Liquidity when it matters: QE and Tobin’s q

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Abstract

How and why do financial conditions matter for real outcomes? The ‘workhorse model of money and liquidity’ of Kiyotaki and Moore (2008) shows how - with full employment maintained by flexible prices – shifting credit constraints can affect investment and future aggregate supply. We show that, when the flex-price assumption is dropped, an adverse but temporary liquidity shock can lead to Keynesian-style demand failure. Optimistic expectations may speed recovery; but prompt liquidity infusion by the central bank – i.e. Quantitative Easing - is the appropriate policy to avert prolonged recession.

Keywords: Credit Constraints; Temporary Equilibrium; Liquidity Shocks.

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‘Surely, the Second Great Contraction – the financial crisis of the late 2000s – will have a profound impact on economics, particularly the study of the linkages between the financial markets and the real economy.’


Introduction

The history of market economies is, according to Reinhart and Rogoff, one of repeated credit booms and busts; and, on occasion, the lessons of history can prove more relevant for policy-makers than sophisticated economic models fitted to periods of economic stability.

In Lords of Finance: the bankers who broke the world, Liaquat Ahamed provides a graphic account of the ill-designed and uncoordinated response by central bankers, in America and elsewhere, to the US stock market collapse of 1929. Prior to the bust, the US had enjoyed a substantial investment boom - with the real capital stock increasing by more than 3 percent a year since 1925: but the value of the stock market, as measured by Tobin’s q\(^1\), had increased much faster, more than doubling over the same period, see Figure 1.

Then, in two short years, the stock market fell by more than 70\%, and the capital stock began literally to contract. These were the years of the Great Depression, when the US banking system collapsed and unemployment grew to over 20\% - leading Roosevelt to declare war on unemployment and Keynes to develop the theory of demand-determined output, published in 1936.

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\(^1\) Tobin’s q is the ratio of the stock market valuation to the current replacement cost of capital, see Blanchard and Fischer (1989, p.62).
Policy-makers have, in his view, learned from past mistakes, however:

In the current crisis, central banks and treasuries around the world, drawing to some degree on the lessons learned during the Great Depression, have reacted with an unprecedented series of moves to inject gigantic amounts of liquidity into the credit market and provide capital to banks. Without these measures, there is little doubt that the world’s financial system would have collapsed as dramatically as it did in the 1930’s. Liaquat Ahamed (2009, p.500)

The slogan - according to Wessel (2009) - was to do ‘whatever it takes’, slashing interest rates to almost zero, providing widespread loan guarantees, recapitalising major banks and buying in vast amounts of frozen money-market assets – so-called Quantitative Easing. As a result central bank balance sheets ballooned sharply as never before - doubling in the US, tripling in the UK, see Figure 2, and treasury backing had to sought for the quasi-fiscal nature of some of these operations. There was in addition a round of fiscal easing, coordinated through the IMF. In the event, GDP did fall in the US and elsewhere: but there was no Great Depression.
Figure 2. Central Bank total liabilities in the crisis (index Aug. 2007 = 100).


Paul Tucker (2009) has noted that, in response to the crisis, Central Banks far exceeded their customary remit, acting not only as Lenders of Last Resort but also as Market Makers and – in conjunction with the Treasury – as Suppliers of Capital too. Taking a historical perspective, Eggertson (2008) argues that it was President Roosevelt’s willingness to challenge the established precepts of a balanced budget and a fixed price of gold that helped the US recover from the Great Depression. Was the willingness of policy-makers to step outside the usual rules of the game to avert market failure the modern equivalent of FDR’s activism? Will they follow this through with structural reforms to the financial system to prevent a recurrence?

We leave these policy issues to one side to ask: what of macroeconomic theory? Unfortunately, the New-Keynesian economic paradigm - widely used by academics and central banks during the period that preceded the crisis - famously neglected the role of financial markets and the danger of shocks emanating from them. It was a model for the Great Moderation, not one for all seasons. So when financial markets
froze, policy-makers had to ‘fly by the seat of their pants’\(^2\) i.e. without the aid of operational macroeconomic models.

In the light of this experience, incorporating financial factors and financial frictions is seen as a key issue for macroeconomics, see for example Blanchard et al. (2009) writing from the IMF and Bean (2009) speaking for the Bank of England. Curdia and Woodford (2008) have already introduced financial frictions in a setting with heterogeneous consumers: and research interest in this topic is widespread. In this paper we turn to the macroeconomic framework of Kiyotaki and Moore (2008) where the focus is on the *ex post* heterogeneity of investors - only some of whom have ideas for investment in any given period. In this setting, investors who expect to be credit-constrained hold money for precautionary reasons: and, if credit constraints unexpectedly tighten, *ad hoc* an open market operation that supplies liquidity in exchange for frozen private sector assets (i.e. Quantitative Easing) is an effective tool of policy.

Researchers at the Federal Reserve Bank of New York, working together with Kiyotaki, have calculated that, when such features are added to a DSGE model including Calvo contracts, changes in credit conditions can have substantial real effects. On the ‘conservative’ assumption that the expected duration of the credit crunch was only 8 quarters, they find that an unanticipated tightening of credit constraints leads to pronounced recession. Specifically, a temporary shock which reduces the re-saleability of equity by about three-quarters, and reduces Tobin’s q by 10 percent would, in the absence of intervention, lead to a roughly proportional cut in investment, consumption and output, as shown by the dashed lines in Figure 3.

\(^{2}\) The words of one central banker closely involved.
Figure 3. Effect of a liquidity shock that is expected to last for eight quarters
(dashed lines without intervention, solid lines with QE.)
Under a more extreme scenario where the liquidity shock is expected to last for 8 years instead of 8 quarters (i.e. to be of similar duration as the shocks to the Japanese economy during the Great Recession or the US during the Great Depression), the authors conclude: ‘Without intervention the equilibrium is a disaster. Output collapses by about 20 percent and deflation reaches double digits. In short, the equilibrium outcome starts looking a bit like the Great Depression.’

The focus of the FRBNY study is a quantitative evaluation of how prompt policy response in the form of cutting interest rates and injecting liquidity of $1 trillion can substantially avert these real effects, as shown by the solid lines in the figure. Rather than examining the precise details of monetary policy in the U.S. using a large-scale calibrated model for the purpose, we focus on the analytical properties of Kiyotaki and Moore’s approach to financial frictions. To this end, we adopt a two-regime approach, with prices and wages rising in response to an unanticipated loosening of credit, but not falling in response to a sudden contraction. The former matches the flex-price analysis adopted by the authors themselves: the latter is much closer to neo-Keynesian approach of the applied study by the FRBNY. Impulse responses are calculated using the parameters from the latter study: but the structure is kept so simple that phase diagrams can be used to illustrate the effects of credit tightening - and of asset swaps offset this.

Like the authors of the FRBNY study, we take a broad interpretation of QE – including open market purchases of private sector paper as well as long-dated government debt. This is not only how the current Chair of the Federal Reserve uses the term, it is also consistent with the actions of his antecedents: the mandate of the Fed at its inception was ‘to regulate the supply of credit ...[with] purchasing trade acceptances’ being among the techniques to be used, Eichengreen (2010,p. 26).

The paper is structured as follows. First, in Section 1, the key features of the approach developed Kiyotaki and Moore (2008), hereafter KM, are presented, together with a summary of the formal model. [In Appendix 1, after linearizing and
imposing the parameters of the FRBNY, the flex-price framework is used to show the positive real effects of ‘Big Bang’ in moving the economy towards the modified Golden Rule; the role of the Pigou effect in stabilising demand in the short run; and the loss of entrepreneurial ‘rents’ as liquidity constraints are eased.] In Section 2, we study an adverse liquidity shock in a fix-price context, analogous to Del Negro et al. (2009). The effect of expectations is highlighted by contrasting a short versus a protracted liquidity squeeze. In Section 3 policy actions are considered, especially the use of open market operations to purchase the assets whose liquidity is temporarily impaired. Section 4 picks up the theme of boom followed by bust emphasized by Reinhart and Rogoff (2009): as historical movements in the stock market are much larger than can be associated with fundamentals, we discuss how a liquidity crisis can be triggered by an asset price correction, especially one that impacts on bank balance sheets. Finally, we offer a wider interpretation of the formal results; and discuss whether this framework might provide a ‘work horse’ macro model which incorporates the missing financial factors, and acts as a bridge between DSGE and the temporary equilibrium of Keynesian economics.

Section 1 Key features of the KM framework: an overview

As an alternative to the representative agent assumption of DSGE models, the key idea is that investors are ex ante identical, but only a fraction actually turn out to have ideas that will generate investment in the current period. This is like the specification of Diamond and Dybvig (1983) in their classic paper on banking, where agents identical ex ante turn out to have patient or impatient consumer preferences. Here, as in the banking paper, there is no insurance market to handle the risk of needing cash in a hurry.

Rational expectations prevail in the stock market; but credit markets are far from perfect. Workers cannot borrow and choose not to hold financial assets with returns that lie below their rate of time preference: so households are income-constrained and all wages are spent on consumption. Entrepreneurs can optimise over time but they
face limits in terms of new equity finance available and in re-selling existing shares to finance investment - and there are no banks to supply loans.

These constraints on inter-temporal arbitrage lead to a Hicksian type of temporary equilibrium, with a precautionary demand for money by entrepreneurs to ensure that investment opportunities are not wasted. As the reformulated relations do include inter-temporal optimising behaviour by entrepreneurs, the KM approach might be characterised as Dynamic Stochastic Temporary Equilibrium. In sharp contrast to the fix-price Hicksian economics, however, prices and wages are perfectly flexible and there is continuous market clearing with full employment due to the operation of a Pigou effect. Conditional on the current capital stock, the clearing of goods and money market determines the aggregate price level and the real price of equity: and the investment equation determines the evolution of the capital stock.

A potential criticism

Before proceeding, consider the objection that this approach ignores the potential role of banks in providing ‘liquidity insurance’, as they do in the Diamond and Dybvig framework. Banks can in principle provide longer term funding as well as desired access to liquidity when needed; but they are themselves subject to spectacular coordination failures in the form of bank runs.\(^3\)

The explicit or implicit promise by the authorities to insure the banks (by FDIC guarantees or lender of last resort facilities) can avert bank runs; but this, in turn, can have perverse effects incentive effects, allowing banks to take on excessive risk, as discussed in Hellman, Murdock and Stiglitz (2000). Such moral hazard problems - particularly in regulatory regimes operating with a ‘light touch’ that allow limited-liability banks to banks to increase leverage and put their capital at risk - may indeed imply (as Sinn(2010) argues forcefully in *Casino Capitalism*) that banks become part of the problem rather than providing the solution! In these circumstances,

\(^3\) For the view that the recent credit crisis was effectively ‘a silent bank run on shadow banks’, see Milne (2009) and Gorton (2010).
looking at the impact of credit constraints on entrepreneurs who supply funds between themselves without recourse to banks seems a reasonable compromise\(^4\).

**Formal structure of KM model**

*Entrepreneurs:*

KM take an economy consisting of entrepreneurs and workers. Entrepreneurs, who own capital and financial assets, are responsible for organising production and for all real investment. Their objective function is to maximise the expected present discounted utility value of current and future consumption, i.e.

\[
E_i \sum_{t} 
\]

with \(\beta (0 < \beta < 1)\) the discount factor. They can employ labour \((l_t)\) and capital \((k_t)\) to produce general output \((y_t)\), using a constant-returns-to-scale Cobb-Douglas production function with capital share \(\gamma\) and productivity parameter \(A_t\)

\[
y_t = \frac{1}{\gamma + 1} A_t^{\gamma} l_t^\gamma k_t^{1-\gamma}.
\]

Entrepreneurs can also invest, i.e. convert general output into capital goods, but are only able to do so when they have ‘an idea’ for an investment project. These arrive randomly, with probability \(\pi\) each period. Given large numbers, it may be assumed that a given fraction \(\pi\) of entrepreneurs receive an idea each period, and the remaining \((1-\pi)\) does not.

Entrepreneurs can finance investment by issuing equity claims to the future returns from newly produced capital; but, owing to limited commitment, they can only do this against a fraction \(\theta\) of the new capital investment they undertake. Because of this ‘borrowing constraint’, entrepreneurs can use their own money holdings, which are perfectly liquid and can be spent immediately, and/or sell the shares they own in existing firms to finance real investment. But access to financial markets is also restricted by a ‘resaleability constraint’ - only a fraction \(\varphi\) of these holdings can be sold each period- representing the illiquidity of equity in the model. (As a simplification, KM assume that after one period, the equity held by an entrepreneur in his own firm is just as liquid as the equity in other firms.)

\(^4\) Tightening credit constraints may, as discussed later, stand as proxy for the contraction of a poorly regulated banking system that is failing.
As a result of this, an entrepreneur who enters the period with holdings of equity $n_t$ and holdings of money $m_t$, and who has an investment idea, can invest an amount $i_t$, which must satisfy the constraints that at least a fraction $(1-\varphi)$ of initial equity (after allowing for depreciation at rate $\lambda$) is retained and at least an amount of new equity $(1-\theta)i_t$ in the new capital is retained. Therefore the entrepreneur holds equity $n_{t+1}$ at the start of the next period satisfying

$$
n_{t+1} \geq \frac{1}{1+\varphi} - \frac{1}{1+\varphi} (1-\lambda) + \frac{1}{1+\varphi} (1-\theta)i_t
$$

and money balances

$$
m_{t+1} \geq 0
$$

The spending of the entrepreneur on consumption $c_t$ and investment $i_t$, together with acquisition of new money balances and equity, satisfies the budget constraint

$$
c_t + i_t - q_t - p_t = 0
$$

In this equation, $q_t$ denotes the price of a unit of equity, and $p_t$ the price (in terms of goods) of one unit of money; and $r_t$ is the rate of return on capital.

**Workers:**

The role of workers, who do not have investment opportunities cannot borrow against future labour income, is much more straightforward. They supply labour and consume goods. In principle they may hold money and equity to smooth consumption and labour supply over time: but they choose not to do so, as the rates of return they earn on these assets are less than their rate of time preference. Workers supply labour as an increasing function of the real wage $w_t$:

$$
I^*_t = \omega \left( \frac{w}{l_t} \right)^{1/\nu}
$$

where $\omega$ and $\nu$ are preference parameters.

**Labour Markets:**

The labour demand of entrepreneurs is determined by the marginal productivity of labour, and, when wages and prices are flexible so that we have labour market clearing, labour supply equals labour demand, and:

$$
\left( \frac{w}{l_t} \right)^{1/\gamma} = \frac{1}{1+\theta}
$$

This ties down the real wage rate and the marginal product of capital as functions of the capital stock:
with \( a_t = \binom{1}{1}^{1-} \) and \( \alpha = \frac{\gamma + \ldots}{\ldots} \), and \( K_t \) is the aggregate capital stock of the economy.

**Real Investment:**

When the value of capital \( q_t \) exceeds one, entrepreneurs who have an investment idea will issue as much equity as they can, and sell as much of their existing equity holdings as possible, given the credit limits given above, and they will use all their holdings of money to invest. Thus their flow of funds is:

\[
 c_i^t + \ldots + \ldots = + \ldots + \ldots
\]

They carry no money forward to the next period. Taking account of the liquidity constraints, the equity held over to the next period satisfies:

\[
 c_i^t + \ldots = \ldots + \ldots + \ldots
\]

with \( q_{i,t}^s = \ldots \), where the right hand side of the equation denotes the entrepreneur’s net worth at the start of period \( t \). With log utility, these entrepreneurs are assumed to consume a fraction \((1-\beta)\) of this each period:

\[
 c_i^t = \ldots + \ldots + \ldots + \ldots
\]

and therefore they invest an amount:

\[
 i_t = \frac{\ldots}{(1-\ldots)}
\]

**Financial Assets:**

Things are different for entrepreneurs who do not have an investment idea. They accumulate money and equity to build up resources for use in future if an investment opportunity comes along. Their flow-of-funds constraint is simply

\[
 c_i^s + \ldots = \ldots + \ldots
\]

showing the value of net worth on the right-hand side. The superscript, \( s \), against their holdings of money and equity and consumption in equation (13) distinguishes these as variables referring to non-investing entrepreneurs. Optimal consumption for these entrepreneurs is once again a fraction \((1-\beta)\) of net worth:

\[
 c_i^s = \ldots + \ldots + \ldots
\]
The non-investing entrepreneur has to decide what fraction of assets to put into money and how much into equity. The marginal utility of consumption in period $t$ has to equal the discounted expected marginal utility of holding additional units of money into period $t+1$ and consuming them then. Also, it must equal the expected discounted utility of holding additional equity into period $t+1$. Thus we have KM’s equation (21) for portfolio balance:

$$u'(c_t) = \left\{ \begin{array}{l}
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array} \right\}$$

$$= - \left\{ \begin{array}{l}
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array} \right\}$$

$$+ \left\{ \begin{array}{l}
\vdots \\
\vdots \\
\vdots \\
\vdots
\end{array} \right\}$$

(15)

**Aggregate relationships:**

The above analysis describes the behaviour of individual entrepreneurs. So it is necessary to aggregate across all entrepreneurs to find how the economy as a whole evolves. The expressions for consumption and investment of each type of entrepreneur are linear in start-of-period holdings of equity and money, which simplifies matters considerably.

As KM note, a fraction $\pi$ of aggregate capital $K_t$ and money $M_t$ is held by investing entrepreneurs, so aggregate investment is:

$$(1 - \theta q_t) \gamma_t = \pi \left\{ \beta \left[ (\gamma_t + \lambda \phi_t q_t) K_t + p_t M_t \right] - (1 - \beta) (1 - \phi_t) \lambda q_t^{\phi} K_t \right\}$$

(16)

where

$$q_t^2 = \frac{1 - \theta q_t}{1 - \beta} < 1 \text{ as } q_t > 1$$

The **aggregate demand equation**, balancing the net output of goods with the demand for investment plus consumption from the two types of entrepreneurs implies:

$$\tau_t K_t = \alpha_t K^{\phi}_t = l_t + (1 - \beta) \left\{ \left\{ \gamma_t + (1 - \pi + \pi \phi_t) \lambda q_t + \pi (1 - \phi_t) \lambda q_t^{\phi} \right\} K_t + p_t M_t \right\}.$$  

(17)

The aggregate portfolio balance equation is obtained by aggregating over the wealth of the non-investing entrepreneurs. They buy equity in the amount $\theta$ from the investing entrepreneurs, and a fraction $\phi$ of their depreciated equity $\pi \lambda$; they also
retain the depreciated equity carried over from the preceding period. Therefore their equity holdings at the start of period t+1 are \( N_{t+1}^{s} \), defined as:

\[
\theta l_{t} + \phi_{t} \pi K_{t} + (1 - \pi) \lambda K_{t} \equiv N_{t+1}^{s}
\]  

(18)

The non-investing entrepreneurs hold all the money stock \( M_{t} \). As utility is logarithmic, marginal utility is the reciprocal of consumption. The portfolio balance equation, (15) above, then becomes, at the aggregate level:

\[
(1 - \lambda) - \frac{p_{M}}{K} = r + \frac{q}{K}
\]

(19)

Finally the aggregate capital stock evolves as:

\[
K_{t+1} = \lambda K_{t} + I_{t},
\]

(20)

where (1-\( \lambda \)) is the rate of depreciation.

To summarize, the model boils down to equations (16) – (20). These equations define the dynamic system, whether in the flexible-price mode of KM or in the fixed-price demand-deficient mode.

**Linear approximation around steady state**

The non-linear dynamics can be solved by linearizing around steady state values for \( K \) and \( q \). We first compute a solution for the steady state, assuming that the liquidity constraints are such that precautionary holding of money is justified. The steady state is obtained from equations (16) to (20) above. These equations can be reduced to three relationships in the steady state, written as follows:

\[
\pi \beta + \frac{1}{\lambda} = \frac{1}{\kappa}
\]

(21)

\[
\beta - \frac{1}{\lambda} = \frac{1}{\kappa}
\]

(22)

\[
r = \frac{1}{\lambda} + \frac{1}{\kappa}
\]

(23)

These three equations determine three unknowns: \( pM/K \), \( r \), and \( q \). The first two can be solved for \( pM/K \) and \( r \) as linear functions of \( q \). When these solutions are substituted
into (23), this can be solved as a quadratic in \( q \), and we select the economically meaningful of the two solutions.

Having found the stationary state, we take linear approximations around it, and reduce the model to a system of two first-order, linear difference equations, one in \( K \) and one in \( q \). Note that the investment equation (16) and the aggregate demand equation (17) can be linearized around the steady state to give two equations that express \( dI_t \) and \( dp_t \) as linear functions of \( dK_t \) and \( dq_t \). These variables are defined as

\[
dK_t = \ldots
\]

where \( \bar{K} \) is the steady state value of the capital stock, and analogously for the others. In interest of analytical clarity, we treat as constant the productivity parameter \( A_t \), the liquidity constraint \( \phi_t \), and the money supply \( M \). The interest rate is just a function of the capital stock, from equation (8).

We totally differentiate the portfolio balance equation (19) around steady state values. This gives a linear expression that relates \( dp_{t+1} \), \( dK_{t+1} \) and \( dq_{t+1} \) to \( dp_t \) and \( dq_t \). In doing this we make use of the definition of \( N_{t+1} \) (18) which expresses it as a function of \( I_t \) and \( K_t \). The capital stock accumulation equation (20) is also linearized around the steady state. Assembling all of these elements, \( dp_{t+1} \), \( dp_t \) and \( dI_t \) can be substituted out, and we are left with a state space representation which is a pair of first-order, linear difference equations in \( dK_t \) and \( dq_t \).

Of the two variables in the state-space system, \( K \) is predetermined, while \( q \) is a non-predetermined ‘jump’ variable. Using the parameters from the FRBNY study, we determine the stable and unstable roots of the system and present impulse responses illustrating these results by familiar phase diagrams\(^5\).

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The flexible-price solution (see Appendix 1)

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\(^5\) Given the discrete dynamics, the paths will consist of discrete points, as shown in Figure 6 for example. Elsewhere, phase diagrams with continuous paths are used as a convenient illustrative device, though the continuity of phase path is only approximately correct in a discrete-time context.
In flexible price mode, the investment equation and the aggregate demand equation determine \( p_t \) and \( I_t \) as functions of \( K_t, q_t, \varphi_t \), and the other parameters of the model \((M, \pi, \theta, \lambda, \beta)\). The return on capital \( r_t \) is moreover a function of the capital stock \( K_t \) and various parameters of the model. These functions can then be substituted into the portfolio balance equation, in place of \( r_{t+1}, p_{t+1}, p_t, \) and \( I_t \), so the portfolio balance equation is reduced to an equation in \( q_{t+1}, K_{t+1}, \varphi_{t+1}, K_t, q_t, \) and \( \varphi_t \). We then have a first order dynamic system in three variables, \( K_t, q_t, \) and \( \varphi_t \).

If, as a further simplification, one fixes the value of \( \varphi \) at a constant level, treating it as one of the model’s fixed parameters, the dynamic system reduces to one of only two variables, \( K_t \) and \( q_t \). The two equations are the capital accumulation equation, (20) above, and the solved-out portfolio balance equation.

**The fixed-price solution**

New Keynesian macro-economists have chosen to capture temporary wage-price stickiness by the analytical device of Calvo contracts, which allow for gradual revision of aggregate wage and price indices in response to expected future events, Woodford (2003); and this is the approach taken in Del Negro *et al.* (2009). Here, in the interest of analytical tractability, we take a two-regime approach, instead, with a fixed price regime in situations where there is excess supply and flex prices otherwise. With fixed prices, there is no Pigou effect to stabilise aggregate demand in the face of a fall of investment, so a contraction of liquidity may lead to failure of market clearing in goods and labour markets -- as in the ‘fix-price macroeconomics’ of the 1970s described in the writings of the French theorists Benassy (1975) and Malinvaud (1977) and of Muellbauer and Portes (1978), economists at Birkbeck College. We assume that the real wage is determined by bargaining, as in Layard and Nickell (1987) and Manning (1990), for example. Shimer(2009) has emphasized the role of real wage rigidity in explaining observed fluctuations of employment and output – and the need for the real wage to lie below the marginal productivity of labour to give firms the incentive to hire. Accordingly we assume that at full employment the

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6 Neary and Stiglitz (1983) also use a fixprice flexprice, two-regime approach.
bargaining wage lies below the marginal product of labour\textsuperscript{7} and, for convenience, that this real wage is maintained even when the demand falls and workers are laid off.

In the fixed-price mode, assuming that there is excess supply of labour and goods, the same equations determine the dynamics of the system around steady state. However, some things change. With prices and wages predetermined, they may be treated as fixed parameters in the analysis. Now aggregate demand from entrepreneurs for consumption and investment determines their income \( r_t K_t \); and the rate of return, \( r_t \), is no longer a simple function of the capital stock \( K_t \): it is the residual after subtracting the wage costs from the receipts obtained from meeting current aggregate demand. Equations (22) and (23) now determine \( r_t \) and \( I_t \) as functions of \( K_t, q_t, \phi_t \) and the other parameters of the model \((M, \pi, \theta, \lambda, \beta)\) – and now we add \( p = p_t = p_{t+1} \) to the list of fixed parameters.

We substitute these functions for \( r_t, r_{t+1}, \) and \( I_t \) into the portfolio balance equation, and impose the fact of \( p \) being fixed. Once again, the portfolio balance equation is reduced to a relation between \( q_{t+1}, K_{t+1}, \phi_{t+1}, K_t, q_t, \) and \( \phi_t \). Our dynamic system is again a non-linear first-order difference equation system in the same three variables as in the flexible price case, \( K_t, q_t, \) and \( \phi_t \). With the further simplification that \( \phi \) is constant, we have a system in two variables, \( K_t \) and \( q_t \).

**Section 2: How an adverse liquidity shock leads to recession**

The behaviour of the flex-price system - and how it responds to liquidity shocks - is analysed by numerical simulation in the original KM paper, so it need not detain us here\textsuperscript{8}. The main focus of this paper, as for the FRBNY study, is to account for the impact of adverse liquidity shocks on the real economy when prices are sticky and credit constraints operative.

\textsuperscript{7} Setting the two equal would lead to excessive fluctuations in employment when demand falls, as the envelope theorem implies.

\textsuperscript{8} For completeness, the effects of easing liquidity – a Big Bang – in a flex-price context are briefly discussed in Appendix 1.
Aggregate demand for net output\(^9\) and goods market equilibrium

Before turning to impulse responses for the complete model in the fixed price case, it may be useful to discuss in broad brush terms how a liquidity contraction can affect entrepreneurial income (and national product) for a given \(K\) and \(q\), i.e. to solve for the rate of return on capital (and output) conditional on \(K\) and \(q\).

First, we note that for a firm with the production function described by equation (2), which adjusts output by varying employment at a constant real wage \(w\), the residual income available to entrepreneurs (\(x\)), the excess of production over the wage bill, varies with production \(y_t\) as follows:

\[
x(y_t; w_t, k_t) = - \left( - \frac{1}{A_t k_t} \right) y_t^{\frac{1}{\gamma}}
\]

Expressed as a rate of return on the (constant) capital employed, this may be written for brevity as:

\[
r = \frac{x_t}{k_t} = r(y_t) = \frac{y_t}{k_t} - w \left( \frac{y_t}{A_t k_t} \right)^{\frac{1}{\gamma - 1}}
\]

which is increasing in \(y_t\) in the range from 0 to the point where the marginal product of labour equals the real wage. Where \(y_t\) is demand determined, the relation between the rate of return on capital and the quantity of capital implied by equation (8), no longer applies: it is replaced by equation (25).

Since the price level is fixed, there will be no Pigou effect to ensure full employment. The level of output (and hence the return on capital) adjusts to bring supply and demand into balance. ‘While workers spend what they earn, entrepreneurs earn what they spend’, as Kalecki put it.

Turning to aggregates, we note that, from equation (16), other things being equal, the marginal effect of an increase in \(r\), as defined in equation (25), on investment demand is:

\[\text{i.e. output less what is consumed by employees, } y_t - w l_t = x_t.\]
and on entrepreneurial consumption is:

\[ dC_t = \ldots \]

Hence the total effect of an increase in \( r_t \) on entrepreneurial income is:

\[ \ldots \]

For stability at an interior solution (with excess supply of labour), we need

\[ \mu = \ldots \quad \text{or} \quad \frac{\pi}{1 - \theta q_t} < 1, \]

where \( \mu \) denotes the marginal propensity to spend out of entrepreneurial income.

As we are assuming \( q > 0 \), a necessary condition is that \( \pi + \theta < 1 \); i.e. there is a stability restriction on ‘induced investment’ such that the fraction of entrepreneurs who have new ideas plus the fraction of new investment they can fund via new equity issues must be less than 1.

The ‘fix-price macroeconomic’ framework used here can be illustrated as in Figure 4. The bottom panel on the left shows how the wage bill varies with employment at the fixed real wage. The bottom right panel shows how profits, \( X \), the residual income available to entrepreneurs, fall away as employment contracts. So too does demand by entrepreneurs as shown in the top panel, where the marginal propensity to spend is \( \mu \). Note that here, for convenience, demand is shown at a constant real share price and constant \( K \).
Figure 4. Short-run determination of entrepreneurial income, $X$, and gross output, $Y$.

The figure illustrates how a fall in investment demand due perhaps to a fall in liquidity - represented by the downward shift in $D(X)$ in the top panel - will lead to a greater contraction of entrepreneurial income, $X$, as equilibrium shifts from $E$ to $D$. The impact on employment is even more pronounced as shown by the shift from $E^*$ to $D^*$ in the lower right panel\(^{10}\).

**Phase diagram in $K$ and $q$**

Our approach is to solve the model by simulation, and illustrate the results using phase diagrams with $K$ predetermined and $q$ a jump variable. Figure 5 shows how the capital and real price of equity evolve, assuming that the model remains in the fixed-

\(^{10}\) To limit the impact on employment in the simulations below, it is assumed that the initial equilibrium is one where the marginal product of labour is five percent above the real wage.
price regime throughout. On the schedule labelled SK in Figure 4, all investment is for replacement, so the capital stock will be stationary: and the parameters of the model confirm that SK slopes upwards.

Figure 5. Capital accumulation and stock market

Likewise, stationary values for the value of the stock price define the asset market equilibrium, given by the downward sloping schedule labelled PB in the figure. Stationary equilibrium is at E where they both intersect. Given the saddle point dynamics, the stable path to equilibrium will slope downwards, see SS in the figure. (The unstable eigenvector has a positive slope). Also shown are integral curves that asymptotically approach SS and UU. This is the ‘workhorse model’ we use to discuss the effects of a negative liquidity shock, and of policy taken to offset this.

\[ \Delta q/\Delta t = 0 \]

\[ \Delta K/\Delta t = 0 \]

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Short and long run impact of a permanent liquidity shock

What are the effects of a negative liquidity shock? Assume that the economy starts at a high employment, steady-state equilibrium E, as depicted in the NW panel of Figure 6, and the shock throws it into a demand-deficient regime. As it tightens the financial constraints on firms who want to invest and since workers are income-constrained (with no Pigou effect to stimulate consumption of entrepreneurs), the impact will be to reduce entrepreneurial income (as discussed above) and the asset price. As this reduces the incentive to invest and the attractiveness of equity, this will shift both schedules for stationarity to the left, moving the long run equilibrium moving from E to E', as shown in Figure 6.

Figure 6 Short run and long run impact of permanent liquidity shock
Because the labour force will in the long run be reduced in line with the capital stock, Tobin’s q is essentially unaffected. On impact, however, when the capital stock is fixed, the fall of demand will lead to a fall in the rate of profit, as indicated in the SE panel, and the asset price will fall from E to R lying on the stable eigenvector that leads to E’.

Also indicated in the figure are the asset returns. Given fixed prices, the (gross) rate of return on money is one. As indicated by the hyperbola labelled \( R^0 \), the rate on equity for savers must in equilibrium be higher than one: while the rate for investors (not shown) will be less than one. In the short run, however, the fall in the yield on equity, see the hyperbola labelled \( R'^0 \), will be balanced by anticipated capital gains as the system moves along the eigenvector SS.

**A temporary decline in liquidity**

The immediate impact of the liquidity squeeze on the stock market will depend on how long the shock is expected to last. If the shock was expected to be permanent, the market would fall to P on the new stable eigenvector S’S’ before converging to E’. But if the liquidity squeeze is only expected to remain in force for T periods, then the relevant trajectory will take the form indicated by EDLE in the Figure 7\(^{12}\). Thus after a smaller initial decline, the asset value begins to recover even while the capital stock contracts as the trajectory follows an integral curve for T periods from D to L on the stable eigenvector SS. Then the ‘overshooting’ of the asset price will gradually subside as capital stock and q proceed back to the original equilibrium.

\(^{12}\) Note that this trajectory is constructed on the assumption that there will be no regime switch when the liquidity is restored, i.e. the relevant stable eigenvector SS that which applies in the fix-price regime. While such regime switches are possible, they do not occur with the parameters used below.
Using parameter values taken from Del Negro et al. (2010) – as detailed in Appendix 2 - the impulse responses for a 20% cut in $\phi$ expected to last for 10 quarters are shown by the solid lines in Figure 8, with the trajectories for K and q just described shown in the top panel, and the other panels showing the sharp fall in profits and associated fall in investment and employment. The impact of QE is discussed in the next section; and the effects of a longer credit squeeze – 8 years instead of 8 quarters.
– are shown in Appendix 2.

Figure 8 – Effects of 8-quarter credit crunch (solid lines) and addition of QE (asterisks) where the Central Bank acquires 2.6 percent of the capital stock

For convenience, the impact effects on the economy are summarised in Table 1. The magnitudes of initial jumps depend on the expected length of liquidity squeeze, whether short, long and permanent, as follows:

<table>
<thead>
<tr>
<th></th>
<th>Short (2 years)</th>
<th>Long (8 years)</th>
<th>Permanent</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>-1.43% (-0.57%)</td>
<td>-2.56% (-1.01%)</td>
<td>-2.62%</td>
</tr>
<tr>
<td>r</td>
<td>-8.26% (-2.85%)</td>
<td>-9.12% (-3.20%)</td>
<td>-9.24%</td>
</tr>
<tr>
<td>X</td>
<td>-8.55% (-2.97%)</td>
<td>-9.45% (-3.34%)</td>
<td>-9.68%</td>
</tr>
<tr>
<td>y</td>
<td>-11.58% (-4.78%)</td>
<td>-14.56% (-5.36%)</td>
<td>-14.78%</td>
</tr>
</tbody>
</table>

Table 1. Impact effects of a 20% cut in $\phi$ for different lengths of time
It turns out that the pattern of events is similar whether the squeeze is expected to last for a long time or not: all variables except for K fall sharply in the first period then recover as the end of the liquidity squeeze is anticipated. The asset price recovers and ‘over-shoots’ before returning to equilibrium. The capital stock remains unchanged in period 1, but then keeps contracting until liquidity is restored. The initial impact on output is surprisingly large for the parameters used by FRBNY, roughly 12 percent for the shorter squeeze rising to around fifteen percent for a prolonged squeeze (see the bottom line of the table).

Section 3 Open Market Operations or Quantitative Easing

Expectations of an early restoration of liquidity do help, but as the simulations indicate, they do not prevent the economy from experiencing pronounced recession. The authorities can, however, take direct action to bring the markets back to life: the central bank can purchase equity with money. For, as KM point out:

When the resaleability of equity falls with an arrival of liquidity shock, the central bank can do [an] open market purchase operation, increasing the liquidity of an investing entrepreneurs. Then the quantities and asset prices will be insulated from the liquidity shock. KM (2008, p. 27).

What if policy makers take such ‘prompt corrective action’ to avert recession? Strictly speaking, with a large enough Open Market Operation (OMO), equilibrium would remain unchanged, with the uptake of capital by the public sector and the easing of liquidity constraints offsetting the leftward shift in the schedules for portfolio balance and of replacement investment due to loss of resaleability. In Figure 9 this would imply a complete reversal of the movement from E to E’, with the schedules labelled PB’ and SK’ shifting back to intersect at the original equilibrium E.

Consider instead the case where the effects of illiquidity are only partially offset, so equilibrium moves part of the way back from E’ to E, as illustrated in the Figure. Assuming that the OMO will be reversed as and when resaleability recovers in T periods time, the analysis is much as before except that the relevant eigenvectors will
be those that characterise the half-cured problem of illiquidity, shown labelled $S^{QE}$ and $U^{QE}$. (It is ‘as if’ the liquidity shock, the fall in $\varphi$ for example, was smaller.) So the starting value of the price of equity at A lies on the integral curve which takes $T$ periods to reach the point B on the saddle path leading to E (when liquidity recovers and the OMO is reversed).

Figure 9. Effect of temporary Open Market Operation (QE)

Using the same parameter values, the impulse responses for a short liquidity shock *partly offset by Quantitative Easing* are shown in Figure 8, where the trajectory marked by asterisks in last panel indicates the increase in the money stock involved in this operation – which involves the temporary purchase of 2.6 percent of the capital stock measured at book value. This open market operation checks the fall in investment, in profits and employment by around two-thirds, so output falls by about 5 percent instead of 12, see Table 1. (Quantitative Easing has much the same
proportionate effects for the longer credit squeeze as shown in the second column and in Appendix 2.)

The beneficial effects of Quantitative Easing in the face of a two-year credit crunch shown in column 1 are broadly analogous\textsuperscript{13} to those obtained by Del Negro et al. (2009). Using their much more complicated calibrated numerical model they reported that an OMO could reduce fall in investment, consumption and output from 10% to 6%, see the solid lines in Figure 3. It is on this basis that the team from FRBNY argue that, by injecting a trillion dollars into the financial markets in 2008-9, the Federal Reserve engineered a ‘Great Escape’ for the US economy.

So far, we have focused on the central bank as ‘market maker of last resort’, Tucker (2009). What about fiscal stimulus? In models of this kind, where credit constraints are prevalent, the Ricardian equivalence theorems used to demonstrate the irrelevance of tax changes will not generally apply. Even with intertemporal optimisation and Ricardian equivalence, with price stickiness and demand deficiency changes in government expenditure can affect aggregate demand, Krugman(1998), Christiano, Eichenbaum and Rebelo(2007).

Active fiscal policy will help recovery, but the effects of public spending will be regime dependent. Extra spending by public agencies, which can lift output and employment when the economy is in a demand-constrained recession, will lead to ‘crowding out’ and inflation in the flex-price fully employed economy. The policy implication is naturally that the restoration of liquidity calls for a scaling back of fiscal stimulus as well as a reversal of the OMO.

\textbf{Section 4. Extensions: Asset Bubbles and Irreversible Investment}

Although it allows for financial frictions, the KM model assumes that assets are correctly priced and, as a result, the variable \( q \) has limited volatility. As is evident from Figure 1 above, however, historical evidence, especially up to and during the Great Depression, paints a very different picture – with Tobin’s \( q \) doubling in the

\textsuperscript{13} No great significance should be attached to the fact that the percentage reduction in the effects of the liquidity squeeze so achieved are somewhat larger than for FRBNY: this depends on the size of the OMO chosen for simulations.
three years before the Wall Street Crash of 1929, and falling by three quarters in the next couple of years.

A run-up in asset prices can, of course, be captured in the KM model by looking at the integral curves that do not satisfy the transversality condition, as in Figure 11 where the integral curve above the stable manifold no longer correctly represents future fundamentals, but is simply a bubble.\textsuperscript{14}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{Bubble collapse preceding liquidity shock: like 1929}
\end{figure}

It is clear from Figure 1 in the introduction that it took some years for asset prices to recover to more normal values in the 1930s. It is worth noting, however, that:

‘Bank panics were a recurrent phenomenon in the United States until 1934… Friedman and Schwartz (1963) enumerate 5 bank panics between 1929 and 1933, the most severe period in the financial history of the United States.’

\textsuperscript{14} Abreu and Brunnermeier (2003) discuss how such mispricing may be sustained for some time by heterogeneous beliefs.
Freixas and Rochet (1997, p.191).

It was, in fact, only after the substantial restructuring of the financial system – including the setting-up of FDIC, passing the Glass-Steagall act, changing in bankruptcy law and strengthening of the security regulation, that asset prices were able to recover.

Perhaps the historical data for the Great Depression could be interpreted as follows: the fall in asset prices after a prolonged and explosive bubble led – in the absence of prompt corrective action by the central bank – to a succession of liquidity shocks whose effects were only finally reversed by the restructuring of the financial system.

Excessively overvalued \( q \) is one of the important features missing from the model: another is the very low values that were observed in the Great Depression. Could this be attributable to the irreversibility of investment? Irreversibility increases the volatility of asset prices in theoretical models because investment is not undertaken until \( q \) exceeds one by a suitable margin, as firms exploit the option value of not investing. When \( q \) falls below one, firms cannot disinvest as fast as they might wish: they are limited to disinvesting at most at the rate at which capital depreciates. Meanwhile \( q \) can fall to low levels.

**Section 5. Conclusion**

In his assessment of factors causing the Great Depression in the US, Milton Friedman emphasized financial factors - and blamed the Federal Reserve for not acting to head off cumulative collapse of hundreds of banks; and the account of central bank mismanagement provided in Ahamed’s ‘*Lords of Finance*’ adds historical weight to Friedman’s perspective. By way of contrast, believers in the Efficient Market Hypothesis and Real Business Cycle theory argue that, in general, financial factors play little or no causal role in economic booms and slumps. With respect to the current financial crisis Eugene Fama, for example, has argued that financial factors were simply reflecting prior deterioration in economic fundamentals.
The simple two-regime model used here endorses Friedman’s perspective and offers analytical support for the results obtained by the Federal Reserve Bank of New York using a much more complex DSGE model. The tractability of the second-order system used allows for qualitative analysis of liquidity shocks, of OMO buffering, and of various expectational effects - including deviations from rational expectations.

The effect of financial accelerators in the credit market, as discussed in Kiyotaki (1998), Bernanke et al. (1999) and in Miller and Stiglitz (2009), are not treated here, where the focus is on illiquidity. It has to be said, however, that the model includes no financial intermediation per se: the liquidity squeeze is a failure lending on a bilateral basis between one set of entrepreneurs and another. It would be preferable to include intermediation explicitly, of course - and this would help link ‘irrational exuberance’ in asset markets to a subsequent liquidity crunch.15

The results obtained in this paper for a liquidity squeeze can, perhaps, best be thought of as a ‘reduced form’ of what happens after a sharp contraction of financial intermediation. The severe economic effects that follow - and the links with asset mispricing that precede - become much more plausible on this interpretation: but any policy conclusions must be treated with considerable care. If the crisis was due to moral hazard problems in intermediaries, for example, then liquidity injections which fix things in the short run may exacerbate problems in the long run – unless financial re-regulation follows.

With flexible prices - and the Pigou effect - a ‘Big-Bang’ can stimulate output and investment and leading in the limit to a modified golden rule equilibrium. But without price flexibility and the Pigou effect, tightening credit constraints can convert a situation of efficient inter-temporal optimisation to one of inefficient temporary equilibrium. In this way, the KM approach, as analysed in this paper, can be seen as a bridge between real business cycle theorising on one hand, and Keynesian macro economics on the other. Will this be the impact of recent events – the development of

15 Before the Great Depression, banks lent heavily to those speculating on shares using the shares themselves as collateral; in the current ‘Sub-prime crisis’, shadow banks have performed a similar role in respect of real estate.
integrated models of heterogeneous agents operating subject to financial constraints, which can encompass different macroeconomic views as special cases?

References
Ahamed, Liaquat (2009), Lords of Finance: The bankers who broke the world. London: William Heinemann


Blanchard, Olivier, Giovanni Dell’Ariccia, and Paolo Mauro (2010), ‘Rethinking Macroeconomic Policy’. IMF Staff Position Note, SPN/10/03, (February).


Appendix 1  Flexprice Regime: a ‘Big Bang’ in financial development

Here we use the flex-price approach of KM to study the impact of an unanticipated loosening of credit constraints – a ‘Big Bang’ in financial development, for example. In Figure A1, E and E’ are the steady state equilibria associated with permanently low and permanently high levels of liquidity, and the corresponding stable and unstable eigenvectors indicate the saddle point dynamics. In equilibrium, higher liquidity leads to more investment, capital and aggregate consumption, so E’ lies to the right of E. Assuming for convenience that the system begins at E, the impact of an unexpected liquidity increase is shown by the jump in asset prices to B, followed by convergence to E’ along the new stable eigenvector. Full employment prevails throughout, by assumption, with the potential excess demand checked by the jump in prices and reduction in real balances, i.e. there is a Pigou effect at work. (Due to the impact of price increases on the real balances held by entrepreneurs, their consumption will fall immediately as φ is increased: for entrepreneurs who take the price of money as parametric, the Pigou effect is a negative externality.)

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17 In a discrete time model, there will be an impact effects on K as well as q; while these are not shown explicitly in the diagram, they appear in the simulation results.
Figure A1. A ‘Big Bang’ in liquidity: an unexpected increase in $\phi$.

Impulse response in the flex-price quarterly model

For an unexpected 20% increase in liquidity, $q$ will immediately jump up by 0.3% percent, converging thereafter back to its original value; $r$, $k$ and $y$, on the other hand, do not jump in the period when the ‘Big-Bang’ occurs, they will start to converge to their new equilibrium level monotonically in the next period, as shown below:
The impulse responses – where variables are measured as deviations from the initial equilibrium - confirm that reducing financial frictions can have substantial real effects even with flexible prices and wages. The added investment leads to an increase of the capital stock and an increase in output; but the rate of return on capital declines. A 20% increase in the resaleability constraint (so 15.6% of shares can be sold per period rather than 13%) increases the equilibrium capital stock by over one percentage point.\textsuperscript{18}

Memo items

Following DelNegro et al. (2010), the parameters for the linearized KM model used here are chosen as: $\varphi = 0.13$; $\beta = 0.99$; $\theta = 0.13$; $\pi = 0.075$ and $\lambda = 0.975$.

Initial (‘base case’) equilibrium values of the variables are then: $q = 1.1175$; $r = 0.0374$; $K = 152.5056$, $y = 17.2644$ and $M_p/K = 0.1171$. The eigen-values for the base case are 0.9837 and 1.1010. Slope of stable eigenvector: -0.0016.

\textsuperscript{18}These effects will be more pronounced than those described in the KM paper, where $\varphi$ follows a two-state stochastic process, so a reversal is anticipated.
Appendix 2

Simulation results in the fix-price quarterly model: a prolonged credit crunch

To complement the results of a short liquidity squeeze discussed in the text, we illustrate the effect of a squeeze more like that of the Great Depression: thus the 20% cut in the liquidity parameter $\phi$, from 0.13 to 0.104, is taken to last for 8 years rather than 8 quarters (with effects of including QE shown with asterisks).

Memo items

Following Del Negro et al.(2010), the parameters for the linearized KM model used here are chosen as: $\phi = 0.13; \beta = 0.99; \theta = 0.13; \pi = 0.075$ and $\lambda = 0.975$.

Initial (‘base case’) equilibrium values of the variables are then:
$q = 1.1175; r = 0.0374; K = 152.5056, y = 17.2644$ and $M_p/K = 0.1171$.

The stable eigenvalue is 0.9969, and the slope of stable eigenvector: -0.0012.