1. Introduction

- Seminal paper by Bils and Klenow (AER, 2000)
1. Introduction

- Seminal paper by Bils and Klenow (AER, 2000)
  - human capital

\[
h(t) = h(s) \times \underbrace{e^{g(t-s)}}_{\text{schooling}} = \underbrace{e^{f(s)+g(t-s)}}_{\text{experience}}
\]
1. Introduction

- Seminal paper by Bils and Klenow (*AER, 2000*)
  - human capital
    \[ h(t) = h(s) \times e^{g(t-s)} = e^{f(s)+g(t-s)} \]
  - schooling
  - experience

- optimal years of schooling
  \[ (1 + \mu) w(s) h(s) = \int_s^T \left[ f'(s) - g'(t-s) \right] e^{-r(t-s)} w(t) h(t) dt \]
  - marginal cost
  - marginal benefit
Figure 1. Years of schooling - 2005
Data versus Bils and Klenow model
1. Introduction

- Issues with Bils and Klenow (AER, 2000)

- Lack of predictability ($R^2 = 0.27$)
  - High rate of discount: $r = 9.5\%$
  - Downplays, by construction, the role of life expectancy
1. Introduction

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- Most recent human capital paper
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  - Erosa, Koreshkova and Restuccia (*RES*, 2010)
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- Most recent human capital paper
  - Erosa, Koreshkova and Restuccia (*RES*, 2010)
  - cannot account for schooling dispersion

![Graph showing the relationship between Ln GDP PPP and Mean schooling years. The graph includes multiple lines for different values of eps, with data points indicating a positive correlation.]
1. Introduction

Objective → build a theory of schooling that better accounts for

- quantities (schooling years)
- prices (interest rate and returns to schooling)
- The theory must also be consistent with other stylized facts:
  - public nature of education
  - more spending per pupil in richer countries
  - demographic (fertility and mortality) differences across countries
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  - public nature of education
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Public education spending per pupil per year relative to the US

PPP Prices

Per capita GDP relative to US
Fertility and life expectancy versus income - 2004

- Fertility data represented by blue diamonds.
- Life expectancy data represented by red squares.

The graph shows a scatter plot with income per capita (relative to US) on the x-axis and fertility and life expectancy at birth on the y-axes.
1. Introduction

- Key features of model →
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  - life-cycle economy with altruistic parents
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1. Introduction

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1. Introduction

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  - credit frictions:
1. Introduction

- Key features of model →
  - life-cycle economy with altruistic parents
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  - public subsidies for education for a certain number of years
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  - credit frictions:
    - benchmark model: borrowing constraints for students
1. Introduction

- Key features of model →
  - life-cycle economy with altruistic parents
  - time and resources in the production of human capital
  - public subsidies for education for a certain number of years
  - private education spending
  - credit frictions:
    - benchmark model: borrowing constraints for students
    - alternative model: non-negative bequest constraint
2. The model
2.1. Human capital

- \( h(a) = \) human capital of an individual of age \( a \) with \( s \) years of schooling

\[
h(a) = h(s) \times e^{\frac{(a-s)}{\beta}} \quad \text{for} \quad a \geq s
\]
2. The model

2.1. Human capital

- \( h(a) = \) human capital of an individual of age \( a \) with \( s \) years of schooling

\[
h(a) = h(s) \times e^{v(a-s)} \quad \text{for } a \geq s
\]

- Students accumulate knowledge in schools according to

\[
h(s) = \left( \int_0^s i(t)^\beta \, dt \right)^{\gamma/\beta} = \left( \int_0^s \left( \frac{e(t)}{pE} \right)^\beta \, dt \right)^{\gamma/\beta}
\]

where \( (\beta, \gamma) \in (0, 1] \)
2. The model
2.1. Human capital

- \( h(a) = \text{human capital of an individual of age } a \text{ with } s \text{ years of schooling} \)
  \[ h(a) = h(s) \times e^{\nu(a-s)} \text{ for } a \geq s \]

- Students accumulate knowledge in schools according to
  \[ h(s) = \left( \int_0^s i(t)^\beta \, dt \right)^\gamma/\beta = \left( \int_0^s \left( \frac{e(t)}{p_E} \right)^\beta \, dt \right)^\gamma/\beta \]
  where \( (\beta, \gamma) \in (0, 1] \)

- \( s \) includes pre-school years
2. The model

2.1. Human capital

- Returns to schooling

\[ r_s(s) = \frac{d \ln (wh(s))}{ds} = \frac{\gamma}{\beta} h(s)^{-\frac{\beta}{\gamma}} \left( \frac{e(s)}{p_E} \right)^\beta \]
2. The model
2.1. Human capital

- **Returns to schooling**

\[
rs(s) = \frac{d \ln (wh(s))}{ds} = \frac{\gamma}{\beta} h(s)^{\frac{\beta}{\gamma}} \left( \frac{e(s)}{p_E} \right)^{\beta}
\]

- **Example → pure public education**

\[
h(s) = \left( \frac{e_p}{p_E} \right)^\gamma s^{\gamma/\beta}
\]

\[
rs(s) = \frac{\gamma}{\beta} \frac{s^{\gamma/\beta}}{s}
\]
2. The model
2.2. Individual’s problem

- Individual’s problem

\[ V(b) = \max_{\{c(a), e_s(a)\}} \int_0^T e^{-\rho a} u(c(a)) \pi(a) \, da + \pi(F) e^{-\rho F} \phi(f) V(b') \]

own consumption

\[ \quad \text{utility children} \]

subject to

\[ \int_0^s (c(a) + e_s(a)) \, q(a) \, da + q(s) \omega(s) \leq b \]

saving

bequest

consumption / education

\[ \int_s^T c(a) \, q(a) \, da + q(F) fb' \leq \int_s^R wh(s) e^{\nu(a-s)}q(a) \, da + q(s) \omega(s) \]

wage earnings

saving

\[ \text{consumption} \]

\[ \text{bequest} \]

\[ \text{saving} \]
2. The model
2.2. Individual’s problem

\[ h(s) = \left( \int_0^s \left( \frac{(e_p(a) + e_s(a))}{p_E} \right)^\beta da \right)^{\gamma/\beta} \]

\[ e_s(a) \geq 0 \]

\[ \omega(s) \geq \omega = 0 \]

\[ 0 \leq s \leq F \]

\[ e_p(a) = \begin{cases} 
  e_p(a) & \text{if } s \leq a \leq \bar{s} \\
  0 & \text{otherwise} 
\end{cases} \]
2. The model
2.2. Individual’s problem

- Functional forms
2. The model

2.2. Individual’s problem

- Functional forms
  - utility

\[ u(c) \equiv \frac{c^{1-\sigma} - 1}{1 - \sigma} \]
2. The model

2.2. Individual’s problem

- Functional forms
  - utility
    \[ u(c) \equiv \frac{c^{1-\sigma} - 1}{1 - \sigma} \]
  - altruism
    \[ \phi(f) \equiv \phi f^\psi \text{ with } 0 < \psi < 1 \]
2. The model

2.2. Individual’s problem

- Functional forms
  - utility
    \[ u(c) \equiv \frac{c^{1-\sigma} - 1}{1 - \sigma} \]
  - altruism
    \[ \phi(f) \equiv \phi f^\psi \text{ with } 0 < \psi < 1 \]
  - age-contingent prices
    \[ q(a) = e^{-ra} \pi(a) \]
2. The model
2.3. Optimality conditions

Optimality for bequest

\[ u'(c(F)) = \frac{\phi(f)}{f} u'(c_{\text{child}}(0)) \]

- Marginal cost
- Marginal benefit
2. The model
   2.3. Optimality conditions

- Optimality for bequest

$$u'(c(F)) = \frac{\phi(f)}{f} u'(c^{\text{child}}(0))$$

- marginal cost
- marginal benefit

- Are bequests large enough to substitute for perfect credit markets?
2. The model

2.3. Optimality conditions

- Optimality for bequest

\[ u'(c(F)) = \frac{\phi(f)}{f} u'(c_{\text{child}}(0)) \]

- marginal cost
- marginal benefit

- Are bequests large enough to substitute for perfect credit markets?
  - limits to altruism

\[ G \equiv \frac{f}{\phi(f)} e^{-(r-\rho)F} > 1 \]

- assumption
2. The model
2.3. Optimality conditions

- Optimality for bequest

\[
\frac{u'(c(F))}{\text{marginal cost}} = \frac{\phi(f)}{f} \frac{u'(c^{\text{child}}(0))}{\text{marginal benefit}}
\]

- Are bequests large enough to substitute for perfect credit markets?
  - limits to altruism

\[
G \equiv \frac{f}{\phi(f)} e^{-(r-\rho)F} > 1
\]

  - shadow price of "credit"

\[
r_b = r + \ln(G)/F = \rho + \frac{(1-\psi)\ln f - \ln \phi}{F} > r
\]
Optimal education spending:

\[
q(a) \geq \frac{1}{G} \int_s^R w \frac{\partial h(s)}{\partial e^*(a)} e^{v(t-s)} q(t) dt
\]

- Marginal cost
- Marginal benefit
2. The model

2.3. Optimality conditions

- Optimal education spending:

  \[ q(a) \geq \frac{1}{G} \int_s^R w \frac{\partial h(s)}{\partial e^*(a)} e^{v(t-s)} q(t) dt \]

  marginal cost

  marginal benefit

- Alternatively:

  \[ e^*(a) = \max \{ \hat{e}^*(a), e_p(a) \} \text{ for } a \in [0, s] \]
Figure 2. Individual expenditures in education: $e^*(a)$

Case 1: Some public school
Case 2: Full public school + some private
Case 3: Full private and public school + some more private
2. The model
2.3. Optimality conditions

- Optimal schooling choice:

\[
e_s(s) + \sigma \frac{\Delta u(s)}{u'(c^S(s))}
\]

\[= \frac{1}{G} \frac{1}{q(s)} \partial \left[ \int_s^R wh(s) e^{v(a-s)} q(a) da \right]
\]

marginal cost

net marginal benefit
3. Calibration

Table 1. Parameters common across countries

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concept</th>
<th>Value</th>
<th>Source / Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>relative risk aversion</td>
<td>1.5</td>
<td>Cooley and Prescott (1995)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>returns to experience</td>
<td>2%</td>
<td>Bils and Klenow (2000a)</td>
</tr>
<tr>
<td>$s$</td>
<td>starting schooling age</td>
<td>6</td>
<td>UNESCO</td>
</tr>
<tr>
<td>$F$</td>
<td>parenthood age</td>
<td>25</td>
<td>Satisfies restriction $s \leq F$</td>
</tr>
<tr>
<td>$R$</td>
<td>retirement age</td>
<td>65</td>
<td>Binding level in richer countries</td>
</tr>
<tr>
<td>$\phi$</td>
<td>level in $\phi(f) = \phi f^\psi$</td>
<td>1</td>
<td>Perfect altruism when $f = 1$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>degree of altruism</td>
<td>0.4</td>
<td>Birchenall and Soares (2009)</td>
</tr>
<tr>
<td>$r$</td>
<td>riskless interest rate</td>
<td>3%</td>
<td>Mehra (2003)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.33</td>
<td>Gollin (2002)</td>
</tr>
</tbody>
</table>
Table 2. Calibrated parameters

<table>
<thead>
<tr>
<th>Concept</th>
<th>Value</th>
<th>Target in OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>4.69%</td>
<td>Average schooling: 16.14 years</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
<td>Private spending % GDP: 0.65%</td>
</tr>
<tr>
<td>$\gamma/\beta$</td>
<td>1.5</td>
<td>Returns to schooling: 8.28%</td>
</tr>
</tbody>
</table>

with years of schooling measured as:

$$SLE_{a}^{t} = \sum_{i=a}^{n} \frac{enrollment_{i}^{t}}{population_{i}^{t}} \times 100$$
3. Calibration

Countries differ in:

- schooling-related variables: $\pi$, $\epsilon$, and $s$ (grade repetition)
- demographics: $\pi(a)$
- prices: $p_E$ and $w$
3. Calibration

Countries differ in:

- schooling-related variables: $e_p$, and $\bar{s}$ (grade repetition)
3. Calibration

Countries differ in:

- schooling-related variables: $e_p$, and $\bar{s}$ (grade repetition)
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- Countries differ in:
  - schooling-related variables: $e_p$, and $\bar{s}$ (grade repetition)
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3. Calibration

- Public education expenditures per pupil $e_p$ are from UNESCO (2005)
3. Calibration

- Public education expenditures per pupil $e_p$ are from UNESCO (2005)
- $\bar{s}$ computed for each country as

\[
\bar{s} = 6 + \text{duration prim&sec} \times \frac{\text{public expenditures prim&sec}}{\text{total expenditures prim&sec}} + (SLE - \text{duration prim&sec}) \times \frac{\text{public expenditures terciary}}{\text{total expenditures terciary}}
\]
3. Calibration

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+ (SLE - \text{duration prim\&sec}) \times \frac{\text{public expenditures terciary}}{\text{total expenditures terciary}}
\]

- Correction for repetition rates $d$:

\[
h(s) = \left( \int_0^s \left( \frac{d \cdot e(t)}{p_E} \right)^\beta dt \right)^{\gamma/\beta}
\]
Figure 3. Maximum public schooling (s upper-bar) versus school life expectancy in the data - 2005
3. Calibration

- Probability of survival:

\[
\pi(a) = \begin{cases} 
  e^{-p_c a} & \text{for } a \leq 5 \\
  \pi(5) e^{-p_s(a-5)} & \text{for } 5 \leq a \leq 25 \\
  \pi(25) \frac{e^{-p(a-5)} - \xi}{1 - \xi} & \text{for } 25 < a \leq T 
\end{cases}
\]

where \(\pi(a)\) for the adult follows Boucekkine, de la Croix and Licandro (2002) and

\[
T = -\frac{\log(\xi)}{p} + 25
\]
3. Calibration

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\[ T = \frac{-\log(\xi)}{p} + 25 \]

- \( p_c, p_s, p \) and \( \xi \) match survival probabilities WHO (2006)
3. Calibration

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- \( f \) from World Development Indicators (2005)
3. Calibration

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    \pi(25) \frac{e^{-p(a-5)} - \zeta}{1 - \zeta} & \text{for } 25 < a \leq T
\end{cases}
\]

where \( \pi(a) \) for the adult follows Boucekkine, de la Croix and Licandro (2002) and

\[
T = -\frac{\log(\zeta)}{p} + 25
\]

- \( p_c, p_s, p \) and \( \zeta \) → match survival probabilities WHO (2006)
- \( f \) from World Development Indicators (2005)
- \( p_E \) proxied by relative price of government spending from PWT
Figure 4. Survival probabilities at different ages
Precited (dashed) and Data (solid)
### Table 3. Model’s performance

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Means</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years of schooling</td>
<td>12.96</td>
<td>13.60</td>
</tr>
<tr>
<td>Returns to schooling</td>
<td>11.2%</td>
<td>8.3%</td>
</tr>
<tr>
<td>Private education spending % GDP</td>
<td>1.2%</td>
<td>1.2%</td>
</tr>
<tr>
<td><strong>Standard deviations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years of schooling</td>
<td>3.35</td>
<td>2.78</td>
</tr>
<tr>
<td>Returns to schooling</td>
<td>2.1%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Private education spending % GDP</td>
<td>1.25%</td>
<td>0.98%</td>
</tr>
<tr>
<td><strong>Correlation between model and data</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Years of schooling</td>
<td>84.7%</td>
<td></td>
</tr>
<tr>
<td>Returns to schooling</td>
<td>86.3%</td>
<td></td>
</tr>
<tr>
<td>Private education spending % GDP</td>
<td>35.0%</td>
<td></td>
</tr>
</tbody>
</table>
Figure 5. School life expectancy in the model and the data
Figure 6. Returns to schooling
Model versus BK Estimates

Per capita GDP relative to US
Figure 7. Private expenditures in education as a % of GDP
Model versus Data - Subset of countries

Per capita GDP relative to US
Figure 8. Quality of human capital
5. Counterfactuals

Table 4. Schooling counterfactuals (% change)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$stdev(s)$</th>
<th>$mean(s)$</th>
<th>$var(ln(b))$</th>
<th>$mean(b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_c$</td>
<td>-3.7</td>
<td>0.4</td>
<td>-6.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$p_s$</td>
<td>-3.5</td>
<td>0.3</td>
<td>-3.0</td>
<td>0.1</td>
</tr>
<tr>
<td>$p$</td>
<td>-22.5</td>
<td>2.4</td>
<td>-16.5</td>
<td>0.7</td>
</tr>
<tr>
<td>$p_c, p_s, p$</td>
<td>-30.8</td>
<td>3.2</td>
<td>-24.9</td>
<td>1.5</td>
</tr>
<tr>
<td>$f$</td>
<td>-56.2</td>
<td>3.5</td>
<td>-60.9</td>
<td>-6.4</td>
</tr>
<tr>
<td>$e_p$</td>
<td>22.7</td>
<td>-7.5</td>
<td>-17.6</td>
<td>2.4</td>
</tr>
<tr>
<td>$\bar{s}$</td>
<td>-35.3</td>
<td>1.8</td>
<td>-11.7</td>
<td>-1.7</td>
</tr>
<tr>
<td>$p_E$</td>
<td>2.0</td>
<td>-0.6</td>
<td>18.5</td>
<td>-13.0</td>
</tr>
<tr>
<td>$w$</td>
<td>-2.7</td>
<td>0.7</td>
<td>-53.7</td>
<td>67.2</td>
</tr>
</tbody>
</table>
Figure 9. Schooling: benchmark and counterfactual

Per capita GDP relative to US
Figure 10. Parental transfers: benchmark and counterfactual
6. Robustness

- Two robustness checks:
6. Robustness

- Two robustness checks:
  - altruistic parameter $\psi$

Two robustness checks:
- altruistic parameter $\psi$

model explains 94% of schooling's standard deviation
- high value for $\psi = 0.58$
- model explains 69% of schooling's standard deviation

fertility and the duration of the public education subsidy are still key
6. Robustness

- Two robustness checks:
  - altruistic parameter $\psi$
  - frictionless version of our model
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- Two robustness checks:
  - altruistic parameter $\psi$
  - frictionless version of our model
- Altruistic parameter
  - low value for $\psi = 0.39$
    - model explains 94% of schooling’s standard deviation
  - high value for $\psi = 0.58$
    - model explains 69% of schooling’s standard deviation
- fertility and the duration of the public education subsidy are still key
6. Robustness

- Frictionless version of our model
6. Robustness

- Frictionless version of our model
  - fertility and bequests do not play a role \((G = 1)\)
6. Robustness

- Frictionless version of our model
  - fertility and bequests do not play a role ($G = 1$)
  - model can explain at most 34% of schooling dispersion
Children live with their parents during school years
7. Alternative model

- Children live with their parents during school years
- Parents:
  - full access to credit markets
  - make optimal consumption and schooling choices on behalf of their children
  - Children become independent upon finishing school and receive a non-negative bequest

Model has identical predictions to our benchmark model
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Individual’s problem:

\[
V(h, s, b) = \max_{\{c(a), e_s(a)\}_{a>0}} \int_s^T e^{-\rho(a-s)} u(c^W(a)) \frac{\pi(a)}{\pi(s)} da
\]

\[+ \phi(f) e^{-\rho(F-s)} \left[ \int_0^{s'} e^{-\rho a} u(c^S(a)) \pi(a) da + e^{-\rho s'} V(h', s', b') \pi(s') \right] \frac{\pi(F)}{\pi(s)} \]

subject to:

\[
\int_s^T c^W(a) q(a) da + \int_0^{s'} f(c^S(a) + e_s(a)) q(F + a) da + q(F + s') fb' \leq \int_s^R wh(s)e^{\nu(a-s)} q(a) da + q(s) b;
\]

\[e_s(a) \geq 0; \quad b' \geq 0; \quad 0 \leq s' \leq F\]
7. Alternative model

Optimal bequest:

\[
\frac{u'(c^S(s))}{u'(c^W(s))} = G \equiv \frac{f}{\phi(f)} e^{-(r-\rho)F} \frac{\pi(s + F)}{\pi(F)\pi(s)} > 1
\]