Habits, Catching Up with the Joneses and International Risk Sharing

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Abstract

A consumption-based asset pricing model which includes preferences with habits and catching up with the Joneses is studied in order to address the international risk sharing puzzle described by Brandt, Cochrane and Santa-Clara (2006). According to their measurements, international risk sharing in the G7 countries is almost complete since the correlation among stochastic discount factors is close to 97% when estimated with asset market data. However, consumption-based asset pricing models with standard preferences show that this correlation should be 25% in average according to consumption growth data. This paper shows that by using preferences with benchmark levels of consumption it is possible to reconcile international risk sharing measurements by obtaining high levels of risk sharing with consumption growth data. Additionally, the calibrated model is consistent with real exchange rate volatility, the average equity premium and the average risk free rate.

Keywords: International risk sharing; Exchange rate volatility; Discount factor; Habits; Catching up with the Joneses

JEL Classification: G12; G15; F31

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1. Introduction

This paper provides an alternative solution to the international risk sharing puzzle described by Brandt, Cochrane and Santa-Clara (2006) by analyzing an N-country version of the benchmark-consumption framework defined by Abel (1990). It is shown that this framework, which includes habits and catching up with the Joneses, is compatible with the high levels of risk sharing implied by asset prices. Furthermore, this model of preferences is consistent with some financial market moments across the G7 countries: real exchange rate volatility, the average equity premium, and the average risk free rate.

Preferences are assumed such that representative consumers in each country have a benchmark consumption level which depends on past world consumption growth as well as on past domestic consumption growth. Utility parameters are then calibrated such that a set of asset pricing and international risk sharing moments accord with data. In particular, one of these moments is the measure of bilateral international risk sharing which is obtained by computing correlation coefficients between stochastic discount factors. The model is calibrated with simulated i.i.d. consumption growth data for 7 countries and by computing world consumption growth as the weighted average across countries with weights corresponding to the relative size of the economies of the G7 countries. While the standard CRRA utility specification implies low degrees of international risk sharing, 25% in average for any value of the risk aversion coefficient, the calibration shows that the new model of preferences is compatible with the high degree of international risk sharing computed with asset market data (97% in average). These results represent a solution to the puzzle described in Brandt, Cochrane and Santa-Clara (2006).

This paper is also a contribution to the debate on the lack of empirical evidence of international consumption risk sharing described by Karen Lewis (1996, 1999) and examined recently by Marianne Baxter (2006). Full international risk sharing is an implication of open economy models with no barriers to trade, no financial frictions and complete markets. Furthermore, asset price data confirm the existence of high levels of risk sharing in developed economies. The insight that this research brings about is that it is possible to measure high levels of international risk sharing with real consumption data if representative consumers have habits and want to catch up with average world consumption.
This paper is organized in the following way. A summary of related research is presented in section 2. The international risk sharing puzzle is described in section 3. A model with N countries and benchmark-consumption preferences is presented in section 4. The calibration of the model and its implications on the puzzle and other moments of international markets are described in Section 5. Finally, section 6 concludes.

2. Related Literature

Although the empirical literature on international risk sharing is quite substantial, this section will focus on papers on international risk sharing which are closely related. The most recent literature review on international risk sharing is presented in Lewis (1999). The most closely related papers are Baxter (2006), Brandt, Cochrane and Santa-Clara (2006) and Colacito and Croce (2007).

Baxter (2006) extends and refines the empirical investigation on international risk sharing for OECD countries. She explores and analyzes several risk sharing testing methods on both short and long run time horizons. Furthermore, Baxter (2006) shows the importance of performing direct and bilateral risk sharing measurements. Direct tests allow observing the precise degree of risk sharing in contrast to indirect tests where it is only decided whether or not full risk sharing holds. By applying bilateral tests it is possible to analyze what kinds of country pairs are more likely to share risks better. Finally, Baxter (2006) shows that most of the positive international risk sharing evidence is concentrated on short run horizons.

In the paper by Brandt, Cochrane and Santa-Clara (2006)\(^1\), a new index for the measurement of international risk sharing is proposed on the basis of asset pricing theory. When this index is computed with asset market data, it implies a high degree of risk sharing between several pairs of developed countries. However, when the index is computed with consumption data assuming CRRA preferences, a low degree of risk sharing is obtained. This puzzle remains true even under reasonable levels of incomplete markets because the level of risk sharing with asset market data remains well above the index measured with

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\(^1\) An early version was published in 2001 in the NBER working paper series #8404
consumption data. The quest for a new specification of preferences is suggested by Brandt, Cochrane and Santa-Clara (2006) as the next step to solve the puzzle. I perform similar computations in Section 3 with updated data from the G7 countries.

A solution to the international risk sharing puzzle is provided by Colacito and Croce (2007) by assuming recursive preferences as in Epstein and Zin (1989), and introducing unobserved long run components to the consumption growth process in which shocks are perfectly cross-country correlated and highly persistent. They show, using US-UK consumption and asset market data, that under these assumptions it is possible to obtain high levels of international risk sharing along with the appropriate levels for several international asset market moments. Below, I show an alternative approach which is based on a natural generalization of CRRA preferences and does not require assuming any specific long run structure on the data generating process of consumption growth.

Colacito and Croce (2007) study the puzzle in the context of complete markets and consumption-based asset pricing with the goal of finding a model such that high levels of international risk sharing are found using both asset prices and consumption data. I follow a similar strategy below. An alternative approach that several recent papers use consists of constructing models with incomplete asset markets, productivity shocks and/or transaction costs such that the implied international risk sharing is low as suggested by measurements with consumption data and CRRA preferences. Under the latter approach, Corsetti, Dedola and Leduc (2008), as well as Benigno and Thoenissen (2008), present models with incomplete markets and shocks to the production structure. Similarly, Basu and Wada (2006) work a model with incomplete markets and heterogeneous consumers. In a slightly different line, Becker and Hoffman (2006) and Lewis (1996) allow for transaction costs and restrictions in international financial markets.

Additionally, there is a recent literature which has explored the use of non standard preferences to study real exchange rate and international finance phenomena other than international risk sharing. Moore and Roche (2007 and 2008) use “deep habits” preferences as in Ravn, Schmitt-Grohe and Uribe (2006) in order to study exchange rate volatility and

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2 Some incomplete market models have the drawback that they imply low realized risk sharing but high conditional risk sharing using forecast data. Devereux, Smith and Yetman (2009) show that this result holds in the models presented by Corsetti, Dedola and Leduc (2008) and by Benigno and Thoenissen (2008).
persistence as well as the forward premium puzzle. Verdelhan (2009) study a model based on external habits as in Campbell and Cochrane (1999) in order to address the uncovered interest parity puzzle as well as to study exchange rate volatility. Finally, Aydemir (2008) uses external habits to explain the countercyclical stock market correlations across countries.

3. The International Risk Sharing Puzzle

In this section first, I describe the method for international risk sharing measurement; next, I analyze the contrasting results with asset market data versus consumption data.

3.1. Measuring International Risk sharing

Following Brandt, Cochrane and Santa-Clara (2006) international risk sharing can be measured on the basis of Equation (1) which relates real exchange rate variations to the differential of Stochastic Discount Factors (SDFs).

\[ q_{i,t+1} - q_{i,t} = m_{i,t+1} - m_{US,t+1} \] (1)

Throughout this paper lower case letters are logs of the original variables. In (1), \( m_{i,t+1} \) and \( m_{US,t+1} \) are the US and country i's log SDFs respectively. This equation says that the log variation in the real exchange rate is equal to the difference between the log SDF in the foreign country and in the US\(^3\).

As shown originally by Backus and Smith (1993), Equation (1) results from a two-country endowment economy with complete markets. Assume that in each country a representative investor has access to a domestic bond that pays off one unit of domestic consumption next period in each state of the world. Additionally, these investors have access to a foreign bond that pays a stochastic return \( R_{i,t+1}^{*} \). Assume as well that the investors choose their optimal portfolios by solving a standard problem of dynamic optimization of utility. Then the Euler equation for a foreign investor buying a foreign bond is

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\(^3\) The real exchange rate is measured as the number of US goods that buys one unit of country i's good. It is measured using the nominal exchange rate and the consumer price level in both countries. Stochastic discount factors are defined as the time preference parameter multiplied by marginal utility growth in each country.
\( E_t(M^*_{t+1} R^*_{t+1}) = 1 \). The Euler equation for a domestic investor buying the same foreign bond is 
\( E_t(M^*_{t+1} r^*_{t+1} Q_{t+1} / Q_t) = 1 \). Since under complete markets SDFs are unique, we must have from these Euler equations: 
\( M^*_{t+1} = M_{t+1} Q_{t+1} / Q_t \). Taking natural logarithms at both sides we obtain Equation (1)\(^4\).

Perfect risk sharing is defined as the right hand side of (1) being equal to 0 as implied by complete market models with no frictions to international trade. Therefore, a simple measure of risk sharing is the correlation between SDFs\(^5\):

\[
\text{corr}(m^i t^{US}, m^j t^{US}) = \frac{\text{cov}(m^i t^{US}, m^j t^{US})}{\sqrt{\text{var}(m^i t^{US}) \text{var}(m^j t^{US})}}
\]  
(2)

The correlation defined in Equation (2) can be interpreted as the percentage of total risk that is actually shared across countries. This result is clearer if we compute the variance at both sides of (1).

\[
\text{var}(q_{t+1} - q_t) = \text{var}(m^j t^{US}) + \text{var}(m^i t^{US}) - 2 \text{cov}(m^i t^{US}, m^j t^{US})
\]  
(3)

If the correlation in (2) is perfect and we assume similar SDF variances for both countries, then the variance of real exchange rate fluctuations is zero in (3). This result means that 100% of risks are shared. But if the correlation in (2) is equal to zero, then real exchange rate fluctuations are as large as the sum of SDF variances in Equation (3). In this case, countries do not share any risks because all available risks are transmitted to the real exchange rate.

3.2. Computation with Asset Market Data

In this section I measure international risk sharing by computing the correlation between SDFs for pairs of G7 countries using data on real exchange rates, real stock returns, real risk free rates and real foreign currency returns. Following Brandt, Cochrane and Santa-Clara (2006), Equation (2) can be computed as in the following formula:

\(^4\) See Cochrane (2005, chapter 4), for a proof of the existence and unicity of the stochastic discount factor in asset pricing economies under complete markets.

\(^5\) The correlation between SDFs is very similar to the international risk sharing index proposed by Brandt Cochrane and Santa-Clara (2006). In fact, both measures are equivalent in the context of the model presented in Section 4. See the Appendix for an algebraic explanation of this result.
In Equation (4), $\mu' \Sigma^{-1} \mu$ and $\mu'^{US} \Sigma'^{-1} \mu^{US}$ are the SDF variances in country $i$ and in the US respectively. Three assets are considered for the computation of (4): domestic stock, foreign currency and foreign stock. Therefore, $\mu$ and $\mu^{US}$ are the vectors of expected excess returns for each one of these assets from the point of view of an investor in country $i$ and in the US, respectively. The covariance matrix for these three excess returns is $\Sigma$ which is equal for both investors because the rows are arranged such that a domestic shock for country $i$’s investor is the foreign shock for the US investor and vice versa. It is important to point out that Equation (4) is computed with the minimum variance SDF which is calculated by using Hansen and Jaganathan’s (1991) bounds. This computation is performed in a continuous time setting which includes domestic and foreign assets and assumes complete markets. The final expressions for the log SDFs are constructed via Ito’s lemma. See Brandt, Cochrane and Santa-Clara (2006, p. 676) for details.

Equation (4) is computed with data for pairs of G7 countries with the US as the base country. Annual data from 1975 through 2006 are used in all computations. Table A1, in the appendix, show summary statistics for mean returns as well as return volatilities and correlations among countries. Table 1 shows results from the computation of the correlation between SDFs with respect to the US.

<table>
<thead>
<tr>
<th></th>
<th>UK</th>
<th>GERMANY</th>
<th>JAPAN</th>
<th>CANADA</th>
<th>FRANCE</th>
<th>ITALY</th>
<th>AVERAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Sharing</td>
<td>0.986</td>
<td>0.964</td>
<td>0.977</td>
<td>0.991</td>
<td>0.961</td>
<td>0.955</td>
<td>0.972</td>
</tr>
<tr>
<td>Standard Deviation of SDFs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>US</td>
<td>0.524</td>
<td>0.406</td>
<td>0.524</td>
<td>0.386</td>
<td>0.396</td>
<td>0.372</td>
<td>0.435</td>
</tr>
<tr>
<td>Foreign Country</td>
<td>0.496</td>
<td>0.404</td>
<td>0.551</td>
<td>0.375</td>
<td>0.369</td>
<td>0.355</td>
<td>0.425</td>
</tr>
<tr>
<td>Standard Deviation of RER</td>
<td>0.090</td>
<td>0.108</td>
<td>0.116</td>
<td>0.051</td>
<td>0.109</td>
<td>0.110</td>
<td>0.097</td>
</tr>
</tbody>
</table>

Risk sharing is measured with correlations between SDFs as described in equation (4). The standard deviation of the SDFs is measured under the assumption that the investable assets are the domestic interest rate, the domestic stock market, the foreign interest rate and the foreign stock market. This is the formula for the SDF variance in each country: $\mu' \Sigma^{-1} \mu$.

Note that these are $(3 \times 1)$ vectors of constants. The risk free rate is subtracted from the gross returns in order to compute excess returns for these three kinds of assets.
From Table 1, it is possible to conclude that for the G7 countries the extent of international risk sharing is very high. This is because only less than 5% of the available market risk is not shared or equivalently, the correlation is higher than 95% in all cases. The intuitive explanation for this result is that SDFs, computed with asset market data, are much more volatile than real exchange rate movements. Thus, while the standard deviation of SDFs averages 43%, the standard deviation of the real exchange rate averages only 10%. It is important to note that these computations do not make any assumption about the functional form of consumer preferences. They only assume complete markets and three assets in each country such that a unique SDF exists and can be recovered from asset market data.

3.3. Computation with Consumption Data
Following Brandt, Cochrane and Santa-Clara (2006), I start by assuming standard CRRA preferences in order to measure risk sharing with consumption data. The marginal utility of consumption in this case is \( U'(C_t) = \gamma^{\alpha} \), where \( \alpha \) is the risk aversion coefficient. Log marginal utility growth is then \(-\alpha(\epsilon_{t+1} - \epsilon_t) \equiv -\alpha \Delta \epsilon_{t+1} \), which is the risk aversion coefficient multiplied by log consumption growth. Therefore, the variance of the log SDF is \( \alpha^2 \text{var}(\Delta \epsilon_{t+1}) \). Since both countries have the same risk aversion coefficient, \( \alpha \) cancels out of the correlation formula. Thus, the correlation between SDFs is equivalent to the correlation between consumption growths across countries:

\[
\text{corr}(m_{US}^{t+1}, m_{i}^{t+1}) = \frac{\text{cov}(\Delta \epsilon_{US}^{t+1}, \Delta \epsilon_{i}^{t+1})}{\sqrt{\text{var}(\Delta \epsilon_{US}^{t+1}) \text{var}(\Delta \epsilon_{i}^{t+1})}} \tag{5}
\]

Equation (5) has the same interpretation as Equation (2); it measures the percentage of total risk which is actually shared between the US and country \( i \). Using annual real consumption per capita data for the G7 countries, Table 2 shows the results of computing Equation (5) as well as the implied volatility between SDFs.

From table 2 it is clear that risk sharing measurements with consumption data are very different to those computed with asset market data. Only in the case of Canada, the correlation is above 0.5. It is close to zero in the case of Germany and slightly negative for Italy. The interpretation of a negative correlation is that the unshared risk is greater than the
This contrasting difference between risk sharing measures is the core of the puzzle described by Brandt, Cochrane and Santa-Clara (2006).

<table>
<thead>
<tr>
<th>TABLE 2 - RISK SHARING INDEX FROM CONSUMPTION DATA WITH CRRA UTILITY FUNCTION</th>
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<tbody>
<tr>
<td>Span of annual data: 1975-2006; Base Country: US.</td>
</tr>
<tr>
<td>UK</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Risk Sharing: Correlation of Consumption Growth</td>
</tr>
<tr>
<td>Standard Deviation of Stochastic Discount Factor</td>
</tr>
<tr>
<td>US Investor</td>
</tr>
<tr>
<td>Foreign Investor</td>
</tr>
</tbody>
</table>

This table shows risk sharing measurements with consumption growth. Consumption is real per-capita and includes non-durables and services. Nominal consumption is deflated with the CPI price index and turned per-capita with population statistics. All data were retrieved from the IFS database. Annual data span 1975 through 2006.

In Table 2 we can see that the SDF volatility for a US investor (3.7%) is very close to the average SDF volatility for the rest of countries (3.8%). These volatilities are entirely driven by the volatility of consumption growth multiplied by a standard value for the coefficient of risk aversion ($\alpha = 2$). Notice that while the average SDF volatility is 3.8% in Table 2, it is 43% in Table 1 which is computed with asset market data. Therefore, in order to reconcile both computations we would need risk aversion coefficients close to 23. These high implied risk aversion coefficients represent the second aspect of the international risk sharing puzzle.

It is important to highlight that the measurement with asset market data does not make any specific assumption about the form of the utility function. In order to compute Equation (4), it is only assumed the presence of complete markets, no arbitrage in asset markets and the law of one price across assets. These general assumptions guarantee the existence and uniqueness of the SDF in each country. Furthermore, Brandt, Cochrane and Santa-Clara (2006) show that high levels of risk sharing are not necessarily incompatible with the home bias puzzle. This is because while the home bias puzzle is related to problem of choosing an optimal portfolio at any given period of time, the risk sharing puzzle is related to the optimal intertemporal allocation of consumption and savings.

Three alternative types of explanations can be provided for the international risk sharing puzzle. First, the complete markets assumption is too strong for actual asset markets. Second, in practice economic agents deviate from the behavior predicted by first order

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Cochrane (2005, chapter 4) presents a detailed explanation of the existence and uniqueness results.
conditions. Third, a CRRA utility function is not an accurate description of agents’ preferences.

Brandt, Cochrane and Santa-Clara (2006) show that it would be necessary to assume extremely volatile non-market risks in order to obtain low levels of risk sharing with asset market data and with incomplete asset markets. Some recent works, however, obtain low consumption risk sharing from combining incomplete markets and supply side shocks. The second alternative solution consists of analyzing different types of deviations from first order conditions by assuming frictions or transaction costs. In this paper, I follow the third solution path by assuming an alternative specification of preferences under the motivation that introducing benchmark consumption levels in the utility function has been useful to solve the equity premium puzzle and the forward premium puzzle as shown by Campbell and Cochrane (1999), Verdelhan (2009) and Abel (1990, 2006) among others.

4. A Framework with Habits and Catching Up with the Joneses

In this section, I initially describe the framework which generalizes a CRRA utility function by including a benchmark level of consumption; this benchmark includes habits and catching up with the Joneses. Then, I analyze the general implications of this model for international risk sharing and for asset pricing. Next, I present a calibration of this framework which allows reconciling international risk sharing measurements with the evidence from asset market data presented in Section 3.2. I also explain how the calibrated model is compatible with some stylized facts in international markets: real exchange rate volatility, average equity premia and average risk free rate.

4.1. The Model Setup

I use a consumption-based asset pricing framework based on Abel (1990, 2006) and extended to include N countries \((i = 1, 2, \ldots, N)\). The representative consumer in each country \(i\) maximizes:

\[ U(c) = \log(c) + \beta \log(\bar{c}) \]

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8 See among others, Corsetti, Dedola and Leduc (2008) and Benigno and Thoenissen (2008).
9 See for example, Lewis (1996) and Becker and Hoffman (2006).
\[ U_i = E_i \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{1}{1-\alpha} \right) \left( \frac{C_{i+j}}{V_i^{\gamma_j}} \right)^{1-\alpha} \right] \]  

In Equation (6), \( \alpha \) denotes the risk aversion coefficient, \( \beta \) is the time discount factor, \( C_i \) is the level of consumption in each country and \( V_i^{\gamma_j} \) is the benchmark level of consumption where the power \( \gamma \) measures the importance of the benchmark in consumer’s preferences. Benchmark consumption includes past domestic consumption and past world consumption and it is defined by:

\[ V_i = \left[ \left( C_{i-j} \right)^D \left( C_{w,j-1} \right)^{1-D} \right] \]  

In Equation (7), \( C_w \) denotes world consumption and \( D \) is a weight that measures the importance of domestic consumption relative to world consumption in the composition of the benchmark level of consumption. World consumption is defined as the geometric weighted average of consumption across countries. The weights \( \omega_i \) in Equation (8) are determined by the relative size of country \( i \):

\[ C_w = \prod_{i=1}^{N} C_i^{\omega_i} \]  

The utility framework in Equations (6) to (8) nests the standard CRRA case when \( \gamma = 0 \) because in this case the benchmark consumption does not have any influence in utility. When \( \gamma > 0 \), utility depends on the ratio between domestic and benchmark consumptions. The presence of \( V_i^{\gamma_j} \) in the utility function captures two effects: habit formation and catching up with the Joneses. By calibrating \( \gamma \) and \( D \), we can have an idea of the importance of allowing for these kinds of consumer effects in explaining stylized facts in asset markets.

From Equation (6), it is possible to compute the marginal utility of consumption in each country.

\[ \frac{\partial U_i}{\partial C_i} = \frac{1}{C_i} E_i \left[ \left( \frac{C_i}{V_i^{\gamma_j}} \right)^{1-\alpha} - \gamma D \beta \left( \frac{C_{i+1}}{V_i^{\gamma_j}} \right)^{1-\alpha} \right] \]  

\[ (9) \]
Note that if \( \gamma = 0 \) then marginal utility in (9) becomes exactly equal to the case of a standard CRRA utility function \( (C_t^{-\alpha}) \). Therefore, it is possible to partition marginal utility in three components: standard CRRA, benchmark consumption and habits. These three components are specified in Equation (10).

\[
\frac{\partial U_t}{\partial C_t} = C_t^{-\alpha} V_t^{\gamma(\alpha-1)} H_t, \tag{10}
\]

The component \( V_t^{\gamma(\alpha-1)} \) measures the effect of benchmark consumption on marginal utility. This effect has a negative as well as a positive component. The former component is the instantaneous drop in utility which occurs when \( V_t \) increases. The positive component is about the higher marginal utility which is possible to obtain with lower ratios \( C_t/V_t \) as a result of the concavity of the utility function. The parameter \( \alpha \) determines the extent of this concavity; therefore when \( \alpha > 1 \), the positive effect dominates such that the net effect of \( V_t \) on marginal utility is positive. In Equation (10) it is also clear that in the log-utility case, \( \alpha = 1 \), both components cancel each other so that the effect is zero. Finally, when the utility function is less concave, \( \alpha < 1 \), the net effect of the benchmark consumption on marginal utility is negative.

The component \( H_t \), defined in Equation (11), measures the effect of habits on marginal utility. It is a number between 0 and 1 which takes into account the fact that a higher consumption today has a negative effect on tomorrow’s utility because it increases benchmark consumption.

\[
H_t = 1 - D\gamma \beta E_t \left( X_{t+1}^{1-\alpha} \right) X_t^{\gamma(\alpha-1)} X_{w,t}^{(1-D)\gamma(\alpha-1)} \tag{11}
\]

In (11), \( X_t \) corresponds to the gross rate of consumption. Therefore, we define:

\[
X_{t+1} = C_{t+1}/C_t \quad \text{and} \quad X_{w,t+1} = C_{w,t+1}/C_{w,t}.
\]

4.2. Asset Pricing Implications

Equation (10) and the definition of benchmark consumption allow computing easily the Stochastic Discount Factor (SDF) or pricing kernel as the product of the time discount factor and marginal utility growth.
The correlation of the natural logarithm of (12) across countries and its volatility are computed in the next section in order to study the international risk sharing puzzle. For additional asset pricing implications it is necessary to insert the definition of SDF in the following Euler equation.

\[ E_t(M_{t+1} R_{t+1}) = 1 \]  \hspace{1cm} (13)

Equation (13) is the fundamental asset pricing equation which results from the first order optimality conditions for intertemporal consumption choice. \(^{10}\) \( R_{t+1} \) is the gross rate of return of any asset that we want to price. In particular, using (12) and (13) we can easily compute the risk free rate of return:

\[ R_f = \frac{H_t}{\beta X_t^{D\gamma(a-1)} X_t^{y(1-D)\gamma(a-1)} E_t \left( X_t^{a} H_{t+1} \right)} \]  \hspace{1cm} (14)

Another asset we want to price using (13) is equity. In this case, the gross return is determined by dividends as well as by market price appreciation. Following Abel (2006) we initially assume the equivalency between consumption and dividends. Therefore, the definition of equity return is given by the following equation in which \( P_t \) is the price of equity and \( \lambda \) is a constant leverage parameter.

\[ R_{t, t+1} = \frac{(P_{t+1} + C_{t+1}^{\lambda})}{P_t} \]  \hspace{1cm} (15)

Following the formulation in Abel (2006), the leverage parameter \( \lambda \) captures the fact that when corporations have outstanding bond debt which must be repaid every period, the coefficient of variation of their levered equity payoff increases by a factor of \( \lambda \) with respect to the unlevered case. That is, the presence of leverage increases the volatility of the equity returns. Leverage coefficients assumed in related papers in the literature are \( \lambda = 3.6 \) and \( \lambda = 3 \) in Abel (2006) and Colacito and Croce (2007) respectively.

\(^{10}\) See Cochrane (2005, chapter 1) for a formal derivation and alternative uses of Equation (13).
Using equations (12), (13) and (15) it is possible to compute a closed form solution for \( R_{t,t+1} \). This is performed by finding a solution for the price-consumption ratio in terms of its value one period ahead and then computing a limiting solution by iterating forward. For the latter step it is necessary to make a distributional assumption such that consumption growth is iid log normal and uncorrelated across countries with mean \( g \) and variance \( \sigma^2 \). Formally: \( \ln(C_{t+1}) - \ln(C_t) \sim iid N\left(g, \sigma^2\right) \). This distribution of consumption growth as well as the mean and the variance are common for all N countries. The resulting equation for equity return takes the following form\(^{11}\):

\[
R_{t,t+1} = J_{t+1}K_t
\]

In (16), \( J_{t+1} \) and \( K_t \) correspond to the unpredictable and predictable components of the equity return respectively. The following equations define these components in terms of the parameters of the models and consumption growth.

\[
J_{t+1} = \frac{X_{t+1}^{xD_y(a-1)+\lambda} X_{x,t+1}^{x(1-D)y(a-1)}}{H_{t+1}} + X_{n}^{x} \frac{1 - A_0}{\beta A_1}
\]

\[
K_t = \frac{H_t}{X_t^{xD_y(a-1)} X_{x,t}^{x(1-D)y(a-1)}}
\]

In (17), the terms \( A_0 \) and \( A_1 \) are unconditional expectations which are constant as a result of the i.i.d. distribution of consumption growth. These constants are defined in the following equations:

\[
A_0 = \beta E\left(X_t^{x(a-1)(D_y-1)+\lambda-1} X_{x,t}^{x(1-D)y(a-1)}\right)
\]

\[
A_1 \equiv E\left(X_t^{x} H_t\right)
\]

The solution for equity returns described in (16), (17) and (18) is valid provided the following restrictions hold: \( 0 < A_0 < 1 \) and \( A_1 > 0 \). These restrictions are further explained in the appendix.

\(^{11}\) The algebraic steps to compute the equity return are described in detail in the appendix.
The equity premium is defined as the ratio between gross equity returns and the gross risk free rate; that is, the equity premium is the result of dividing (16) by (14). Using the expressions defined in (17), (18) and (20), the final expression for the equity premium can be simplified in the following way:

\[ EP_{t+1} = \beta J_{t+1} E_t \left( X_t^{\alpha H_{t+1}} \right) \]  

(21)

In order to compute the implied rate of real exchange rate appreciation, I use Equation (1) in levels and I denote US variables with their respective subscript. The implied appreciation of any country’s real exchange rate with respect to the US is computed with the following expression:

\[ \frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}}{M_{US,t+1}} \]  

(22)

Using the definition of SDF for each country as in Equation (12), it is possible to obtain an expression for the real exchange rate appreciation in terms of parameters, consumption growth and the habit components of marginal utility as defined in (11).

\[ \frac{Q_{t+1}}{Q_t} = \left( \frac{X_{t+1}}{X_{US,t+1}} \right)^{-\alpha} \left( \frac{X_t}{X_{US,t}} \right)^{D\gamma(\alpha-1)} \left( \frac{H_{t+1}}{H_{US,t+1}} \right) \left( \frac{H_{US,t}}{H_t} \right) \]  

(23)

Note that the expression in (23) nests the standard CRRA utility case (\( \gamma = 0 \)) which implies no habit effects (\( H_t = 1 \forall t \)). Therefore, in the CRRA case real exchange rate appreciation is entirely determined by the first parenthesis in the right hand side of (23) which describes relative consumption growth with a negative exponential given by the risk aversion coefficient. This result was shown for the first time by Backus and Smith (1993).

4.3. Simplified Closed-Form Expressions

As a first step to understand a solution for the international risk sharing puzzle, I compute closed form expressions for the correlation between SDFs and for their volatility. In order to simplify the interpretation, I assume no habit effects (\( D = 0 \)) so that \( H_t = 1 \) and only average world consumption remains as benchmark consumption. For the final
calibration of the model this assumption will be dropped. Similarly to Section 4.2, consumption growth is i.i.d. lognormally distributed: \( \ln(C_{i,t+1}) - \ln(C_i) \sim iid N(g, \sigma^2) \).

Assuming equal parameters in the utility function for all countries, the expression in (24) is the implied correlation of log SDFs between any country and the US. This correlation is our measure of international risk sharing degree\(^{12} \).

\[
Corr(m_{US,t}, m_t) = \frac{\gamma^2 (\alpha - 1)^2 \bar{\omega}^2}{\alpha^2 + \gamma^2 (\alpha - 1)^2 \bar{\omega}^2} \tag{24}
\]

In Equation 24, \( \bar{\omega} \) corresponds to the sum of squared weights which are used to compute consumption growth; therefore, \( \bar{\omega} \) is a positive fraction.

\[
\bar{\omega} = \sum_{i=1}^{N} \omega_i^2 \tag{25}
\]

Several interesting results can be deduced from Equation (24). First, the correlation is zero whenever \( \gamma = 0 \) or \( \alpha = 1 \), that is, under CRRA or log utility. Second, under the presence of benchmark consumption (\( \gamma > 0 \)) and assuming \( \alpha > 1 \), the correlation increases as \( \gamma \) and/or \( \alpha \) increase. Third, as the number of countries (\( N \)) increases, the fraction \( \bar{\omega} \) gets closer to zero and then the correlation decreases\(^{13} \). The reason for the latter result is that world consumption volatility shrinks as \( N \) increases.

Under the same assumptions as Equation (24), the expression corresponding to the variance of any country’s SDF is the following:

\[
\text{var}(m_{t+1}) = \alpha^2 \sigma^2 + \gamma^2 (\alpha - 1)^2 \sigma^2 \bar{\omega}^2 \tag{26}
\]

Equations (24) and (26) show that in order to reconcile international risk sharing measurements, as described in Section 3, it is necessary to have strong enough benchmark consumption effects (\( \gamma > 0 \)) and possibly risk averse consumers (\( \alpha > 1 \)).

Equation (23) shows the general solution for the real exchange rate appreciation. If we assume \( D = 0 \), it is easy to see that the unconditional expectation of the appreciation rate (\( \log Q_{t+1} - \log Q_t \)) is zero as a result of the symmetry in parameters and consumption growth.

\(^{12} \)The appendix contains a description of the derivation of these results.

\(^{13} \)This result assumes that the additional countries are neither too big nor too small.
distributions. Interestingly, its variance only depends on $\alpha$ and $\sigma^2$; it does not depend on the degree of catching up with the Joneses.

$$\text{var}(\log Q_{t+1} - \log Q_t) = 2\alpha^2 \sigma^2$$  \hspace{1cm} (27)

For the risk free rate case, Equation (14) is the most general solution. The unconditional expectation of the log risk free rate is, however, much easier to interpret by assuming $D = 0$:

$$E(\log(R_t)) = -\log(\beta) - g(\gamma(\alpha - 1) - \alpha) - \alpha^2 \sigma^2 / 2$$  \hspace{1cm} (28)

Equation (28) shows that the presence of catching up with the Joneses effects ($\gamma > 0$) is equivalent to a lower expected value for the risk free rate. Another effect of the presence of a benchmark consumption level in the utility function is that the risk free rate is no longer constant and its volatility is directly related to the parameter $\gamma$. The variance of the risk free rate shows this effect:

$$\text{var}(\log(R_t)) = \gamma^2 (\alpha - 1)^2 \sigma^2 \sigma^2$$  \hspace{1cm} (29)

Equations (17), (20) and (21) allow writing the general solution for the equity premium. In order to understand better the effect of catching up with the Joneses, I assume no habits ($D = 0$) as well as that $A_0 = 1$ where $A_0$ is defined in (19). The latter assumption is reasonable since in practice $A_0$ shows values very close to 1 because a successful calibration needs high values for $\gamma$ which are compensated with lower values for $\beta$ such that the solution for the equity premium in (21) is valid.

Using these assumptions, the following equation describes the expected log equity premium:

$$E(\log(EP_t)) = \log(\beta) + g(\lambda - \alpha + \gamma(\alpha - 1)) + 0.5\alpha^2 \sigma^2$$  \hspace{1cm} (30)

From (30), it is clear that the presence of catching up with the Joneses in the utility function implies a higher equity premium as long as $\alpha > 1$. This result is important because, as shown by Abel (1990, 2006), this utility framework can be parameterized to solve the equity premium puzzle. The expression in (31) shows the implied variance of the equity premium. Derivations of Equations (30) and (31) are described in detail in the appendix.
\[
\text{var}(\log(EP)) = \sigma^2 \left[ \lambda^2 + \gamma(\alpha - 1)^2 \bar{\omega}_i^2 + 2\lambda\gamma(\alpha - 1)\omega_i \right] \tag{31}
\]

Similarly to the risk free rate case, Equation (31) shows that the presence of a benchmark consumption level increases the volatility of the equity premium as long as \( \alpha > 1 \). Interestingly, this volatility is increasing in the size of the country under consideration which is measured by the fraction \( \omega_i \).

As a summary of this subsection, the presence of catching up with the Joneses allows computing more correlated and volatile SDFs with consumption data. Section 5 shows how this feature is useful to reconcile the international risk sharing puzzle. Additionally, when \( \gamma > 0 \) the expected risk free rate is lower and the expected equity premium is higher than in the CRRA case. The latter effect is consistent with the equity premium puzzle. A potential drawback of this framework is that it may imply a huge increase in the volatility of returns.

5. Reconciling International Risk Sharing Measurements

The goal in this section is finding a parameterization for the framework with habits and catching up with the Joneses such that high levels of international risk sharing become compatible with consumption data. This parameterization is then applied to the most general asset pricing formulas described in 4.2 to study the implications for the first and second moments of the risk free rate, the equity premium and real exchange rate appreciation.

The calibration is performed by simulating 1000 consumption growth observations for 7 countries and then choosing parameters in order to match some implied asset market moments with their observed values. For each country, consumption growth has a log normal distribution: \( \ln(C_{it}) - \ln(C_i) \sim i.i.d. N\left( g, \sigma^2 \right) \) where the mean and variance are assumed equal across countries: \( g = 2.5\% \) and \( \sigma = 2.5\% \). These values correspond, respectively, to the average and standard deviation of annual consumption growth across the G7 countries. In line with its definition in Equation (8), world consumption growth is computed as the weighted average for 7 countries by using the same relative weights as the G7 countries' real GDP. These weights are shown in Table 3.
The parameters to calibrate are those corresponding to the framework described in Section 4.1: $\beta$, $\alpha$, $\gamma$, $D$ and $\lambda$. The goal is matching the stylized facts on international risk sharing described in Section 3.2 (correlation and volatility of Stochastic Discount Factors), as well as the following moments of international markets: real exchange rate volatility, average risk free rate, average equity premium, and volatility of the equity premium. While Table 3 summarizes the resulting parameter values, Table 4 compares implied versus empirical values for the mentioned moments.

The calibrated model implies highly correlated SDFs in line with the stylized facts computed with asset market data. Thus, this parameterization allows obtaining high levels of international risk sharing with consumption data which represents a solution to the puzzle described in Section 3. This parameterization also allows obtaining values for the real exchange rate volatility, the average equity premium, the volatility of the equity premium and the average risk free rate which are close to those observed in data from asset markets in the G7 countries.

### TABLE 3 - Features of the Model Calibration

<table>
<thead>
<tr>
<th>A. Weights Used to Compute Total Consumption Growth in G7 countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$ United States                                   48.6%</td>
</tr>
<tr>
<td>$\omega_2$ Japan                                           15.5%</td>
</tr>
<tr>
<td>$\omega_3$ Germany                                         9.6%</td>
</tr>
<tr>
<td>$\omega_4$ United Kingdom                                  7.9%</td>
</tr>
<tr>
<td>$\omega_5$ France                                         7.6%</td>
</tr>
<tr>
<td>$\omega_6$ Italy                                          6.6%</td>
</tr>
<tr>
<td>$\omega_7$ Canada                                         4.1%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Calibrated Value of Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$ Risk Aversion Coefficient                          0.85</td>
</tr>
<tr>
<td>$\gamma$ Benchmark Consumption                              7.42</td>
</tr>
<tr>
<td>$\beta$ Time Discount Factor                                1.00</td>
</tr>
<tr>
<td>$D$ Habit Weight                                           12.9%</td>
</tr>
<tr>
<td>$\lambda$ Leverage Factor                                   3.0</td>
</tr>
</tbody>
</table>

Panel A presents the weights for the computation of total consumption growth. These weights are computed from real GDP data which is adjusted by Purchasing Power Parity (PPP) as of 2006 and retrieved from World Development Indicators 2008. Panel B shows the final calibrated value for the parameters of the utility framework with habits and catching up with the joneses described in Section 4.
The implied SDF volatility in the parameterized model in Table 4 is lower than that computed with asset market data. As shown in Equation (26), this volatility is directly linked to the calibrated value for $\gamma$. Therefore, a higher value for this benchmark consumption parameter would increase the equity premium as well as its volatility pushing them away from their observed values as is clear in (30) and (31). The implied SDF volatility is, however, high enough to be consistent with high levels of risk sharing.

The only drawback in the parameterization presented in Tables 3 and 4 is the implied risk free rate volatility which is much higher than the empirical one. This result is a negative feature of the utility model which was already identified in Abel (1991). The reason for this implied volatility is that its value is directly related to the high value for $\gamma$ which is necessary in order to match the stylized facts on international risk sharing as shown in equations (24) and (29). Finally, it is important to note that the calibrated values for all parameters are consistent with the pair of constraints that the model must satisfy: $H_j > 0$ and $0 < A_0 < 1$.

It is important to understand the contribution of both types of habits for the results presented in Table 4. This analysis is performed in Table 5 by comparing the results from

---

The former constraint prevents marginal utility from being zero or negative as explained in Equations (10) and (11). The latter constraint is a necessary condition for the equity premium solution to be valid as explained in Section 4.2.
the calibrated model with those from the model without any type of habits and the model with no internal habits.

<table>
<thead>
<tr>
<th>TABLE 5 - Moments Implied by Alternative Specifications of the Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>-------------------</td>
</tr>
<tr>
<td>A. Stylized Facts on International Risk Sharing</td>
</tr>
<tr>
<td>Corr($m_{iS}$, $m_i$)</td>
</tr>
<tr>
<td>$\text{var}(m_{iS}, m_i)^{0.5}$</td>
</tr>
<tr>
<td>B. International Market Moments</td>
</tr>
<tr>
<td>$\text{var}(\log Q_{t+1} - \log Q_t)^{0.5}$</td>
</tr>
<tr>
<td>$E(\log(EP))$</td>
</tr>
<tr>
<td>$\text{var}(\log(EP))^0.5$</td>
</tr>
<tr>
<td>$E(\log(R_t))$</td>
</tr>
<tr>
<td>$\text{var}(\log(R_t))^0.5$</td>
</tr>
</tbody>
</table>

This table compares the implied moments from alternative parametrizations of the model with the moments implied by the calibration presented in Table 3. Column I shows the results from assuming a CRRA utility function with the remaining parameters as in Table 3. Column II shows the results from assuming parameter values as in Table 3 but with no internal habit formation. Column III shows the results from the complete calibrated model with both internal and external habits. Panel A compares the calibrated model with stylized facts for international risk sharing. Panel B makes a similar comparison with moments in international financial markets.

In Table 5, it is clear that the presence of habit formation is crucial in order to obtain high levels of risk sharing and volatile SDFs. In particular, the presence of both type of habits, ($\gamma > 0$ and $0 < D < 1$), allows reaching the stylized facts on the correlation of SDFs. It is important to highlight that the presence of internal habits ($D > 0$) is key in order to obtain international market moments which are consistent with data. In Table 5, internal habits allow increasing the volatility of the real exchange rate as well as the first and second moment of the equity premium. Additionally, they decrease the risk free rate so that it gets closer to its empirical value.

In summary, the calibration presented in Table 3 allows incorporating habit formation in such a way that international risk sharing measurements, with uncorrelated consumption growth data, are consistent with stylized facts. These parameters also manage to control habits as a new source of volatility so that the implied values for additional market moments are consistent with data. The only exception is the volatility of the risk free rate which is found to be closely related to the volatility of the SDF in contrast with observed data.
Finally, it is interesting to point out that country by country implied asset pricing and risk sharing moments are not very different from each other except in the US case. For the remaining countries, most of the results are very similar due to the assumption on symmetric parameters across countries as in Table 3. Since the US is the biggest country, it has a more volatile SDF, a higher equity premium and a lower risk free rate. Table A2 in the appendix shows in detail these country by country results.

6. Conclusions

International risk sharing is better than usually thought of when it is measured with asset market data from the G7 countries and under the assumption of complete markets. The results show that in average 97% of total risks are shared among countries. In contrast, when it is measured with real consumption data and standard preferences, risk sharing levels are much lower (26% in average). Additionally, total risk levels, which are measured by the volatility of Stochastic Discount Factors (SDFs), are much larger when computed with asset market data (43%) than with consumption data and standard preferences (4%). This international risk sharing puzzle was originally described by Brandt, Cochrane and Santa-Clara (2006).

This puzzle is studied in this paper by measuring international risk sharing in the context of a consumption-based asset pricing framework which includes habits and catching up with the Joneses. This framework is an open economy extension of the model studied by Andrew Abel (1991, 2006). Namely, consumers have a benchmark level of consumption to compare their own consumption with; this benchmark consists of their own past level of consumption as well as the past average consumption around the world. This assumption captures the intuitive idea that consumers not only have habits but also look at foreign countries when evaluating their own standards of consumption.

By calibrating the model with a strong influence from a benchmark level of consumption and using simulated consumption growth data, it is possible to obtain highly correlated SDFs between the US and the G7 countries. Furthermore, the implied volatilities of SDFs are much higher than those implied by the standard utility function. The calibrated
model also is consistent with the empirical real exchange rate volatilities, the average risk free rate and the first and second moments of the equity premium.

These computations, using the calibrated framework, represent an alternative explanation for the puzzle described by Brandt, Cochrane and Santa-Clara (2006). These results are also a contribution to the debate on the lack of evidence on consumption-based international risk sharing as described by Karen Lewis (1996, 1999). Colacito and Croce (2007) also addressed this puzzle for UK-US data, by assuming recursive preferences as in Epstein and Zin (1989), and assuming a long run component in the data generating process of consumption growth. However, an essential requirement for their results is that consumption growth has a long run component which is highly correlated among countries. This feature is difficult to prove with observed data.
### Appendix A: Additional Tables

#### Table A1 - Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>US Stock</th>
<th>UK</th>
<th>Germany</th>
<th>Japan</th>
<th>Canada</th>
<th>France</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
<td>Stock X-rate</td>
</tr>
<tr>
<td>mean</td>
<td>6.78</td>
<td>7.56</td>
<td>1.87</td>
<td>8.44</td>
<td>0.7</td>
<td>6.62</td>
<td>5.32</td>
</tr>
<tr>
<td>Std. Dev</td>
<td>13.06</td>
<td>11.25</td>
<td>9.72</td>
<td>21.72</td>
<td>11.69</td>
<td>18.39</td>
<td>17.24</td>
</tr>
<tr>
<td>Return Correlations (1 = 100%)</td>
<td>0.66</td>
<td>0.04</td>
<td>0.52</td>
<td>0.03</td>
<td>0.33</td>
<td>0.17</td>
<td>0.61</td>
</tr>
</tbody>
</table>

This table is a replication of "Table 1" in Brandt, Cochrane and Santa-Clara (2006) but adding more countries and using a longer span of data. It shows summary statistics for real excess returns on stock indices and exchange rates for G7 countries. The stock indices are total market returns from Datastream, the interest rates are for one-month Eurocurrency deposits from Datastream, and the CPI as well as exchange rates are from the International Monetary Fund's IFS database. The stock returns (Stock) are excess returns over the same country's one-month interest rates. The exchange rate returns (X-rate) are excess returns for borrowing in dollars, converting to the foreign currency, lending at the foreign interest rates, and converting the proceeds back to dollars. Annual data from 1975 through 2006.

#### Table A2 - Moments Implied by the Calibrated Model: Country by Country

<table>
<thead>
<tr>
<th></th>
<th>US</th>
<th>UK</th>
<th>GERMANY</th>
<th>JAPAN</th>
<th>CANADA</th>
<th>FRANCE</th>
<th>ITALY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation of SDFs</td>
<td>1,000</td>
<td>960</td>
<td>961</td>
<td>961</td>
<td>960</td>
<td>960</td>
<td>965</td>
</tr>
<tr>
<td>SDF Volatility</td>
<td>34.3%</td>
<td>29.3%</td>
<td>29.7%</td>
<td>30.7%</td>
<td>28.4%</td>
<td>29.3%</td>
<td>28.8%</td>
</tr>
</tbody>
</table>

A. Stylized Facts on International Risk Sharing

B. International Market Moments

<table>
<thead>
<tr>
<th></th>
<th>Real Exchange Rate Volatility</th>
<th>Risk Free Rate</th>
<th>Risk Free Rate Volatility</th>
<th>Equity Premium</th>
<th>Equity Premium Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NA</td>
<td>2.4%</td>
<td>25.8%</td>
<td>7.1%</td>
<td>19.9%</td>
</tr>
<tr>
<td></td>
<td>10.2%</td>
<td>3.1%</td>
<td>21.7%</td>
<td>6.9%</td>
<td>19.6%</td>
</tr>
<tr>
<td></td>
<td>10.1%</td>
<td>3.0%</td>
<td>22.2%</td>
<td>6.9%</td>
<td>19.6%</td>
</tr>
<tr>
<td></td>
<td>9.8%</td>
<td>2.9%</td>
<td>23.0%</td>
<td>7.0%</td>
<td>19.5%</td>
</tr>
<tr>
<td></td>
<td>10.7%</td>
<td>3.1%</td>
<td>21.1%</td>
<td>6.7%</td>
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<tr>
<td></td>
<td>10.3%</td>
<td>2.9%</td>
<td>21.9%</td>
<td>6.7%</td>
<td>19.5%</td>
</tr>
<tr>
<td></td>
<td>10.0%</td>
<td>3.0%</td>
<td>21.4%</td>
<td>6.7%</td>
<td>19.6%</td>
</tr>
</tbody>
</table>

Panel A shows the implied values for international risk sharing related moments country by country. Panel B shows implied asset market moments country by country. These implied values are computed with the model by simulating consumption growth and by using parameters values calibrated as in Table 3.

NA: Not Available

### Appendix B: Algebraic Proofs (Work in Progress)

- Correlation=index
- Derivation of EP and implied restrictions
- Derivation of simplified expressions
References


