Depressions in the Colombian economic growth during the XX century: A Markov Switching Regime Model

Martha Misas  
Central Bank of Colombia  
mmisasar@banrep.gov.co

María Teresa Ramírez  
Central Bank of Colombia  
mramirgi@banrep.gov.co

April, 2005

ABSTRACT

In this paper, we modeled the Colombian long run economic growth (1925-2003) using a two-regime first order Markov switching model. We found evidence of non-linearity in the annual rate of economic growth. The results show that changes between regimes are sudden and sporadic. The Colombian economy remains in the sustainable growth regime most of the time. The turning points from the Markov switching model capture very well the behavior of real output through time. In fact, they identify the four main depressions of the century.

JEL classification: O40, C22, E32, N16  
Keywords: Markov switching regime model, economic growth, fluctuations, Colombia.

Please address all correspondence concerning this paper to:  
María Teresa Ramírez  
Subgerencia de Estudios Económicos  
Banco de la República  
Carrera 7 # 14-78 Piso 11  
Bogotá, COLOMBIA  
E-mail: mramirgi@banrep.gov.co  
FAX: (+571) 2841686  
Phone: (+571) 343 0564

1 The authors are senior researchers from the Economic Research Department of the Central Bank of Colombia. We are grateful to Juan Nicolas Hernández for excellent research assistance. We thank Luis Eduardo Arango, Munir Jalil and Carlos Esteban Posada for their comments. The views expressed are those of the authors and not of the Banco de la República (Central Bank of Colombia) or of its Board of Directors. The usual disclaimer applies to any remaining errors or omissions.
Depressions in the Colombian economic growth during the XX century:  
a Markov Switching Regime Model

1. Introduction

Nonlinearities in the business cycle have become an important topic in the economic literature. Researchers have found that real output responds very differently to a shock if the economy is in an expansion or in a recession. Therefore, it is intuitive to think that the amplitude and duration of the cycle’s phases are asymmetric, indicating that nonlinearities are important in the growth process. Recent studies include Buckle et al, 2004; Pok-Sank, 2004; Breuning and Stegman, 2003; Mills and Wang, 2003; Kim et al, 2002; Filardo and Gordon, 1998, among others, for developed countries and Moolman, 2004; Soto, 2002 and Bautista 2000 for less developed countries.

Since Hamilton (1989), the Markov switching regime model (MSRM) has become increasingly used to analyze nonlinearities in economic growth, since linear models are not able to capture such asymmetries. The Markov switching model allows the economy to be in different regimes (i.e. slow or fast growth, in the case of two regimes) with the switch between regimes governed by the outcome of a Markov process. In general, the MSRM allows asymmetric reactions of real output to different shocks depending on the regime in which the economy is, and models the transition between different phases of the cycle as a regime switch. Also, the MSRM provides a link between the transition probabilities of moving from one regime to the other and the expected durations of the cycle’s phases. The model can also date the beginning and ending of each stage of the business cycle.

---

3 Threshold models and smooth transition autoregressive models are also widely used to study nonlinearities in macroeconomic variables. All of the papers mentioned above employ Markov switching regime models to capture nonlinearities in economic growth.
4 See Hamilton, 1994, pp. 677-701
5 One advantage of this model, as Moolman 2004 underlines, is that no previous information concerning the dates when the economy was in each regime or the size of these regimes are required. Also, the probability of being in a specific regime is inferred from the data.
The majority of Colombian’ business cycle studies assume that the growth rate follows a linear process. One exception is Arango and Melo (2005) who studied the nonlinear business cycle (proxied by the Colombian industrial production index) properties over the last two decades using the smooth transition autoregressive STAR model. They found evidence of nonlinearities and asymmetric behavior in the Colombian business cycle.

In this paper, we employ the Markov switching regime techniques to model the Colombian long run economic growth. This is the first paper that models the Colombian long run economic growth using the MSRM. In particular, we study the existence of different regimes in the real GDP annual growth rate for the period 1925-2003, the probabilities of moving from one regime to the other, the expected length and the dates of the beginning and ending of each cycle’ phase. We find four main results. First, the results indicate that non-linearities in the Colombian economic growth are important. Second, changes between regimes are sudden and infrequent, indicating that there is no evidence of smooth movements from one regime to the other. Third, the Colombian economy remains in the sustainable growth regime most of the time. Fourth, the turning points from the Markov switching model identify very well the behavior of real output through time, capturing the four main depressions of the century.

The rest of the paper is organized as follows. Section 2 presents some stylized facts of Colombian GDP growth. Section 3 briefly describes the Markov switching regime model proposed by Hamilton (1989). Section 4 reports and discusses the results, and section 5 concludes.

### 2. Some stylized facts of Colombian GDP growth

Graph 1 presents the evolution of Colombian real GDP growth for the period between 1925 and 2003. During these years the economy grew on average 4.3%. As it can be observed, the Colombian economy has presented long periods of stable economic growth. The longer one occurred between 1951 and 1974, in which the economy grew

---


8 Besides Colombia, they also studied the business cycle of Brazil, Mexico, Chile and Venezuela.
on average 5.2%, and the second one during 1984-1997, with an average growth of 4.1%. On the other hand, the largest fluctuations took place between 1925 and 1950, in which the economy was characterized by large capital inflows followed by the Great depression, then by an economic recovery and after that, by the Second World War, and finally by an expansion.

Graph 1
Real GDP growth: 1925-2003

In particular, throughout 1925 and 2003 the Colombian economy has experienced four main slowdowns. The first one occurred between 1930 and 1931, in which the Colombian economy was affected by the World Great Depression and its effects on the international capital market. Also, the international coffee prices and the terms of trade collapsed. All these factors produced a severe monetary and fiscal contraction. As a result, the Colombian economy declined 0.9% in 1930 and 1.6% in 1931. The second slowdown occurred between 1940 and 1943, period in which the economy grew on average only 1%. The World War II considerably reduced the international trade flows affecting Colombian exports, imports, the terms of trade and consequently the country’s economic growth. In particular, the restrictions imposed on imports by the United

Source: GRECO and DANE

---

9 During 1951-1974 the standard deviation of the annual rate of growth was 1.5% and during 1984-1997 it was 1.2% been the lowest of the period under analysis.
10 During 1925-1950 the standard deviation of the annual rate of growth was 3.25%, the largest of the period under analysis.
States, the use of their commercial vessels as military floats and the German submarine campaign (1942-1943) reduced the international flows exchange.\textsuperscript{11}

The third period 1981-1982 corresponds to the Debt Crisis. In the middle of 1982 started the worst internal financial crisis since the thirties\textsuperscript{12}. One cause of this internal crisis was the Latin-American Debt Crisis that temporarily closed the international capital markets in 1982. In addition, the Colombian economy was affected by the decline of the international coffee price in 1980. As a result, in 1982 the economy only grew 0.9%. However, the main slowdown in economic activity took place between 1998 and 1999, in the latter year the economy declined 4.2%. This recession was originated by a number of external and internal causes. The external cause was the international financial crisis of 1998 and its negative effects on neighboring countries, on the terms of trade, and on the capital markets. The internal causes included the fragility in the financial sector, a fall in private saving, macroeconomic imbalances produced by excessive aggregate demand during the nineties that comprise an increasing deterioration of public finances and a high current account deficit, among others\textsuperscript{13}.

Regarding expansions, during 1925 and 1928 the economy presented high rates of economic growth that were originated by the access to the international capital markets and the resources from the American reparations for Panama. These money inflows allowed the government to increase its investments, especially in public infrastructure.\textsuperscript{14} The economy also registered expansions after the Second World War with the restoration of international trade and the increase in international coffee prices. Other periods of high growth occurred between 1966 and 1973, when the economy grew on average 6.1%, and the years 1978 and 1986 with the significant increase in external coffee prices.

In short, as we mentioned above for long periods of time the economy has remained in a band of stable growth that we called sustainable growth, although real output has

\textsuperscript{11} See Ocampo, 1987.
\textsuperscript{12} See Ocampo, 1987 for a complete description of this crisis.
\textsuperscript{13} See The Boards of Directors’ Report to the Congress of Colombia, Banco de República, March 2000.
\textsuperscript{14} See Ramirez (2004).
presented some fluctuations through time. It is important to highlight that such fluctuations have depended mainly on external shocks that have affected the evolution of the terms of trade, capital inflows and international coffee prices, among others.

3. The Markov switching regime model applied to the Colombian long run economic growth

The switching regime model allows the economy to be in different states, each of them characterized by different rates of growth. In other words, real output changes stochastically from one regime to the other. In this paper, we considered two regimes: sustainable growth and depressions\textsuperscript{15}. In particular, the sustainable growth regime also includes periods of booms.

Let $y_t$ be the real GDP annual growth rate such as:

$$y_t = \beta_s X_t + \varepsilon_t \quad s_t = 0, 1$$

(1)

$$\varepsilon_t \sim N(0, \sigma^2_{s_t})$$

(2)

$$\sigma^2_{s_t} = \sigma^2_0 (1 - s_t) + \sigma^2_1 s_t$$

(3)

$$\beta_{s_t} = \beta_0 (1 - s_t) + \beta_1 s_t$$

(4)

$$P[s_t = 0|s_{t-1} = 0] = p \quad P[s_t = 1|s_{t-1} = 1] = q$$

$$P[s_t = 1|s_{t-1} = 0] = 1 - p \quad P[s_t = 0|s_{t-1} = 1] = 1 - q$$

(5)

Equation (1) is a regression model\textsuperscript{16}, where the growth rate ($y_t$) depends on $X_t$, which includes lags of the dependent variable and on $\varepsilon_t$ an iid random variable which follows a normal distribution with mean zero and regime or state ($s_t$) dependent variance (equation 3). $s_t$ is an unobserved discrete variable that represents the state of the economy. In this case, it takes two values (0 and 1, sustainable growth and depression, respectively), and constitutes the non-linear component of the equation. As described by equation (4), the parameter $\beta$ depends on the regime or state ($s_t$) in which the economy

\textsuperscript{15} We also considered three different regimes: depression, sustainable growth and booms, which were not supported by the data.

\textsuperscript{16} This regression set up follows closely Soto (2002).
is in time \( t \). Finally, equation 5 expresses the switching of regimes as a first order Markov-chain. It means that the current regime \( s_t \) is determined only by the preceding regime \( s_{t-1} \) and the realization of the stochastic process that leads the evolution of states. \( p \) is the probability of being in state 0 at time \( t \) given that the economy is in state 0 at time \( t-1 \), \( q \) is the probability of being in state 1 at time \( t \) given that the economy is in state 1 at time \( t-1 \) and \( 1-p \) and \( 1-q \) are the transition probabilities from one regime to the other.

In a first step, we considered \( X_t \) to be conformed by an intercept and the first four lags of the dependent variable, \( X_t = \{1, y_{t-1}, \ldots, y_{t-4}\} \), and the random variable \( \varepsilon_t \) presents a state dependent variance. The estimation results show that these lags of GDP growth are not statistically significant. Therefore, we proceed to perform the estimation including only the intercept in \( X_t \). The results of this specification indicate that the random variable \( \varepsilon_t \) is not a state dependent variance. We set up a third specification in which \( X_t \) is conformed by an intercept and the first four lags of the dependent variable, \( X_t = \{1, y_{t-1}, \ldots, y_{t-4}\} \) but now we consider that the random variable \( \varepsilon_t \) is a state independent variance. The results are very similar to those found in the first specification. Finally, we conclude that the adequate model for the Colombian economic activity is the following:

\[
\begin{align*}
    y_t &= \mu_{s_t} + \varepsilon_t \quad s_t = 0, 1 & (6) \\
    \mu_{s_t} &= \mu_0 (1-s_t) + \mu_1 s_t & (7) \\
    \varepsilon_t &\sim \mathcal{N}(0, \sigma^2) & (8) \\
    P[s_t = 0|s_{t-1} = 0] &= p \quad P[s_t = 1|s_{t-1} = 1] = q \\
    P[s_t = 1|s_{t-1} = 0] &= 1-p \quad P[s_t = 0|s_{t-1} = 1] = 1-q & (9)
\end{align*}
\]

Where the GDP growth rate \( (y_t) \) depends on its average level \( \mu_{s_t} \) which follows a Markov process (equation 7), and on a random variable \( \varepsilon_t \), which variance is
statistically not different across regimes. Similarly to the first model, equation (9) considers that switching of regimes follows a first order Markov-chain. Finally, $s_t = 0,1$ where $s_t = 0$ corresponds to sustainable growth and $s_t = 1$ to depression, and $p$ and $q$ are the transition probabilities.

4. Results

A first-order two-state Markov switching model was estimated for the Colombian economic growth, using annual data of the first difference of the logarithm of real GDP for the period 1925-2003.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimation</th>
<th>Standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>0.0492</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.0114</td>
<td>0.0069</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0004</td>
<td>6.994E-5</td>
</tr>
<tr>
<td>$p = P_{11}$</td>
<td>0.9199</td>
<td>0.0476</td>
</tr>
<tr>
<td>$q = P_{22}$</td>
<td>0.6872</td>
<td>0.1497</td>
</tr>
</tbody>
</table>

$\rho = 0.7961$

$P(S_t = 1|y_1, \ldots, y_T; \theta) = 0.9914$

Table 1 presents the maximum likelihood estimates of parameters in the selected model. All the parameters are significant at 5%. The results support the assumption that two different levels are presented in the data, $\mu_0$ and $\mu_1$ are statistically different. In particular, the estimation reports an average annual economic growth of 4.92% in

---

17 However, when the variances are estimated individually, the variance of the depression regime is numerically higher than the variance of the sustainable growth regime.

18 A similar model is presented in Hamilton (1990).

19 $(DLGDP = (1 - L)LogGDP)$
regime 0 (sustainable growth) and 1.14% in regime 1 (depression). Given the facts discussed in section 2, it is no surprising that the probability \( p=P_{11} \) of staying in sustainable growth at time \( t \) given that the economy is in sustainable growth at time \( t-1 \) is very large. In fact, the probability of being in regime 0 is 0.92. On the other hand, the probability \( q=P_{22} \) of being in depression in time \( t \) given that the economy was in the same state at time \( t-1 \) is 0.69, lower than \( p=P_{11} \). These high probabilities indicate that if the economy is in either sustainable growth or slow growth, it is likely to remain in such regime.

Table 2 shows that the probability of switching from a sustainable growth state to depression \( (1-p=P_{21}) \) is 0.08 while the probability of changing from depression to sustainable growth \( (1-q=P_{12}) \) is 0.31. This result indicates that it is more likely to pass from depression to sustainable growth than enter in depression being in sustainable growth. The latter fact could be the result of the economic policies and measures that arise when the economy is in recession in order to move it out of that regime. On the other hand, if the economy is in sustainable growth it is unlikely to enter in depression unless some negative shocks affect the economic activity.

**Table 2**

Transition Matrix

\[
\begin{bmatrix}
P_{11} & P_{21} \\
P_{12} & P_{22}
\end{bmatrix} = \begin{bmatrix}0.9199 & 0.3127 \\0.0801 & 0.6873\end{bmatrix}
\]

Given the estimated transition probabilities, we can infer the average length of each state through the following equations. Equation (10) is the expected duration of state 1 and equation (11) the expected duration of state 0\(^{21}\).

\[
\sum_{j=1}^{\infty} i \times q^{(j-1)} (1-q) \quad (10)
\]

\(^{20}\) Similar magnitudes are found in Hamilton (1989) for the US’s GDP rate of growth and Soto (2002) for the Chilean rate of growth.

\(^{21}\) For practical reasons, equations (10) and (11) are truncated in 100,000.
\[
\sum_{i=1}^{(i-1)} i \times p \times (1 - p)
\]  

(11)

We obtain that the average length of being in sustainable growth is 12 years whereas the expected duration of a depression regime is approximately 3 years. This result could suggest that the Colombian business cycle lasts 15 years. However, it is important to recall that in this paper we considered as \textit{sustainable growth} the time in which the economy experienced persistent growth, including periods of booms and very small slowdowns\textsuperscript{22}.

Graph 2 shows the evolution of the smoothed probabilities of state 0, \( P(s_t = 0|y_1, \ldots, y_T; \hat{\Theta}) \), through time. In other words, the graph plots the probability of being in sustainable growth at each date in the sample. This inference is based on the full sample and the estimated maximum likelihood parameters, which are presented in table 1. In the graph we point out the years in which the economy has switched of regimes, based on \( P(s_t = 0|y_1, \ldots, y_T; \hat{\Theta}) \leq 0.5 \).

In general, the results show that changes between regimes are sudden, deeper and sporadic. As expected, the Colombian economy remains in the sustainable growth regime most of the time\textsuperscript{23}. The economy only departs from the sustainable growth regime when a major external shock affects the Colombian economic activity. The high probabilities of staying in regime 0 are between 1950 and 1980, period in which the imports substitution program was fully implemented. Two questions emerge here: First, was the import substitution a successful policy during this period? Or was the post-war favorable international environment that led this behavior? In a future research we empirically pretend to answer these questions.

In particular, the turning points from the Markov switching model capture very well the behavior of real output through time. In fact, they identify the four main depressions of

\textsuperscript{22} Previous studies have found that the average length of the Colombian business cycle is approximately 8 years; see for example, Posada, 1999; and Fernández and Gonzáles, 2000.

\textsuperscript{23} Similar results were found by Arango L. and Melo, L (2005) for the Colombian industrial production index, their proxy for economic activity.
the century that were described above. Our results improve previous studies such as Arango, L and Melo, L (2005) whose *STAR* model fails to identify the important crisis of 1982-1983.

**Graph 2**

Probabilities of being in sustainable growth (state 0)

As we can observe, the graph shows four switches from sustainable growth to depression in the sample: in 1930, 1940, 1981 and 1997. The first period of recession is 1930-1931, in which the economy was affected by the Great Depression. Then, 1940-1943 which corresponds to the World War II period. After these years the economy experimented a long sustained growth process in which the probability of remaining in such state was greater than 0.8. One exception was 1950 in which the probability of being in sustainable growth was 0.69. In 1950, the rate of economic growth was 1.1%, very close to the average annual growth rate of 1.14%, that the model estimates for regime 1 (depression). However, the model fails to identify this year as a recession given that in the preceding years the economy experimented higher rates of growth and following 1950 the economy presented a fast recovery.\(^{24}\)

\(^{24}\) In 1949, the economy grew 8.7%, in 1948, 4% and in 1946, 9.6%. The economic slowdown of 1950 was the consequence of tight economic policies (See Ocampo, 1987).
As mentioned before, the period 1981-1983 was characterized by an internal financial crisis, after which, the economy stayed in a period of sustainable growth, with the probability of remaining in such state, higher than 0.85 until 1995. In 1996, the probability of being in regime 0 declined to 0.55 due to a reduction of economic activity. In that year the economy grew only 2%. The last period 1997-2002 captured the largest depression of the XX century that took place between 1998 and 1999. However, the model gives a wrong indication of recession in 1997 despite the fact that the economy grew more than 3.4%. The relatively low probability of being in sustainable growth (0.4) in 1997 perhaps is reflecting the slowdown that the economy started experimenting since 1996. The graph also shows that the probability of being in sustainable growth considerably decreased after such depression and only until 2003 the probability of being in regime 0 was higher than 0.5.

Finally, table 3 presents some specification tests proposed by Hamilton (1996) in order to verify the performance of the model. First, the White autocorrelation test\(^{25}\) suggests that there is no evidence of autocorrelation. Second, the White specification test\(^{26}\) indicates that the Markov model can not be rejected against the alternative that there are no changes in regime. Therefore, evidence of non-linearity in the Colombian economic growth is found. Regarding LM tests, they confirm the results of no autocorrelation. Similar results are also obtained when we examine each regime separately, and the LM test on ARCH effects shows that there is no indication of the presence of such effects. Summing up these tests suggest that there is no evidence of model misspecification.

\(^{25}\) White autocorrelation test verifies the score correlation at time \((t)\) with respect to \(\mu_i\) and the score of time \((t - 1)\) with respect to \(\mu_j\) with \(i, j = 1, 2\).

\(^{26}\) The Markov assumption that \(P(s_t = i)\) depends only of the state in \((t - 1)\) can be tested against two alternatives hypothesis: (i) \(P(s_t = i)\) depends on several previous states or (ii) \(P(s_t = i)\) depends on the realization of \(y_{t-1}\). The test verifies if the score with respect to the transition probabilities can be forecasted by its lags or by the score with respect to the average.
Table 3
Specification tests

<table>
<thead>
<tr>
<th>Test Description</th>
<th>Test Statistic</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>White autocorrelation test ( \chi^2(4) )</td>
<td>3.674</td>
<td>0.518</td>
</tr>
<tr>
<td>White Markov specification test ( \chi^2(4) )</td>
<td>8.467</td>
<td>0.076</td>
</tr>
<tr>
<td>LM test on autocorrelation in state 0, ( \chi^2(1) )</td>
<td>0.499</td>
<td>0.479</td>
</tr>
<tr>
<td>LM test on autocorrelation in state 1, ( \chi^2(1) )</td>
<td>0.134</td>
<td>0.714</td>
</tr>
<tr>
<td>LM test on autocorrelation across states, ( \chi^2(1) )</td>
<td>0.555</td>
<td>0.456</td>
</tr>
<tr>
<td>LM test on ARCH effects, ( \chi^2(1) )</td>
<td>0.267</td>
<td>0.606</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, we modeled the Colombian long run economic growth using a two-regime first order Markov switching model. We found four main results. First, the estimations show evidence of non-linearity in the Colombian economic growth series. Second, the results confirmed that the Colombian economy experienced a sustained growth most of the time. Third, the estimations indicate that changes between regimes are sudden and sporadic, and periods of sustainable growth are longer and more persistent than depression periods. Fourth, the turning points from the Markov switching model identify very well the four main depressions of the century, that were mainly associated with negative external shocks.

According to the probabilities of being in the sustainable growth regime, we identified three main periods. The first one is between 1925 and 1950, in which the probability of staying in such regime is volatile. This fact can be explained by several international shocks that affected the Colombian economy in that time. The second period, between
1951 and 1980, is characterized by high probabilities of remaining in the sustainable growth regime. This could be the result of the economic policy implemented in those years or the post war favorable international economic behavior which influenced the Colombian economy. Finally, the third period comprises the years 1981-2003, in which the probability of staying in stable growth varies over time. The question here is whether the economic policy implemented in the previous years was obsolete or were the international shocks the ones that moved out the economy from the stable growth path. These are the issues that we empirically pretend to answer in a second paper. It is important to understand how policy measures can affect the path of economic growth through time. To this end, we will model the Colombian economic growth with time varying transitional probabilities, allowing them to be affected by policy shocks, both internal and external.

Finally, in a third paper we will compare the behavior of the Colombian economic growth during the XX century with other Latin American countries, using a Markov Switching regime model.
References


Posada, C., 1999. “Los ciclos económicos en el siglo XX”. *Borradores de Economía* # 126, Banco de la República


Soto, R., 2002. “Switching regimes, macroeconomic policies and sustained growth”. Instituto de Economía, Pontificia Universidad Católica de Chile, Manuscript.