## A Macro CGE Model for the Colombian Economy Banco de la República's Internal Seminar

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#### Outline

Introduction Social Accounting Matrix The Model Supply Side Factor Demand Problem. Added Value and GDP Supply Formation Supply Distribution Income Distribution **Factor Remuneration** (Net) Rents and Transfers Direct Taxes Demand Side Domestic Demand External Demand Closure of the model Parameter Calibration Model Summary Macro CGEM usage: An example

#### Motivation

- Macro CGE models acknowledge the links between National Accounts and Balance of Payments and Fiscal Accounts.
- Allow for Taxation and Sectoral analyses.
- CGEM are NOT intended for Policy Recommendations but are mostly used to present the economy's outcomes after assessing different *alternative* scenarios.

## The Economy I



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#### Social Accounting Matrix:

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## The Model: Supply Side

Production factors, Indirect taxes, Intermediate consumption and Imports are combined to create total supply of the representative good (Activity).

This process involves solving three different cost minimization problems:

- Factor Demand Problem.
- ► GDP Intermediate Consumption Problem.
- Output Imports Problem.

Firm also maximizes its revenue by optimally solving:

- FC and IC Distribution Problem.
- FC components Distribution Problem (link to demand side).

#### Factor Demand Block



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#### The Model: Factor Demand Problem I

- Three factors combined + Production Taxes = Value Added.
- Firms solve the Factors Demand Problem (minimizes expenditure subject to production):

$$\min_{\{L,K,Z\}} p_L L + p_K K + p_Z Z,$$

s.t.

$$FAC = \theta_F \left( \pi_L L^{\frac{\sigma_F - 1}{\sigma_F}} + \pi_K K^{\frac{\sigma_F - 1}{\sigma_F}} + \pi_Z Z^{\frac{\sigma_F - 1}{\sigma_F}} \right)^{\frac{\sigma_F}{\sigma_F - 1}}$$

Elasticity of substitution among factors satisfies σ<sub>F</sub> > 0.

#### The Model: Factor Demand Problem II

From the FOCs we derive the optimal demand of factors:

$$L = \left(\theta_F \pi_L \frac{p_F}{p_L}\right)^{\sigma_F} \frac{FAC}{\theta_F} , \ K = \left(\theta_F \pi_K \frac{p_F}{p_K}\right)^{\sigma_F} \frac{FAC}{\theta_F} ,$$
$$Z = \left(\theta_F \pi_Z \frac{p_F}{p_Z}\right)^{\sigma_F} \frac{FAC}{\theta_F} ,$$

where the aggregated price of factors  $p_F$  is expressed as

$$p_F = \frac{1}{\theta_F} \left( \pi_L^{\sigma_F} p_L^{1-\sigma_F} + \pi_K^{\sigma_F} p_K^{1-\sigma_F} + \pi_Z^{\sigma_F} p_Z^{1-\sigma_F} \right)^{\frac{1}{1-\sigma_F}}$$

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#### The Model: Indirect taxes and GDP

 Added value, AV, is completed once indirect production taxes are acknowledged (nominal terms):

$$p_{AV}AV = p_FFAC + TX_{va} \; ,$$

where tax revenue in production  $(TX_{va})$  is given by

$$TX_{va} = tx_{va}p_FFAC.$$

► GDP (Y) supply is obtained by adding up AV, indirect (net) taxes over products (TX<sub>YY</sub>) and import tariffs (TR<sub>ff</sub>):

$$p_Y Y = p_{AV} AV + T X_{yy} + T R_{ff} ,$$

where indirect product taxes and tariffs are given by

$$TX_{yy} = tx_{yy}p_{AV}AV$$
, and  $TR_{ff} = tr_{ff}p_MM$ 

#### Domestic Supply Block



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#### The Model: Domestic Output Problem I

- GDP and IC combined yield total domestic supply of the representative good.
- The firm solves for optimal combination of Y and IC in the domestic output's second-level cost minimization problem:

$$\min_{\{Y,IC\}} p_Y Y + p_{IC} ICD$$

s.t.

$$OUT = \theta_O \left( \pi_Y Y^{\frac{\sigma_O - 1}{\sigma_O}} + \pi_{ICD} ICD^{\frac{\sigma_O - 1}{\sigma_O}} \right)^{\frac{\sigma_O}{\sigma_O - 1}}$$

Again, elasticity of substitution between Y and IC satisfy  $\sigma_O > 0$ , however, these goods are more complementary than substitutes ( $0 < \sigma_O < 1$ ).

#### The Model: Domestic Output Problem II

 From FOCs, optimal GDP and Intermediate Consumption demands are

$$Y = \left(\theta_O \pi_Y \frac{p_O}{p_Y}\right)^{\sigma_o} \frac{OUT}{\theta_O} \text{ , and } ICD = \left(\theta_O \pi_{ICD} \frac{p_O}{p_{IC}}\right)^{\sigma_O} \frac{OUT}{\theta_O},$$

where the aggregated price of domestic output  $p_O$  is

$$p_{O} = \frac{1}{\theta_{O}} \left( \pi_{Y}^{\sigma_{O}} p_{Y}^{1-\sigma_{O}} + \pi_{ICD}^{\sigma_{O}} p_{IC}^{1-\sigma_{O}} \right)^{\frac{1}{1-\sigma_{O}}}$$

With Y's Demand and Supply equations, one can solve for the price of GDP, p<sub>Y</sub>:

$$p_{Y} = \left[\frac{p_{AV}AV + TX_{yy} + TR_{ff}}{\left(\theta_{O}\pi_{Y}p_{O}\right)^{\sigma_{O}}\frac{OUT}{\theta_{O}}}\right]^{\frac{1}{1-\sigma_{O}}}$$

## Total Supply Block



୬ ୯ ୯ 14 / 49 The Model: Activity (total supply) I

When The firm when it solves the first-level cost minimization problem given by:

$$\min_{\{OUT,M\}} p_OOUT + p_MM$$

s.t.

$$ACT = \theta_A \left( \pi_O OUT^{\frac{\sigma_A - 1}{\sigma_A}} + \pi_M M^{\frac{\sigma_A - 1}{\sigma_A}} \right)^{\frac{\sigma_A}{\sigma_A - 1}},$$

• Elasticity of substitution between OUT and M is  $\sigma_A > 0$ .

#### The Model: Activity (total supply) II

 From FOCs, optimal domestic output and imports demands are, respectively

$$OUT = \left(\theta_A \pi_O \frac{p_A}{p_O}\right)^{\sigma_A} \frac{ACT}{\theta_A}, \text{ and } M = \left(\theta_A \pi_M \frac{p_A}{p_M}\right)^{\sigma_A} \frac{ACT}{\theta_A}$$

with aggregated price of ACT,  $p_A$  given by

$$p_{A} = \frac{1}{\theta_{A}} \left( \pi_{O}^{\sigma_{A}} p_{O}^{1-\sigma_{A}} + \pi_{M}^{\sigma_{A}} p_{M}^{1-\sigma_{A}} \right)^{\frac{1}{1-\sigma_{A}}}$$

 OUT demand and supply equations are solved for price of OUT, p<sub>O</sub>:

$$p_{O} = \theta_{A} \pi_{O} \left[ \frac{\frac{ACT}{\theta_{A}}}{\theta_{O} \left( \pi_{Y} Y^{\frac{\sigma_{O}-1}{\sigma_{O}}} + \pi_{ICD} ICD^{\frac{\sigma_{O}-1}{\sigma_{O}}} \right)^{\frac{\sigma_{O}}{\sigma_{O}-1}}} \right]^{\frac{1}{\sigma_{A}}} p_{A}$$

The Model: Activity (total supply) III

Additional considerations on ACT formation:

The clearing market condition assures that

$$p_A A C T = p_O O U T + p_M M.$$

 RW provides all demand for imports inelastically at the international price p<sup>\*</sup><sub>M</sub>, and therefore

$$p_M = e p_M^*$$

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where  $\bar{e}$  is the nominal exchange rate.

### Supply Distribution Block



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#### The Model: Supply Distribution I

Total supply, ACT, is distributed between intermediate and final consumption.

 The firm determines distribution of ACT between intermediate (IC) and final consumption (FC) by maximizing revenue from sales:

$$\max_{\{IC,FC\}} p_{IC}ICS + p_{FC}FC$$

s.t. a CET technology of distribution

$$ACT = \theta_{AD} \left( \pi_{ICS} ICS^{\frac{\tau_A - 1}{\tau_A}} + \pi_{FC} FC^{\frac{\tau_A - 1}{\tau_A}} \right)^{\frac{\tau_A}{\tau_A - 1}}$$

► The elasticity of transformation between intermediate and final consumption is \u03c6<sub>A</sub> < 0.</p>

#### The Model: Supply Distribution II

From the FOCs, optimal FC and IC supplies are

$$ICS = \left(\theta_{AD}\pi_{ICS}\frac{p_A}{p_{IC}}\right)^{\tau_A}\frac{ACT}{\theta_{AD}}, \text{ and } FC = \left(\theta_{AD}\pi_{FC}\frac{p_A}{p_{FC}}\right)^{\tau_A}\frac{ACT}{\theta_{AD}}$$

with aggregated price of activity,  $p_A$ , given by

$$p_{A} = \frac{1}{\theta_{AD}} \left( \pi_{ICS}^{\tau_{A}} p_{IC}^{1-\tau_{A}} + \pi_{FC}^{\tau_{A}} p_{FC}^{1-\tau_{A}} \right)^{\frac{1}{1-\tau_{A}}}$$

However, p<sub>A</sub> is determined through the market clearing condition:

$$p_A A C T = p_{IC} I C S + p_{FC} F C$$

With IC supply and demand equations, we have

$$p_{IC} = \left[\frac{\left(\theta_{O}\pi_{ICD}p_{O}\right)^{\sigma_{o}}\frac{OUT}{\theta_{O}}}{\left(\theta_{AD}\pi_{ICS}p_{A}\right)^{\tau_{A}}\frac{ACT}{\theta_{AD}}}\right]^{\frac{1}{\sigma_{o}-\tau_{A}}}$$

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#### FC Distribution Block



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#### The Model: Final Consumption Distribution I

- The firm determines distribution of FC supply between Consumption (C), Investment (I), Government Expenditure (G) and Exports (X). X are classified between traditional, X<sub>T</sub> (which are assumed as exogenous); and non-traditional, X<sub>N</sub>.
- Firm maximizes its revenue from selling final consumption:

$$\max_{\{C,I,G,X_N\}} p_C C + p_I I + p_G G + p_{X_T} \bar{X}_T + p_{X_N} X_N$$

s.t.

$$FC = \bar{X}_{T} + \theta_{FC} \left( \pi_{C} C^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_{I} I^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_{G} G^{\frac{\tau_{FC}-1}{\tau_{FC}}} + \pi_{XN} X_{N}^{\frac{\tau_{FC}-1}{\tau_{FC}}} \right)^{\frac{\tau_{FC}-1}{\tau_{FC}}}$$

 Elasticity of transformation between types of final consumption satisfies τ<sub>FC</sub> < 0.</li> ---

#### The Model: Final Consumption Distribution II

From FOCs we derive the optimal supply of each of the FC components:

$$Z = \left[\theta_{FC}\pi_{Z}\frac{p_{FC}FC - p_{XT}\bar{X}_{T}}{p_{Z}\left(FC - \bar{X}_{T}\right)}\right]^{\tau_{FC}}\frac{FC - \bar{X}_{T}}{\theta_{FC}} \text{, with } Z \in \{C, G, I, X_{N}\}$$

With  $p_{FC}$  given by

$$p_{FC}FC = \frac{1}{\theta_{FC}} \left( \sum \pi_Z^{\tau_{FC}} p_Z^{1-\tau_{FC}} \right)^{\frac{1}{1-\tau_{FC}}} \left( FC - \bar{X}_T \right) + p_{XT} \bar{X}_T$$

 FC supply or FC demand equations can be placed in the latter expression in order to solve for p<sub>FC</sub>.

#### Factor Remuneration Block



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#### The Model: Income Distribution I

- Factor supply is assumed to be exogenous (completely inelastic): *L*, *K*, and *Z*.
- Then it holds:

$$p_L \bar{L} = REM = REM_{HH} + F_L$$
  
 $p_K \bar{K} = EBE = EBE_{HH} + EBE_{FR} + EBE_{GV}$   
 $p_Z \bar{Z} = MIX = MIX_{HH}$ 

 Given the supplies of factors, and the demands (AV production), we derive the factor prices:

$$p_W = heta_F \pi_W \left( rac{FAC}{ heta_F ar W} 
ight)^{rac{1}{\sigma_F}} p_F$$
 , for  $W \in \{L, K, Z\}$ .

#### The Model: Income Distribution II

Distribution of factor remunerations and rents in the model are paid according to fixed coefficients:

- Factor remunerations:
  - $REM_{HH} = \pi_{HH}^{REM} REM$  and  $F_L = \pi_{RW}^{REM} REM$ . •  $EBE_{HH} = \pi_{HH}^{EBE} EBE$ ,  $EBE_{FR} = \pi_{FR}^{EBE} EBE$  and
  - $EBE_{HH} = \pi_{HH}^{EBE} EBE$ ,  $EBE_{FR} = \pi_{FR}^{EBE} EBE$  and  $EBE_{GV} = \pi_{GV}^{EBE} EBE$ .
  - $MIX = MIX_{HH}$ .
- Rents:
  - ► Payments:  $R^{HH} = \pi_R^{HH} EBE_{HH}$ ,  $R^{FR} = \pi_R^{FR} EBE_{FR}$  and  $R^{GV} = \pi_R^{GV} EBE_{GV}$ . ►  $R = R^{HH} + R^{FR} + R^{GV} = R_{HH} + R_{FR} + R_{GV} + F_K$ . ► Recipients:  $R_{HH} = \pi_{HH}^{R}R$ ,  $R_{FR} = \pi_{FR}^{R}R$ ,  $R_{GV} = \pi_{GV}^{R}R$ , and
    - $F_{K} = \pi_{RW}^{R} R.$

#### The Model: Direct Taxes

• Households' income:  $Y_{HH} = REM_{HH} + EBE_{HH} + MIX_{HH} + (R_{HH} - R^{HH}).$ 

- Firm's income:  $Y_{FR} = EBE_{FR} + (R_{FR} R^{FR}).$
- Government's income:  $Y_{GV} = EBE_{GV} + (R_{GV} R^{GV}).$

Assuming no tax evasion and perfect fiscal compliance, institutional agents pay direct taxes as a constant fraction of their income:

$$TXhh = txhhY_{HH}, TXac_{FR} = txac_{FR}Y_{FR},$$

and 
$$TXac_{GV} = txac_{GV}Y_{GV}$$
.

Total direct taxes are given by

$$T = TXhh + TXac_{HH} + TXac_{GV}$$
.

#### The Model: Transfers I

There are four types of transfers: social contributions (SC), social benefits (SB), current transfers (CT), and product transfers (PT).

We assume exogenous payments of social contributions SC<sup>HH</sup> by HH, which is distributed FR and GV:

$$SC_{FR}^{HH} = \pi_{FR}^{SC} \overline{SC}^{HH}$$
, and  $SC_{GV}^{HH} = \pi_{GV}^{SC} \overline{SC}^{HH}$ 

► HH receive exogenously assumed social benefits,  $SB = \overline{SB}_{HH}$ , from FR and GV:

$$SB_{HH}^{FR} = \pi_{FR}^{SB}\overline{SB}_{HH}$$
, and  $SB_{HH}^{GV} = \pi_{GV}^{SB}\overline{SB}_{HH}$ .

FR and RW pay CT exogenously,  $\overline{CT}^{RW} + \overline{CT}^{FR} = CT$ , which is distributed to HH and GV as:

$$CT_{HH} = \pi_{HH}^{CT}CT$$
, and  $CT_{GV} = \pi_{GV}^{CT}CT$ .

#### The Model: Transfers II

- We also assume exogenous product transfers from the Government to households, PT<sub>HH</sub>.
- Net transfers are then represented by the following equations:

$$NT_{HH} = -\overline{SC}^{HH} + \overline{SB}_{HH} + CT_{HH} + \overline{PT}_{HH}^{GV}$$
$$NT_{FR} = SC_{FR}^{HH} - SB_{HH}^{FR} - \overline{CT}^{FR}$$
$$NT_{GV} = SC_{GV}^{HH} - SB_{HH}^{GV} + CT_{GV} - \overline{PT}_{HH}^{GV}$$
$$NT_{RW} = -\overline{CT}^{RW}$$

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#### **Demand Block**



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#### The Model: Domestic Demand I

 HH have standard well-behaved preferences (e.g. Cobb-Douglas with savings in utility), which yield final consumption demand and savings as:

$$C = \alpha \frac{DY_{HH}}{p_C}$$
, and  $S_{HH} = DY_{HH} - p_C C$ 

with  $DY_{HH} = Y_{HH} + NT_{HH} - TXhh$ . HH's marginal propensity to consume (MPC) satisfies  $0 < \alpha < 1$ .

• HH and FR investment form private investment,  $I_{PR}$ , as

$$I_{HH} = \beta I_{PR}$$
  
 $I_{FR} = (1 - \beta) I_{PR}$   
 $I_{PR} = I_{HH} + I_{FR}$ 

with  $0 < \beta < 1$ .

#### The Model: Domestic Demand II

Accordingly, we have FR savings given by

$$S_{FR} = Y_{FR} + NT_{FR} - TX_{ac_{FR}}$$

Total demand for investment, I, is

$$I = I_{PR} + I_{GV}$$

which along with investment supply yields the price of investment equation,  $p_I$ 

$$p_{I} = \theta_{FC} \pi_{I} \frac{p_{FC}FC - p_{XT}\bar{X}_{T}}{\left(FC - \bar{X}_{T}\right)} \left[\frac{FC - \bar{X}_{T}}{\theta_{FC}\left(I_{PR} + \bar{I}_{GV}\right)}\right]^{\frac{1}{\tau_{FC}}}$$

#### The Model: Domestic Demand III

▶ We assume GV's expenditure, G, and investment, I<sub>GV</sub>, to be exogenous:

$$G = \overline{G}$$
, and  $I_{GV} = \overline{I}_{GV}$ 

 GV expenditure price is jointly determined by its supply and demand functions

$$p_{G} = \theta_{FC} \pi_{G} \frac{p_{FC}FC - p_{XT}\bar{X}_{T}}{\left(FC - \bar{X}_{T}\right)} \left(\frac{FC - \bar{X}_{T}}{\theta_{FC}\bar{G}}\right)^{\frac{1}{\tau_{FC}}}$$

Accordingly, GV savings are given by

$$S_{GV} = Y_{GV} + NT_{GV} + Tx + T - TXac_{GV} - p_G\bar{G}$$

with indirect taxes  $Tx = TX_{va} + TX_{yy} + TR_{ff}$ .

#### The Model: External Demand I

RW demand for X<sub>NT</sub> is defined according to:

$$X_{N} = \left(\theta_{M^{*}}\pi_{COL}\frac{e\bar{p}_{M^{*}}^{*}}{p_{XN}}\right)^{\sigma_{p}^{*}}\frac{\bar{M}^{*}}{\theta_{M^{*}}}$$

where  $\theta_{M^*}$  and  $\pi_{COL}$  are scale and Colombian share parameter in the aggregation of RW imports,  $\overline{M}^*$  which are assumed to be exogenous, such as their price,  $\overline{p}_{M^*}^*$ .

► Total exports quantities must satisfy  $X = X_N + \overline{X}_T$ , and their price is determined by

$$p_X = \frac{p_{X_T} \bar{X}_T + p_{X_N} X_N}{X}$$

#### The Model: External Demand II

Price of X<sub>N</sub>, p<sub>X<sub>N</sub></sub>, is determined by its supply and demand equilibrium:

$$p_{X_N} = \left\{ \frac{\left(\theta_{M^*} \pi_{COL} e \bar{p}_{M^*}^*\right)^{\sigma_M^*} \frac{\bar{M}^*}{\theta_{M^*}}}{\left[\theta_{FC} \pi_{XN} \frac{p_{FC} F C - p_{XT} \bar{X}_T}{(FC - \bar{X}_T)}\right]^{\tau_{FC}} \frac{FC - \bar{X}_T}{\theta_{FC}}} \right\}^{\frac{1}{\sigma_M^* - \tau_{FC}}}$$

► RW demands X<sub>T</sub> at the international price p<sup>\*</sup><sub>X<sub>T</sub></sub>, which means that the internal price of X<sub>T</sub> is given by

$$p_{X_T} = e p_{X_T}^*$$

#### The Model: Closure Equations I

We set 
$$Y_{RW} = F_L + F_K - \overline{CT}^{RW}$$
.

#### Private Investment Closure

- Exogenous exchange rate:  $e = \bar{e}$ .
- Exogenous  $I_{PR}$ :  $I_{PR} = \overline{I}_{PR}$ .
- Endogenous S<sub>RW</sub>:

$$-CC = S_{RW} = Y_{RW} + p_M M - p_X X$$

► S-I balance depends on Endogenous  $p_C$  (replacing  $S_{HH}$ ):

$$p_I \overline{I} = S_{HH} + S_{FR} + S_{GV} + S_{RW}$$
$$p_C = \frac{DY_{HH} + S_{FR} + S_{GV} + S_{RW} - p_I \overline{I}}{C}$$

#### The Model: Closure Equations II

#### **RW Savings Closure**

- Exogenous consumption price:  $p_C = \bar{p}_C$ .
- Exogenous External Savings:  $S_{RW} = \bar{S}_{RW}$ .
- Endogenous exchange rate, e (derived from the following equation):

$$\overline{S}_{RW} = Y_{RW} + p_M(e)M(e) - p_X(e)X(e)$$

► *I<sub>PR</sub>* is determined by the S-I balance:

$$I_{PR} = \frac{S_{HH} + S_{FR} + S_{GV} + \bar{S}_{RW}}{p_I} - \bar{I}_{GV}$$

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#### Parameter Calibration: An example (I)

Using information from the Macro-SAM constructed for the model, we show an example of how share and scale parameters are calibrated. All parameters can be calibrated following the same steps.

Share parameters: We have that

$$\pi_{K} = \pi_{L} \frac{p_{K}}{p_{L}} \left(\frac{K}{L}\right)^{\frac{1}{\sigma_{F}}}, \ \pi_{Z} = \pi_{L} \frac{p_{Z}}{p_{L}} \left(\frac{Z}{L}\right)^{\frac{1}{\sigma_{F}}} \text{ and}$$
$$\pi_{L} + \pi_{K} + \pi_{Z} = 1,$$

which yields

$$\pi_L = \frac{p_L L^{\frac{1}{\sigma_F}}}{p_L L^{\frac{1}{\sigma_F}} + p_K K^{\frac{1}{\sigma_F}} + p_Z Z^{\frac{1}{\sigma_F}}}$$

all other parameters can be calibrated analogously.

Parameter Calibration: An example (II)

Scale parameters: Using the share parameters and FAC, we have

$$\theta_{F} = FAC \left( \frac{p_{L}L^{\frac{1}{\sigma_{F}}} + p_{K}K^{\frac{1}{\sigma_{F}}} + p_{Z}Z^{\frac{1}{\sigma_{F}}}}{p_{L}L + p_{K}K + p_{Z}Z} \right)^{\frac{\sigma_{F}}{\sigma_{F}-1}}$$

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## Model Summary

A grand total of 99 variables:

- ▶ 73 endogenous variables.
- 22 exogenous variables.
- 4 closure variables:
  - 2 endogenous variables (depending on which closure we choose).
  - 2 exogenous remaining variables: i) A nominal anchor, and ii) a real quantity.

#### **Endogenous Variables**

#### Endogenous Variables List (73)

FAC	p <sub>F</sub>	AV	$TX_{va}$	$p_{AV}$	$TX_{yy}$	$TR_{ff}$	Y
ICD	$p_Y$	OUT	М	р <sub>О</sub>	ACT	р <sub>М</sub>	$IC_S$
FC	$p_A$	ріс	I	$X_N$	<i>p<sub>FC</sub></i>	REM	EBE
MIX	$p_L$	р <sub>К</sub>	pz	REM <sub>HH</sub>	$F_L$	EBE <sub>HH</sub>	$EBE_{FR}$
EBE <sub>GV</sub>	R <sup>HH</sup>	R <sup>FR</sup>	R <sup>GV</sup>	R <sub>HH</sub>	R <sub>FR</sub>	$R_{GV}$	F <sub>K</sub>
R	$Y_{HH}$	$Y_{FR}$	$Y_{GV}$	TXhh	TXac <sub>FR</sub>	TXac <sub>GV</sub>	Т
SC <sup>HH</sup>	$SC_{GV}^{HH}$	$SB_{HH}^{FR}$	$SB_{HH}^{GV}$	СТ	СТ <sub>НН</sub>	$CT_{GV}$	ΝΤ <sub>ΗΗ</sub>
NT <sub>FR</sub>	$NT_{GV}$	С	S <sub>HH</sub>	DY <sub>HH</sub>	I <sub>HH</sub>	I <sub>FR</sub>	$S_{FR}$
p <sub>l</sub>	$S_{GV}$	Тx	p <sub>G</sub>	Х	$p_X$	$p_{X_N}$	$p_{X_T}$
$Y_{RW}$ .							

Exogenous Variables: A list (I)

- Factors:  $\overline{L}$ ,  $\overline{K}$ ,  $\overline{Z} \longrightarrow \text{DPI-BR}$
- Total Factor Productivity:  $\theta_F \longrightarrow \text{DPI-BR}$
- ▶ Indirect Taxes Rates:  $tx_{va}$ ,  $tx_{yy}$ ,  $tr_{ff}$   $\longrightarrow$  Calibrated (ftc)
- ► International Price of Imports: p<sup>\*</sup><sub>M</sub> → BoP / ToT (Imports Index)
- Traditional Export Quantities:  $\bar{X}_T \longrightarrow BOP$
- ▶ Direct Taxes Rates: txhh,  $txac_{FR}$ ,  $txac_{GV}$  → Calibrated (ftc)

#### Exogenous Variables: A list (II)

► HH payments to SC: SC<sup>HH</sup> → Pension Funds Financial Statements.

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- SB payments to HH: SB payments
- ► FR payments to CT: *CT*<sup>FR</sup>
- RW payments to CT:  $\overline{CT}^{RW} \longrightarrow BOP$
- GV payments to PT:  $\overline{PT}_{HH}^{GV}$

#### Exogenous Variables: A list (III)

- GV Investment:  $\bar{I}_{GV} \longrightarrow \text{DPI-BR}$
- GV Spending:  $\bar{G} \longrightarrow \text{DPI-BR}$
- ▶ Price of RW Imports:  $\bar{p}^*_{M^*} \longrightarrow WEO$  (External Inflation)
- RW Imports Quantities:  $\bar{M}^* \longrightarrow \text{BOP}$
- ► Traditional Exports Prices: p<sub>X<sub>T</sub></sub> → BoP / ToT (Exports Index)

Investment Closure

- Nominal Exchange Rate:  $\bar{e} \longrightarrow BOP$
- Private Investment:  $\bar{I}_{PR} \rightarrow \text{DPI-BR}$

Savings Closure

• Consumption Good Price:  $\bar{p}_C \longrightarrow \text{DPI-BR}$ 

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• External Savings:  $\bar{S}_{RW} \longrightarrow BOP$ 

#### Macro CGEM usage: An example

Using observed information from BOP, National Accounts and other relevant variables, we replicate 2012 economy taking as a starting point 2011 SAM and our model.

Main Results - 2012											
Variable	Observed	CGEM									
Valiable	Observed	Average	Investment	Savings							
GDP	4.0	4.1	4.2	4.1							
C	4.4	4.0	4.5	3.5							
G	5.7	5.7	5.7	5.7							
I	4.6	4.5	5.0	4.0							
I <sub>PR</sub>	4.9	4.4	4.9	3.8							
$I_{GV}$	5.3	5.3	5.3	5.3							
Х	6.1	5.0	4.6	5.3							
М	8.9	5.4	7.2	3.5							
CAD (%GDP)	3.1	3.6	4.0	3.1							

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#### Consistency of the model



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This model can be further extended along the following lines:

- Demand driven economy.
- Assuring BoP matching with the model.
- Multi-sector CGE model.
- Extension of Fiscal Block.
- Money in CGEM (anchor to Monetary accounts).

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# THE END

## Thank You.

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