
<td>$p_{X_T}$</td>
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<tr>
<td>$p_I$</td>
<td>$S_{GV}$</td>
<td>Tx</td>
<td>$p_G$</td>
<td>$X$</td>
<td>$p_X$</td>
<td>$p_{X_N}$</td>
<td>$p_{X_T}$</td>
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<tr>
<td>$Y_{RW}$</td>
<td>.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
Exogenous Variables: A list (I)

- Factors: $\bar{L}, \bar{K}, \bar{Z} \rightarrow \text{DPI-BR}$
- Total Factor Productivity: $\theta_F \rightarrow \text{DPI-BR}$
- Indirect Taxes Rates: $tx_{va}, tx_{yy}, tr_{ff} \rightarrow \text{Calibrated (ftc)}$
- International Price of Imports: $p^*_M \rightarrow \text{BoP / ToT (Imports Index)}$
- Traditional Export Quantities: $\bar{X}_T \rightarrow \text{BOP}$
- Direct Taxes Rates: $tx_{hh}, tx_{ac}_{FR}, tx_{ac}_{GV} \rightarrow \text{Calibrated (ftc)}$
Exogenous Variables: A list (II)

- HH payments to SC: $SC^{HH} \rightarrow$ Pension Funds Financial Statements.
- SB payments to HH: $SB_{HH}$
- FR payments to CT: $CT^{FR}$
- RW payments to CT: $CT^{RW} \rightarrow$ BOP
- GV payments to PT: $PT^{GV}_{HH}$
Exogenous Variables: A list (III)

- GV Investment: $\bar{I}_{GV} \rightarrow$ DPI-BR
- GV Spending: $\bar{G} \rightarrow$ DPI-BR
- Price of RW Imports: $\bar{p}_{M^*} \rightarrow$ WEO (External Inflation)
- RW Imports Quantities: $\bar{M}^* \rightarrow$ BOP
- Traditional Exports Prices: $p_{X_T} \rightarrow$ BoP / ToT (Exports Index)
Closure Variables

Investment Closure
▶ Nominal Exchange Rate: $\bar{e} \rightarrow \text{BOP}$
▶ Private Investment: $\bar{I}_{PR} \rightarrow \text{DPI-BR}$

Savings Closure
▶ Consumption Good Price: $\bar{p}_C \rightarrow \text{DPI-BR}$
▶ External Savings: $\bar{S}_{RW} \rightarrow \text{BOP}$
Macro CGEM usage: An example

- Using observed information from BOP, National Accounts and other relevant variables, we replicate 2012 economy taking as a starting point 2011 SAM and our model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Observed</th>
<th>Average</th>
<th>CGEM Investment</th>
<th>Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>4.0</td>
<td>4.1</td>
<td>4.2</td>
<td>4.1</td>
</tr>
<tr>
<td>C</td>
<td>4.4</td>
<td>4.0</td>
<td>4.5</td>
<td>3.5</td>
</tr>
<tr>
<td>G</td>
<td>5.7</td>
<td>5.7</td>
<td>5.7</td>
<td>5.7</td>
</tr>
<tr>
<td>I</td>
<td>4.6</td>
<td>4.5</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$I_{PR}$</td>
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<td>4.4</td>
<td>4.9</td>
<td>3.8</td>
</tr>
<tr>
<td>$I_{GV}$</td>
<td>5.3</td>
<td>5.3</td>
<td>5.3</td>
<td>5.3</td>
</tr>
<tr>
<td>X</td>
<td>6.1</td>
<td>5.0</td>
<td>4.6</td>
<td>5.3</td>
</tr>
<tr>
<td>M</td>
<td>8.9</td>
<td>5.4</td>
<td>7.2</td>
<td>3.5</td>
</tr>
<tr>
<td>CAD (%GDP)</td>
<td>3.1</td>
<td>3.6</td>
<td>4.0</td>
<td>3.1</td>
</tr>
</tbody>
</table>
Consistency of the model

Current Account Deficit vs. Rest of the World's Savings (% of GDP)


-5 -4 -3 -2 -1 0 1 2

(% of GDP)

Current Account Deficit

Negative of RWSV

(=Rest of the World's Savings)
What’s Next?

This model can be further extended along the following lines:

- Demand driven economy.
- Assuring BoP matching with the model.
- Multi-sector CGE model.
- Extension of Fiscal Block.
- Money in CGEM (anchor to Monetary accounts).
THE END

Thank You.