

Welfare gains of the poor:  
An endogenous Bayesian approach with spatial random effects\*

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**Abstract**

We introduce a Bayesian instrumental variable procedure with spatial random effects that handles endogeneity, and spatial dependence with unobserved heterogeneity. We take into account the endogeneity through a system of simultaneous equations where a conditional correlation between the stochastic errors captures the endogeneity, and exclusion restrictions are used to treat endogenous regressors. In addition, we propose a Bayesian hierarchical spatial framework to model spatial dependence and heterogeneity. A Gibbs sampling algorithm is used to draw samples from all our conditional posterior distributions. We find through a limited Monte Carlo experiment that our proposal works well in terms of point and interval estimates, as well as prediction, compared with other possible alternatives. Taking advantage of the fact that a Bayesian framework permits easily performing statistical inference related to complicated non-linear functions of parameter estimates, we apply our method to analyze the welfare effects on the poorest households generated by a process of electricity tariff unification. In particular, we deduce an Equivalent Variation measure from a logarithmic demand function and a budget constraint for a two-tiered pricing scheme. We find the posterior distribution of the Equivalent Variation, and estimate the welfare implications in a context where electricity tariffs decreased by as much as 17.53%. We find that 10% of the poorest municipalities attained welfare gains above 2% of their initial income.

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# Introduction

We propose a Bayesian simultaneous equations system with spatial random effects suited to handling spatial dependence and heterogeneity, endogeneity, and statistical inference associated with complicated non-linear functions of the parameter estimates. In particular, we define a system of simultaneous equations where a conditional correlation between the stochastic errors captures the endogeneity, and instrumental variables are used to model the endogenous variables. In addition, we employ a Bayesian hierarchical spatial framework, based on a Conditional Autoregressive (CAR) spatial prior, to structure the unobserved heterogeneity and the spatial dependence. After model specification, we find the conditional posterior distributions of all the parameter sets, thus we can use Gibbs sampling algorithms to draw simulations of all our posterior distributions.

We perform a limited Monte Carlo simulation exercise where we find that our proposal obtains good outcomes regarding point and interval estimation compared with competing alternatives. In addition, prediction is substantially improved introducing spatial effects.

Establishing a Bayesian approach allows performing statistical inference related to functions of the parameter estimates using simple rules of probability theory. Thus, we apply our methodology to evaluate the welfare implications for poor households, measured through the Equivalent Variation, caused by the electricity price changes which took place in the province of Antioquia (Colombia), after *Empresas Públicas de Medellín* (EPM) acquired *Empresa Antioqueña de Energía* (EADE) in 2006. The Equivalent Variation is a non-linear function of parameter estimates of a demand function, which we estimate using data at the municipality level. Therefore, in our empirical exercise, we should take into consideration spatial effects, endogeneity between price and electricity demand, and unobserved heterogeneity due to latent economical, cultural and geographical factors. Finally, we want to perform a statistical inference regarding the Equivalent Variation. This application is interesting in itself because electricity services represent a significant share of households' budgets, and this fact is prominent for the poor population ([Gomez-Lobo, 1996](#), [Ruijs, 2009](#), [You and Lim, 2013](#)). As a consequence, small variations in

electricity prices may have relevant impacts on the welfare of households.

Consideration of spatial effects while performing statistical inference based on cross-sectional areal data has a long tradition in spatial statistics ([Cressie, 1993](#), [Ripley, 2005](#)), and more recently in spatial econometrics ([Anselin, 1988](#)). Most of the methods in spatial econometrics have been based on the frequentist approach, although there are some remarkable exceptions founded on the Bayesian framework ([LeSage, 1997, 2000](#), [Parent and LeSage, 2008](#), [LeSage and Pace, 2009](#), [LeSage and Llano, 2013](#)).

The issue of endogeneity emerges naturally due to the presence of the spatial lag of the dependent variable in Spatial Autoregressive (SAR) models, and spatial econometric estimators have taken this problem into consideration since its beginning ([Anselin, 1990](#), [Kelejian and Prucha, 1998, 1999](#)). However, the treatment of endogeneity due to other regressors has only recently been analyzed ([Rey and Boarnet, 2004](#), [Kelejian and Prucha, 2004](#), [Fingleton and Le Gallo, 2008](#), [Drukker et al., 2013](#), [Liu and Lee, 2013](#)). Thus, these new kind of spatial estimators take into consideration spatial dependence and feedback endogeneity simultaneously. However, since most of these methods rely on two-stage least squares estimation, recovering the parameters of the auxiliary, first stage regressions is usually not possible, given the complexity of implementing maximum likelihood estimation even in single equation settings ([Kelejian and Prucha, 2004](#), [Anselin and Lozano-Gracia, 2008](#)). At the same time, they fail to take into account unobserved heterogeneity, and have to rely on asymptotic methods, like the Delta method, to perform statistical inference regarding complicated non-linear functions of the parameter estimates.

Unobserved heterogeneity is another issue that may arise with cross-sectional areal data ([Parent and LeSage, 2008](#)). Unfortunately, to the best of our knowledge, there is only limited spatial econometric literature regarding this issue. This fact may be due to the difficulty of introducing unobserved heterogeneity in cross-sectional areal data using frequentist methods. However, [LeSage \(2000\)](#), [Smith and LeSage \(2004\)](#), [LeSage et al. \(2007\)](#), [Parent and LeSage \(2008\)](#), [Seya et al. \(2012\)](#) and [LeSage and Llano \(2013\)](#) have tackled unobserved heterogeneity using a Bayesian approach, simultaneously including

spatial effects and unobserved heterogeneity, but they do not take into consideration recursive endogeneity: an issue that has been considered from a Bayesian perspective in [Drèze \(1976\)](#), [Kleibergen and Van Dijk \(1998\)](#), [Zellner \(1998\)](#), [Kleibergen and Zivot \(2003\)](#).

Applications in the literature that examine the welfare effects of price changes in utilities, either through price variations within an existing structure, or different pricing schemes, are plentiful, but fail to take into account some important factors. [Acton and Mitchell \(1983\)](#) for example, look at the welfare effects on both consumers and producers from moving to a time-of-use price structure. The authors conclude that consumers attain a positive welfare gain from the new scheme, especially if they are highly price responsive. Another conclusion is that the effects are greater for those with larger levels of consumption. Their results, however, are based on the consumer surplus welfare measure, and therefore leave income effects out of the analysis. [Gomez-Lobo \(1996\)](#) estimate the welfare implications of tariff rebalancing for British energy markets, presenting some scenarios of price variations and showing that increased competition in the gas market can benefit most households, but also coming to the conclusion that the households in lower income groups stand to lose the most. Although the study makes use of Heckman's two-step estimator to correct for the selection bias of households' gas connectivity and some of the non-linearity in prices, the endogeneity between consumption and price is not resolved. Furthermore, the use of the Compensating Variation as a measure of welfare, given that it ranks price alternatives incorrectly, can lead to problems with the main conclusions. Finally, when working with pooled household data instead of panel data, controlling for unobserved heterogeneity is difficult and can significantly damage the estimated parameters and the inference. [You and Lim \(2013\)](#) suffers from the same issue with unobserved heterogeneity, and although they reject the use of instrumental variables in favor of ordinary least squares, the set of instruments they propose is very limited. The authors evaluate the welfare of households in different income deciles under several alternative price schemes. Non-linearity in prices for the Equivalent Variation is

taken into account in a fashion similar to ours for a linear demand function. The analysis concludes that price structures with lesser blocks are preferred when society is concerned about inequality, but flat rates perform better if lower-income households can be compensated through other means. [Ruijs \(2009\)](#) also establishes the Equivalent Variation measure with increasing-block pricing present in water consumption in Brazil, through a linear demand function, using aggregated data for 39 municipalities of the Metropolitan Region of São Paulo, estimating the aggregate demand function, and determining the welfare effects associated with several possible scenarios, including price changes in both blocks. He emphasizes the effects that different policies can have on different income groups and how important it is to look at these instead of the implications for the simple average welfare. He also notes how substantial the effects can be in terms of disposable income for the poorest households, even with slight price variations. These, and other studies, however, exhibit how welfare implications based on estimates that take into account spatial characteristics, endogeneity, and unobserved heterogeneity, are scarce in the literature (for example [Dodonov et al., 2004](#), [Lundgren, 2009](#)).

To conclude our literature review, we find that few authors have studied welfare effects due to changes within block price systems, and even fewer have introduced spatial random effects within an endogenous framework to analyze these welfare implications. From a theoretical standpoint, the main consideration for adopting the Bayesian approach is that it allows us to establish a statistical framework that simultaneously unifies decision theory, statistical inference, and probability theory under a single philosophically and mathematically consistent structure. From an empirical perspective, the Bayesian approach has some advantages in the present setting compared with the frequentist framework. In particular, we can easily make statistical inferences associated with the Equivalent Variation, which is a complicated function of the parameter estimates, using simple rules of probability theory, which could prove difficult with a frequentist statistical approach. In addition, our econometric approach takes into account the endogeneity issue that is present, where the Bayesian paradigm is less affected by the presence of weak instruments, allowing us

to identify the structural parameters from the reduced form in our empirical exercise. The frequentist procedures, however, deal with more severe identification problems in the presence of weak instruments. Finally, a Bayesian framework permits us to introduce spatial random effects in our cross-sectional areal data structure, and control the unobserved heterogeneity and autocorrelation that can arise in spatial settings. On the other hand, a frequentist approach does not allow us to easily take this phenomenon into account.

Using data at the municipality level for the province of Antioquia, and different spatial contiguity criteria, we find that the posterior mean of the price, income, substitute and urbanization rate demand elasticities are  $-0.88$ ,  $0.30$ ,  $0.12$  and  $0.57$ , respectively. In addition, the posterior mean of the semi-elasticity of electricity demand associated with a sea level dummy, which is equal to one when the municipality is located less than 1000 meters above sea level, and zero otherwise, is approximately  $0.14$ . With these estimates as inputs, we calculate the posterior distribution of the Equivalent Variation welfare measure as a share of income for each municipality. We deduce this measure using a logarithmic demand function, and taking into account a budget constraint for a two-tiered pricing scheme. We find that the average household enjoys a mean welfare gain of approximately  $0.87\%$  of their initial income, which can be considerable when taking their socioeconomic situation into account. However, these results depend heavily on whether the municipalities are part of the Metropolitan Area or not, on their average electricity consumption levels, and on other geographical and economic factors. For example, Medellín, the capital of the province, and its main center of economic activity, presented a mean welfare gain equal to  $0.14\%$ , which is approximately equal to the average improvement for all Metropolitan Area municipalities. On the other hand, municipalities located outside of the Metropolitan Area had, in total, mean welfare gains equal to  $0.94\%$ . In particular, 11 of the less urban municipalities, which are also the poorest, had welfare gains above  $2\%$  of their initial income. Just to serve as a reference, low income households in Colombia expend on average  $1.13\%$ ,  $2.04\%$ , and  $4.79\%$  of their income on pension, health care, and education, respectively (DANE, 2015). This illustrates how important are the welfare

implications of utility regulation: price changes in this sector may have huge effects on households' welfare, especially for the poorest.

The remainder of this paper is organized as follows. Section 1 outlines the complete endogenous Bayesian modeling strategy, the likelihood formulation, our prior specification, the deduction of the conditional posterior densities, as well as the results of our simulation exercises. Section 2 addresses the generalities of the Colombian energy market that are fundamental to the understanding of our application. Section 3 deals with the microeconomic foundation of the Equivalent Variation welfare measure, its ties to the econometric specification of the system of equations, and derives the measure for the specific case of a logarithmic demand function taking into consideration a budget constraint for a two-tiered pricing scheme. Section 4 is divided into four subsections. The first presents summary statistics for the data used in the econometric exercise. The second presents the specific characteristics of our econometric specification for the application. The third presents a summary of the results of our demand equation estimation, with some robustness checks regarding the spatial structure. The fourth presents the main findings for our application: the analysis of the posterior distribution of the welfare effects and its geographical patterns. Finally, Section 5 presents our conclusions.

## 1 Econometric Approach

We propose an endogenous Bayesian approach using simultaneous equations with spatial random effects, which takes into account unobserved heterogeneity and spatial dependence, in a context where there is recursive endogeneity. In particular, we employ an instrumental variable approach to handle the endogeneity issue. The specification of the model is

$$y_i = \pi_0 + \mathbf{z}'_{1i}\boldsymbol{\pi}_1 + \alpha x_i + u_{1i} + v_i \quad (1)$$

where  $y_i$  is the variable of interest that depends on a set of  $K_1$  exogenous controls

$\mathbf{z}_{1i}$ , and an endogenous regressor  $x_i$  such that  $E(x_i u_{1i}) \neq 0$ . Omitting this fact would generate biased and inconsistent parameter estimates.

In addition,  $u_{1i}$  is an idiosyncratic stochastic shock, and  $v_i$  is a spatial random effect to control for spatial heterogeneity and spatial dependence between cross-sectional units. This dependence emerges due to clusters and/or spillover effects between neighboring regional units, and allow us to control for unobservable spatial heterogeneity.

Given that we implement an instrumental variable approach to handle endogeneity, we set some exclusion restrictions in the main equation. These are associated with  $K_2$  instrumental variables  $\mathbf{z}_{2i}$  that do not affect  $y_i$  if  $x_i$  is held constant. Then,

$$x_i = \phi_0 + \mathbf{z}'_{1i}\phi_1 + \mathbf{z}'_{2i}\alpha_s + u_{2i} \quad (2)$$

where  $u_{2i}$  is an idiosyncratic stochastic shock such that  $(u_{1i}, u_{2i})' \sim \mathcal{N}(\mathbf{0}, \Omega)$ ,  $\Omega = \{\omega_{ij}\}$ . Thus,  $\omega_{12}$  captures the endogeneity of the system (Greenberg, 2008).

At this point, we should mention that an instrumental variable approach in the Bayesian framework has advantages compared with the frequentist framework. For instance, two-stage least squares and limited information maximum likelihood have some difficulties dealing with weak instruments and small samples (Angrist and Pischke, 2008), whereas the Bayesian approach does not require asymptotic criteria, and works well using weak instruments due to the fact that the likelihood function and its identification are less important for deriving estimates in Bayesian models (Zellner, 1996, Imbens and Rubin, 1997, Zellner, 1998, Crespo-Tenorio and Montgomery, 2013).

One of the most popular spatial econometric alternatives to our specification is an SAR model. For instance, Ohtsuka et al. (2010) use it in a Bayesian econometric framework to model electricity demand in Japan in a spatiotemporal setting without taking into consideration endogeneity issues. Unfortunately, an SAR specification cannot be used in a two-component disturbances decomposition (Parent and LeSage, 2008), like the one that we propose. In addition, parameter estimates do not have an easy interpretation in SAR models due to the presence of the spatial lags (Elhorst, 2014).



We should keep in mind that our final objective is to carry out a statistical inference related to complicated non-linear functions of the parameter estimates, for instance the Equivalent Variation Equations (20) and (21) in Section 3, which can be troublesome using a frequentist approach. Therefore, this is another argument in favor of using a Bayesian framework to accomplish this task. In particular, using frequentist methods would require estimating Equation (1) by means of instrumental variables (Rey and Boarnet, 2004, Kelejian and Prucha, 2004, Fingleton and Le Gallo, 2008, Drukker et al., 2013, Liu and Lee, 2013) or the generalized method of moments (Fingleton and Le Gallo, 2008, Drukker et al., 2013), and implementing spatial resampling algorithms (Lahiri et al., 2006) or the Delta method (Casella and Berger, 2002) to find the standard errors associated with functions of the parameter estimates. These tasks are difficult and tedious, require extra computational effort, and, more importantly, are based on asymptotic results. On the other hand, the Bayesian framework allows us to obtain full posterior distributions on all the parameters from Equation (1), and using simple probabilistic rules, we obtain the posterior distributions of the functions of parameter estimates without any additional computational effort nor assumptions regarding asymptotic outcomes (Bernardo, 2003).

The likelihood function of the system is

$$f(\mathbf{y}, \mathbf{x} | \mathbf{z}_1, \mathbf{z}_2; \Omega, \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}) = \frac{|\Omega \otimes \mathbf{I}_N|^{-1/2}}{(2\pi)^{N/2}} \times \exp \left\{ -\frac{1}{2} \sum_{i=1}^N (y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i, x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi}) \Omega^{-1} \begin{pmatrix} y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i \\ x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi} \end{pmatrix} \right\} \quad (3)$$

where  $\mathbf{w}'_i = [\mathbf{z}'_{1i}, x_i]$ ,  $\mathbf{z}'_i = [\mathbf{z}'_{1i}, \mathbf{z}'_{2i}]$ ,  $\boldsymbol{\pi}' = [\boldsymbol{\pi}'_1, \alpha]$  and  $\boldsymbol{\phi}' = [\boldsymbol{\phi}'_1, \boldsymbol{\alpha}'_s]$ .

Observe that the introduction of the spatial random effects,  $v_i, i = 1, 2, \dots, N$ , is another argument in favor of the Bayesian approach. In particular, the additional  $N$  parameters cannot be estimated by means of maximum likelihood methods due to the limited number of degrees of freedom (Seya et al., 2012). Thus, we follow a Bayesian hierarchical approach to model the spatial random effects where each unit is associated with a particular  $v_i$ , and the conditional distributions of these parameters depend on their

neighbors, through a contiguity matrix and a precision parameter that is drawn from a Gamma distribution.

In particular, we assume that each spatial random effect has as prior distribution an improper (intrinsic) conditionally autoregressive (CAR) structure (Besag et al., 1991):

$$v_i | \mathbf{v}_{i \sim j} \sim \mathcal{N} \left( \sum_{i \sim j} \frac{w_{ij} v_j}{\sum_{i \sim j} w_{ij}}, \frac{\sigma_v^2}{\sum_{i \sim j} w_{ij}} \right) \quad (4)$$

where  $\mathbf{v}_{i \sim j}$  is a vector composed of the spatial components of the stochastic error of the neighbors  $j$  of  $i$  ( $i \sim j$ ), and  $w_{ij}$  are the elements of the contiguity matrix which defines the spatial structure of the model. The joint distribution of the improper CAR is  $\mathbf{v} \sim \mathcal{N}_N(\underline{\mathbf{v}}, \sigma_v^2(\mathbf{D}_W - \mathbf{W}_N)^{-1})$  where  $\mathbf{W}_N$  is the contiguity matrix and  $\mathbf{D}_W = \text{diag}(\sum_{i \sim j} w_{ij})$  (Banerjee et al., 2004, Wall, 2004). Despite the fact that  $\mathbf{v}$  has an improper distribution, Theorem 2 in Sun et al. (1999) guarantees that a proper posterior distribution exists if  $(\mathbf{D}_W - \mathbf{W}_N)$  is nonnegative definite, the precision parameters have Gamma prior distributions, and the intercepts have diffuse prior distributions. We satisfy all these criteria.

The contiguity relation is binary, that is, if region  $i$  and  $j$  are neighbors, the element  $ij$  is equal to 1 and 0 otherwise, thus the contiguity matrix is symmetric, which is a requirement of the CAR model. By definition, the elements on the main diagonal of the contiguity matrix are set equal to 0.  $\sigma_v^2$  is a parameter that defines the conditional variance of the spatial process, where the conditional variance must be inversely proportional to the number of neighbors.

There is a spatial literature that favors CAR priors (Banerjee et al., 2004, Parent and LeSage, 2008, Darmofal, 2009, Chakraborty et al., 2013), and another that supports SAR prior specifications (Smith and LeSage, 2004, LeSage et al., 2007, LeSage and Llano, 2013). Our decision to use a CAR prior distribution to model the spatial random effects, instead of an SAR prior, is due to the fact that heteroscedasticity is inherent to the CAR specification, and we achieve a higher level of heterogeneity (Cressie, 1993). In addition, the CAR specification is a Markovian process in space, that is, the spatial heterogeneity

is due to local variation, rather than a global spatial pattern, which is present in SAR specification (Anselin, 2003). Our intuition is that the unobserved heterogeneity present in our application, which is related to residential electricity consumption in a municipality, is affected by the first order neighbors (see Section 4, Maps (2) and (3) and their comments). Finally, a CAR prior offers computational convenience because we just need to work with its conditional distributions, avoiding matrix inversion. On the other hand, SAR models do not have full conditional distributions with a convenient form, and this increases the computational burden (Banerjee et al., 2004).

It is well known that the joint distribution of a CAR process is improper, and although we can obtain a proper CAR process by just introducing a single parameter, we work with an improper rather than a proper prior because the latter includes the spatial autocorrelation parameter that needs to lie in a specific region, usually between  $-1$  and  $1$ . As a consequence, the final solution using MCMC techniques becomes more complicated and computationally expensive, and we would need to use a Gibbs sampler with some steps of the Metropolis–Hastings algorithm (Greenberg, 2008). Additionally, this spatial autocorrelation term also limits the set of spatial patterns that the distribution can replicate and becomes much less intuitive (Banerjee et al., 2004).

We should bear in mind that the improper CAR is identified only up to an additive constant, thus to identify the intercepts in our model, it is necessary to add the constraint  $\sum_{i=1}^N v_i = 0$ . As a consequence, it is necessary to use improper uniform priors for the constant terms ( $\pi_0$  and  $\phi_0$ ) in both equations.

To complete our Bayesian specification, we set the remaining priors as follows:  $\boldsymbol{\pi} \sim \mathcal{N}_{K_1+1}(\boldsymbol{\pi}, \mathbf{\Pi})$  and  $\boldsymbol{\phi} \sim \mathcal{N}_{K_1+K_2}(\boldsymbol{\phi}, \mathbf{\Phi})$ . In our application, we set  $\boldsymbol{\pi} = \mathbf{0}_{K_1+1}$ ,  $\boldsymbol{\phi} = \mathbf{0}_{K_1+K_2}$ ,  $\mathbf{\Pi} = 1000\mathbf{I}_{K_1+1}$  and  $\mathbf{\Phi} = 1000\mathbf{I}_{K_1+K_2}$ . This implies vague prior information where there is no effect of each control variable on the dependent variables.

In addition, we assume a Wishart distribution for  $\Omega^{-1}$ , that is,  $\Omega^{-1} \sim \mathcal{W}_2(\underline{\omega}, \underline{\Omega})$ . In particular, we set  $\underline{\omega} = 3$  and  $\underline{\Omega} = \mathbf{I}_2$ , where setting the degrees of freedom to  $p + 1$ , where  $p$  is the dimension of the covariance matrix, the Wishart form reduces

to  $\pi(\Omega^{-1}) \propto |\Omega^{-1}|^{-(N+1)/2}$ , which is a diffuse prior used by Savage that emerges using Jeffrey’s invariance theory (Zellner, 1996). Thus, a priori, there is no endogeneity, and the fat-tailed prior will guarantee the robustness of the outcomes regarding this distribution (Berger, 1985).

To specify the prior distribution of the precision parameter of the CAR component, we must take into account that there are two different sources of stochastic variability in our main equation,  $u_{1i}$  and  $v_i$ . As a consequence, both sets of hyperparameters of the prior distributions of these random effects cannot imply arbitrarily large variability, since these effects would be unidentifiable. We try to identify two random effects using a single observation at each spatial unit. Therefore, we cannot use arbitrarily vague prior distributions in our hierarchical approach. We propose a *fair* argument to construct the prior distribution of the precision parameter of the CAR component (Banerjee et al., 2004). Specifically, we posit a priori that the proportion of the variability due to spatial effects is 0.5, that is, we set  $Var(u_{1i}) = Var(v_i)$ . Thus, taking into consideration that  $u_{1i} \sim \mathcal{N}(0, \omega_{11})$  where  $\omega_{11} \sim \mathcal{IG}((\underline{\omega} - 1)/2, 1/2)$  due to our prior assumptions, and  $Var(u_{1i}) = \omega_{11} \approx \frac{\sigma_v^2}{0.72(\sum_{i \sim j} w_{ij})^{Ave}} \approx Var(v_i)$  (Bernardinelli et al., 1995) where  $(\sum_{i \sim j} w_{ij})^{Ave}$  is the average number of neighbors, we obtain that the prior distribution of  $1/\sigma_v^2$  is approximately proportional to  $\mathcal{G}(\frac{\underline{\omega}-1}{2}, 1/2)$ . Moreover, we find in our application that the posterior parameter estimates are robust to changes of the hyperparameters of the CAR’s precision (available upon request).

We assume that the prior distributions are independent, that is,

$$\pi(\Omega, \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}_i, \sigma_v^2) = \pi(\Omega)\pi(\boldsymbol{\pi})\pi(\boldsymbol{\phi})\pi(\pi_0)\pi(\phi_0)\pi(\mathbf{v}_i|\sigma_v^2)\pi(\sigma_v^2) \quad (5)$$

The full conditional posteriors for all parameters are

$$\begin{aligned} \Omega | \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}, Data &\sim \mathcal{IW}_2(\bar{\omega}, \bar{\Omega}) \\ \bar{\omega} &= \underline{\omega} + N \\ \bar{\Omega} &= \left[ \underline{\Omega}^{-1} + \sum_{i=1}^N \begin{pmatrix} y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i \\ x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi} \end{pmatrix} (y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i, x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi}) \right] \end{aligned} \quad (6)$$

To sample  $\boldsymbol{\pi}$ , we use  $f(y_i, x_i | \boldsymbol{\Theta}) = f(y_i | x_i, \boldsymbol{\Theta}) f(x_i | \boldsymbol{\Theta})$  where  $\boldsymbol{\Theta} = (\Omega, \boldsymbol{\pi}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v})$ . In particular,  $y_i | x_i, \boldsymbol{\Theta} \sim \mathcal{N}(\pi_0 + \mathbf{w}'_i \boldsymbol{\pi} + v_i + \frac{\omega_{12}}{\omega_{22}}(x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi}), \psi_{11})$  where  $\psi_{11} = \omega_{11} - \frac{\omega_{12}^2}{\omega_{22}}$ . Then,

$$\begin{aligned} \boldsymbol{\pi} | \Omega, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}, Data &\sim \mathcal{N}_{K_1+1}(\bar{\boldsymbol{\pi}}, \bar{\boldsymbol{\Pi}}) \\ \bar{\boldsymbol{\Pi}} &= [\underline{\boldsymbol{\Pi}}^{-1} + \psi_{11}^{-1} \mathbf{W}' \mathbf{W}]^{-1} \\ \bar{\boldsymbol{\pi}} &= \bar{\boldsymbol{\Pi}} [\underline{\boldsymbol{\Pi}}^{-1} \boldsymbol{\pi} + \psi_{11}^{-1} \mathbf{W}' \mathbf{y}_1] \end{aligned} \quad (7)$$

where  $\mathbf{W}$  is an  $N \times (K_1 + 1)$  matrix whose rows are  $\mathbf{w}'_i$  and  $\mathbf{y}_1$  is an  $N \times 1$  vector whose elements are  $y_i - \frac{\omega_{12}}{\omega_{22}}(x_i - \phi_0 - \mathbf{z}'_i \boldsymbol{\phi}) - \pi_0 - v_i$ .

We follow the same procedure to deduce the conditional posterior distribution of  $\boldsymbol{\phi}$ , that is, we use  $f(y_i, x_i | \boldsymbol{\Theta}) = f(x_i | y_i, \boldsymbol{\Theta}) f(y_i | \boldsymbol{\Theta})$ . In particular,  $x_i | y_i, \boldsymbol{\Theta} \sim \mathcal{N}(\phi_0 + \mathbf{z}'_i \boldsymbol{\phi} + \frac{\omega_{12}}{\omega_{11}}(y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i), \psi_{22})$  where  $\psi_{22} = \omega_{22} - \frac{\omega_{12}^2}{\omega_{11}}$ . Thus,

$$\begin{aligned} \boldsymbol{\phi} | \boldsymbol{\pi}, \Omega, \pi_0, \phi_0, \mathbf{v}, Data &\sim \mathcal{N}_{K_1+K_2}(\bar{\boldsymbol{\phi}}, \bar{\boldsymbol{\Phi}}) \\ \bar{\boldsymbol{\Phi}} &= [\underline{\boldsymbol{\Phi}}^{-1} + \psi_{22}^{-1} \mathbf{Z}' \mathbf{Z}]^{-1} \\ \bar{\boldsymbol{\phi}} &= \bar{\boldsymbol{\Phi}} [\underline{\boldsymbol{\Phi}}^{-1} \boldsymbol{\phi} + \psi_{22}^{-1} \mathbf{Z}' \mathbf{y}_2] \end{aligned} \quad (8)$$

where  $\mathbf{Z}$  is an  $N \times (K_1 + K_2)$  matrix whose rows are  $\mathbf{z}'_i$  and  $\mathbf{y}_2$  is an  $N \times 1$  vector whose elements are  $x_i - \frac{\omega_{12}}{\omega_{11}}(y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i) - \phi_0$ .

Regarding the posterior distribution of the constant term  $\pi_0$ , using as prior an im-

proper uniform distribution and given  $f(y_i, x_i | \Theta) = f(y_i | x_i, \Theta) f(x_i | \Theta)$ , we obtain

$$\begin{aligned} \pi_0 | \phi, \boldsymbol{\pi}, \Omega, \phi_0, \mathbf{v}, Data &\sim \mathcal{N}(\bar{\pi}_0, \psi_{11}/N) \\ \bar{\pi}_0 &= \frac{1}{N} \sum_{i=1}^N \left\{ y_i - \frac{\omega_{12}}{\omega_{22}} (x_i - \phi_0 - \mathbf{z}'_i \phi) - \mathbf{w}'_i \boldsymbol{\pi} - v_i \right\} \end{aligned} \quad (9)$$

In a similar way, using as prior an improper uniform distribution for  $\phi_0$ , and the fact that  $f(y_i, x_i | \Theta) = f(x_i | y_i, \Theta) f(y_i | \Theta)$ , we obtain

$$\begin{aligned} \phi_0 | \pi_0, \phi, \boldsymbol{\pi}, \Omega, \mathbf{v}, Data &\sim \mathcal{N}(\bar{\phi}_0, \psi_{22}/N) \\ \bar{\phi}_0 &= \frac{1}{N} \sum_{i=1}^N \left\{ x_i - \frac{\omega_{12}}{\omega_{11}} (y_i - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi} - v_i) - \mathbf{z}'_i \phi \right\} \end{aligned} \quad (10)$$

As we mentioned, we just need to use the conditional prior distribution to obtain the posterior distribution of the spatial random effects. In particular, using the fact that  $f(y_i, x_i | \Theta) = f(y_i | x_i, \Theta) f(x_i | \Theta)$ , we find that

$$\begin{aligned} v_i | \mathbf{v}_{-i}, \phi_0, \pi_0, \phi, \boldsymbol{\pi}, \Omega, \sigma_v^2, Data &\sim \mathcal{N}(\bar{\xi}_i, \bar{\eta}_i) \\ \bar{\eta}_i &= \left[ \psi_{11}^{-1} + \left( \frac{\sigma_v^2}{w_{i+}} \right)^{-1} \right]^{-1} \\ \bar{\xi}_i &= \bar{\eta}_i \left[ \left( \frac{\sigma_v^2}{w_{i+}} \right)^{-1} \left( \sum_{i \sim j} \frac{w_{ij}}{w_{i+}} v_j \right) + \psi_{11}^{-1} v_i^0 \right] \end{aligned} \quad (11)$$

where  $\mathbf{v}_{-i}$  is the set of spatial random effects excluding region  $i$ ,  $w_{i+} = \sum_{i \sim j} w_{ij}$  and  $v_i^0 = y_i - \frac{\omega_{12}}{\omega_{22}} (x_i - \phi_0 - \mathbf{z}'_i \phi) - \pi_0 - \mathbf{w}'_i \boldsymbol{\pi}$ . To identify  $\pi_0$ , we must add the constraint  $\sum_{i=1}^N v_i = 0$ . Therefore, this constraint must be imposed numerically by recentering each  $\mathbf{v}$  vector around its own mean following each Gibbs iteration.

In addition,

$$\begin{aligned}\mathbf{v}|\phi_0, \pi_0, \boldsymbol{\phi}, \boldsymbol{\pi}, \Omega, \sigma_v^2, \text{Data} &\sim \mathcal{N}_N(\bar{\mathbf{v}}, \bar{\mathbf{V}}) \\ \bar{\mathbf{V}} &= [\psi_{11}^{-1}\mathbf{I}_N + \sigma_v^{-2}(\mathbf{D}_W - \mathbf{W}_N)]^{-1} \\ \bar{\mathbf{v}} &= \bar{\mathbf{V}}[\psi_{11}^{-1}\mathbf{v}^0]\end{aligned}$$

where  $\mathbf{v}^0$  is an  $N \times 1$  vector whose elements are  $v_i^0$ .

Finally,

$$\begin{aligned}1/\sigma_v^2|\mathbf{v} &\sim \mathcal{G}(\bar{\alpha}, \bar{\beta}) \\ \bar{\alpha} &= \frac{\omega - 1}{2} + N/2 \\ \bar{\beta} &= 1/2 + \frac{\mathbf{v}'(\mathbf{D}_W - \mathbf{W}_N)\mathbf{v}}{2}\end{aligned}\tag{12}$$

An unfortunate consequence of introducing spatial random effects is a reduction of the efficiency of MCMC sampling schemes. This in turn generates poor mixing and slow convergence (Best et al., 1999). To mitigate this problem, we draw multivariate blocks from distributions (6)–(12), whenever possible, using the Gibbs sampler algorithm (Geman and Geman, 1984).

## 1.1 Simulation Exercises

In this subsection we present the results of a limited Monte Carlo experiment comparing Bayesian and frequentist estimators applied to a very simple two-equation simultaneous model with spatial effects. The main objective of these exercises is to illustrate the consequences for parameter estimates and prediction of omitting important factors of the data generating process of the estimators.

In particular, we implement five estimators: our proposed Bayesian Instrumental Variable with Spatial Effects, two instrumental variable approaches without spatial effects (one Bayesian and one frequentist), a maximum likelihood estimator with conditional

autoregressive spatial effects, and ordinary least squares.

Regarding the estimation of the endogenous Bayesian model with spatial effects, we implement the Gibbs sampler algorithm using one million iterations and a burn-in of 500,000. Then, we draw an observation every 50 iterations to have an effective sample size of 10,000. This last step is done to mitigate the autocorrelation of the chains. All the chains seem stable, and different diagnostics indicate that the chains converge to stationary distributions (outcomes available upon request). The same procedure was followed to implement the Bayesian instrumental model without spatial effects, except that in this case it was only necessary to iterate 100,000 times with a burn-in period of 50,000, drawing every 5 iterations, to achieve convergence.

The formulation of our model is

$$y_i = \pi_0 + \pi_1 x_i + v_i + u_{1i}$$

$$x_i = \phi_0 + \phi_1 z_{1i} + \phi_2 z_{2i} + u_{2i}$$

where  $\mathbf{v} \sim \mathcal{N}_N(\mathbf{0}, \sigma_v^2(\mathbf{D}_W - \mathbf{W}_N)^{-1})$ ,  $\begin{bmatrix} u_{1i} \\ u_{2i} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix}\right)$ ,  $z_{1i} \sim \mathcal{N}(0, \sigma_z^2)$  and  $z_{2i} \sim \mathcal{N}(0, \sigma_z^2)$ .

We generate four possible scenarios under the conditions summarized in Table (1). In all four scenarios the parameters  $\pi_0$ ,  $\pi_1$ ,  $\phi_0$  and  $\phi_1$  were set to 0.7,  $-1.2$ , 0.5 and 0.8, respectively. On the other hand,  $\phi_2$  was set to 0 for runs I and III and  $-1.0$  for runs II and IV. This is to illustrate the consequences on just-identified and over-identified cases. In runs I to IV, the covariance structure of the stochastic shocks is the same as well as the spatial random effects, which are based on a rook binary contiguity criterion. In addition, the variance parameter of the CAR effect is such that 50% of the variance of the main interest variable  $y_i$  is explained by the spatial effect (Bernardinelli et al., 1995). Finally, the variance of the instruments is equal to 0.2 in scenarios I and II and 2 in scenarios III and IV. The main idea is to show the consequences associated with using weak instruments.



**Table 1:** Conditions under which data were generated

Run I	Run II	Run III	Run IV
$\pi_0 = 0.7$	$\pi_0 = 0.7$	$\pi_0 = 0.7$	$\pi_0 = 0.7$
$\pi_1 = -1.2$	$\pi_1 = -1.2$	$\pi_1 = -1.2$	$\pi_1 = -1.2$
$\phi_0 = 0.5$	$\phi_0 = 0.5$	$\phi_0 = 0.5$	$\phi_0 = 0.5$
$\phi_1 = 0.8$	$\phi_1 = 0.8$	$\phi_1 = 0.8$	$\phi_1 = 0.8$
$\phi_2 = 0.0$	$\phi_2 = -1.0$	$\phi_2 = 0.0$	$\phi_2 = -1.0$
$\sigma_{11} = 1$	$\sigma_{11} = 1$	$\sigma_{11} = 1$	$\sigma_{11} = 1$
$\sigma_{22} = 1$	$\sigma_{22} = 1$	$\sigma_{22} = 1$	$\sigma_{22} = 1$
$\sigma_{12} = -0.5$	$\sigma_{12} = -0.5$	$\sigma_{12} = -0.5$	$\sigma_{12} = -0.5$
$\mathbf{W}_N$ is rook $\{0, 1\}$	$\mathbf{W}_N$ is rook $\{0, 1\}$	$\mathbf{W}_N$ is rook $\{0, 1\}$	$\mathbf{W}_N$ is rook $\{0, 1\}$
$\sigma_v^2 = 0.7^2 \left( \sum_{i \sim j} w_{ij} \right)^{Ave}$	$\sigma_v^2 = 0.7^2 \left( \sum_{i \sim j} w_{ij} \right)^{Ave}$	$\sigma_v^2 = 0.7^2 \left( \sum_{i \sim j} w_{ij} \right)^{Ave}$	$\sigma_v^2 = 0.7^2 \left( \sum_{i \sim j} w_{ij} \right)^{Ave}$
$\sigma_z^2 = 0.2$	$\sigma_z^2 = 0.2$	$\sigma_z^2 = 2$	$\sigma_z^2 = 2$

Observe that if  $\sigma_{12} = 0$  and  $\mathbf{v} = \mathbf{0}$ , then there are no endogeneity or spatial effects, so we should use OLS, whereas if  $\sigma_{12} = 0$  but  $\mathbf{v} \neq \mathbf{0}$ , the MLE with CAR effects is appropriate. In addition,  $\sigma_{12} \neq 0$  and  $\mathbf{v} = \mathbf{0}$  requires taking into account endogeneity issues without spatial effects, so IV estimators (frequentist and Bayesian) are good alternatives. Finally, an IV with spatial effects is required if both  $\sigma_{12}$  and  $\mathbf{v}$  are not equal to zero. Therefore, we implement different estimators designed to take into account different nested models.

For all scenarios, we generate samples of size 49, 100 and 144. However, we do not use repeated trials to assess the estimators' performance, which is the common approach employed in the frequentist framework, because this methodology is not the most appropriate for judging alternative approaches, and additionally, it is not epistemologically consistent with the Bayesian statistical framework (Zellner, 1996).

We present in Table (2) the posterior means and the highest probability density (HPD) 90% credible intervals for the Bayesian estimators of  $\pi_1$ . Observe that the posterior conditional distribution of this parameter is normal, so the mean, median and mode are the same, and the symmetric credible interval is equal to the HPD interval as well. In addition, for the frequentist estimators, this table shows the point estimates and the 90% confidence intervals under the assumption of asymptotic normality. As we can see in Table

(2), OLS and MLE have a poor performance in general. On the other hand, instrumental variable techniques (Bayesian and frequentist) generate good results regarding point estimates using strong instruments and relatively large sample sizes (100 and 144). Although not directly comparable but with some similarities using non-informative priors (Zellner, 1996), the highest probability density interval at 90% is narrower than the confidence interval at 90%. We can see in this table that in the case of exactly identified models, the intervals are wider than in the over-identified case. Moreover, weak instruments generate poor point estimates (Kleibergen and Van Dijk, 1998, Angrist and Pischke, 2008), although 90% intervals of instrumental estimators always embrace the true value  $-1.2$ , and the HPD credible interval of the endogenous Bayesian approach with spatial effects are considerably narrower compared to alternative approaches. In addition, small sample sizes generate problems in the point estimates and confidence intervals of the frequentist instrumental variable estimator, whereas the Bayesian approach renders less critical the issue of how large is large, (Zellner, 1996, 1998).

We use the mean squared prediction error and the mean absolute prediction error to assess the forecasting performance of the estimators. As can be seen in Table (3), the Bayesian Instrumental Variable with Spatial Effects obtains by far the lowest MSPE and MAPE, followed by the maximum likelihood estimator with CAR effects. Taking into account spatial effects substantially improves the prediction performance (Reich et al., 2006).

## 2 The Colombian energy market

To better understand our application and its microeconomic foundation, there are some characteristics of the Colombian electricity market that must be taken into consideration. In particular, we must explain the price changes, and thus, their welfare implications. First, Colombian law divides its population into socioeconomic strata. This segmentation is defined as “an instrument that allows a municipality or district to classify its population

**Table 2:** Simulation results: Point Estimates, 90% Highest Probability Density Intervals and Confidence Intervals

Instrument Type	Sample Size	Exactly Identified															
		Bayesian						Frequentist									
		IV CAR		90% HPD		Mean	IV		90% HPD		Est.	OLS		IV		ML CAR	
Weak Instruments	49	-1.50	(-1.96,-1.03)	-1.73	(-2.44,-0.96)	-1.74	(-2.05,-1.42)	-1.70	(-3.80,-0.40)	-1.70	(-1.97,-1.42)	-1.70	(-3.80,-0.40)	-1.70	(-1.97,-1.42)	-1.63	(-1.84,-1.41)
	100	-0.66	(-1.23,-0.09)	-0.41	(-1.13,0.30)	-1.64	(-1.87,-1.42)	0.37	(-2.02,2.76)	-1.63	(-1.84,-1.41)	-1.75	(-5.69,2.19)	-1.72	(-1.89,-1.54)	-1.72	(-1.89,-1.54)
	144	-1.69	(-2.54,-0.89)	-1.75	(-2.89,-0.62)	-1.74	(-1.94,-1.53)	-1.74	(-1.94,-1.53)	-1.75	(-1.89,-1.54)	-1.24	(-1.45,-1.02)	-1.32	(-1.46,-1.17)	-1.22	(-1.34,-1.09)
Strong Instruments	49	-1.19	(-1.27,-1.10)	-1.25	(-1.34,-1.16)	-1.38	(-1.55,-1.20)	-1.24	(-1.45,-1.02)	-1.32	(-1.46,-1.17)	-1.22	(-1.19,-0.84)	-1.22	(-1.34,-1.09)	-1.37	(-1.47,-1.26)
	100	-1.03	(-1.12,-0.94)	-1.03	(-1.11,-0.94)	-1.22	(-1.35,-1.08)	-1.02	(-1.19,-0.84)	-1.22	(-1.34,-1.09)	-1.38	(-1.49,-1.26)	-1.22	(-1.35,-1.07)	-1.37	(-1.47,-1.26)
	144	-1.20	(-1.27,-1.13)	-1.22	(-1.29,-1.14)	-1.38	(-1.49,-1.26)	-1.38	(-1.49,-1.26)	-1.22	(-1.35,-1.07)	-1.22	(-1.35,-1.07)	-1.22	(-1.35,-1.07)	-1.37	(-1.47,-1.26)
Over Identified																	
Instrument Type	Sample Size	Exactly Identified															
		Bayesian						Frequentist									
		IV CAR		90% HPD		Mean	IV		90% HPD		Est.	OLS		IV		ML CAR	
Weak Instruments	49	-1.14	(-1.74,-0.51)	-0.31	(-1.11,0.43)	-1.74	(-2.19,-1.28)	0.48	(-1.61,2.56)	-1.69	(-2.05,-1.32)	-1.69	(-2.05,-1.32)	-1.69	(-2.05,-1.32)	-1.58	(-1.77,-1.39)
	100	-1.49	(-1.93,-1.02)	-1.37	(-1.95,-0.79)	-1.60	(-1.81,-1.38)	-1.29	(-2.34,-0.23)	-1.58	(-1.77,-1.39)	-1.33	(-2.09,-0.56)	-1.67	(-1.81,-1.52)	-1.67	(-1.81,-1.52)
	144	-1.49	(-1.83,-1.15)	-1.38	(-1.84,-0.91)	-1.59	(-1.78,-1.40)	-1.11	(-1.27,-0.94)	-1.19	(-1.32,-1.05)	-1.25	(-1.33,-1.16)	-1.19	(-1.28,-1.09)	-1.26	(-1.33,-1.18)
Strong Instruments	49	-1.13	(-1.22,-1.04)	-1.00	(-1.09,-0.93)	-1.11	(-1.27,-0.94)	-1.11	(-1.17,-0.82)	-1.19	(-1.32,-1.05)	-1.27	(-1.34,-1.19)	-1.22	(-1.30,-1.13)	-1.28	(-1.34,-1.21)
	100	-1.20	(-1.25,-1.16)	-1.19	(-1.22,-1.15)	-1.25	(-1.33,-1.16)	-1.19	(-1.28,-1.09)	-1.26	(-1.33,-1.18)	-1.27	(-1.34,-1.19)	-1.22	(-1.30,-1.13)	-1.28	(-1.34,-1.21)
	144	-1.22	(-1.26,-1.18)	-1.22	(-1.25,-1.18)	-1.27	(-1.34,-1.19)	-1.27	(-1.34,-1.19)	-1.22	(-1.30,-1.13)	-1.22	(-1.30,-1.13)	-1.22	(-1.30,-1.13)	-1.28	(-1.34,-1.21)

Source: Author's calculations

**Table 3:** Simulation results: Mean Squared Prediction Error and Mean Absolute Prediction Error

		Exactly Identified													
Instrument Type	Sample Size	Bayesian						Frequentist							
		IV CAR		IV		OLS		IV		ML CAR		MAPE			
		MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE		
Weak Instruments	49	0.42	0.53	1.45	0.98	1.45	0.98	1.45	0.98	1.45	0.98	1.45	0.98	1.03	0.88
	100	2.28	1.16	3.28	1.39	1.87	1.07	5.65	1.81	1.57	0.99	1.93	1.05	1.51	0.95
	144	1.20	0.86	1.93	1.05	1.93	1.05	1.93	1.05	1.93	1.05	1.93	1.05	1.51	0.95
Strong Instruments	49	0.58	0.62	1.65	1.01	1.60	1.00	1.65	1.01	1.65	1.01	1.65	1.01	1.19	0.93
	100	1.3	0.89	2.17	1.11	2.05	1.09	2.18	1.11	1.75	1.04	2.18	1.11	1.75	1.04
	144	1.30	0.88	2.18	1.14	2.10	1.12	2.18	1.14	1.70	1.02	2.18	1.14	1.70	1.02
Over Identified															
Instrument Type	Sample Size	Bayesian						Frequentist							
		IV CAR		IV		OLS		IV		ML CAR		MAPE			
		MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE	MSPE	MAPE		
Weak Instruments	49	1.04	0.78	4.30	1.65	2.72	1.27	6.54	2.04	1.72	1.05	2.04	1.06	1.35	0.93
	100	0.28	0.43	1.69	1.05	1.63	1.03	1.73	1.06	1.46	0.98	1.73	1.06	1.35	0.93
	144	0.58	0.61	2.26	1.17	2.21	1.16	2.29	1.18	1.46	0.98	2.29	1.18	1.46	0.98
Strong Instruments	49	1.27	0.86	2.97	1.33	2.90	1.30	2.98	1.33	1.88	1.14	2.98	1.33	1.88	1.14
	100	0.61	0.64	1.81	1.09	1.78	1.09	1.81	1.09	1.48	1.00	1.81	1.09	1.48	1.00
	144	0.81	0.70	2.37	1.20	2.35	1.20	2.37	1.20	1.73	1.05	2.37	1.20	1.73	1.05

Source: Author's calculations

in distinct groups or strata with similar social and economic characteristics.” (Bushnell and Hudson, 1996). This classification was actually initiated to establish cross-subsidies that would help the lower socioeconomic classes to pay for utilities such as electricity. Housing characteristics are the main criteria used for classifying the population into six strata: one represents lower-low, two is low, three is upper-low, four is medium, five is medium-high, and six is high. Second, the Colombian energy regulator establishes a subsistence electricity consumption that is subsidized for strata one, two and three. The regulator determines the maximum subsidy percentage, and each municipality defines its own measure within this limit. In addition, the subsistence consumption level depends on whether the altitude of the municipality exceeds one thousand meters above sea level or not, due to weather conditions that may affect electricity consumption. Municipalities located near sea level have higher temperatures, and as a consequence they present a higher electricity consumption. Specifically, the subsistence consumption is 173 kWh a month per household for the municipalities below this threshold and 130 kWh for the ones above it.<sup>1</sup> Third, the Colombian energy regulator stipulates that each electric company must have the same reference tariff throughout its entire market, which involves many municipalities. And fourth, there are basically four components to establish the reference electricity tariff for each company: electricity generation, transport at the country level, distribution at the market level, and commercialization. As a consequence of this regulation framework, we should bear in mind that although there is just one reference tariff for each electric company, there are different average electricity prices among the consumers of different strata and municipalities.

The acquisition of EADE by EPM led to a tariff unification process that generated welfare effects, especially for the households belonging to stratum one. Such a household’s electricity consumption is approximately 5% of its income in the province of Antioquia, whereas this share is less than 1% for stratum six. In particular, there is EPM, whose market was characterized as an urban region with high population density, and on the other

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<sup>1</sup>Resolution 0355 of the Mining/Energy Planning Unit (UPME).

hand there is EADE, whose market was a rural area with low population density. Under the Colombian electricity regulation framework, *ceteris paribus*, these market structure differences imply a higher reference tariff in the latter company than in the former. This is because of the third and four components of the reference tariff: distribution at the market level and commercialization. Thus, the acquisition of EADE by EPM implied that the stratum one electricity consumers of the former company experienced a huge decrease in their electric bills, while the consumers of the latter company faced a slight increase.<sup>2</sup> As a consequence, these changes generated considerable welfare impacts on the poorest inhabitants of the province of Antioquia, who live in the rural areas.

### 3 Microeconomic Foundations: Equivalent Variation

We apply our proposed methodology to analyze the welfare changes arising from the tariff unification in the municipalities of Antioquia using an Equivalent Variation (EV) approach. The Equivalent Variation measures the “amount that the consumer would be indifferent about accepting in lieu of the price change; that is, it is the change in her wealth that would be equivalent to the price change in terms of its welfare impact” (Mas-Colell et al., 1995). The EV presents several advantages over other welfare measures used in applied economic work, such as the Compensating Variation (CV) and consumer surplus (CS). In particular, Chipman and Moore (1980) and Mas-Colell et al. (1995) show that the EV is the appropriate measure to correctly order different pricing policies in welfare analysis. The CV orders alternatives correctly only when consumers exhibit homothetic preferences and income remains unchanged. However, in our empirical application, tariff unification translates into implied subsidies for some consumers, and therefore, changes in income. Another argument in favor of the EV is that, by definition, it is an ex-post measure of welfare based on the Hicksian demand function. It takes into account income effects associated with price changes, which are ignored by the Marshallian demand

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<sup>2</sup>Regulations stipulate that strata one and two cannot have a tariff increase higher than the inflation rate.

function on which the CS is based on. Furthermore, [Hausman \(1981\)](#) showed that it was possible to derive EV as a product of observable Marshallian demand functions. His method can be applied to the case of linear budget constraints, and both [Reiss and White \(2006\)](#) and [Ruijs \(2009\)](#) extend it to the case of budget constraints generated by block-pricing systems using linear demand functions.

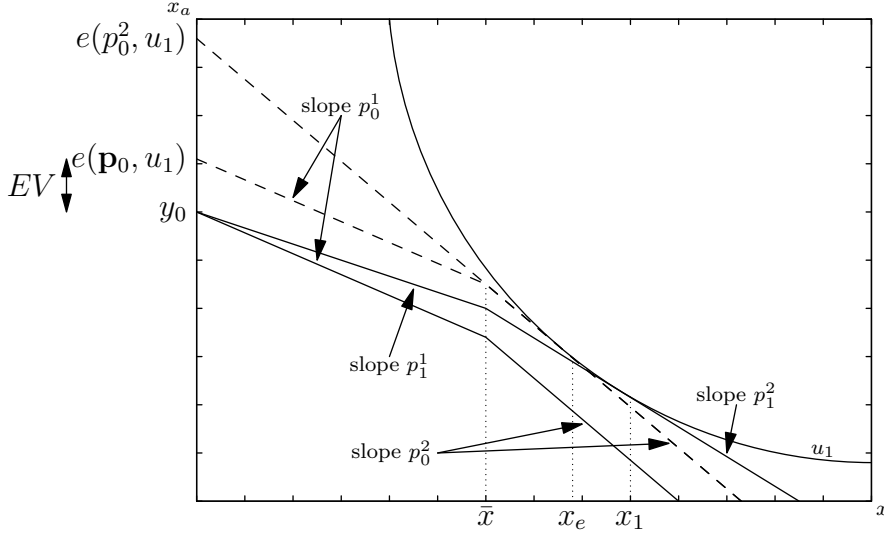
To build our application, we consider the two-good case in which a representative agent consumes a good  $x$ , say electricity, and an aggregate good as a numeraire ( $x_a$ ). We note that, for our application, the representative agent assumption is not as restrictive as it may seem. In particular, given that we work with the stratum one population at the municipality level, a fairly homogeneous group within each polygon, the assumption that agents with similar preferences can be aggregated into a single agent per municipality is not unthinkable. This could be thought of as a case of dispersion in preferences and income where, although individuals may present erratic utility functions, the aggregate demand for the commodities of interest are well-behaved ([Trockel, 1987](#)). In addition, representative agent models dominate microfounded macroeconomics due to their simplicity and tractability ([Acemoglu, 2008](#)). One final argument is the impossibility of obtaining data at the micro-level to correct for the bias raised by agent heterogeneity. Therefore, although we are aware of the disadvantages of the representative agent ([Kirman, 1992](#), [Reiss and White, 2006](#)), we will continue to work under this assumption.

Throughout this paper we will indicate a situation before or after tariff unification with the subscripts 0 or 1, respectively. Subsistence consumption will be denoted by  $\bar{x}$ . This subsistence consumption divides demand into two possible tiers, denoted by superscript 1 when the consumer demands a quantity less than  $\bar{x}$  and 2 when it is greater than  $\bar{x}$ . As shown in [Varian \(1992\)](#), EV can be represented in terms of the expenditure function as

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(\mathbf{p}_0, u_1) - e(\mathbf{p}_1, u_1) = e(\mathbf{p}_0, u_1) - y_0 \quad (13)$$

with  $\mathbf{p}_i = (p_i^1, p_i^2, 1)$ ,  $i = 0, 1$  and  $e(\mathbf{p}_0, u_1)$  the expenditure needed to reach the utility level  $u_1$  when facing prices  $\mathbf{p}_0$ . Call  $x_1$  the new demand at prices  $\mathbf{p}_1$  and income  $e(\mathbf{p}_1, u_1) = y_0$ .

**Figure 1:** Example of Equivalent Variation with price changes on both tiers, and new and virtual consumption on the second tier



Tangency between  $u_1$  and the budget curve characterized by  $\mathbf{p}_0$  and income  $e(\mathbf{p}_0, u_1)$  is referred to as *virtual* consumption ( $x_e$ , see Figure (1)). As in Ruijs (2009) and You and Lim (2013), we assume that new consumption  $x_1$  and virtual consumption  $x_e$  fall on the same tier. Therefore, the utility level  $u_1$  and the expenditures  $e(\mathbf{p}_0, u_1)$  are such that (i)  $x_1, x_e < \bar{x}$  or (ii)  $x_1, x_e > \bar{x}$ . For the first case:

$$u_1 = V((p_1^1, 1), y_0)$$

with  $V((p_1^1, 1), y_0) = \max_{x_1, x_a} \{u(x) : p_1^1 x_1 + x_a \leq y_0\}$ , the indirect utility function. At this utility level,  $e(\mathbf{p}_0, u_1) = e(p_0^1, u_1)$  and Equivalent Variation can be written as

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(p_0^1, u_1) - y_0 \quad (14)$$

For the second case, when  $x_1$  is in the second tier, the utility level associated with this consumption is

$$u_1 = V((p_1^2, 1), y_0 + (p_1^2 - p_1^1)\bar{x})$$



A consumer pays  $p_1^1$  for consumption up to  $\bar{x}$  and  $p_1^2$  after, up to  $x_1$ , which means consumers save  $(p_1^2 - p_1^1)\bar{x}$ . The expenditures  $e(\mathbf{p}_0, u_1)$  are also subject to change since an amount needed to acquire  $x_e$  is lessened by  $(p_0^2 - p_0^1)\bar{x}$ , and therefore  $e(\mathbf{p}_0, u_1) = e(p_0^2, u_1) - (p_0^2 - p_0^1)\bar{x}$ . In this context,  $e(p_0^2, u_1)$  is the virtual expenditure under the assumption of a flat rate of  $p_0^2$  throughout the whole system, and  $(p_0^2 - p_0^1)\bar{x}$  is the implicit subsidy perceived by consumers in the second block. Under these conditions, the Equivalent Variation turns out to be

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = e(p_0^2, u_1) - (p_0^2 - p_0^1)\bar{x} - y_0 \quad (15)$$

In this paper we ignore the possibility that  $x_e$  is located exactly at the kink  $\bar{x}$  since this is not the case for any of the municipalities we analyze.<sup>3</sup> Given that any alternative represents a convex budget set and that we estimate a quasi-concave demand function, we can focus all of our attention on the two cases stated above (Hausman, 1985, Moffitt, 1990).<sup>4</sup>

Hausman (1981) developed a method that relates the econometric specification of a Marshallian demand function to the definitions stated above. He starts with Roy's identity:

$$x(p, y) = -\frac{\partial V(p, y)/\partial p}{\partial V(p, y)/\partial y} \quad (16)$$

For our case,  $x(p, y)$  is specified as an exponential function

$$x(p, y) = p^\alpha y^{\delta_1} e^{z'\delta} \quad (17)$$

with  $\alpha$ ,  $\delta_1$  and  $\delta$ , the price elasticity of demand, the income elasticity of demand, and the vector of coefficients for the covariates  $\mathbf{z}$ , respectively. The next step is to solve the differential equation implicit in (16), by using separation of variables, to find

$$V(p, y) = c = -e^{z'\delta} \frac{p^{1+\alpha}}{1+\alpha} + \frac{y^{1-\delta_1}}{1-\delta_1} \quad (18)$$

---

<sup>3</sup>See Ruijs (2009) and You and Lim (2013) for EV under this condition.

<sup>4</sup>A kinked budget constraint represents a convex budget set when  $p^2 \geq p^1$ .

Setting the constant of integration  $c = \bar{u}$  and solving for  $y$  results in the expenditure function in terms of  $p$  and  $\bar{u}$

$$e(p, \bar{u}) = \left[ (1 - \delta_1) \left( \bar{u} + e^{z'\delta} \frac{p^{1+\alpha}}{1 + \alpha} \right) \right]^{\frac{1}{1-\delta_1}} \quad (19)$$

Now all we need to do is plug Equations (18) and (19) into (14) and (15) using the appropriate prices and income to find expressions for the Equivalent Variation in terms of the parameters of our proposed model. For case (i) with  $u_1 = V(p_1^1, y_0)$ ,

$$u_1 = -e^{z'\delta} \frac{p_1^{1(1+\alpha)}}{1 + \alpha} + \frac{y_0^{1-\delta_1}}{1 - \delta_1}$$

$$e(p_0^1, u_1) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{1(1+\alpha)} - p_1^{1(1+\alpha)} \right) e^{z'\delta} + y_0^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}}$$

and the Equivalent Variation is

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{1(1+\alpha)} - p_1^{1(1+\alpha)} \right) e^{z'\delta} + y_0^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}} - y_0 \quad (20)$$

For case (ii) with  $u_1 = V(p_1^2, y_0 + (p_1^2 - p_1^1)\bar{x})$ ,

$$u_1 = -e^{z'\delta} \frac{p_1^{2(1+\alpha)}}{1 + \alpha} + \frac{(y_0 + (p_1^2 - p_1^1)\bar{x})^{1-\delta_1}}{1 - \delta_1}$$

$$e(p_0^2, u_1) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{2(1+\alpha)} - p_1^{2(1+\alpha)} \right) e^{z'\delta} + (y_0 + (p_1^2 - p_1^1)\bar{x})^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}}$$

and the Equivalent Variation is

$$EV(\mathbf{p}_0, \mathbf{p}_1, y_0) = \left[ \frac{1 - \delta_1}{1 + \alpha} \left( p_0^{2(1+\alpha)} - p_1^{2(1+\alpha)} \right) e^{z'\delta} + (y_0 + (p_1^2 - p_1^1)\bar{x})^{1-\delta_1} \right]^{\frac{1}{1-\delta_1}} - (p_0^2 - p_0^1)\bar{x} - y_0 \quad (21)$$

We can see from Equations (20) and (21) that a price decrease of an inelastic necessity good produces welfare gains that can be quantified through a positive Equivalent Variation.

Equation (21) takes into consideration that the subsidy has effects on the expenditure function as well as on the agent’s income.

## 4 Results

### 4.1 Data

Data was collected for the average individual of stratum one in all 125 municipalities of the department of Antioquia (Colombia) in 2005. Table (A.1) in the Appendix lists all the variables, their measurement units, and sources. Stratum level income was constructed from regional participation in the total production of the province and population (Ramírez and Londoño, 2009). We standardized both electricity and substitute good prices to US\$/kWh by taking their calorific power into account. For municipalities in the Metropolitan Area, the substitute good was natural gas. For the other municipalities, it was liquefied petroleum gas due to absence of natural gas. In addition, we have to mention that by construction, the average price is affected by electricity consumption because the average electricity tariff is a weighted average between the tariffs in the first and second tiers, where the weights depend on the observed and subsistence consumption levels. This generates the endogeneity problem in our application.

We present descriptive statistics in Table (4). The average annual electricity consumption is 234.87 kWh with a standard deviation of 117.81 kWh. The prices for electricity and the substitute good averaged 6.10 and 3.00 cents a kWh, respectively. Additionally, the average annual per capita income was US\$397, with a standard deviation of US\$95.24. The maximum values for both electricity consumption and average annual income were found at the municipality of Envigado, which is located in the Metropolitan Area. The lowest value for consumption was found at Vigía del Fuerte, a municipality that faces many adversities due to its geographical location. El Bagre was found to have the lowest per capita income. Regarding electricity and substitute prices, the maximum values were found at Enterríos and Chigorodó, respectively. On the other hand, the minimum values

were found at Don matías and Venecia. Approximately 29% of the municipalities in the province of Antioquia are located less than 1000 meters above sea level, the average urbanization rate is 45.8%, and 77.4% of the municipalities used to be covered by EADE prior to its acquisition by EPM.

**Table 4:** Descriptive Statistics

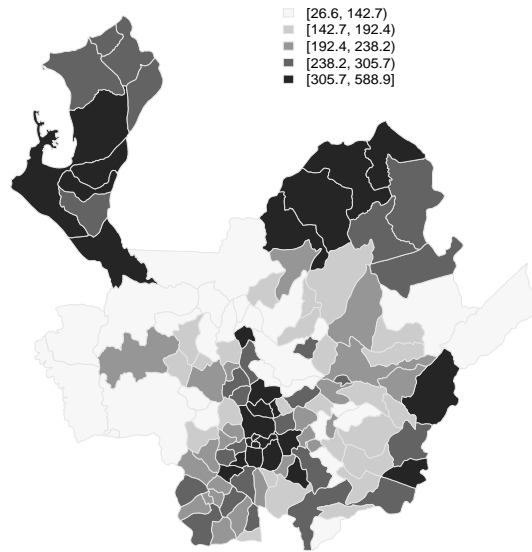
Variable	Mean	Std. Dev.	Min	Max
Consumption (kWh)	234.874	117.811	26.595	588.937
Electricity Price (US\$)	0.061	0.024	0.039	0.240
Income (US\$)	397.085	95.242	230.514	619.227
Substitute Price (US\$)	0.030	0.006	0.016	0.056
Sea level	29.032%	45.575%	0.000	1.000
Urbanization	45.876%	19.917%	10.700%	98.247%
Coverage (EADE)	77.419%	41.981%	0.000	1.000

*Source:* Author’s calculations

We can observe the geographical distribution of the electricity consumption in Map (2). In particular, the average consumption of electricity tends to be higher in regions that are located less than one thousand meters above sea level (the Northern and Eastern regions). Consumption is also exceptionally high in the Metropolitan Area of Antioquia (South-Central region), where most of the population and economic activity of the province are focused.

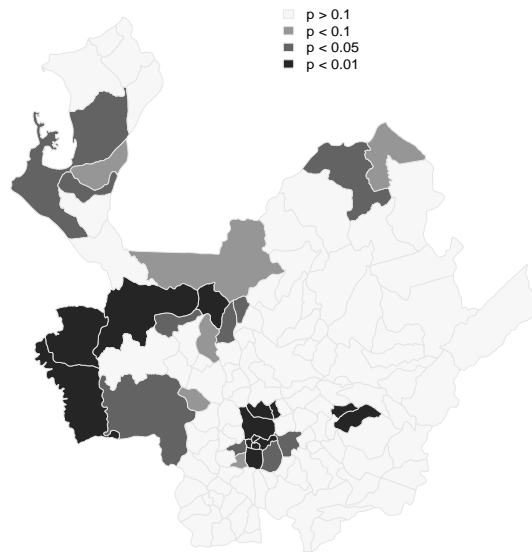
Map (3) shows that most of the spatial autocorrelation is due to local clusters. This obeys unobserved social, cultural, economical and geographical restrictions, like the limited and bad roads between municipalities or constrained budgets that poor households face in these municipalities. Avoiding a global spatial effect regarding electricity consumption for the inhabitants of the province appears to be the most natural approach, as is provided by the CAR specification.

**Figure 2:** Average Annual Electricity Consumption per Household (kWh): Province of Antioquia (Colombia) in 2005, Stratum One



*Source:* Empresas Públicas de Medellín

**Figure 3:** Local Moran's I test  $p$ -values of Average Annual Electricity Consumption per Household (kWh): Province of Antioquia (Colombia) in 2005, Stratum One



*Source:* Authors' calculations

## 4.2 Model Specification

We need to estimate the electricity demand function to perform a statistical inference of the Equivalent Variation. However, it is necessary to take into account the endogeneity issue between price and consumption to avoid biased and inconsistent parameter estimates; it is also necessary to introduce spatial effects in order to have a good municipality electricity prediction. We realize that both elements are crucial to obtaining a reliable Equivalent Variation measure in light of Equations (20) and (21).

As instrument, we use a dummy variable that is equal to 1 if the municipality was serviced by EADE, and 0 otherwise. The argument behind this instrument is that the national electricity regulations generate restrictions that imply that the only effect of the electricity supplier on average consumption in each municipality is through price. However, the regional market reference tariff, and as a consequence the average electricity price of the low strata in each municipality, is drastically affected by each supplier.

The structural specification of our system is

$$\ln x_i = \pi_0 + \mathbf{z}'_{1i}\boldsymbol{\pi}_1 + \alpha \ln p_i + u_{1i} + v_i \quad (22)$$

$$\ln p_i = \phi_0 + \mathbf{z}'_{1i}\boldsymbol{\phi}_1 + \alpha_s z_{2i} + u_{2i} \quad (23)$$

where  $x_i$  and  $p_i$  are the electricity consumption and price,  $\mathbf{z}'_{1i} = (\ln y_i, \ln p_i^s, alt_i, \ln urb_i)$  is a vector of exogenous covariates that affects the system (income, substitute price, sea level dummy, and urbanization rate) and  $z_{2i} = EADE_i$  is our instrument. Additionally,  $\pi_0$ ,  $\boldsymbol{\pi}'_1 = (\pi_1, \pi_2, \pi_3, \pi_4)$ ,  $\phi_0$ ,  $\boldsymbol{\phi}'_1 = (\phi_1, \phi_2, \phi_3, \phi_4)$ ,  $\alpha$  and  $\alpha_s$  are parameters to be estimated. Finally,  $u_{1i}$  and  $u_{2i}$  are the idiosyncratic error terms associated with the demand and price of each municipality, and  $v_i$  are spatial random effects to control for spatial heterogeneity and spatial dependence between cross-sectional units that is present in our application (see Map (3)). These emerge due to clusters and/or spillover effects between neighboring municipalities, and allow us to control for unobservable spatial heterogeneity. Omitting this last component can cause a loss of good statistical properties of estimators (Anselin,

1988).

Unfortunately, we find just one available instrument: however, we can use this situation to illustrate that our econometric framework encompasses the more simple technique of multivariate regression models when there is an exactly identified system. In particular, we estimate the reduced model that results from substituting (23) into (22) in our application.

$$\begin{aligned}\ln x_i &= \delta_0 + \delta_1 \ln y_i + \delta_2 \ln p_i^s + \delta_3 \text{alt}_i + \delta_4 \ln \text{urb}_i + \gamma \text{EADE}_i + \mu_{1i} + v_i \\ \ln p_i &= \phi_0 + \phi_1 \ln y_i + \phi_2 \ln p_i^s + \phi_3 \text{alt}_i + \phi_4 \ln \text{urb}_i + \alpha_s \text{EADE}_i + \mu_{2i}\end{aligned}\quad (24)$$

where  $\mu_{1i} = u_{1i} + \alpha u_{2i}$  and  $\mu_{2i} = u_{2i}$ , such that  $(\mu_{1i}, \mu_{2i})' \sim \mathcal{N}(\mathbf{0}, \Sigma)$ ,  $\Sigma = \{\sigma_{ij}\}$ . The structural parameters can be recovered using  $\alpha = \gamma/\alpha_s$  and  $\pi_l = \delta_l - \phi_l \alpha$ ,  $l = \{0, 1, \dots, 4\}$ .

Setting  $\mathbf{z}'_i = (\ln y_i, \ln p_i^s, \text{alt}_i, \ln \text{urb}_i, \text{EADE}_i)$ ,  $\boldsymbol{\delta}' = (\delta_1, \delta_2, \delta_3, \delta_4, \gamma)$ ,  $\boldsymbol{\phi}' = (\phi_1, \phi_2, \phi_3, \phi_4, \alpha_s)$  and  $\mathbf{v}' = (v_1, v_2, \dots, v_n)$ , and taking into consideration that the determinant of the Jacobian matrix of the transformation is 1, the likelihood function of the system is

$$\begin{aligned}f(\ln \mathbf{x}, \ln \mathbf{p} | \mathbf{z}; \Sigma, \boldsymbol{\delta}, \boldsymbol{\phi}, \pi_0, \phi_0, \mathbf{v}) &= \frac{|\Sigma|^{-N/2}}{(2\pi)^{N/2}} \times \\ \exp \left\{ -\frac{1}{2} \sum_{i=1}^N (\ln x_i - \mathbf{z}'_i \boldsymbol{\delta} - \delta_0 - v_i, \ln p_i - \mathbf{z}'_i \boldsymbol{\phi} - \phi_0) \Sigma^{-1} \begin{pmatrix} \ln x_i - \mathbf{z}'_i \boldsymbol{\delta} - \delta_0 - v_i \\ \ln p_i - \mathbf{z}'_i \boldsymbol{\phi} - \phi_0 \end{pmatrix} \right\}\end{aligned}\quad (25)$$

We can estimate our reduced form model with spatial random effects using this likelihood, and prior independent distributions such that

$$\pi(\Sigma, \boldsymbol{\delta}, \boldsymbol{\phi}, \delta_0, \phi_0, \mathbf{v}_i, \sigma_v^2) = \pi(\Sigma) \pi(\boldsymbol{\delta}) \pi(\boldsymbol{\phi}) \pi(\delta_0) \pi(\phi_0) \pi(\mathbf{v}_i | \sigma_v^2) \pi(\sigma_v^2) \quad (26)$$

where  $\Sigma^{-1} \sim \mathcal{W}_2(3, \mathbf{I}_2)$ ,  $\boldsymbol{\delta} \sim \mathcal{N}_5(\mathbf{0}, 1000\mathbf{I}_5)$ ,  $\boldsymbol{\phi} \sim \mathcal{N}_5(\mathbf{0}, 1000\mathbf{I}_5)$ ,  $\pi(\delta_0) \propto 1$ ,  $\pi(\phi_0) \propto 1$ ,  $v_i | \mathbf{v}_{i \sim j} \sim \mathcal{N}\left(\sum_{i \sim j} \frac{w_{ij} v_j}{\sum_{i \sim j} w_{ij}}, \frac{\sigma_v^2}{\sum_{i \sim j} w_{ij}}\right)$  and  $1/\sigma_v^2 \sim \mathcal{G}(1, 1/2)$ . As we can see, most of the priors are vague or diffuse, except the Gamma distribution, which was set to reflect the prior belief that 50% of the variability of the reduced model are subject to spatial effects. However, we find in our application that the posterior parameter estimates are robust to

changes in the hyperparameters of the CAR’s precision (available upon request).

We should bear in mind that the posterior framework that was deduced in Section (1) applies to our empirical exercise. We just need to take into consideration that in this subsection,  $\mathbf{w}_i = \mathbf{z}_i$  and we treat  $\boldsymbol{\delta}$  in a similar way as  $\boldsymbol{\pi}$ .

We must bear in mind two aspects that are important in our application. First, a just-identified model allows  $\boldsymbol{\delta}$  to have independent variation, a situation that is not possible with over-identified models in reduced form. Second, under independent prior distributions, we can express the posterior as  $\pi(\boldsymbol{\pi}|\boldsymbol{\delta}, Data) \propto \pi(\boldsymbol{\pi}|\boldsymbol{\delta})$ , that is, the conditional posterior of the structural parameters is unaffected by the observations once the reduced form parameters are taken into account (Zellner, 1996).

### 4.3 Estimation Results

Since the solution for the model depends on the selection of the contiguity matrix, we will test our specification under three different matrices. The first one uses the road lengths between each municipality, regarding two regions to be neighbors if the roads connecting them are less than 300 kilometers long, which ensures that each region has at least one neighbor. The second uses the queen criterion, where two regions are considered as neighbors if they share at least a single border point. The third one uses the rook criterion, where regions are considered neighbors if they share more than one border point.

We estimate each of our models using Markov chain Monte Carlo techniques (MCMC, see Robert and Casella, 2004, for details). In particular, we use the Gibbs sampling algorithm, due to the availability of all the conditional posterior distributions (Geman and Geman, 1984). After running the chains for 10 million iterations, we discard the first 5 million and draw an observation every 500 iterations to get an effective sample size of 10,000. This procedure allows us to keep autocorrelation to a minimum and ensure the convergence of the chains (see Table (B.1) in the Appendix, subsection B).<sup>5</sup>

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<sup>5</sup>All the simulation exercises and posterior analyses were performed using R (R Core Team, 2014).



The correlation of the instrument with the logarithm of the price is approximately  $-0.46$ , its variability is very low due to its being a dummy variable, its standard deviation is equal to  $0.42$ ,<sup>6</sup> and its 90% probability highest density interval in the price equation is  $(-0.48, -0.22)$  with a mean and median equal to  $-0.35$ . However, even if this instrument were weak, the Bayesian approach works well in this context using proper priors due to the fact that the likelihood function and its identification are less important for deriving estimates in Bayesian models (Zellner, 1996, Imbens and Rubin, 1997, Zellner, 1998, Crespo-Tenorio and Montgomery, 2013).

We present in Table (5) the main outcomes associated with the structural parameters of our estimation exercises, using different contiguity criteria to check their robustness. And as we can see, all contiguity criteria yield similar results. In particular, we present the posterior mean and median that minimize the quadratic and absolute value loss functions under a decision theory framework. In addition, in order to describe the inferential content of the posterior distributions of the parameters, we present the 90% highest probability density credible intervals for each parameter of interest. Finally, to test whether the microeconomic restrictions are compatible with the observed data, we calculate the odds ratio in favor of the null hypothesis  $H_0 : \theta \in (0, \infty)$  versus  $H_1 : \theta \in (-\infty, 0]$ , using 0.5 as the prior probability for each of these hypotheses. This procedure is consistent with a symmetric loss function, for instance a zero-one loss function (Berger, 1985, Zellner, 1996). Testing microeconomic restrictions is very important in this setting because our main objective is to carry out a statistical inference regarding Equivalent Variation, and so there are some implicit restrictions placed on the parameter estimates. Thus, we follow a statistical decision theory framework, where an action regarding the domain of the posterior densities must be made. These actions are based on prior and sample information. Kleit and Terrell (2001) reiterate the importance of placing restrictions on Bayesian models and priors based on microeconomic theory.

Regarding endogeneity in our application, we find that the posterior median esti-

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<sup>6</sup>As this instrument is a Bernoulli random variable, its possible maximum standard deviation is equal to 0.5. This would be a limitation of using dummy variables as instruments in a frequentist approach.

mates of  $\sigma_{12}$  are approximately  $-0.06$  using different contiguity criteria, and the highest probability intervals at 90% of credibility are  $(-0.100, -0.022)$ ,  $(-0.088, -0.043)$ , and  $(-0.096, -0.019)$  using roads, queen, and rook contiguity criteria, respectively. This evidence suggests that there is endogeneity between electricity consumption and price.

**Table 5:** Summary of structural parameter posterior estimates

Road Length Contiguity					
Parameter	Mean	Median	90% HPD Interval		$R_{01} = \frac{P(\theta \in (0, \infty))}{P(\theta \in (-\infty, 0])}$
			Lower	Upper	
Constant	1.913	1.940	-1.539	5.393	4.663
Price	-0.886	-0.882	-1.449	-0.278	0.014
Income	0.301	0.297	-0.054	0.636	11.920
Subs. Price	0.123	0.120	-0.215	0.449	2.560
Altitude	0.139	0.137	-0.041	0.304	9.235
Urbanization	0.571	0.566	0.410	0.724	908.090
Queen Contiguity					
Parameter	Mean	Median	90% HPD Interval		$R_{01} = \frac{P(\theta \in (0, \infty))}{P(\theta \in (-\infty, 0])}$
			Lower	Upper	
Constant	1.873	1.961	-2.081	5.704	4.038
Price	-0.877	-0.876	-1.570	-0.200	0.027
Income	0.308	0.298	-0.068	0.701	9.941
Subs. Price	0.117	0.117	-0.276	0.488	2.331
Altitude	0.130	0.135	-0.097	0.391	5.826
Urbanization	0.575	0.566	0.375	0.751	139.840
Rook Contiguity					
Parameter	Mean	Median	90% HPD Interval		$R_{01} = \frac{P(\theta \in (0, \infty))}{P(\theta \in (-\infty, 0])}$
			Lower	Upper	
Constant	1.956	1.965	-2.126	5.662	4.061
Price	-0.818	-0.876	-1.567	-0.189	0.027
Income	0.297	0.298	-0.095	0.674	9.834
Subs. Price	0.084	0.117	-0.283	0.482	2.328
Altitude	0.178	0.135	-0.104	0.384	5.775
Urbanization	0.575	0.565	0.377	0.756	124.000

*Source:* Author's calculations

Given that we obtain robust outcomes regarding the contiguity criteria, we discuss the results associated with the road length criterion. This criterion better illustrates the connectivity between municipalities in a province that is characterized by irregular

geographical conditions and poor roads. Thus, when we observe the posterior mean and median, we see that all the point estimates have the expected signs. Electricity behaves as both an ordinary and a normal good, given the negative price-demand elasticity and positive income-demand elasticity. For instance, the average as well as the median price demand elasticity is  $-0.88$ , that is, an increase of 1% in the price of electricity implies a reduction of 0.88% in consumption. In addition, the average and median income elasticity is approximately 0.30, which implies that a 1% income increase means a 0.30% increase in electricity consumption. Regarding the highest probability density credible intervals of these parameters, we have that these are  $(-1.45, -0.28)$  and  $(-0.05, 0.64)$  for the price elasticity and income elasticity, respectively. In addition, we calculate the inverse odds ratio in favor of the null hypothesis  $H_0 : \alpha \in (-\infty, 0]$  to check the microeconomic restriction of a negative price elasticity. This is equal to 0.014 that means an odds ratio supporting  $H_0$  equal to 71.42, which implies that  $\log_{10}(R_{01}) = 1.85$ . Thus, we have very strong evidence for  $H_0$  following Jeffreys's guidelines (Greenberg, 2008). Regarding the null hypothesis of a positive income elasticity,  $H_0 : \pi_1 \in (0, \infty)$ , which is suggested by most of the literature on electricity demand (Hsiao and Mountain, 1985, Dergiades and Tsoulfidis, 2008), we have  $\log_{10}(R_{01}) = 1.07$ , indicating strong evidence for  $H_0$ .

Regarding cross elasticity with the substitute good, although this is positive on average, a 1% increase in the price of the substitute implies a 0.12% increase in electricity demand, so there is weak evidence for  $H_0 : \pi_2 \in (0, \infty)$  due to the fact that  $\log_{10}(R_{01}) = 0.40$ . Probably this is because of the lack of electricity substitutes in rural areas, or the fact that the demand for electricity is derived for most household appliances which cannot function with anything but electricity. For this parameter, we observe a HPD credible interval between  $-0.21$  and  $0.45$ . The mean altitude semi-elasticity is equal to 0.14, which means that municipalities located at lower altitude demand approximately 14% more electricity, *ceteris paribus*. In this case, we have  $\log_{10}(R_{01}) = 0.97$ , which is substantial evidence for  $H_0 : \pi_3 \in (0, \infty)$ . Finally, there is the urbanization rate, which has a strong positive effect on electricity consumption, as one would expect:  $\log_{10}(R_{01}) = 2.96$  for  $H_0 : \pi_4 \in (0, \infty)$ ,

which is decisive support for  $H_0$ . The median and mean urbanization rate elasticity is approximately 0.57 with a highest probability density credible interval equal to (0.41, 0.72).

Despite the fact that our prior assumption regarding the participation of the spatial effects on electricity consumption variability is 50%, we find that the posterior mean proportion is 8.56% with a standard deviation equal to 9.55% and the HPD at 90% equal to (0.40%, 21.11%). This outcome is robust to many hyperparameter combinations of the prior distribution of the precision parameter of the CAR component (available upon request).

Finally, we compute several diagnostics to assess the convergence and stationarity of the chains. In particular, we employ the method due to [Heidelberger and Welch \(1983\)](#), the mean difference test proposed by [Geweke \(1992\)](#), and the diagnostic from [Raftery et al. \(1992\)](#). We show that in general all the chains under different contiguity criteria achieve convergence and stationarity in the Table (B.1) in the Appendix, subsection B.

#### 4.4 Welfare Implications

The tariff unification procedure brought about by the acquisition of EADE by EPM created tier price variations that depended on whether the municipality was part of the Metropolitan Area or not. In particular, by the end of this process,  $p_1^1$  and  $p_1^2$  changed according to the values in Table (6) with respect to their pre-unification values. We expect to see that the municipalities which consumed less than the subsistence consumption and are not part of the Metropolitan Area have the largest welfare gains, followed by those that are not part of the Metropolitan Area and had average consumption higher than the subsistence consumption. The welfare effects in the municipalities that belong to the Metropolitan Area are not clear and will depend on whether they consumed more than the subsistence consumption or not, and how much of their consumption was above this threshold, among other factors ([Ramírez and Londoño, 2009](#)). Here, we note that subsistence consumption is measured in kilowatts/hour a month per household. Therefore, in order to make it comparable with our measure of income, we work with an annual per

capita consumption for each altitude.<sup>7</sup>

**Table 6:** Tariff variations due to unification

Location	$p_1^1$	$p_1^2$
Metropolitan Area	-0.33%	8.12%
Rest	-17.53%	-0.95%

*Source:* Author's calculations

To compute the posterior distribution of the Equivalent Variation, we follow the guidelines of the Bayes theorem, and renormalize the unrestricted posterior distribution of each parameter according to the outcomes of the microeconomic restrictions in Table (5), where the statistical evidence suggests the fulfillment of those restrictions (Berger, 1985, Bernardo, 2003). This allows us to obtain sensible results, based on a statistical decision theory framework, regarding the Equivalent Variation, which is calculated for each municipality at each observation of these new chains, through Equations (20) and (21). This procedure leaves an effective sample size of 5410 with which to make the computations.

Table (7) lists the mean, median, and 90% highest probability density interval for the total, for the Metropolitan Area, and for the rest of Antioquia, as a share of original income  $y_0$ . As can be observed, the median Equivalent Variation in the whole province is approximately 0.63%, and its standard error is approximately 0.67%. This welfare gain is lower in the Metropolitan Area (0.14%) and greater in the rest of the province (0.65%). The 90% HPD interval is equal to (0.0%, 1.9%), and the posteriors tend to be skewed to the right, as the mean is greater than the median. For stratum one households, this impact can be very substantial, especially for those who are located in regions other than the Metropolitan Area of Antioquia.

Table (8) shows the same summary statistics as before for a few representative municipalities. Each of these four municipalities presents a different price variation, according to their location and consumption levels. Medellín, the capital of Antioquia, located in the

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<sup>7</sup>The original levels were multiplied by 12 to obtain an annual measure and then divided by an average of 4.04 people per household in stratum one to get our variable of interest.

**Table 7:** Equivalent Variation as share of income by Total, Metropolitan Area and Rest

Road Length Contiguity				
Equivalent Variation	Mean	Median	90% HPD Interval	
			Lower	Upper
M. Area	0.126%	0.141%	0.006%	0.209%
Rest	0.940%	0.655%	0.257%	2.006%
Total	0.874%	0.630%	0.005%	1.913%
Queen Contiguity				
Equivalent Variation	Mean	Median	90% HPD Interval	
			Lower	Upper
M. Area	0.127%	0.141%	0.005%	0.212%
Rest	0.937%	0.653%	0.256%	2.003%
Total	0.872%	0.628%	0.004%	1.907%
Rook Contiguity				
Equivalent Variation	Mean	Median	90% HPD Interval	
			Lower	Upper
M. Area	0.127%	0.141%	0.005%	0.212%
Rest	0.936%	0.653%	0.257%	2.000%
Total	0.871%	0.627%	0.005%	1.905%

*Source:* Author's calculations

Metropolitan Area, is characterized by high consumption, high urbanization levels, and a high altitude. Its average and median welfare gains are approximately 0.14%. Barbosa, a member of the Metropolitan Area but with low consumption, presents the lowest of all average welfare gains, along with Girardota (0.00%), since they received the smallest price reduction and did not receive any implicit subsidy from their consumption; all located in tier 1 on average. San José de la Montaña is a special case, exhibiting higher than subsistence consumption but being located outside of the Metropolitan Area. The welfare increase for this municipality is greater than for those of the Metropolitan Area (0.31%). Santa Fe de Antioquia, whose welfare gain is on average 2.29%, is part of the largest group of municipalities: those with lower consumption located outside of the Metropolitan Area. These municipalities tend to have large welfare gains, but they vary greatly depending on their initial price levels, urbanization rates, and altitude. Caucasia, part of the same group as Santa Fe de Antioquia, had the largest average increase in welfare with 3.30%

as a share of income.

**Table 8:** Equivalent Variation as share of income for representative municipalities

Road Length Contiguity				
Municipality	Mean	Median	90% HPD Interval	
			Lower	Upper
Medellín	0.137%	0.139%	0.081%	0.194%
Barbosa	0.008%	0.008%	0.006%	0.009%
San José de la Montaña	0.313%	0.313%	0.307%	0.319%
Santa Fe de Antioquia	2.297%	2.297%	1.956%	2.619%
Queen Contiguity				
Municipality	Mean	Median	90% HPD Interval	
			Lower	Upper
Medellín	0.139%	0.140%	0.073%	0.199%
Barbosa	0.008%	0.008%	0.006%	0.009%
San José de la Montaña	0.313%	0.313%	0.306%	0.320%
Santa Fe de Antioquia	2.285%	2.284%	1.918%	2.658%
Rook Contiguity				
Municipality	Mean	Median	90% HPD Interval	
			Lower	Upper
Medellín	0.139%	0.140%	0.075%	0.200%
Barbosa	0.008%	0.008%	0.006%	0.009%
San José de la Montaña	0.313%	0.313%	0.306%	0.320%
Santa Fe de Antioquia	2.281%	2.281%	1.913%	2.647%

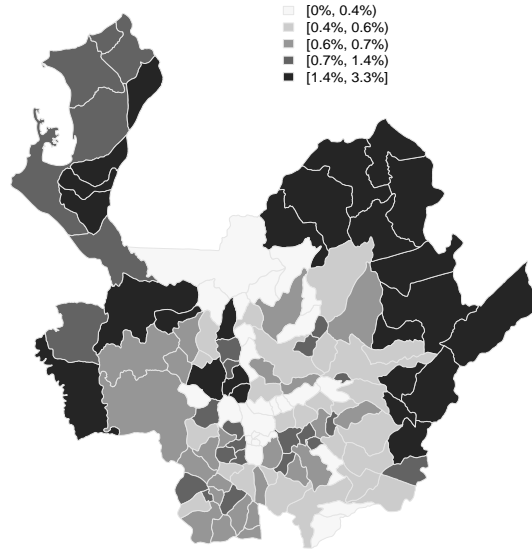
*Source:* Author's calculations

Finally, Map (4) presents the median Equivalent Variation as a share of income in the province. The spatial distribution is low around the metropolitan area and high in the more rural areas, which received the greatest improvement and benefits from the tariff unification.

## 5 Concluding Remarks

In this paper, we introduced spatial random effects into an endogenous Bayesian framework with simultaneous equations and deduced the complete conditional posterior distributions. Thus, we were able to draw observations from the model using a Gibbs sampler

**Figure 4:** Median Equivalent Variation by Municipality



*Source:* Authors' calculations

algorithm. This approach allows dealing simultaneously with three shortcomings, which would be quite difficult to manage simultaneously with a frequentist approach. First, it permits taking into account the endogeneity issues in our estimation procedure. Second, we can carry out statistical inferences of complicated non-linear functions of the parameter estimates in our application. Third, it allows controlling for the non-observable heterogeneity and spatial autocorrelation present in cross-sectional data.

We performed simple Monte Carlo simulation exercises which show that our econometric approach handles the endogeneity and spatial effects well. In particular, the posterior point and credible interval estimates are sensible, and the prediction is significantly improved by introducing the spatial effects.

Using these features of the Bayesian framework to our advantage, we estimated the Equivalent Variation welfare measure, as a share of mean income, that stemmed from a process of electricity tariff unification in the province of Antioquia (Colombia), with data at the municipality level. We estimated a demand function for electricity and found the average price, income, substitute, and urbanization rate demand elasticities to be,



respectively,  $-0.88$ ,  $0.30$ ,  $0.12$ , and  $0.57$ . The semi-elasticity associated with a dummy for the altitude of the municipalities was approximately  $0.14$ . Using this information as input, we found the Equivalent Variation for the province as a whole to be  $0.87\%$  on average and with a median of  $0.63\%$ . When taking into account the welfare gains of the municipalities of the Metropolitan Area, these amount only to  $0.13\%$ . However, the municipalities that are not part of the Metropolitan Area gained on average  $0.94\%$ , while the  $10\%$  of the municipalities with the least urbanization and least income increased their welfare by an amount well above  $2\%$  of their initial income. Comparing these figures with the the amount that low income households expend on pensions ( $1.13\%$ ), health care ( $2.04\%$ ), and education ( $4.79\%$ ) illustrates the huge effect of electricity regulation on the welfare of the poor.

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# Appendix

## A Variable Definitions

**Table A.1:** Variable definitions and sources

Variable	Definition	Source
Consumption ( $x$ )	Average annual electricity consumption per household in kilowatts hour (kWh)	EPM <sup>a</sup>
Price ( $p$ )	Average annual electricity price in US\$ by kilowatt hour (US\$/kWh)	EPM
Income ( $y$ )	Average annual per capita income in US\$	Authors' calculations
Substitute price ( $p^s$ )	Average annual price of the substitute good in US\$ by kilowatt hour (US\$/kWh)	CREG <sup>b</sup>
Urbanization ( $urb$ )	Ratio of urban to total population	DANE <sup>c</sup>
Altitude ( $alt$ )	Dummy variable taking on 1 when the municipality is located less than 1000ms above sea level	Anuario Estadístico de Antioquia <sup>d</sup>
Coverage ( $EADE$ )	Dummy variable taking on 1 when municipality used to be covered by EADE and 0 otherwise	SUI <sup>e</sup>

*Notes:* <sup>a</sup> Empresas Públicas de Medellín, <sup>b</sup> Comisión de Regulación de Energía y Gas, <sup>c</sup> Departamento Administrativo Nacional de Estadística, <sup>d</sup> Antioquia's Statistical Yearbook compiled by the Government of Antioquia, <sup>e</sup> Sistema Único de Información

## B Diagnostics

**Table B.1:** Stationarity and Convergence diagnostics

Road Length Contiguity				
Parameter	Heidelberger (1st Part/p-value) <sup>a</sup>	Heidelberger (2nd Part) <sup>b</sup>	Geweke <sup>c</sup>	Raftery <sup>d</sup>
Constant	0.887	0.064	0.758	1.46
Price	0.923	-0.047	0.820	1.10
Income	0.414	0.035	-0.740	2.74
Subs. Price	0.909	0.067	-0.311	1.11
Altitude	0.581	0.052	-0.515	1.08
Urbanization	0.871	0.024	-0.407	1.05
Queen Contiguity				
Parameter	Heidelberger (1st Part/p-value) <sup>a</sup>	Heidelberger (2nd Part) <sup>b</sup>	Geweke <sup>c</sup>	Raftery <sup>d</sup>
Constant	0.830	0.085	-0.079	2.37
Price	0.142	-0.054	-0.238	1.12
Income	0.578	0.037	-0.399	2.71
Subs. Price	0.530	0.111	-0.450	1.16
Altitude	0.266	0.216	1.083	1.34
Urbanization	0.126	0.013	0.550	1.21
Rook Contiguity				
Parameter	Heidelberger (1st Part/p-value) <sup>a</sup>	Heidelberger (2nd Part) <sup>b</sup>	Geweke <sup>c</sup>	Raftery <sup>d</sup>
Constant	0.604	0.208	-0.930	2.39
Price	0.280	-0.155	-0.967	1.16
Income	0.226	0.095	0.866	2.72
Subs. Price	0.634	0.455	-0.745	1.14
Altitude	0.935	0.356	0.541	1.33
Urbanization	0.894	0.034	0.034	1.24

*Notes:* <sup>a</sup> Null hypothesis is stationarity of the chain, <sup>b</sup> Half-width to mean ratio (threshold of 0.1), <sup>c</sup> Mean difference test z-score, <sup>d</sup> Dependence factor (threshold of 5)