# A Macro CGE Model for the Colombian Economy Banco de la República's Internal Seminar 

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## Outline

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## Motivation

- Macro CGE models acknowledge the links between National Accounts and Balance of Payments and Fiscal Accounts.
- Allow for Taxation and Sectoral analyses.
- CGEM are NOT intended for Policy Recommendations but are mostly used to present the economy's outcomes after assessing different alternative scenarios.


## The Economy



## Social Accounting Matrix:

|  |  |  | Factors |  |  | Production and products |  |  |  |  |  |  | Distribution |  |  |  |  |  |  |  |  |  |  |  | Agents |  |  |  | Rents taxes and transfers |  |  |  |  |  |  |  |  |  | Bal. | тот. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | F |  |  | P |  |  |  |  |  |  | D |  |  |  |  |  |  |  |  |  |  |  | A |  |  |  | T |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  | L | K | Z | FAC: | AV: | Y | !ICD | OUT | M | ACT | ICS | FC | C | 1 | Ihh | Ifr I l | g 1 | Ipr | G | X | Xt | Xn | HH: | FR | GV: | RW | R | TXv | TXy\% | TRf | Thh | Tac: | SC: | SB: | CT | PT | B ${ }^{\text {SI }}$ |  |
|  | LABOUR | L |  |  |  | 196 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 196 |
| F | CAPITAL | $K$ |  |  |  | 223 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 223 |
|  | MIXED | z |  |  |  | 135 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 135 |
|  | PRODUCTION | FAC |  |  |  |  | 554 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 554 |
|  | ADDED VALUE | AV |  |  |  |  |  | 567 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 567 |
|  | GROSS DOMESTIC PRODUCT | Y |  |  |  |  |  |  |  | 622 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 622 |
| P | DEMAND OF INT. CONSUMP. | ICD |  |  |  |  |  |  |  | 479 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 479 |
|  | OUTPUT | OUT |  |  |  |  |  |  |  |  |  | 1100 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1100 |
|  | IMPORTS | M |  |  |  |  |  |  |  |  |  | 123 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 123 |
|  | ACTIVITY | ACT |  |  |  |  |  |  |  |  |  |  | 479 | 745 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1223 |
|  | SUPPLY OF INT. CONSUMP. | ICS |  |  |  |  |  |  | 479 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 479 |
|  | FINAL CONSUMPTION | FC |  |  |  |  |  |  |  |  |  |  |  |  | 423 | 148 |  |  |  |  | 56 |  | 81 | 37 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 745 |
|  | PRIVATE CONSUMPTION | C |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 423! |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 423 |
|  | INVESTMENT | 1 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 35 | 92 | 21 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 148 |
|  | HOUSEHOLDS INVESTMENT | inh |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 35 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 35 |
|  | FIRMS INVESTMENT | Ifr |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 92 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 92 |
| D | GOVERNMENT INVESTMENT | $\lg v$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 21 | 21 |
|  | PRIVATE INVESTMENT | Ipr |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 126 | 126 |
|  | GOVERNMENT EXPENDITURE | G |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 56 |  |  |  |  |  |  |  |  |  |  |  |  | 56 |
|  | EXPORTS | X |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 118 |  |  |  |  |  |  |  |  |  |  |  | 118 |
|  | TRADITIONAL EXPORTS | Xt |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 81 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 81 |
|  | NON-TRADITIONAL EXPORTS | Xn |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 37 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 37 |
|  | HOUSEHOLDS | HH | 198 | 26 | 135 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 59 |  |  |  |  |  |  | 51 | 25 | 43 |  | 538 |
|  | FIRMS | FR |  | 191 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 46 |  |  |  |  |  | 18 |  |  |  |  | 254 |
| A | GOVERNMENT | GV |  | 6 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 25 | 13 | 49 | 5 | 7 | 31 | 36 |  | 3 |  |  | 176 |
|  | REST OF THE WORLD | RW | -2 |  |  |  |  |  |  |  | 123 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 27 |  |  |  |  |  |  |  |  |  |  | 148 |
|  | RENTS | R |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 11 | 130 | 16 |  |  |  |  |  |  |  |  |  |  |  |  | 157 |
|  | PRODUCTION TAXES | TXva |  |  |  |  | 13 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 13 |
|  | PRODUCT TAXES | TXy |  |  |  |  |  | 49 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 49 |
|  | IMPORT TARIFFS | TRff |  |  |  |  |  | 5 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 5 |
|  | DIRECT TAXES TO HH | Thh |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 7. |
| T | DIR TAXES TO FR \& GV | Tac |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 31 | 0 |  |  |  |  |  |  |  |  |  |  |  |  | 31 |
|  | SOCIAL CONTRIBUTIONS | SC |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 53 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 53 |
|  | SOCIAL BENEFITS | SB |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 5 | 46 |  |  |  |  |  |  |  |  |  |  |  |  | 51 |
|  | CURRENT TRANSFERS | CT |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 18 |  | 11 |  |  |  |  |  |  |  |  |  |  |  | 28 |
|  | PRODUCT TRANSFERS | PT |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 43 |  |  |  |  |  |  |  |  |  |  |  |  | 43 |
| B | SAVINGS-INVESTMENT BAL. | SI |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | ! |  |  |  | 43 | 70 | 15 | 20 |  |  |  |  |  |  |  |  |  |  |  | 148 |

TOTAL


## The Model: Supply Side

Production factors, Indirect taxes, Intermediate consumption and Imports are combined to create total supply of the representative good (Activity).

This process involves solving three different cost minimization problems:

- Factor Demand Problem.
- GDP - Intermediate Consumption Problem.
- Output - Imports Problem.

Firm also maximizes its revenue by optimally solving:

- FC and IC Distribution Problem.
- FC components Distribution Problem (link to demand side).


## Factor Demand Block



## The Model: Factor Demand Problem I

- Three factors combined + Production Taxes = Value Added.
- Firms solve the Factors Demand Problem (minimizes expenditure subject to production):

$$
\min _{\{L, K, Z\}} p_{L} L+p_{K} K+p_{Z} Z
$$

s.t.

$$
F A C=\theta_{F}\left(\pi_{L} L^{\frac{\sigma_{F}-1}{\sigma_{F}}}+\pi_{K} K^{\frac{\sigma_{F}-1}{\sigma_{F}}}+\pi_{Z} Z^{\frac{\sigma_{F}-1}{\sigma_{F}}}\right)^{\frac{\sigma_{F}}{\sigma_{F}-1}}
$$

- Elasticity of substitution among factors satisfies $\sigma_{F}>0$.


## The Model: Factor Demand Problem II

From the FOCs we derive the optimal demand of factors:

$$
\begin{gathered}
L=\left(\theta_{F} \pi_{L} \frac{p_{F}}{p_{L}}\right)^{\sigma_{F}} \frac{F A C}{\theta_{F}}, K=\left(\theta_{F} \pi_{K} \frac{p_{F}}{p_{K}}\right)^{\sigma_{F}} \frac{F A C}{\theta_{F}}, \\
Z=\left(\theta_{F} \pi_{Z} \frac{p_{F}}{p_{Z}}\right)^{\sigma_{F}} \frac{F A C}{\theta_{F}}
\end{gathered}
$$

where the aggregated price of factors $p_{F}$ is expressed as

$$
p_{F}=\frac{1}{\theta_{F}}\left(\pi_{L}^{\sigma_{F}} p_{L}^{1-\sigma_{F}}+\pi_{K}^{\sigma_{F}} p_{K}^{1-\sigma_{F}}+\pi_{Z}^{\sigma_{F}} p_{Z}^{1-\sigma_{F}}\right)^{\frac{1}{1-\sigma_{F}}}
$$

## The Model: Indirect taxes and GDP

- Added value, $A V$, is completed once indirect production taxes are acknowledged (nominal terms):

$$
p_{A V} A V=p_{F} F A C+T X_{v a}
$$

where tax revenue in production $\left(T X_{v a}\right)$ is given by

$$
T X_{v a}=t x_{v a} p_{F} F A C
$$

- GDP $(Y)$ supply is obtained by adding up AV, indirect (net) taxes over products ( $T X_{y y}$ ) and import tariffs ( $T R_{f f}$ ):

$$
p_{Y} Y=p_{A V} A V+T X_{y y}+T R_{f f},
$$

where indirect product taxes and tariffs are given by

$$
T X_{y y}=t x_{y y} p_{A V} A V, \text { and } T R_{f f}=\operatorname{tr}_{f f} p_{M} M
$$

## Domestic Supply Block



## The Model: Domestic Output Problem I

- GDP and IC combined yield total domestic supply of the representative good.
- The firm solves for optimal combination of Y and IC in the domestic output's second-level cost minimization problem:

$$
\min _{\{Y, I C\}} p_{Y} Y+p_{I C} I C D
$$

s.t.

$$
O U T=\theta_{O}\left(\pi_{Y} Y^{\frac{\sigma_{O}-1}{\sigma_{O}}}+\pi_{I C D} I C D^{\frac{\sigma_{O}-1}{\sigma_{O}}}\right)^{\frac{\sigma_{O}}{\sigma_{O}-1}}
$$

- Again, elasticity of substitution between Y and IC satisfy $\sigma_{O}>0$, however, these goods are more complementary than substitutes $\left(0<\sigma_{O}<1\right)$.


## The Model: Domestic Output Problem II

- From FOCs, optimal GDP and Intermediate Consumption demands are
$Y=\left(\theta_{O} \pi_{Y} \frac{p_{O}}{p_{Y}}\right)^{\sigma_{O}} \frac{O U T}{\theta_{O}}$, and $I C D=\left(\theta_{O} \pi_{I C D} \frac{p_{O}}{p_{I C}}\right)^{\sigma_{O}} \frac{O U T}{\theta_{O}}$,
where the aggregated price of domestic output $p_{O}$ is

$$
p_{O}=\frac{1}{\theta_{O}}\left(\pi_{Y}^{\sigma_{O}} p_{Y}^{1-\sigma_{O}}+\pi_{I C D}^{\sigma_{O}} p_{I C}^{1-\sigma_{O}}\right)^{\frac{1}{1-\sigma_{O}}}
$$

- With Y's Demand and Supply equations, one can solve for the price of GDP, $p_{Y}$ :

$$
p_{Y}=\left[\frac{p_{A V} A V+T X_{y y}+T R_{f f}}{\left(\theta_{O} \pi_{Y} p_{O}\right)^{\sigma_{O} O} \frac{O U T}{\theta_{O}}}\right]^{\frac{1}{1-\sigma_{O}}}
$$

## Total Supply Block



## The Model: Activity (total supply) I

- When The firm when it solves the first-level cost minimization problem given by:

$$
\min _{\{O U T, M\}} p_{O} O U T+p_{M} M
$$

s.t.

$$
A C T=\theta_{A}\left(\pi_{O} O U T^{\frac{\sigma_{A}-1}{\sigma_{A}}}+\pi_{M} M^{\frac{\sigma_{A}-1}{\sigma_{A}}}\right)^{\frac{\sigma_{A}}{\sigma_{A}-1}}
$$

- Elasticity of substitution between OUT and M is $\sigma_{A}>0$.


## The Model: Activity (total supply) II

- From FOCs, optimal domestic output and imports demands are, respectively

$$
\text { OUT }=\left(\theta_{A} \pi_{O} \frac{p_{A}}{p_{O}}\right)^{\sigma_{A}} \frac{A C T}{\theta_{A}} \text {, and } M=\left(\theta_{A} \pi_{M} \frac{p_{A}}{p_{M}}\right)^{\sigma_{A}} \frac{A C T}{\theta_{A}}
$$

with aggregated price of ACT, $p_{A}$ given by

$$
p_{A}=\frac{1}{\theta_{A}}\left(\pi_{O}^{\sigma_{A}} p_{O}^{1-\sigma_{A}}+\pi_{M}^{\sigma_{A}} p_{M}^{1-\sigma_{A}}\right)^{\frac{1}{1-\sigma_{A}}}
$$

- OUT demand and supply equations are solved for price of OUT, po:

$$
p_{O}=\theta_{A} \pi_{O}\left[\frac{\frac{A C T}{\theta_{A}}}{\theta_{O}\left(\pi_{Y} Y^{\frac{\sigma_{O}-1}{\sigma_{O}}}+\pi_{I C D} I C D^{\frac{\sigma_{O}-1}{\sigma_{O}}}\right)^{\frac{\sigma_{O}}{\sigma_{O}-1}}}\right]^{\frac{1}{\sigma_{A}}} p_{A}
$$

## The Model: Activity (total supply) III

Additional considerations on ACT formation:

- The clearing market condition assures that

$$
p_{A} A C T=p_{O} O U T+p_{M} M .
$$

- RW provides all demand for imports inelastically at the international price $p_{M}^{*}$, and therefore

$$
p_{M}=e p_{M}^{*}
$$

where $\bar{e}$ is the nominal exchange rate.

## Supply Distribution Block



## The Model: Supply Distribution I

Total supply, ACT, is distributed between intermediate and final consumption.

- The firm determines distribution of ACT between intermediate (IC) and final consumption (FC) by maximizing revenue from sales:

$$
\max _{\{I C, F C\}} p_{I C} I C S+p_{F C} F C
$$

s.t. a CET technology of distribution

$$
A C T=\theta_{A D}\left(\pi_{I C S} I C S^{\frac{\tau_{A}-1}{\tau_{A}}}+\pi_{F C} F C^{\frac{\tau_{A}-1}{\tau_{A}}}\right)^{\frac{\tau_{A}}{\tau_{A}-1}}
$$

- The elasticity of transformation between intermediate and final consumption is $\tau_{A}<0$.


## The Model: Supply Distribution II

- From the FOCs, optimal FC and IC supplies are
$I C S=\left(\theta_{A D} \pi_{I C S} \frac{p_{A}}{p_{I C}}\right)^{\tau_{A}} \frac{A C T}{\theta_{A D}}$, and $F C=\left(\theta_{A D} \pi_{F C} \frac{p_{A}}{p_{F C}}\right)^{\tau_{A}} \frac{A C T}{\theta_{A D}}$
with aggregated price of activity, $p_{A}$, given by

$$
p_{A}=\frac{1}{\theta_{A D}}\left(\pi_{I C S}^{\tau_{A}} p_{I C}^{1-\tau_{A}}+\pi_{F C}^{\tau_{A}} p_{F C}^{1-\tau_{A}}\right)^{\frac{1}{1-\tau_{A}}}
$$

- However, $p_{A}$ is determined through the market clearing condition:

$$
p_{A} A C T=p_{I C} I C S+p_{F C} F C
$$

- With IC supply and demand equations, we have

$$
p_{I C}=\left[\frac{\left(\theta_{O} \pi_{I C D} p_{O}\right)^{\sigma_{O}} \frac{O U T}{\theta_{O}}}{\left(\theta_{A D} \pi_{I C S} p_{A}\right)^{\tau_{A}} \frac{A C T}{\theta_{A D}}}\right]^{\frac{1}{\sigma_{O}-\tau_{A}}}
$$

## FC Distribution Block



## The Model: Final Consumption Distribution I

- The firm determines distribution of FC supply between Consumption (C), Investment (I), Government Expenditure (G) and Exports (X). X are classified between traditional, $\bar{X}_{T}$ (which are assumed as exogenous); and non-traditional, $X_{N}$.
- Firm maximizes its revenue from selling final consumption:

$$
\max _{\left\{C, l, G, X_{N}\right\}} p_{C} C+p_{I} I+p_{G} G+p_{X_{T}} \bar{X}_{T}+p_{X_{N}} X_{N}
$$

s.t.
$F C=\bar{X}_{T}+\theta_{F C}\left(\pi_{C} C^{\frac{\tau_{F C}-1}{\tau_{F C}}}+\pi_{I} I^{\frac{\tau_{F C}-1}{\tau_{F C}}}+\pi_{G} G^{\frac{\tau_{F C}-1}{\tau_{F C}}}+\pi_{X N} X_{N}^{\frac{\tau_{F C}-1}{\tau_{F C C}}}\right)^{\frac{\tau_{F C}}{\tau_{F C}-1}}$

- Elasticity of transformation between types of final consumption satisfies $\tau_{F C}<0$.


## The Model: Final Consumption Distribution II

- From FOCs we derive the optimal supply of each of the FC components:

$$
Z=\left[\theta_{F C} \pi_{Z} \frac{p_{F C} F C-p_{X T} \bar{X}_{T}}{p_{Z}\left(F C-\bar{X}_{T}\right)}\right]^{\tau_{F C}} \frac{F C-\bar{X}_{T}}{\theta_{F C}} \text {, with } Z \in\left\{C, G, I, X_{N}\right\}
$$

With $p_{F C}$ given by

$$
p_{F C} F C=\frac{1}{\theta_{F C}}\left(\sum \pi_{Z}^{\tau_{F C}} p_{Z}^{1-\tau_{F C}}\right)^{\frac{1}{1-\tau_{F C}}}\left(F C-\bar{X}_{T}\right)+p_{X T} \bar{X}_{T}
$$

- FC supply or FC demand equations can be placed in the latter expression in order to solve for $p_{F C}$.


## Factor Remuneration Block



## The Model: Income Distribution I

- Factor supply is assumed to be exogenous (completely inelastic): $\bar{L}, \bar{K}$, and $\bar{Z}$.
- Then it holds:

$$
\begin{aligned}
p_{L} \bar{L} & =R E M=R E M_{H H}+F_{L} \\
p_{K} \bar{K} & =E B E=E B E_{H H}+E B E_{F R}+E B E_{G V} \\
p_{Z} \bar{Z} & =M I X=M I X_{H H}
\end{aligned}
$$

- Given the supplies of factors, and the demands (AV production), we derive the factor prices:

$$
p_{W}=\theta_{F} \pi_{W}\left(\frac{F A C}{\theta_{F} \bar{W}}\right)^{\frac{1}{\sigma_{F}}} p_{F} \quad, \text { for } W \in\{L, K, Z\}
$$

## The Model: Income Distribution II

Distribution of factor remunerations and rents in the model are paid according to fixed coefficients:

- Factor remunerations:
- $R E M_{H H}=\pi_{H H}^{R E M} R E M$ and $F_{L}=\pi_{R W}^{R E M} R E M$.
- $E B E_{H H}=\pi_{H H}^{E B E} E B E, E B E_{F R}=\pi_{F R}^{E B E} E B E$ and $E B E_{G V}=\pi_{G V}^{E B E} E B E$.
- $M I X=M I X_{H}$.
- Rents:
- Payments: $R^{H H}=\pi_{R}^{H H} E B E_{H H}, R^{F R}=\pi_{R}^{F R} E B E_{F R}$ and $R^{G V}=\pi_{R}^{G V} E B E_{G V}$.
- $R=R^{H H}+R^{F R}+R^{G V}=R_{H H}+R_{F R}+R_{G V}+F_{K}$.
- Recipients: $R_{H H}=\pi_{H H}^{R} R, R_{F R}=\pi_{F R}^{R} R, R_{G V}=\pi_{G V}^{R} R$, and $F_{K}=\pi_{R W}^{R} R$.


## The Model: Direct Taxes

- Households' income:

$$
Y_{H H}=R E M_{H H}+E B E_{H H}+M I X_{H H}+\left(R_{H H}-R^{H H}\right) .
$$

- Firm's income: $Y_{F R}=E B E_{F R}+\left(R_{F R}-R^{F R}\right)$.
- Government's income: $Y_{G V}=E B E_{G V}+\left(R_{G V}-R^{G V}\right)$.

Assuming no tax evasion and perfect fiscal compliance, institutional agents pay direct taxes as a constant fraction of their income:

$$
\begin{gathered}
T X h h=t \times h h Y_{H H}, T X a c_{F R}=t \times a c_{F R} Y_{F R} \\
\text { and } T X_{a c_{G V}}=t \times a c_{G V} Y_{G V} .
\end{gathered}
$$

Total direct taxes are given by

$$
T=T X h h+T X^{\prime} c_{H H}+T X_{G} c_{G V} .
$$

## The Model: Transfers I

There are four types of transfers: social contributions (SC), social benefits (SB), current transfers (CT), and product transfers (PT).

- We assume exogenous payments of social contributions $\overline{S C}^{H H}$ by HH , which is distributed FR and GV:

$$
S C_{F R}^{H H}=\pi_{F R}^{S C} \overline{S C}^{H H} \text {, and } S C_{G V}^{H H}=\pi_{G V}^{S C} \overline{S C}^{H H} \text {. }
$$

- HH receive exogenously assumed social benefits, $S B=\overline{S B}_{H H}$, from FR and GV:

$$
S B_{H H}^{F R}=\pi_{F R}^{S B} \overline{S B}_{H H}, \text { and } S B_{H H}^{G V}=\pi_{G V}^{S B} \overline{S B}_{H H} .
$$

- FR and RW pay CT exogenously, $\overline{C T}^{R W}+\overline{C T}^{F R}=C T$, which is distributed to HH and GV as:

$$
C T_{H H}=\pi_{H H}^{C T} C T \text {, and } C T_{G V}=\pi_{G V}^{C T} C T .
$$

## The Model: Transfers II

- We also assume exogenous product transfers from the Government to households, $\overline{P T}_{H H}$.
- Net transfers are then represented by the following equations:

$$
\begin{aligned}
N T_{H H} & =-\overline{S C}^{H H}+\overline{S B}_{H H}+C T_{H H}+\overline{P T}_{H H}^{G V} \\
N T_{F R} & =S C_{F R}^{H H}-S B_{H H}^{F R}-\overline{C T}^{F R} \\
N T_{G V} & =S C_{G V}^{H H}-S B_{H H}^{G V}+C T_{G V}-\overline{P T}_{H H}^{G V} \\
N T_{R W} & =-\overline{C T}^{R W}
\end{aligned}
$$

## Demand Block



## The Model: Domestic Demand I

- HH have standard well-behaved preferences (e.g. Cobb-Douglas with savings in utility), which yield final consumption demand and savings as:

$$
C=\alpha \frac{D Y_{H H}}{p_{C}}, \text { and } S_{H H}=D Y_{H H}-p_{C} C
$$

with $D Y_{H H}=Y_{H H}+N T_{H H}-T X h h$. HH's marginal propensity to consume (MPC) satisfies $0<\alpha<1$.

- HH and FR investment form private investment, $I_{P R}$, as

$$
\begin{aligned}
I_{H H} & =\beta I_{P R} \\
I_{F R} & =(1-\beta) I_{P R} \\
I_{P R} & =I_{H H}+I_{F R}
\end{aligned}
$$

with $0<\beta<1$.

## The Model: Domestic Demand II

- Accordingly, we have FR savings given by

$$
S_{F R}=Y_{F R}+N T_{F R}-T X a c_{F R}
$$

- Total demand for investment, I, is

$$
I=I_{P R}+I_{G V}
$$

which along with investment supply yields the price of investment equation, $p_{l}$

$$
p_{I}=\theta_{F C} \pi_{I} \frac{p_{F C} F C-p_{X T} \bar{X}_{T}}{\left(F C-\bar{X}_{T}\right)}\left[\frac{F C-\bar{X}_{T}}{\theta_{F C}\left(I_{P R}+\bar{I}_{G V}\right)}\right]^{\frac{1}{\tau_{F C}}}
$$

## The Model: Domestic Demand III

- We assume GV's expenditure, G, and investment, $I_{G V}$, to be exogenous:

$$
G=\bar{G}, \text { and } I_{G V}=\bar{I}_{G V}
$$

- GV expenditure price is jointly determined by its supply and demand functions

$$
p_{G}=\theta_{F C} \pi_{G} \frac{p_{F C} F C-p_{X T} \bar{X}_{T}}{\left(F C-\bar{X}_{T}\right)}\left(\frac{F C-\bar{X}_{T}}{\theta_{F C} \bar{G}}\right)^{\frac{1}{\tau_{F C}}}
$$

- Accordingly, GV savings are given by

$$
S_{G V}=Y_{G V}+N T_{G V}+T_{X}+T-T X_{a c_{G V}}-p_{G} \bar{G}
$$

with indirect taxes $T x=T X_{v a}+T X_{y y}+T R_{f f}$.

## The Model: External Demand I

- RW demand for $X_{N T}$ is defined according to:

$$
X_{N}=\left(\theta_{M^{*}} \pi_{\operatorname{COL}} \frac{e \bar{p}_{M^{*}}^{*}}{p_{X N}}\right)^{\sigma_{P}^{*}} \frac{\bar{M}^{*}}{\theta_{M^{*}}}
$$

where $\theta_{M^{*}}$ and $\pi_{\text {COL }}$ are scale and Colombian share parameter in the aggregation of RW imports, $\bar{M}^{*}$ which are assumed to be exogenous, such as their price, $\bar{p}_{M^{*}}^{*}$.

- Total exports quantities must satisfy $X=X_{N}+\bar{X}_{T}$, and their price is determined by

$$
p_{X}=\frac{p_{X_{T}} \bar{X}_{T}+p_{X_{N}} X_{N}}{X}
$$

## The Model: External Demand II

- Price of $X_{N}, p_{X_{N}}$, is determined by its supply and demand equilibrium:

$$
p_{X_{N}}=\left\{\frac{\left(\theta_{M^{*}} \pi_{C O L} e \bar{p}_{M^{*}}^{*}\right)^{\sigma_{M}^{*}} \frac{\bar{M}^{*}}{\theta_{M^{*}}}}{\left[\theta_{F C} \pi_{X N} \frac{p_{F C} F C-p_{X T} \bar{X}_{T}}{\left(F C-\bar{X}_{T}\right)}\right]^{\tau_{F C}} \frac{F C-\bar{X}_{T}}{\theta_{F C}}}\right\}^{\frac{1}{\sigma_{M}^{*}-\tau_{F C}}}
$$

- RW demands $X_{T}$ at the international price $p_{X_{T}}^{*}$, which means that the internal price of $X_{T}$ is given by

$$
p_{X_{T}}=e p_{X_{T}}^{*}
$$

## The Model: Closure Equations I

We set $Y_{R W}=F_{L}+F_{K}-\overline{C T}^{R W}$.
Private Investment Closure

- Exogenous exchange rate: $e=\bar{e}$.
- Exogenous $I_{P R}: I_{P R}=\bar{I}_{P R}$.
- Endogenous $S_{R W}$ :

$$
-C C=S_{R W}=Y_{R W}+p_{M} M-p_{X} X
$$

- S-I balance depends on Endogenous $p_{C}$ (replacing $S_{H H}$ ):

$$
\begin{aligned}
& p_{I} \bar{l}=S_{H H}+S_{F R}+S_{G V}+S_{R W} \\
& p_{C}=\frac{D Y_{H H}+S_{F R}+S_{G V}+S_{R W}-p_{I} \bar{l}}{C}
\end{aligned}
$$

## The Model: Closure Equations II

RW Savings Closure

- Exogenous consumption price: $p_{C}=\bar{p}_{C}$.
- Exogenous External Savings: $S_{R W}=\bar{S}_{R W}$.
- Endogenous exchange rate, e (derived from the following equation):

$$
\bar{S}_{R W}=Y_{R W}+p_{M}(e) M(e)-p_{X}(e) X(e)
$$

- $I_{P R}$ is determined by the S-I balance:

$$
I_{P R}=\frac{S_{H H}+S_{F R}+S_{G V}+\bar{S}_{R W}}{p_{I}}-\bar{I}_{G V}
$$

## Parameter Calibration: An example (I)

Using information from the Macro-SAM constructed for the model, we show an example of how share and scale parameters are calibrated. All parameters can be calibrated following the same steps.

- Share parameters: We have that

$$
\begin{gathered}
\pi_{K}=\pi_{L} \frac{p_{K}}{p_{L}}\left(\frac{K}{L}\right)^{\frac{1}{\sigma_{F}}}, \pi_{Z}=\pi_{L} \frac{p_{Z}}{p_{L}}\left(\frac{Z}{L}\right)^{\frac{1}{\sigma_{F}}} \text { and } \\
\pi_{L}+\pi_{K}+\pi_{Z}=1
\end{gathered}
$$

which yields

$$
\pi_{L}=\frac{p_{L} L^{\frac{1}{\sigma_{F}}}}{p_{L} L^{\frac{1}{\sigma_{F}}}+p_{K} K^{\frac{1}{\sigma_{F}}}+p_{Z} Z^{\frac{1}{\sigma_{F}}}}
$$

all other parameters can be calibrated analogously.

## Parameter Calibration: An example (II)

- Scale parameters: Using the share parameters and FAC, we have

$$
\theta_{F}=F A C\left(\frac{p_{L} L^{\frac{1}{\sigma_{F}}}+p_{K} K^{\frac{1}{\sigma_{F}}}+p_{Z} Z^{\frac{1}{\sigma_{F}}}}{p_{L} L+p_{K} K+p_{Z} Z}\right)^{\frac{\sigma_{F}}{\sigma_{F}-1}}
$$

## Model Summary

A grand total of 99 variables:

- 73 endogenous variables.
- 22 exogenous variables.
- 4 closure variables:
- 2 endogenous variables (depending on which closure we choose).
- 2 exogenous remaining variables: i) A nominal anchor, and ii) a real quantity.


## Endogenous Variables

Endogenous Variables List (73)

| FAC | $p_{F}$ | AV | $T X_{v a}$ | $p_{A V}$ | $T X_{y y}$ | $T R_{f f}$ | Y |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $I C_{D}$ | $p_{Y}$ | OUT | M | $p_{O}$ | ACT | $p_{M}$ | $I C_{S}$ |
| FC | $p_{A}$ | $p_{I C}$ | I | $X_{N}$ | $p_{F C}$ | REM | EBE |
| MIX | $p_{L}$ | $p_{K}$ | $p_{Z}$ | $R E M_{H H}$ | $F_{L}$ | $E B E_{H H}$ | $E B E_{F R}$ |
| $E B E_{G V}$ | $R^{H H}$ | $R^{F R}$ | $R^{G V}$ | $R_{H H}$ | $R_{F R}$ | $R_{G V}$ | $F_{K}$ |
| R | $Y_{H H}$ | $Y_{F R}$ | $Y_{G V}$ | $T X h h$ | $T X_{a C_{F R}}$ | $T X_{a C_{G V}}$ | T |
| $S C_{F R}^{H H}$ | $S C_{G V}^{H H}$ | $S B_{H H}^{F R}$ | $S B_{H H}^{G V}$ | CT | $C T_{H H}$ | $C T_{G V}$ | $N T_{H H}$ |
| $N T_{F R}$ | $N T_{G V}$ | C | $S_{H H}$ | $D Y_{H H}$ | $I_{H H}$ | $I_{F R}$ | $S_{F R}$ |
| $p_{I}$ | $S_{G V}$ | TX | $p_{G}$ | X | $p_{X}$ | $p_{X_{N}}$ | $p_{X_{T}}$ |
| $Y_{R W}$. |  |  |  |  |  |  |  |

## Exogenous Variables: A list (I)

- Factors: $\bar{L}, \bar{K}, \bar{Z} \longrightarrow$ DPI-BR
- Total Factor Productivity: $\theta_{F} \longrightarrow$ DPI-BR
- Indirect Taxes Rates: $t x_{v a}, t x_{y y}, t r_{f f} \longrightarrow$ Calibrated (ftc)
- International Price of Imports: $p_{M}^{*} \longrightarrow \mathrm{BoP} / \mathrm{ToT}$ (Imports Index)
- Traditional Export Quantities: $\bar{X}_{T} \longrightarrow$ BOP
- Direct Taxes Rates: $t \times h h, t x a c_{F R}, t x a c_{G V} \longrightarrow$ Calibrated (ftc)


## Exogenous Variables: A list (II)

- HH payments to SC: $\overline{S C}^{H H} \longrightarrow$ Pension Funds Financial Statements.
- SB payments to $\mathrm{HH}: \overline{S B}_{H H}$
- FR payments to CT: $\overline{C T}^{F R}$
- RW payments to CT: $\overline{C T}^{R W} \longrightarrow \mathrm{BOP}$
- GV payments to PT: $\overline{P T}_{H H}^{G V}$


## Exogenous Variables: A list (III)

- GV Investment: $\bar{I}_{G V} \longrightarrow$ DPI-BR
- GV Spending: $\bar{G} \longrightarrow$ DPI-BR
- Price of RW Imports: $\bar{p}_{M^{*}}^{*} \longrightarrow$ WEO (External Inflation)
- RW Imports Quantities: $\bar{M}^{*} \longrightarrow \mathrm{BOP}$
- Traditional Exports Prices: $p_{X_{T}} \longrightarrow$ BoP / ToT (Exports Index)


## Closure Variables

Investment Closure

- Nominal Exchange Rate: $\bar{e} \longrightarrow$ BOP
- Private Investment: $\bar{I}_{P R} \rightarrow \mathrm{DPI}-\mathrm{BR}$

Savings Closure

- Consumption Good Price: $\bar{p}_{C} \longrightarrow$ DPI-BR
- External Savings: $\bar{S}_{R W} \longrightarrow$ BOP


## Macro CGEM usage: An example

- Using observed information from BOP, National Accounts and other relevant variables, we replicate 2012 economy taking as a starting point 2011 SAM and our model.

Main Results - 2012

| Variable | Observed |  | CGEM |  |
| ---: | :---: | :---: | :---: | :---: |
|  | Average | Investment | Savings |  |
| GDP | 4.0 | 4.1 | 4.2 | 4.1 |
| C | 4.4 | 4.0 | 4.5 | 3.5 |
| G | 5.7 | 5.7 | 5.7 | 5.7 |
| I | 4.6 | 4.5 | 5.0 | 4.0 |
| $\boldsymbol{I}_{P R}$ | 4.9 | 4.4 | 4.9 | 3.8 |
| $I_{G V}$ | 5.3 | 5.3 | 5.3 | 5.3 |
| X | 6.1 | 5.0 | 4.6 | 5.3 |
| M | 8.9 | 5.4 | 7.2 | 3.5 |
| CAD (\%GDP) | 3.1 | 3.6 | 4.0 | 3.1 |

## Consistency of the model

## Current Account Deficit vs. Rest of the World's Savings



## What's Next?

This model can be further extended along the following lines:

- Demand driven economy.
- Assuring BoP matching with the model.
- Multi-sector CGE model.
- Extension of Fiscal Block.
- Money in CGEM (anchor to Monetary accounts).


## THE END

Thank You.


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