Optimal Monetary Policy and Asset Prices: the case of Colombia

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Abstract

The unfolding of the 2007 world financial and economic crisis has highlighted the vulnerability of real economic activity to strong fluctuations in asset prices. Which is the optimal monetary policy in an economy like the Colombian that is exposed to swings in asset prices? What is the implication in terms of Central Bank losses when it follows a standard simple rule instead of the optimal monetary policy? To answer these questions we use a Dynamic Stochastic General Equilibrium (DSGE) model with physical capital and sticky wages for the Colombian economy and derive the optimal monetary policy. Then, we explore the dynamic effects of news about a future technology improvement which turns out ex post to be overoptimistic under the optimal policy rule and alternative specifications of simple rules and definitions of output gap.

Key Words: DSGE model, Optimal Monetary Policy, Asset Price boom-busts, Colombia.

JEL Classification: E44, E52, E61.

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1 Introduction

During the last couple of decades, many monetary authorities around the world have achieved the goal of a low and stable inflation rate. However, this price stability has not came hand-in-hand with higher asset price stability. Borio and Filardo (2003), among others, document the emergence of asset prices, credit and investment booms and bust which have become a more important source of macroeconomic instability in both developed and developing countries. Financial unbalances are of great concern because when they unwind, the real economy is exposed to a substantial economic downturn and very frequently to recession. For example, many economist attribute at least some part of the 1990 recession in the United States to the preceding decline in commercial real estate prices (Bernanke and Gertler (1999).

The Colombian economy, like many other developing economies has experienced very strong asset prices and output fluctuations. Figure 1 displays the cyclical component of economic activity and asset prices for the Colombian economy during 1970-2005. Two boom-bust episodes are evident, the first during the eighties and the second during the nineties. Since 2004 there was a boom phase that has been followed by an economic downturn triggered by the 2007 global financial crisis. The close correlation between asset prices cycles and output cycle and the evidence of a financial accelerator mechanism in the Colombian economy that was found by López, Prada and Rodriguez (2008), rises the question if the nature of monetary policy is able to explain the behavior of both variables. Would the boom-bust cycles be smoother if the monetary authority incorporates a response to asset prices in the simple monetary policy rule? How costly, in terms of central bank loss function, is a monetary policy that reacts only to inflation and output gap instead of taking into account asset prices?

To answer these questions, we set up a model for the Colombian economy where, as in Cristiano, Ilut, Motto and Rostagno (2008), the boom phase is triggered by a signal which leads agents to rationally expect an improvement in technology in the future but the signal turns out to be false and the bust phase of the cycle begins when people finds this out. We explore the effects of these news about a future technology improvement which turns out ex post to be overoptimistic under the optimal policy rule and alternative specifications of simple rules.

By optimal monetary policy we mean policy that minimizes an intertemporal loss

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1 asset prices correspond to a weighted average of equity prices and real state prices
function under commitment. The intertemporal loss function is a discounted sum of expected future period losses. We choose two alternative welfare criteria. The first is a quadratic period loss function that corresponds to flexible inflation targeting and is the weighted sum of two terms: the squared inflation gap between inflation and the inflation target and the squared output gap between output and potential output. The second measure of loss that we consider is a utility-based loss function.

Like in Svensson et al. (2008) a key issue for a flexible inflation targeting central bank is which measure of output gap should try to stabilize. We report results from three alternative concepts of gaps used in the loss functions and the simple policy rules. One concept is deviations of output and asset prices from the hypothetical level that would exist if the economy would have had flexible prices and wages. The second is deviations from steady-state values. The third concept (used only in the simple rules) corresponds to growth rates.

The model we use is a DSGE model for a small open economy like Colombia. The model distinguishes households and entrepreneurs. Households consume and work, while entrepreneurs produce an homogeneous intermediate good using capital bought from capital producers and labor supplied by households. Entrepreneurs take bank loans facing borrowing constraints, tied to the value of collateral. In addition, there are banks who offer two types of financial assets to agents: saving and loans; retailers who set the final price of output goods; workers who supply their differentiated labor services through a union which sets wages to maximize member’s utility, generating a nominal rigidity in wages à la Calvo. There is also a foreign sector which provides assets at the foreign interest rate which is positively related to the net foreign asset position of the domestic economy. Finally, there are capital producers who transform output goods into capital goods, a government and a central bank which conducts monetary policy.

The remainder of the paper is as follows. Section 2 describes the model. Section 3 presents the optimal policy problem, the different simple rules and the alternative results of a boom-bust episode. Section 4 concludes.
2 The model

2.1 Households and Wage Setting

2.1.1 Consumption and saving decisions

The domestic economy is inhabited by a continuum of households indexed by \( i \in [0, 1] \). The representative agent \( i \) maximizes the following utility function

\[
E_t \sum_{s=0}^{\infty} \beta^s \frac{N_{t+s}}{N_t} u \left( c_{t+s}^{pc}(i), l - h_{t+s}^{pc}(i) \right)
\]

where \( c_{t}^{pc}(i) \) per-capita consumption, \( h_{t}^{pc}(i) \) is per-capita hours worked \( h_{t}^{pc}(i) \) is per-capita leisure time, which satisfies \( h_{t}^{pc}(i) = l - h_{t}^{pc}(i) \), with \( l > 0 \) being the total endowment of time. \( N_t \) is total population which follows a stochastic process.

The discounted utility is given by

\[
u(\cdot) = \frac{1 - \phi}{1 - \sigma} \chi_t^u \left[ c_t^{pc}(i) - \frac{\phi A_t c_{t-1}^{pc}}{1 - \phi} \right]^{1-\sigma} - \frac{\chi_t^h}{1 + \varsigma} l^{\sigma - \varsigma} A_t^{1-\sigma} (h_{t}^{pc}(i))^{1+\varsigma}
\]

with \( \sigma > 0, \varsigma > 0 \) and \( \phi > 0 \). Parameter \( \varsigma \) is the inverse elasticity of labor supply with respect to real wages. Parameter \( \sigma \) is the constant relative risk aversion coefficient. Preferences display habit formation in consumption governed by parameter \( \phi \). \( \chi_t^{u,h} \) are preferences shocks that shifts the consumption demand and leisure, \( A_t \) represents productivity which follows the process

\[
\ln \left( \frac{A_t}{A_{t-1}} \right) = \rho_a \ln \left( \frac{A_{t-1}}{A_{t-2}} \right) + (1 - \rho_a) \ln (1 + a) + \epsilon_t^A
\]

where \( \epsilon_t^A \) is a white noise variable.

Following Prada (2008) we assume that there exist transaction costs in the economy. The exchange process requires real resources. In this process, the more transactions the higher the transaction cost and the higher the deposits held by households the lower the transaction cost:

\[
v_t(i) = \frac{c_t(i)}{d_{t-1}^h(i)} \frac{A_t N_t}{A_{t-1} N_{t-1}}
\]

where \( v_t(i) \) is deposits velocity and \( d_{t-1}^h(i) \) deposits held by household \( i \).
Cost per unit of transaction is given by \( \vartheta(v_t(i)) \), an increasing, positive, twice differentiable, convex function. In particular we assume that

\[
\vartheta(v_t) = \vartheta_0 v_t^{\vartheta_1}
\] (3)

with \( \vartheta_0 > 0 \) y \( \vartheta_1 > 1 \).

Households decisions have to match the following budget constraint

\[
\begin{align*}
c_t(i) (1 + \vartheta(v_t(i))) + \int p^a_{i}(i) a_t(i) dw_t(i) + \tau_t + d^h_t(i) \leq \quad \text{(4)} \\
\end{align*}
\]

where \( (a_t(i)) \) represents Arrow-Debreu assets with price \( p^a_{i}(i) \), \( (d^h_t(i)) \) deposits, \( (\tau_t) \) lump-sum taxes, \( (w_t) \) real wage, \( (tr_t) \) foreign transfers, \( (\Pi_t) \) total profits from firms and banks ownership, \( (i^d_{t-1}) \) interest on bank deposits and \( (\pi^c_t) \) CPI inflation rate.

Households choose consumption and the composition of their portfolios by maximizing (1) subject to (4). Given that we are assuming the existence of Arrow-Debreu assets, consumption is equalized across households and the first order conditions can be expressed in terms of effective worker:

\[
\lambda_t \left(1 + (1 + \vartheta_1) \vartheta_0 (v_t)^{\vartheta_1}\right) = \lambda_t^u \left[ \frac{c_t - \phi c_{t-1}}{1 - \phi} \right]^{-\sigma} \quad \text{(5)}
\]

\[
\lambda_t = \beta E_t \left( \frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \left( \frac{1 + i^d_t}{1 + \pi^c_{t+1}} \right) \quad \text{(6)}
\]

along with (4), where \( \lambda_t \) is the budget constraint Lagrange multiplier.

2.1.2 Labor supply and wage setting

Following Erceg et al. (2000), we assume that a continuum of monopolistically competitive households supply differentiated labor services to the production sector as an imperfect substitute for the labor services of other households. There is a set of perfect competitive labor service assemblers that combines household’s labor hours in the same proportions as firms would choose. The aggregator’s demand for each household’s labor
demand is defined as

\[ h_t^d = \left[ \int_0^1 h_t(i) \frac{\theta_w - 1}{\theta_w - 1} \, di \right]^{\theta_w - 1} \] (7)

The optimal composition of this labor service unit is obtained by minimizing its cost, given the different wages set by different households. The demand for each differentiated variety of labor is given by

\[ h_t(i) = \left( \frac{w_t(i)}{\bar{w}_t} \right)^{-\theta_w} h_t^d \] (8)

where \( \bar{w}_t \equiv \left[ \int_0^1 w_t(i)^{1-\theta_w} \, di \right]^{\frac{1}{1-\theta_w}} \) is an aggregate wage index and \( \theta_w > 0 \) is the elasticity of substitution among labor varieties.

We assume that wage setting is subject to a nominal rigidity à la Calvo (1983). The duration of each wage contract is randomly determined: in any given period, the household is allowed to reset its wage contract with probability \( (1 - \epsilon) \), the household is not allowed to reset its wage contract. We assume there is an updating rule for all those households that cannot re-optimize their wages. In particular, if a household cannot re-optimize during \( i \) periods between \( t \) and \( t + i \), then its wage at \( t + i \) is given by

\[ w_t^{\text{rule\_pc}}(i) = w_{t-1}^{\text{pc}}(i) \frac{A_t}{A_{t-1}} \left( \frac{\prod_{k=1}^n (1 + \pi_t^k)^\gamma_k (1 + \pi_t)^{1-\sum_{m=1}^n \gamma_m}}{1 + \pi_t^k} \right) \] (9)

where \( n \in \mathbb{N} \) is the indexation horizon, \( \gamma_k \geq 0 \) is the weight assigned to inflation rate \( k \) periods earlier and \( 1 - \sum_{m=1}^n \gamma_m \geq 0 \) is the weight assigned to the target inflation set by the monetary authority \( \pi_t \). This adjustment rule implies that workers who do not optimally reset their wages update them by using a geometric weighted average of past CPI inflation and the inflation target set by the Central Bank, \( \pi_t \).

In any period of time \( t \) in which a household is able to reset its wage contract solves the problem

\[
\max_{w_t(i)} \quad E_t \sum_{i=0}^{\infty} (\beta^{\epsilon_w})^i \frac{N_{t+i}}{N_t} u (c_{t+i}(i), 1 - h_{t+i}(i))
\]

subject to the labor demand (8), the updating rule for the nominal wage (9) and the budget constraint (4).
2.2 Entrepreneurs

Entrepreneurs purchase capital in each period, \((k_{t-1} \frac{N_{t-1}}{A_{t-1} N_{t-1}})\), and use it in combination with hired labor, \(h_t\) to produce the intermediate product, \(q_t^s\), following a constant-returns-to-scale technology

\[
q_t^s = \chi_t^q \left[ \alpha_q^s \left( k_t^s \right)^{\rho q - 1} + (1 - \alpha_q^s) \left( h_t^d \right)^{\rho d - 1} \right]^{\frac{1}{\rho q - 1}}
\]  

(10)

where \(k_t^s = k_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t}\). The intermediate product is sold in a competitive market at wholesale price \(p_t^q\). Following Christiano et al. (2008) we assume that technology, \(\chi_t^q\), follows the exogenous process given by

\[
\ln(\chi_t^q) = \rho^q \ln(\chi_{t-1}^q) + (1 - \rho^q) \ln(\chi_{t-1}^q) + \epsilon_t + \epsilon_{t-p}
\]

where \(\epsilon_t\) and \(\epsilon_{t-p}\) are uncorrelated over time and with each other. This simple process allows to incorporate a boom-bust episode in the model. Throughout the analysis, we consider the following impulse. Up until period 1, the economy is in steady state. In period \(t = 1\), a signal occurs which suggests \(\ln(\chi_t^q)\) will be high in period \(1 + p\). But, when period \(1 + p\) occurs, the expected rise in technology in fact does not happen.

Capital stock depreciates at the rate \(\delta > 0\). Following Gerali et al. (2008) we assume that to finance capital purchases entrepreneurs have access to loan contracts offered by banks. The amount of resources that banks are willing to lend to entrepreneurs, \(z_t^f\), is constrained by the value of their collateral, which is given by their holdings of physical capital. The borrowing constraint is

\[
E_t \left( \frac{1 + \pi_t^c}{1 + \pi_{t+1}^c} \right) z_t^f \leq m_t^f E_t \left( p_{t+1}^k k_t (1 - \delta) \right)
\]

(11)

where \(m_t^f\) is the ‘loan-to-value’ and \(\pi_t^c\) is the interest rate paid on loans, \(z_t^f\). Entrepreneur’s budget constraint is

\[
p_t^q q_t^s + p_t^k (1 - \delta) k_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t} + z_t^f = w_t h_t^d + p_t^k k_t + \left( \frac{1 + \pi_{t-1}^c}{1 + \pi_t^c} \right) z_{t-1}^f \frac{A_{t-1} N_{t-1}}{A_t N_t} + \Pi_t^q
\]

(12)

where \(\Pi_t^q\) represents the flow of profits that will be transferred to households.

Given labor demand, the representative firm purchase \(k_{t+1}^s\) units of capital at price
\( p_t^k \), to maximize its expected sum of profits flows, using \( \Lambda_{t+1} \frac{N_{t+1}}{N_t} \) as the appropriate discount factor. The optimality conditions are given by

\[
p_t^k \lambda_t = \lambda_t m_t m_{t+1} E_t p_{t+1}^k (1 - \delta) + \beta E_t \left( \frac{A_{t+1}}{A_t} \right)^{-\sigma} p_{t+1}^k \chi_{t+1} \left( \frac{\alpha q_{t+1}^s}{\chi_{t+1} h_{t+1}^{s+k}} \right)^{\frac{1}{\beta}} + p_t^k (1 - \delta) \tag{13}
\]

\[
\lambda_t = \lambda_t m_f E_t \left( \frac{1 + i_t^f}{1 + \pi_{t+1}^c} \right) + \beta E_t \left( \frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \left( \frac{1 + i_t^f}{1 + \pi_{t+1}^c} \right) \tag{14}
\]

\[
w_t = p_t^q \chi_t^q \left( \frac{(1 - \alpha) q_s^q}{\chi_t^q h_t^q} \right)^{\frac{1}{\rho}} \tag{15}
\]

### 2.3 Retailers and Price Setting

Retailers buy output from entrepreneurs and slightly differentiate it at no resource cost. The differentiation of output gives the retailers some market power. Households and firms then purchase CES aggregates of these retail domestic good. Retailers are introduced to motivate sticky prices and we follow Calvo (1983) in introducing price inertia. Each retailer faces a demand for variety \( j \) given by

\[
q_t(j) = \left( \frac{\chi_t^{qd} q_t^q(j)}{p_t^{qd}} \right)^{-\theta^q} q_t^d \tag{16}
\]

where \( q_t^d = \chi_t^{qd} \int_0^1 (q_t(j))^{\theta^q - 1} \frac{\theta^q}{\theta^q - 1} dj \) and \( p_t^{qd} = \left( \chi_t^{qd} \right)^{-1} \int_0^1 \left( p_t^q(j) \right)^{1 - \theta^q} \frac{1}{\theta^q} dj \). While \( \chi_t^{qd} \) is an exogenous technological factor, \( p_t^{qd} \) is the output price of the aggregate basket \( q_t^d \) and \( \theta^q \) the price elasticity of demand for variety \( j \). This parameter also define the flexible price equilibrium markup charged by firms.

Following Calvo (1983), we assume that only a fraction \((1 - \epsilon^q)\) of sellers are allowed to reset their prices. In particular, if a firm cannot set an optimal price, then it follows a non-optimal price rule

\[
p_t^{q_{\text{rule}}}(j) = p_t^q(j) \prod_{k=1}^n \left( 1 + \pi_{t-k}^{qd} \right)^{\gamma_{qk}} (1 + \pi^1)^{-\sum_{m=1}^n \gamma_{qm}}
\]

where \( n \in \mathbb{N} \) is the indexation horizon, \( \gamma_k \geq 0 \) is the weight assigned to inflation rate \( k \) periods earlier and \( 1 - \sum_{m=1}^n \gamma_{qm} \geq 0 \) is the weight assigned to the target inflation.
set by the monetary authority $\pi$.

If the firm receive a signal to optimally adjust its price it will choose $p_t^j (j)$ to maximize

$$\max_{p_t^j(j)} E_t \sum_{i=0}^{\infty} (e^q)^i \Lambda_{t+i,t}^j [p_{t+i}^q (j) q_{t+i} (j) - p_{t+i}^{q_p} q_{t+i} (j)]$$

(17)

subject to the demand for variety $j$, (16), using $\Lambda_{t+i,t}^j = \beta^i (\frac{A_{t+i}}{A_i})^{1-\sigma} \frac{N_{t+i}}{N_i} \frac{\lambda_{t+i}}{\lambda_t}$ as the appropriate discount factor.

### 2.4 Capital Producers

Capital producers purchase consumption goods as a material input, $x_t$, and combine it with the existing capital stock ($(1-\delta) k_{t-1}\frac{A_{t-1}N_{t-1}}{A_tN_t}$), to produce new capital. We assume that capital producers are subject to quadratic capital adjustment cost. The price of capital is determined by a $q$-theory of investment.

The aggregate capital stock evolves according to

$$k_t = (1-\delta) k_{t-1}\frac{A_{t-1}N_{t-1}}{A_tN_t} + \chi_t^k x_t$$

(18)

where $\chi_t^k$ is the marginal efficiency of investment following Greenwood et al. (1988).

Capital producers’ optimization problem, in real terms, consists of choosing the quantity of investment to maximize profits, so that

$$\max_{x_t} p_t^k k_t - p_t^k (1-\delta) k_{t-1}\frac{A_{t-1}N_{t-1}}{A_tN_t} - p_t^x x_t - \frac{\psi X}{2} (k_t - k_{t-1})^2$$

(19)

subject to (18). The $k_{t-1}$ first order condition is

$$p_t^x = \chi_t^k (p_t^k - \psi X (k_t - k_{t-1}))$$

(20)

### 2.5 Banks

The banking industry is assumed to be perfectly competitive. Since economic agents require deposits and credit, banks produce the financial services through a production technology that uses real resources from the economy as an input. Following Edwards
and Vegh (1997), the production technology for banks is given by the cost function

$$\xi_t \eta (z^f_t, d_t)$$

which is positive for $z^f_t, d_t > 0$, convex, continuously differentiable, increasing in all arguments and homogeneous of degree one.

$\xi_t$ represents an inverse measure of the total productivity of the banking intermediation sector. It is a cost scale factor exclusive of the banking sector that follows that process

$$\ln (\xi_t) = (1 - \rho_\xi) \ln (\bar{\xi}) + \rho_\xi \ln (\xi_{t-1}) + \epsilon_t^\xi$$

where $\bar{\xi}$ is the expected value of the cost scale factor, $\rho_\xi \in [0,1)$ and $\epsilon_t^\xi$ is white noise variables with variance $\sigma^2_\xi$.

The policy of the Central Bank and the banking sector is related through the reserve requirement which is a fixed proportion $\tau^d_t > 0$ of total deposits, so the bank reserves, $rb_t$, satisfies the constraint

$$rb_t \geq \tau^d_t d_t \quad (21)$$

Banks can borrow from the central bank at a nominal rate $i^{bc}_t$. The net debt of a private bank with the central bank is $b_t$. The banks also finance themselves through foreign debt $f_t$ and they pay the interest rate $i^f_t$ set in the foreign market. It is assumed that the banks are the only private agents that have access to foreign resources.

The representative bank seeks the maximization of the discounted sum of profits $(\Pi^b_t)$. The bank’s resource constraint is given by

$$\left(1 + \frac{i^{zh}_t}{1 + \pi^c_t}\right) z^h_{t-1} \frac{A_{t-1} N_{t-1}}{N_t} + \left(1 + \frac{i^{zd}_t}{1 + \pi^c_t}\right) d_{t-1} \frac{A_{t-1} N_{t-1}}{N_t} + z^h_t$$

$$+ \left(1 + \frac{i^{zf}_t}{1 + \pi^c_t}\right) z^f_{t-1} \frac{A_{t-1} N_{t-1}}{N_t} \frac{S^*_t P^*_c f_t}{A_t N_t} + d_t + \left(1 + \frac{i^{be}_t}{1 + \pi^c_t}\right) b_{t-1} \frac{A_{t-1} N_{t-1}}{N_t} + rb_t$$

$$+ b_t + rb_{t-1} \frac{A_{t-1} N_{t-1}}{A_t} \frac{A_{t-1} N_{t-1}}{N_t} + p^{qd}_t \xi_t \eta \left(z^h_t, z^f_t, d_t\right) + \Pi^b_t$$

Bank’s income is given by credit interest payments at a nominal rate $i^{zf}_{t-1}$, foreign
debt accumulation $f_t$, deposits accumulation $d_t$, accumulation of debt with the central bank $b_t$ and the returned reserve from the central bank $rb_{t-1}$. These revenues are used to pay for deposits at an interest rate $i^d_t$, to accumulate credit $z^f_t$, to pay foreign debt at the interest rate $i^f_{t-1}$, to pay the interest to the central bank $i^c_{t-1}$, to accumulate new reserves, to pay the real cost of the financial intermediation and to make profit transfers to households $\Pi^b_t$. $1 + \pi^*_t$ represents foreign inflation rate.

The production technology of the financial services is represented with the cost function

$$
\eta \left( z^f_t, d_t \right) = \left[ \nu_z \left( z^f_t \right)^\nu + \nu_d \left( d_t \right)^\nu \right]^{\frac{1}{\nu}}
$$

where $\nu > 1$, $\nu_z, \nu_d > 0$.

Bank’s optimization problem is a dynamic process. Banks maximize expected value of the discounted sum of profits flows. The relevant discount factor is $\Lambda^b_{t+1,t} = \beta^i \left( \frac{A_{t+1}}{A_t} \right)^{1-\sigma} \frac{A_{t+1}}{A_t} \lambda_{t+1}$. The first-order conditions for domestic, foreign debt accumulation, deposits and credit are

$$
\lambda_t = \beta E_t \left( \frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \left( \frac{1 + i^{bc}_t}{1 + \pi^{c}_t} \right)
$$

(23)

$$
\lambda_t = \beta E_t \left( \frac{A_{t+1}}{A_t} \right)^{-\sigma} \lambda_{t+1} \left( \frac{1 + i^f_t}{1 + \pi^{c}_t} \right)
$$

(24)

(25)

(26)

### 2.6 Foreign Sector

Following Schmitt-Grohé and Uribe (2003) we assume that the foreign sector provides resources to the economy at the interest rate $i^f_t$ that depends on total net foreign indebtedness, $f - a^bc_t$, as a percentage of GDP, $y_t$, as follows
\[ (1 + i^f_t) = (1 + i^*) \chi'^f_t \exp \left( \Omega_u \left( \frac{(f_t - a^{bc}_t)}{y_t} - FE \right) \right) \] (27)

where \( i^* \) is the risk free foreign interest rate, \( \chi'^f_t \) is an foreign interest rate shock, \( a^{cb}_t \) are foreign assets held by the central bank, \( FE \) is the steady state value of net foreign assets and \( \Omega_u > 0 \) is a scale parameter. We close the model in this way because without it net foreign indebtedness may be non-stationary, complicating the analysis of local dynamics. In steady state \( \frac{(f_t - a^{bc}_t)}{y_t} = FE \) and \( 1 + i^f = (1 + i^*) \chi'^f_t \).

### 2.7 Central Bank

Monetary authority is able to set the nominal interest rate prevailing in the interbank market \( i^{bc}_t \) following a Taylor-type rule

\[ (1 + i^{bc}_t) = (1 + i^{bc}_{t-1})^{\rho_i} \left( 1 + \pi_t \right) \left( 1 + \pi^*_t \right)^{\rho_{\pi}} \left( \frac{y_t}{y^{flex}_t} \right)^{\rho_y} \exp(\epsilon_i^t) \] (28)

where \( \rho_{\pi} \) and \( \rho_y \) are the weights assigned to inflation and output stabilization, respectively, \( \epsilon_i^t \) is an exogenous shock to monetary policy, and \( y^{flex}_t \) represents the hypothetical output level that would exist if the economy would have had flexible prices and wages.

The resource constraint of the Central Bank is given by

\[ \left( \frac{1 + i^f_{t-1}}{1 + \pi^*_t} \right) a^{bc}_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t} + \left( \frac{1 + i^{bc}_{t-1}}{1 + \pi^*_t} \right) b_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t} + \frac{rb_t}{\rho_y} = a^{bc}_t + rb_{t-1} \frac{A_{t-1} N_{t-1}}{A_t N_t} + b_t + \Pi^{bc}_t \] (29)

where \( a^{bc}_t \) is the exogenous stock of foreign net assets and \( \Pi^{bc}_t \) are transfers to the government.

### 2.8 Government

The government obtains resources from lump-sum taxes \( \tau_t \), net transfers from the central bank, the transaction costs and capital adjustment and uses this to finance public expenses \( g_t \), that follows the process

\[ \ln \left( g_t \right) = (1 - \rho_g) \ln \left( g_{t-1} \right) + \rho_g \ln \left( g_{t-1} \right) + \epsilon_i^g \]
where $\bar{g}$ is the expected value of the government expenditure, $\rho_g \in (0, 1)$ and $\epsilon^g$ are white noise with variance $\sigma^2_g$.

2.9 National accounts

Real GDP, $y_t$, the final domestic income of the households

$$y_t = c_t + g_t + x_t + \xi_t \eta(z_t^I, d_t)$$

$$- \left(\frac{1 + \nu_{t-1}^I}{1 + \pi_t^I}\right) \left(a_{t-1}^{bc} - f_{t-1}\right) \frac{A_{t-1} N_{t-1}}{A_t N_t} - tr_t + (f_t - a_t^{db})$$

from which we can define trade balance as

$$XN_t = \left(f_t - a_t^{db}\right) - \left(\frac{1 + \nu_{t-1}^I}{1 + \pi_t^I}\right) \left(a_{t-1}^{bc} - f_{t-1}\right) \frac{A_{t-1} N_{t-1}}{A_t N_t} - tr_t$$

where $tr_t$ represents foreign transfers.

2.10 Model Parametrization

The model is calibrated to match key steady-state ratios of Colombia. A period in the model corresponds to one quarter.

2.10.1 Long-run parameters

Following Mahadeva and Parra (2008), the annualized foreign steady-state real interest rate faced by the colombian economy is set at 3.42%. This implies a discount factor of $\beta = 0.999$. Following Prada (2008), the value of $n$ is set to match the average annual rate of growth of the total population in Colombia (this rate is 1.22%), and the parameter $a$ is calibrated to obtain an annual rate of growth of the labour-augmenting productivity of 1.5%. A value of $\sigma = 2$ is used as the constant relative risk aversion coefficient, Arias (2000).

The steady-state foreign annual inflation rate is set at 2% and the domestic annual rate is set at 3%, the long-run target of the central bank in Colombia. The parameter $\varsigma$ is set at 3 to obtain a Frisch elasticity of 0.33, near the value found by Prada and Rojas (2009).
The model is calibrated to produce a steady state value of $h = 0.294$, the share of time dedicated to the labour market. This implies a value of $\chi^h = 146.90$. We assume that the banking costs are quadratic, and set $\nu = 2$. To match the average annualized real lending rate (7.92%) and the average annualized real deposit rate (2.01%) reported in Prada (2008), we set $\nu_d = 6.284 \times 10^{-5}$ and $\nu_z = 1.324 \times 10^{-4}$.

The level of real GDP the steady-state is normalized to unity. This is achieved by setting $\chi^{qs} = 0.524$. The exogenous public expenditure parameter $g$ is calibrated to obtain a steady-state ratio of government expenditure to GDP of 0.178, equal to the average of that ratio in the period 1994 : 1 – 2007 : 4.

Following Mahadeva and Parra (2008) the value of total foreign net assets to GDP is set to 1.20, and this implies a value of 1.20 for the parameter $FE$. The average ratio of net foreign assets of the central bank to GDP (net foreign assets, monetary sectorization - Banco de la República) is 0.454 in the period 2005 : 1 – 2007 : 4, and the parameter $a^{cb}$ is set to match this ratio.

The average ratio of net foreign transfers to GDP is 0.0351 and the parameter $tr$ is set to this value. We assume quadratic transaction costs and set $\vartheta_1 = 2$. The parameter $\vartheta_0$ is calibrated to match the value of the average ratio of deposits which generate costs to the banks to GDP (1.20). This implies a value of $\vartheta_0 = 0.0126$. The parameter $\alpha = 0.456$ is calibrated to get the average ratio of investment to GDP (0.215) reported in Prada (2008). The steady-state leverage ratio $m^f$ is calibrated to match the average ratio of credit to GDP (2.10). This implies $m^f = 0.33$. Following Prada (2008), $\tau^d$ is set at 0.062 and $\alpha^{ch}$ is set at 0.454.

2.10.2 Short run and additional parameters

Following Arango et al. (1998) the mark-up on production marginal cost is set at 25%, and this implies a value of $\theta^q = 5$. The same mark-up is assumed for the wage setting process. Following Bonaldi et al. (2009), the Calvo parameters that measure the degree of price stickiness are selected in such a way that, on average, the final good price is adjusted once each year ($\epsilon^q = 0.75$) and the wage rate is adjusted once each four months ($\epsilon^w = 0.25$). The elasticity of substitution between labour and capital is set at $\rho = 0.84$, as in Bonaldi et al. (2009).

In the baseline calibration it is assumed that there is no monopolistic competition in the financial system, because this assumption is not needed to explain the spread
between interest rates. Then \( \theta^d \rightarrow \infty \) and \( \theta^z \rightarrow \infty \). The habit persistence \( \phi \) is set at 0.5. The parameter of the adjustment cost of investment \( \Psi \) is set at 0.7. The persistence of the exogenous processes is 0.6. The parameters of the policy rule are standard: \( \rho_i = 0.75, \rho_{\pi} = 1.25 \) and \( \rho_y = 0.50 \).

3 Optimal Monetary Policy and Simple Policy Rules

We find the Ramsey-optimal allocations for our economy using the computer code and strategy used in Levin, Lopez-Salido (2004) and Levin, et al. (2005). The Central Bank minimizes an intertemporal loss function at time \( t \):

\[
L_t = E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{N_{t+s}}{N_t} \right) \left( \frac{A_{t+s}}{A_t} \right)^{1-\sigma} \ell^\text{obj}_{t+s}
\]

where

\[
\ell^\text{obj}_t = \begin{cases} 
\gamma_\pi (\pi_t - \bar{\pi})^2 + \gamma_y (y_t - 1)^2 & \text{if } obj = ss \\
\gamma_\pi (\pi_t - \bar{\pi})^2 + \gamma_y (y_t - y^\text{flex}_t)^2 & \text{if } obj = flex \\
-\ell^\text{util}_t & \text{if } obj = util
\end{cases}
\]

where \( flex \) represents the flexible price equilibrium variables and \( ss \) stands for steady state values. The first two losses are often used as a metric for capturing policymaker’s preferences in studies that attempt to evaluate the trade-off between inflation variability and output variability. In addition to these losses, we consider a second measure of loss, i.e. a utility-based loss function, which we denote \( -\ell^\text{util}_t \). Following Woodford (2001), we derive \( \ell^\text{util}_t \) by taking a second order log-linearization of the utility function around the steady state. We ignore the constant and first-order terms (the latter are zero in unconditional expectation) and focus on the unconditional expectation of the second-order terms. The result is

\[
\ell^\text{util}_t = \frac{1}{2} \chi_u c^{1-\sigma} \left( \left( 1 - \sigma \right)\hat{c}_t^2 - \left( \sigma \phi^2 \frac{1}{1 - \phi} + \phi \right) \hat{c}_{t-1}^2 \right) + \chi_u c^{1-\sigma} \hat{\chi}_u \hat{c}_t - \phi \chi_u \hat{c}_{t-1} + \phi \sigma \frac{1}{1 - \phi} \hat{c}_t \hat{c}_{t-1} \\
- \chi_h (h)^{1+\varsigma} \left( \hat{x}_t^h E_t \left( \hat{h}_t (i) \right) - \frac{1}{2} (1 + \varsigma) \int_0^1 \left( \hat{h}_t (i) \right)^2 \, dt \right)
\]

14
The terms that appear in the utility-based loss function, are directly related to the distortions present in our model, the welfare of the representative consumer is adversely affected by variability in consumption and the dispersion of hours worked between households (similarly to Levin, et al. (2005)).

The minimization of the loss function is subject to the DSGE model described before. The optimization results in a set of first order conditions, which combined with the model equations yields a system of difference equations that can be solved with several alternative algorithms.

On the other hand, we close the model with alternative simple rules and compare the results when a bubble shock occurs. The first policy rule that we examine is the flexible price rule eq.(28), where the central bank responds only to inflation and output gap (defined as deviations from the flexible price equilibrium). The second policy rule used in the simulations is a rule where monetary policy also reacts directly to asset prices:

\[
(1 + i_{t}^{bc}) = (1 + i_{t-1}^{bc})^{\rho_i} \left( 1 + \bar{\pi} \left( \frac{1+\bar{\pi}}{1+\pi} \right)^{\rho_{\pi}} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{y}} \left( \frac{p_t}{p_{t-1}} \right)^{\rho_{p}} \right)^{1-\rho_i} \exp (\epsilon_t^i) \tag{30}
\]

The third and fourth rules are similar to simple the rule eq.(28) and eq.(30), but instead of using deviations of output and asset prices from the flexible price equilibrium we use the output growth rate and asset prices growth rate as follows:

\[
(1 + i_{t}^{bc}) = (1 + i_{t-1}^{bc})^{\rho_i} \left( 1 + \bar{\pi} \left( \frac{1+\bar{\pi}}{1+\pi} \right)^{\rho_{\pi}} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{y}} \left( \frac{p_t}{p_{t-1}} \right)^{\rho_{p}} \right)^{1-\rho_i} \exp (\epsilon_t^i) \tag{31}
\]

and

\[
(1 + i_{t}^{bc}) = (1 + i_{t-1}^{bc})^{\rho_i} \left( 1 + \bar{\pi} \left( \frac{1+\bar{\pi}}{1+\pi} \right)^{\rho_{\pi}} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{y}} \left( \frac{p_t}{p_{t-1}} \right)^{\rho_{p}} \right)^{1-\rho_i} \exp (\epsilon_t^i) \tag{32}
\]

Finally, we use two simple rules where output and asset prices gap are defined as deviations from steady-state values (ss).

\[
(1 + i_{t}^{bc}) = (1 + i_{t-1}^{bc})^{\rho_i} \left( 1 + \bar{\pi} \left( \frac{1+\bar{\pi}}{1+\pi} \right)^{\rho_{\pi}} \left( \frac{y_t}{y_{t-1}} \right)^{\rho_{y}} \left( \frac{p_t}{p_{t-1}} \right)^{\rho_{p}} \right)^{1-\rho_i} \exp (\epsilon_t^i) \tag{33}
\]
3.1 Results for Boom-Bust

The results in Figure 2-4 show the dynamic response of our model to a $\epsilon_t$ shock that occurs in period 1, followed by $\epsilon_t = -\epsilon_{t+p}$ for $p = 5$. Thus, there is a signal that technology will improve in the future but in the end turns out to be false. A positive signal arriving in $t - p$ indicates households that the economy is likely to be more productive $p$ periods ahead. Anticipating this, they try to bring to the present the future value of more production. They increase consumption and investment, in preparation for the future expected increase in productivity. To finance these activities, households increase their demand for credit and assets. Capital price rises because of the expected need for new capital in the future. This constitutes the boom stage of the cycle, based solely on expectations. But $p$ periods ahead, when productivity is supposed to change, a surprise shock $\epsilon_t$ may occur. If for instance, $\epsilon_t = -\epsilon_{t-p}$, then productivity stays still and the expected productivity change was not realized. This may happen for instance, if a new technology resulted less efficient than expected, or if a production policy failed after generating good signals. Then households face the consequences of higher consumption and investment financed through credit, without real support. The economy enters a recession: consumption, investment, asset prices and general economic activity fall. The boom has been burst.

We compare the dynamic properties of output, consumption, investment, asset prices, nominal interest rate, real wages, deposits, credit and inflation in the Ramsey equilibrium with the behavior of these variables when we close the model with alternative simple policy rules. Figure 2 shows the dynamic response of these variables for the Ramsey equilibrium and for the model closed with the simple rule that reacts to output and inflation growth rate and the rule that besides reacts to asset prices growth rate, with $\rho_p = 0.5$. With a monetary authority that follows a simple rule, a minor fluctuation is transformed into a substantial boom-bust cycle. This happens first because the real wage rises during the boom in the Ramsey equilibrium so an efficient way to achieve a higher real wage is to let inflation drop. But, the monetary authority who follows the inflation-targeting strategy is reluctant to allow this to happen. Such a monetary authority responds to inflation weakness by shifting to a looser monetary policy stance and second when the productivity shock is not realized the central bank does not react fast enough relative to the optimal policy causing higher volatility.

Letting a reaction from central bank to asset prices gap does not improve very much the dynamics of the variables, but as we will see later when we compare the rules in
terms of central bank losses there exist an important difference.

Figure 3 plots the results of the policy rule that takes into account output and asset prices deviations from the flexible economy. The boom-bust is smoother in this case because the boom is shorter than in the case of the flexible prices rules shown in Figure 2. The worse scenario occurs in the case were the monetary authority uses an instrument rule that reacts to deviations of output and asset prices from steady state values, Figure 4. In this case, the dynamic of the series is much more volatile. In addition, when the productivity shock turns out to be false, the monetary authority reacts too slowly relative to the flexible price rule. In terms of these responses this is the less desirable type of rule. The most suited policy rule, that is closer to the optimal policy, is the simple rule that reacts to output gap and asset prices gap using deviations from the flexible prices economy.

Something worth noting is that if the monetary policy is more aggressive ($\rho_{\pi} = 2.25$) than accommodative ($\rho_{\pi} = 1.25$) in terms of targeting inflation in the rule that uses deviations from the flexible equilibrium economy, the volatility of output, and inflation is reduced as can be seen in Figure 5. Therefore, we compute the losses for the different types of rules for both cases, the accommodative and the aggressive monetary policy.

Table 1 below shows the results for the three alternative criteria of welfare for the alternative simple rules under accommodative and aggressive policy rules. The optimal policy using deviations from the flexible prices in the loss function is the one that delivers the lower losses.

As can be seen, the lower losses are obtained with the flexible price rules with an aggressive monetary policy. Rules that perform the worst are those where the monetary authority responds to deviations of output and asset prices from steady-state values.

When the central bank follows a policy rule, an aggressive stance against inflation seems to control better the effects of the bubble, in terms of central bank losses. This happens because an aggressive stance allows a lower variability of inflation. A tighter control of prices does not allow the bubble to build up, so the relevant gap of asset prices is lower in the aggressive case. This in turn reflects in a slower growth of investment and output when the bubble is building up and generates a deeper fall of the relevant gap of these aggregates when the bubble bursts.

If the central bank does not follow an optimal policy, for the three objective functions the best results are achieved when the bank follows a rule that takes into account deviations of output and asset prices with respect to their hypothetical paths in an econ-
omy with flexible prices. Since the expectational shock is real by nature, the economy with flexible prices has similar effects: an increase in gross production, consumption, investment and domestic and foreign debt. The central bank that takes into account that the flexible-price real variables are deviated as well will try harder to control prices and to make real variables behave as in the flexible-price economy. Therefore it allows a lower variability of prices and allows a faster fall of consumption, investment and credit when the productivity shock is not realized. This fast adjustment is reflected in less variability of real GDP and generates a smaller loss.

We must note that the dynamics of the economy do not change by much if the central bank takes or not into account the asset prices in the policy rule. The only case in which targeting the price of asset decreases the loss of the central bank for the unrealized productivity shock is when the policy rule looks at the flexible-prices economy. In this case the relative improvement from including asset prices is of 32 percent when the loss function uses flexible equilibrium variables. For all the remaining rules, targeting the asset prices do not decreases the loss. Just as before, if the central bank targets deviations of asset prices, then it will not allow for a fast adjustment. In the case of the flexible-prices economy the asset prices fall sharply, and the rule that follows this information will do a fast adjustment.

In conclusion, to minimize the loss of the central bank, a fast adjustment of the economy is needed when it is obvious that the productivity shock really did not happen.

4 Conclusions

We calibrated a DSGE model for the Colombian economy that incorporates features such as sticky prices and wages, a banking sector and a financial fragility describing balance sheet effects. We use the model to compute the optimal policy response of the economy under an expectations shock of improvement in technology that turns out to be false. The benchmark optimal-Ramsey equilibrium is used to compare simple policy rules that monetary authorities might use in the implementation of monetary policy. We find out that the simple policy rule that reacts to deviations of output from potential output defined as the hypothetical output level that would exist if the economy would have had flexible prices, is the one that delivers the lower central bank losses. This, because a fast adjustment of the economy is needed when it is obvious that the productivity shock did not happen. Adding asset prices gaps to the policy
rule do not improve much the dynamics of the economy unless the central bank is able to identify asset prices misalignments. Finally, an aggressive monetary policy in terms of fighting inflation rate reduces central bank losses given that output and inflation variability are reduced.
<table>
<thead>
<tr>
<th>Model</th>
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<th>Optimal Flexible Gaps</th>
<th>Optimal utility approx</th>
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References


Levin, A. T. and D. Lopez-Salido (2004). Optimal monetary policy with endogenous capital accumulation. manuscript.


Figure 1: Asset prices and economic activity

Source: Banco de la Republica and DANE
Figure 2: Expectation of Technology Shock in period 5 Not Realized: Optimal Vs Simple with Growth rates
Figure 3: Expectation of Technology Shock in period 5 Not Realized: Optimal Vs Simple with deviations from Flexible Equilibrium
Figure 4: Expectation of Technology Shock in period 5 Not Realized: Optimal Vs Simple with deviations from Steady State
Figure 5: Expectation of Technology Shock in period 5 Not Realized: Taylor Accommodative vs Aggressive